

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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Lecture 18: Radiated Emission Model Subproblem I

Welcome to the 18th lecture of the course on EMI/EMC and signal integrity principles, techniques and applications. We were discussing radiated emission model, pre-emission model we will be discussing. Today we will formally discuss the radiation emission model which is actually if you remember in the I think after introduction we have discussed the four sub problems of EMI. So the first one was radiated emission, so that model we will discuss in this class. So I refer to yesterday's drawing that current element. So now I will write that there are what are the fields, these are available in any textbooks or even my NPTEL lecture. So there will be a ER field,  $\frac{2 \pi \eta \sin \theta}{r^2} \cos \theta$  by  $\frac{1}{r^3} \sin \theta$  by  $\frac{1}{r^3} \cos \theta$  component that is  $\frac{2 \pi \eta \sin \theta}{r^2} \sin \theta$  by  $\frac{1}{r^3} \cos \theta$ . Thank you. So this is the thing actually. This is obtained by solving the Helmholtz equation in free space and obviously putting the that is a differential equation. So it is complementary function and one, so you know how to solve second order differential equation so by that these solutions are obtained. So here I think all of you know that here there are this free space impedance  $\eta$  that is given by the material parameters. So this is here we have assumed basically that this is the current element is radiating in free space so everything we are writing as  $\eta$  so  $\mu$  by  $\epsilon$  then  $\beta$  is the phase constant of the wave so it is  $2 \pi$  by  $\lambda$ ,  $\lambda$  is the free space wavelength and that is  $c$  by  $f$  where  $c$  is the velocity of light,  $c$  also can be related to the material parameters that  $1$  by root over  $\mu \epsilon$  and  $f$  is the frequency of the exciting current and so  $c$  is you can write  $1$  by root over  $\epsilon \mu$  in free space you know its value  $3$  into  $10$  to the power  $8$  meter per second et cetera. So you see we have deliberately written the fields in a structure that all fields are functions of electrical distance of the point from the antenna, electrical distance means  $r$  by  $\lambda$  and  $\beta$  is  $2 \pi$  by  $\lambda$  so it is basically  $r$  by  $\lambda$  just multiplied by  $2 \pi$  so all the electric fields are functions of this electrical distance of the point observation point from the antenna.

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\beta_0 = \frac{2\pi}{\lambda_0}$$

$$\lambda_0 = \frac{c}{f}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\tilde{E}_r = 2 \frac{\tilde{I} dl}{4\pi} \eta_0 \beta_0^2 \cos \theta \left( \frac{1}{\beta_0^2 r^2} - j \frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$\tilde{E}_\theta = \frac{\tilde{I} dl}{4\pi} \eta_0 \beta_0^2 \sin \theta \left( j \frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j \frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$\tilde{E}_\phi = 0$$

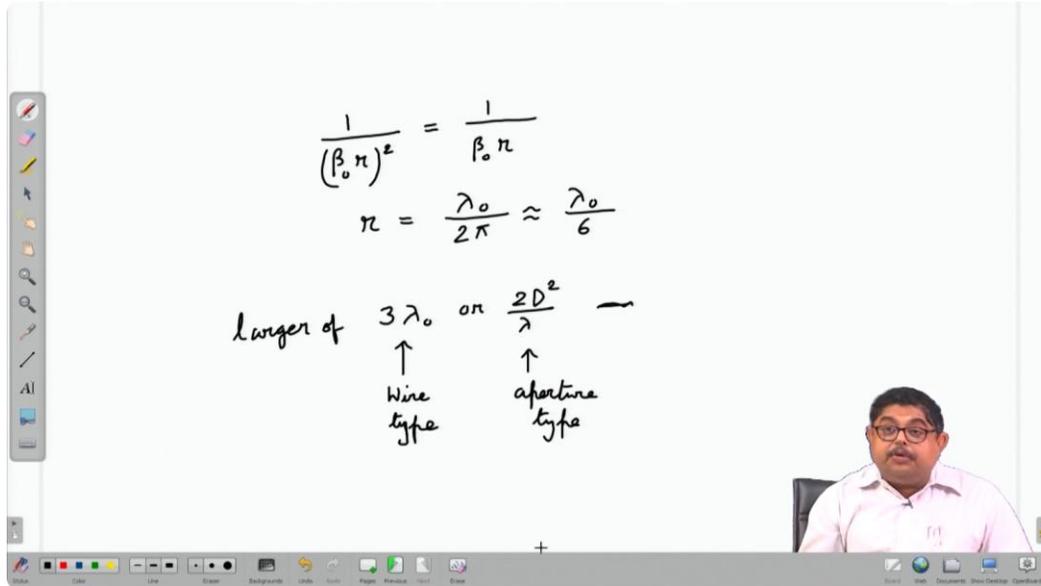
$$\tilde{H}_r = 0$$

$$\tilde{H}_\theta = 0$$

$$\tilde{H}_\phi = \frac{\tilde{I} dl}{4\pi} \beta_0^2 \sin \theta \left( \frac{j}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} \right) e^{-j\beta_0 r}$$

At small  $r$  if you look at, at small  $r$   $1$  by  $r$  square and  $1$  by  $r$  cube or  $1$  by beta naught square  $r$  square and  $1$  by beta naught cube  $r$  cube these terms will dominate but at large  $r$  the  $1$  by  $r$  term begins to dominate. The point where  $1$  by  $r$  square and  $1$  by  $r$  cube terms become insignificant compared to  $1$  by  $r$  terms is referred as the boundary that boundary is called far field of the antenna. So when  $1$  by  $r$  square and  $1$  by  $r$  cube terms dominate that is called near field of the antenna when  $1$  by  $r$  terms began to dominate that is called far field of the antenna. So in the far field of the antenna there are various properties of the antenna which we will try to see. So first what is the limit of  $1$  by what is the boundary of far field we can say that when beta naught  $r$  whole square becomes  $1$  by beta naught  $r$  so that means this gives you  $r$  is equal to lambda naught by  $2\pi$  so roughly we can say it is lambda naught by  $6$  so that means the boundary is lambda naught by  $6$  so note that this is for our current element for other antennas these values will be different because the fields are may not be of this a more realistic choice for this boundary is the larger of people have researched on these that there are also other criteria to characterize far field basically we want that the angular distribution of energy from that point should be stable in near field the angular distribution of energy is not so stable but at far field the angular distribution of energy is stable whatever distance from that boundary wherever you go the angular distribution of energy becomes stable that is why we call that the beam is formed there. Now obviously the values are different at different points but the angular distribution that remains fixed that is the property of the antenna. So a more realistic choice is you can take between  $3$  lambda naught or  $2d$  square by lambda naught whichever is larger so I can say larger of this where  $d$  is the maximum dimension of

larger dimension of the antenna typically this is used for where type of antennas where type that means for current element we can use this because that is a where type antenna and this is generally for aperture type of antenna so horn etcetera for them this point can be used on parabolic reflector etcetera they are this type.



So now we have got a boundary so we can see what is far field so let us now say what is the far field electromagnetic fields so if you look at the expressions you see that ER does not have any 1 by R term so that means it will become 0 at far field E theta has one term as 1 by R the other terms will become insignificant at far field so it will have one term now H phi that also has a 1 by R term so the terms that will be or the fields that will be remaining in far field is one is E theta another is H phi. So let me write what is the expression for E theta I can write that it is part that is J eta naught beta naught I dl by 4 pi sin theta e to the power minus J beta naught R by R so this I can bring in this eta naught place the frequency I want to see directly because frequency is generally known that the current element the currents frequencies so much so let me then just manipulate this eta naught beta naught in terms of phi how I can do that so let me inside write what is eta naught beta naught, eta naught is as we have seen it is mu naught by epsilon naught and beta naught is 2 pi by lambda naught and or let me do eta naught beta naught by 4 pi by 4 pi so it will be 1 by 4 pi so that will be so this 2 pi and 4 pi so here will be 2 lambda naught and this mu 1 what I can do here I can write it as mu naught if I multiply with root over mu naught then here also I will have to multiply by root over mu naught so that mu naught will come and here I can write that this is mu naught into C so now you know that I can easily write by mu naught C is F lambda naught by 2 lambda naught

so that will give me  $\mu_0 \beta_0$  by  $2 F$  so I can write  $J \mu_0 \beta_0$  by  $2 I \, dl \sin \theta e^{-j\beta_0 r}$  to the power minus  $J \beta_0 \lambda$  by  $r$ . So similarly I can write  $H_{\phi}$  by  $r$  so that will be  $J \beta_0 \lambda$  by  $2 I \, dl \sin \theta e^{-j\beta_0 r}$  to the power minus  $J \beta_0 \lambda$  by  $r$ . So again with this I can write  $F_{\theta}$  by  $2 \eta_0 \beta_0 I \, dl \sin \theta e^{-j\beta_0 r}$  to the power minus  $J \beta_0 \lambda$  by  $r$ . So let us see the time domain fields because there is one conclusion I want to draw from there so this is the phasor fields so how to get the time domain fields .

Farfield

$$\tilde{E}_{\theta, \text{far}} = j \eta_0 \beta_0 \frac{\tilde{I} \, dl}{4\pi} \sin \theta \frac{e^{-j\beta_0 r}}{r}$$

$$= j \mu_0 \frac{c}{2} \tilde{I} \, dl \sin \theta \frac{e^{-j\beta_0 r}}{r}$$

$$\tilde{H}_{\phi, \text{far}} = j \beta_0 \frac{\tilde{I} \, dl}{4\pi} \sin \theta \frac{e^{-j\beta_0 r}}{r}$$

$$= j \frac{\mu_0}{2\eta_0} (\tilde{I} \, dl) \sin \theta \frac{e^{-j\beta_0 r}}{r}$$

Side calculation:

$$\frac{\eta_0 \beta_0}{4\pi} = \sqrt{\frac{\mu_0}{\epsilon_0} + \frac{2\pi}{\lambda_0}} \frac{1}{4\pi}$$

$$= \frac{\mu_0 c}{2\lambda_0}$$

$$= \frac{\mu_0 k \lambda_0}{2\lambda_0}$$

$$= \frac{\mu_0 k}{2}$$

so we will have to multiply this expression so et to the power  $J \omega t$  and take the real part. So I can say that  $e^{-j\beta_0 r}$  for this is time domain I am using no more data so this is real of  $e^{-j\beta_0 r}$  to the power for  $e^{-j\beta_0 r}$  to the power  $J \omega t$  so that is real  $F_{\theta}$  by  $2 I \, dl \sin \theta e^{-j\beta_0 r}$  to the power that  $J \omega t$  I am observing here  $J \omega t$  by  $2$   $e^{-j\beta_0 r}$  to the power minus  $J \beta_0 \lambda$  by  $r$  so I am writing like this  $\beta_0 \lambda$  by  $r$  is  $r$  by  $\lambda$   $e^{-j\beta_0 r}$  to the power  $J \omega t$  then this is sorry I got it  $u \theta$  so  $u \theta$  yes I will have to close the third bracket so that is  $F_{\theta}$  by  $2 I \, dl \sin \theta$  where is the there should be the  $r$  term here so that let me take  $\sin \theta$  by  $r$  real of this  $e^{-j\beta_0 r}$  to the power  $J \omega t$  plus  $\pi$  by  $2$  minus  $2\pi r$  by  $\lambda$   $u \theta$  is equal to  $F_{\theta}$  by  $2$  into  $e^{-j\beta_0 r}$  to the power  $J \omega t$  plus  $\pi$  by  $2$   $I \, dl \sin \theta$  by  $r$  so real part if I take that will give me  $\cos \omega t$  minus  $2\pi r$  by  $\lambda$   $u \theta$  plus  $\pi$  by  $2$   $u \theta$  so this is equal to minus  $F_{\theta}$  by  $2 I \, dl \sin \theta$  by  $r$   $\sin(\omega t - r/c - u \theta)$  this comes because  $\cos(\pi/2 + \text{something})$  is minus  $\sin$  so that is why this minus  $\sin$  and if I take this part common if you put the value of  $\lambda$   $\lambda = c/\omega$  you can see that we have put it here yes  $\lambda$  is  $c/\omega$  so by putting that we have got this part. So, you see that in time there is a delay. actually to brought this I brought it here so this can be now written as some constant minus  $E_m$  by  $r \sin(\omega t - r/c - u \theta)$  where  $E_m$  is  $F_{\theta}$  by  $2 I \, dl \sin \theta$ .

$$\begin{aligned}
 \vec{E}_{\text{far}} &= \text{Re} \left[ \vec{E}_{\text{far}} e^{j\omega t} \right] \\
 &= \text{Re} \left[ \frac{\mu_0}{2} (\tilde{I} dl) \sin \theta \left\{ \frac{e^{j\frac{\pi}{2}} e^{-j2\pi(\frac{r}{\lambda_0})} e^{j\omega t}}{r} \right\} \hat{u}_\theta \right] \\
 &= \frac{\mu_0}{2} (\tilde{I} dl) \frac{\sin \theta}{r} \text{Re} \left\{ e^{j(\omega t + \frac{\pi}{2} - 2\pi(\frac{r}{\lambda_0}))} \right\} \hat{u}_\theta \\
 &= \frac{\mu_0}{2} (\tilde{I} dl) \frac{\sin \theta}{r} \cos \left( \omega t - \frac{2\pi r}{\lambda_0} + \frac{\pi}{2} \right) \hat{u}_\theta \\
 &= -\frac{\mu_0}{2} (\tilde{I} dl) \frac{\sin \theta}{r} \sin \left\{ \omega \left( t - \frac{r}{c} \right) \right\} \hat{u}_\theta \\
 &= -\frac{E_m}{r} \sin \left\{ \omega \left( t - \frac{r}{c} \right) \right\} \hat{u}_\theta
 \end{aligned}$$

where  $E_m = \frac{\mu_0}{2} (\tilde{I} dl) \sin \theta$

Similarly, I am not now going into details by the same way you can do h bar and so just I am writing the first part L h bar e to the power j omega t u phi so that in the similar way if you manipulate you will get minus E m by eta naught r sin omega t minus r by c u phi. So, from this we can make some important characteristic of the far field. So, there are the first characteristic is that the so characteristics of current elements far field very very important for us for EMC class. The first property is the fields are proportional to a first thing is 1 by r proportional to I proportional to dl the elemental length proportional to sin theta .

$$\begin{aligned}
 \vec{H}_{\text{far}} &= \text{Re} \left[ \vec{H}_{\text{far}} e^{j\omega t} \right] \hat{u}_\phi \\
 &= -\frac{E_m}{\eta_0 r} \sin \left\{ \omega \left( t - \frac{r}{c} \right) \right\} \hat{u}_\phi
 \end{aligned}$$

Characteristics of current element's farfield

a) The fields are proportional to

- $\frac{1}{r}$
- $\tilde{I}$
- $dl$
- $\sin \theta$

b) The magnitude of  $E_{far}$  by  $H_{far}$  is  $\eta_0$ .  $H_{far}$  is  $\hat{u}_\phi$  and  $E_{far}$  is  $\hat{u}_\theta$  that means they are orthogonal to each other, but you remember that in spherical coordinate these  $\hat{u}_r$ ,  $\hat{u}_\theta$ ,  $\hat{u}_\phi$  all are changing with position. So, this  $\hat{u}_r$ ,  $\hat{u}_\theta$ ,  $\hat{u}_\phi$  are not fixed in spherical coordinate, but in Cartesian coordinate they are fixed always at any point  $\hat{u}_z$  is fixed the unit vectors they are fixed in direction. So, that is why we will call that this  $E_{far}$  field and  $H_{far}$  magnetic field are locally orthogonal locally always they are orthogonal to each other, but their direction is changing everywhere, but they are having a mutually orthogonal relationship at every location that is why you are calling locally orthogonal. Also, let us see this that what is  $E_{far} \times H_{far}$ . So,  $E_{far} \times H_{far}$  means  $E_{far}$  is  $\hat{u}_\theta$   $H_{far}$  is  $\hat{u}_\phi$ . So,  $\hat{u}_\theta \times \hat{u}_\phi$  will give us  $\hat{u}_r$  and the value will be  $E_m^2$  by  $\eta_0 r^2 \sin^2 \omega t - r/c$ . And another characteristic is a phase term  $e^{-j\beta_0 r}$  to the power minus  $j\beta_0 r$  you see this was present in all the phasors and this one in time if we go to the time if we introduce time by multiplying. So, from phasor if we come to the time that means if we include the time variation then it translates to a time delay of  $r/c$  that means the effects are all delayed. So, if I have a disturbance at this point the field will be created after a delay given by the observation distance by  $c$ . So, the time the electromagnetic wave takes to reach there that is the delay when the field gets created. Now, what is the implication of property A? So, the implication is far fields can be extrapolated by using inverse distance relationship you see far field. So, let us see what is property A far field is proportional to  $1/r$  that means at far field the current elements electric field can be inverse can be described by or extrapolated by using that relationship.

b)  $|\vec{E}_{far}| / |\vec{H}_{far}| = \eta_0$

c)  $\vec{E}_{far}$  and  $\vec{H}_{far}$  are locally orthogonal

d)  $\vec{E}_{far} \times \vec{H}_{far} = \frac{E_m^2}{\eta_0 r^2} \sin^2 \left\{ \omega \left( t - \frac{r}{c} \right) \right\} \hat{u}_r$

e) A phase term  $e^{-j\beta_0 r}$  translates to a time delay of  $\frac{r}{c}$

So, the electric field at distance  $d_1$  and  $d_2$  are related by in the far field are related by  $E_{d_2}$  is equal to  $d_1$  by  $d_2$   $E_{d_1}$  obviously in the far field not in near field both  $d_1$  and  $d_2$  should be at the far field. So, if either  $d_1$  or  $d_2$  are at near field this formula is invalid. So, if a hertzian dipole has a length of 1 centimeter length of 1 centimeter carries a 1 ampere 100 megahertz current then determine the magnitude and phase of the electric and magnetic fields at a distance 1 kilometer away and broadside to the antenna. Broadside means theta is 90 degree broadside means that current element this is current element theta is 90 degree means broadside. So, you see the if you look at the line of the antenna anything that is 90 degree to that axis is called broadside and in the same line if it is there that is called end fire this is the antenna terminology. So, today time is up next day we will see this problem and try to see some exemplary things. Thank you.

$$|\vec{E}_{d_2}| = \left(\frac{d_1}{d_2}\right) |\vec{E}_{d_1}|$$

- 1 cm
- 1 A, 100 MHz

+