

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

Professor name: Prof. Amitabha Bhattacharya

Department name: Electronics and Electrical Communication Engineering

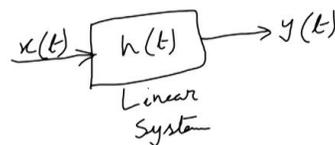
Institute name: IIT Kharagpur

Week :03

Lecture 15: Spectral Bounds for Linear System Output

Welcome to the 15th lecture of the course on EMIEMC and Signal Integrity Principles, Techniques and Applications. We were seen the spectral bounds for a clock output, today we will see that that clock if it is applied to a linear system then what is the spectral bound of that output. So, as before we assume that I have a linear system characterized by $h(t)$. So, this is a linear system and I am giving an basically this $x(t)$ is the clock and I am getting the output $y(t)$. So, what is the upper bound on this $y(t)$. Now, we know that we will have to go to Fourier domain. So, in Fourier domain this discrete spectrum will be $Y(j\omega)$ is equal to $H(j\omega)$ into $X(j\omega)$. So, we are concerned with the magnitude part. So, we know that $|Y(j\omega)|$ is equal to $|H(j\omega)|$ and $|X(j\omega)|$. Obviously, that phase will be just sum of these two phases. So, on a bode plot or log log plot we get $20 \log_{10} |Y(j\omega)|$ is $20 \log_{10} |H(j\omega)|$ plus $20 \log_{10} |X(j\omega)|$. So, now we know this part we know that this is basically the whatever we have got for the clock. So, this part we will have to add.

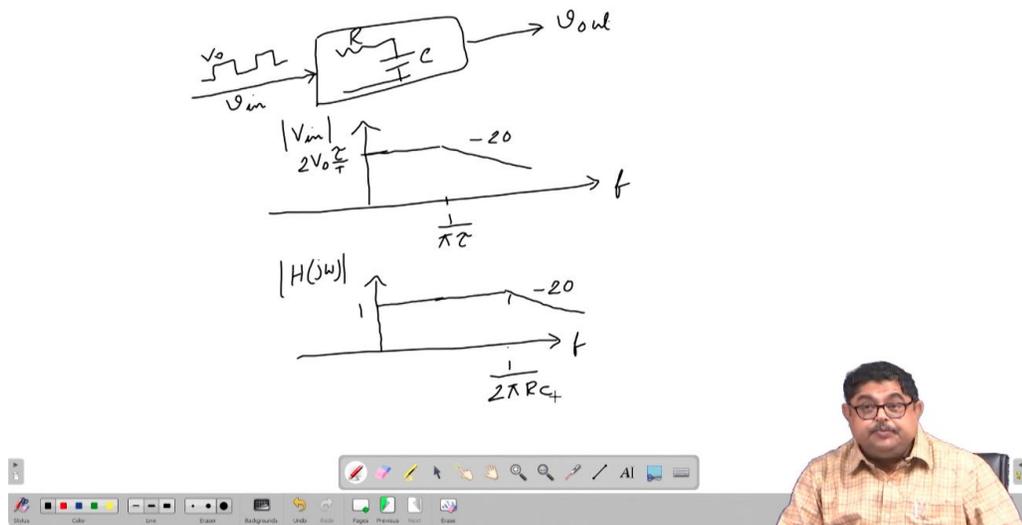
LECTURE 15: Spectral bounds for linear system output



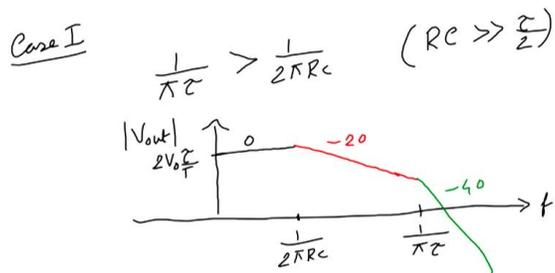
$$Y(j\omega) = H(j\omega) X(j\omega)$$
$$|Y(j\omega)| = |H(j\omega)| |X(j\omega)|$$
$$20 \log_{10} |Y(j\omega)| = 20 \log_{10} |H(j\omega)| + 20 \log_{10} |X(j\omega)|$$



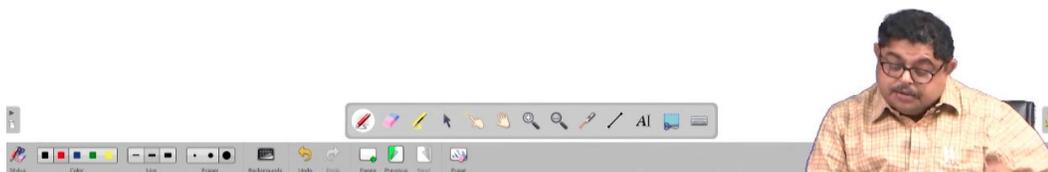
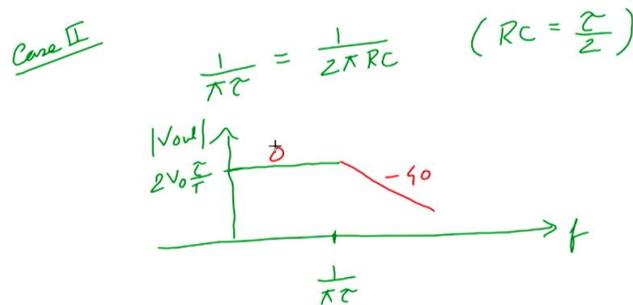
So, let us consider as before a you know as a linear system and R C circuit. So, I have a R C filter R C and to this I am applying let us say square wave and I am getting output. So, let us call it V_{out} in time domain. So, we know that for this clock waveform. So, this let us call it V_{in} . So, for this clock we know the upper bound. So, V_{in} for it we know that if the voltage is V or V_{naught} let us say. So, $2 V_{naught} \tau$ by T then it will go up to 1 by $\pi \tau$ and again assuming a square pulse. If for trapezoidal pulse you can easily modify that and then there will be a minus 20 dB per decade slope line. So, this is the V_{in} spectrum. Now, this R C filter also we have seen before and you know that this R C filter spectrum will be $H(j\omega)$ thing. So, that spectrum is also like this. So, this break point occurs at 1 by $2\pi R C$ and this part is 1 and this is minus 20 because it is nothing, but 1 by 1 plus $j\omega R C$. So, you can do this is also with F .



Now, the question is this break points where which break point will come earlier 1 by $\pi \tau$ or 2π by $R C$. So, depending on that three cases may appear let us call the first one as case 1 where 1 by $\pi \tau$ that means, box frequency break point is coming later than the break point of the system. So, that means, we have $R C$ the time constant of the system that is greater than τ by 2. So, then we can draw the output spectrum V_{out} will be $2 V_0 \tau$ by T and it will go to 0 up to what up to 1 by $2\pi R C$ let us say that 1 by $\pi \tau$ is this is my F . So, between this it will come with a minus 20 dB per decade slope it will go here and then from there it will start a journey with a minus 40 dB per decade slope. So, this is our case 1.

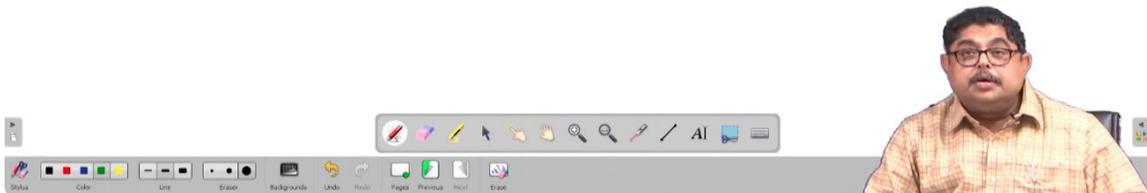
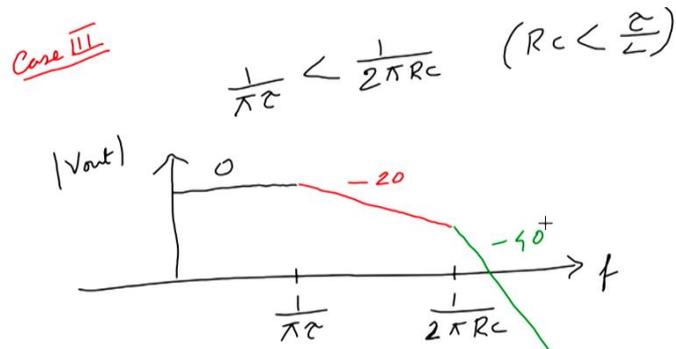


Let us come to case 2 that $\frac{1}{\pi\tau}$ is equal to $\frac{1}{2\pi RC}$ that means, RC is equal to τ by 2. So, in that case we can easily draw that this will be our V_{out} and this is as we call $2V_0\tau$ by T and here let me locate that this is the break point $\frac{1}{\pi\tau}$ same as this. So, up to this it will go and then from here both the break points are falling coinciding. So, from here I will have a minus 40 dB slope.



Then come to case 3. So, here we have $\frac{1}{\pi\tau}$ is less than $\frac{1}{2\pi RC}$ that is RC is less than τ by 2. So, if I draw this time this is my V_{out} this is my F my first break point will be coming by contributing by $\frac{1}{\pi\tau}$ and second break point will be coming by $\frac{1}{2\pi RC}$. So, I will get up to here a 0 dB slope then from here to here there will be a minus 20 dB slope and from there I will have a minus 40 dB slope. So, from here it is. So, now, the question is you see the three cases. So, if we want to reduce the high frequency components of the output then which case we should take? Obviously, you see that here the minus 40 dB is coming much later. Let us see the

previous one, here it is coming 1 by pi tau, but here it is coming from 1 by pi tau itself.



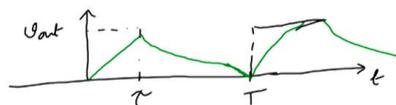
So, obviously, it is clear that we will choose the case 1. So, that means, case 1 means it is 1 by pi tau is we should choose it is to be much greater than 1 by 2 pi R C. That means, our time constant of the circuit should be greater than tau by 2 or it is R C should be greater than tau. So, what does that mean physically? Physically it means capacitor has not had sufficient time to charge up to its final value before the pulse switches off. So, the output will appear. So, if you have a square block you see the tau before that the output is that means, before the capacitor could charge R C is much higher. So, R C is not that means, the this transition when it has come the capacitor is charging, but before the clock switches it has not reached a steady state. That means, it will be something like this exponential then if this is your t then the clock takes a time here also it has not reached again it will rise. So, basically you get a saw tooth type of waveform.

Case I

$$\frac{1}{\pi\tau} >> \frac{1}{2\pi RC}$$

$$RC >> \frac{\tau}{2}$$

$$RC >> \tau$$



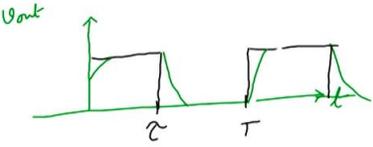
On the other hand if we choose RC is less than τ that means, the third case then let us see what will be the waveform. So, this is for case 1. Case 1 output will look like saw tooth that means, this you see saw tooth compared to a square wave it has much smoother transition that is why its high frequency components will reduce. So, if your aim is to reduce the high frequency components choose that RC time constant should be higher than the your pulse width. On the other hand if we choose case 3 then the output will how it looks like in time domain. So, this is your V out now versus time that we said versus timer this thing I should say that it is versus time this is V out versus t .

So, in this case you see capacitor case 3 means my RC is less than τ by 2 so that means, if I put τ like this t like this. So, my original waveform is this, but this one. So, it charges very fast because RC time constant is very small. So, it is something like this then it is this charges also very fast. So, this green one this is the output. So, output will resemble the input with smooth transition. So, in this case the high frequency components above $1/\pi\tau$ will be diminished by only minus 20 dB per decade slope. So, let us take a problem that suppose I have a 10 megahertz square wave and it is passed through a LR filter LR means the output I am taking across R the duty cycle is 50 percent. So, you will have to determine the value of R what is the value of R so that the output voltage is not significantly distorted that means, you should take case 3. So, you see basically the circuit is something like this that you have V in t here. So, it is a pulsed waveform you have a LR circuit and this is your V out and your V in t is square wave. So, you have V naught you have 0 τ t . So, this you are giving you will have to have the output not much distorted we have seen that in case 1 a square wave becomes a triangular wave saw tooth wave whereas, here so we should choose case 3.

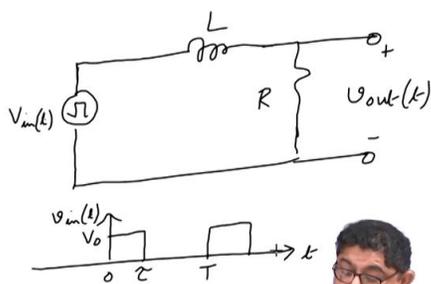
Case III

V_{out}

$RC < \frac{\tau}{2}$



10 MHz square wave
LR filter
50% duty cycle
 $R = ?$




Software interface showing drawing tools: Erase, Backgrounds, Undo, Fill, Pages, Previous, Next, Erase.

So, what is an LR circuit $H(j\omega)$ it is R by $R + j\omega L$ that is 1 by $1 + j\omega L/R$ that means, instead of an RC circuit the time constant of a LR circuit is L/R . So, to prevent distortion we want case 3 that means, L/R should be much less than $\tau/2$. So, R should be greater than $2L/\tau$. Now, given f_0 is 10 megahertz 50 percent. So, τ is equal to 0.1 micro second with 50 percent duty cycle.

So, τ will be 0.05 micro second and so, R should be much greater than $2L/\tau$. I have not said the value of L . So, L let us take 1 microhenry so that means, R should be greater than $2 \times 10^{-6} / 0.05 \times 10^{-6}$ by 0.05 into 10^{-6} so that means, R should be greater than 40 ohm. So, now you can easily choose any value greater than 40 ohm greater than equal to 40 ohm that will have your that effect that no discharge or minimum distortion. But if our motive was to reduce the high frequency components we are not so bothered about distortion let us recalculate R in that case we should choose case 1 that is L/R should be greater than $\tau/2$.

$$H(j\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega \frac{L}{R}}$$

$$\frac{L}{R} \ll \frac{\tau}{2} \quad L = 1 \mu H$$

$$R \gg \frac{2L}{\tau}$$

$$f_0 = 10 \text{ MHz}, \quad T = 0.1 \mu \text{ sec.}$$

$$\tau = 0.05 \mu \text{ sec.}$$

$$R \gg \frac{2 \times 10^{-6}}{0.05 \times 10^{-6}} = 40 \Omega$$



So, R should be now less than 40 ohm in that case you may get a saw tooth wave form that means, output may be distorted, but you will reduce the high frequency component ok. So, that is our signal integrity thing.

$$\frac{L}{R} \gg \frac{Z}{Z}$$

$$R \ll 40 \Omega$$

Measurement Bandwidth

RBW \rightarrow Resolution Bandwidth

0.02 Hz



Now, how to measure all these things that measurement device is a signal analyzer a spectrum analyzer. Let at one go you can see the whole spectrum suppose you want to see as we have seen that what is the spectrum of a trapezoidal clock from 1 megahertz to 100 megahertz. Now, if I want to see that I want a device or a equipment measurement equipment that is called spectrum analyzer. There in the x axis the frequencies plot and in y axis you get the spectral coefficients magnitude ok. So, that spectrum analyzer is a very good instrument you know nowadays it is quite cheaper also. So, generally all the colleges they have at least one spectrum analyzer. So, you can measure it by that actually EMC regulatory authorities also specified that by a spectrum analyzer you should measure that various high frequency components. Now, this is nothing, but a super heterodyne receiver where the incoming signal is mixed. So, how you can see all the frequencies together that actually the local oscillator that is not fixed that is actually changing its frequency. So, that means, your IF frequency is also changing. So, that at now your super heterodyne means in two stage you are doing that. So, finally, you are getting the lower fixed IF and then it is detected and magnitude determined. Then the those values of IF at various frequencies that means, at various times the instrument has already collected the various components. So, it is displaying that ok. So, now, but we need some more understanding of this working of this device for our purpose that for our spectrum measurement that we will see that regulatory authorities has said something actually spectrum analyzer has got two types of bandwidth. One is the its radio bandwidth. So, radio bandwidth or sweeping bandwidth that means, from which frequency start frequency to which end frequency you want to see the spectrum. So, that is its measurement bandwidth. So, in our that case of trapezoidal waveform that I want to see what is the spectrum from 1 megahertz to 100 megahertz. So, that that means, our start frequency will be 1 megahertz, the stop frequency will be 100 megahertz. These are the terminologies of spectrum analyzer. But there is another bandwidth which is very

very important in spectrum analyzer specifications that is called RBW resolution bandwidth. Actually, we want to discuss this resolution bandwidth to understand various EMC requirements. Resolution means what? What is the resolution? In measurement this resolution is very important. Resolution means that what is the minimum separation in any physical quantity whatever you are measuring. So, that you can distinguish two different things. Suppose when a radar measures a echo and suppose it is measuring range. So, what is range resolution? Suppose two objects they are at 1 kilometer distance in range from the radar. One is suppose at 100 kilometer away another is 101 kilometer away. So, if the radar can say that there is one object at 100 kilometer, another object at 101 kilometer, then the radar has a resolution better than 1 kilometer. But suppose two objects they are at 1 meter distance, will the radar still able to say that 101 object is at 100 kilometer, another object is at 100.001 kilometer. If that is there, then the resolution is better than 1 meter. Now, if you go on reducing this, so at one point suppose there two objects are separated by 1 millimeter. Generally no radar will be able to detect it unless and until it is a very sophisticated imaging type of radar. So, what is the minimum distance or minimum separation of two objects or two quantities in the measuring physical quantity that a measuring instrument can say that there are two different objects that is the resolution. So, in terms of resolution bandwidth means if you have two spectral components, what is the minimum separation between them, so that the spectrum analyzer will be able to say that there are two different spectral components that is called resolution bandwidth. Nowadays, the spectrum analyzer can go up to gigahertz, 10 gigahertz, 20 gigahertz, 30 gigahertz is there, measurement bandwidth that can be. But resolution bandwidth even they can be 0.02 hertz, so that means, one frequency component is at suppose 1 megahertz, another frequency component is at 1.00002 megahertz. The spectrum analyzer, modern spectrum analyzers can measure that. Now, cost obviously will increase if you go with that one, but still today's prices, today's economy this is possible. So, now, this resolution bandwidth where we will set for a measurement that is an important topic that we will take up in the next class. Thank you.