

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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Week :03

Lecture 14: Effect of Ringing on Spectral Bounds

Welcome to the 14th lecture of the NPTEL course on EMI EMC and Signal Integrity Principles, Techniques and Applications. Now, in yesterday's class we were discussing the effect of ringing on spectral bounds, we will continue that. So, we have seen that a ringing waveform can be broken into three parts, we have seen how to write these spectral components using linearity. So, this equation you have seen. Now, we will try to evaluate the part. So, let us take that this integral  $0$  to  $T$  by  $2$   $e$  to the power minus  $\alpha$   $t$   $\sin$  of  $\omega$   $R$   $T$  plus  $\theta$   $e$  to the power minus  $j$   $n$   $\omega$   $\text{naught}$   $t$   $dt$ . So, because this is the actual a thing apart from  $k$ ,  $k$  we have just kept. So, how to tackle this nothing just you see all are  $e$  to the power terms only the  $\sin$  is there. So,  $\sin$  by Euler's identity we can break. So, we can write this as a thing only difficulty is you see that this term is  $e$  to the power minus  $j$  something, this term  $\sin$  term will also generate  $e$  to the power  $j$  terms, but this term is a real term, this is not a complex term  $e$  to the power minus  $\alpha$   $t$ . So, the whole thing will be tackled as a complex number not pure imaginary number that is the only difficulty in this one. So, you just break this sign and that will give you  $1$  by  $2$   $j$  will come to have those things and  $e$  to the power minus  $\alpha$  plus  $p$  minus  $j$   $\omega$   $r$  into  $T$  by  $2$ .... Here what I have done for man production one thing is I have put  $\theta$  is equal to  $0$  because  $\theta$  is a phase otherwise it will come here always. So, it is immaterial because it is not a  $T$  dependent quantity is a initial phase or epoch. So, I have put assumption such  $\theta$  is equal to  $0$  also you see I have introduced a quantity  $p$  which actually is an pure imaginary thing and it is nothing, but  $j$   $n$   $\omega$   $\text{naught}$ . So, though it looks like  $\alpha$  and  $p$  I am doing, but actually  $p$  is an purely imaginary number  $\alpha$  is a purely real number.  $\alpha$  is the damping constant  $p$  is the dependent on the frequency  $j$   $\omega$   $r$  is the imaginary number  $j$  multiplied by the ring mean angular frequency. Now here I can make an realistic assumption that  $\alpha$  is of substantial value actually later we will see it is  $10$  to the power  $5$   $10$  to the power  $6$  etcetera. So, this  $e$  to the power minus  $\alpha$   $T$  by  $2$  that will be much much less than  $1$ . The moment that is there you see  $e$  to the power minus  $\alpha$   $T$  by  $2$  if it is much larger less. So, I want to say  $\alpha$  is large. So,  $e$  to the power minus  $\alpha$   $T$  by  $2$  is much less than  $1$ . So, this under the second bracket the quantity that will be remaining that is only minus one thing.

## LECTURE 14: Effect of Ringing on Spectral bounds

$$I = \int_0^{T/2} e^{-\alpha t} \sin(\omega_n t + \theta) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{2j} \left[ \frac{e^{-(\alpha + p - j\omega_n) \frac{T}{2}} - 1}{(-\alpha + p - j\omega_n)} - \frac{e^{-(\alpha + p + j\omega_n) \frac{T}{2}} - 1}{-(\alpha + p + j\omega_n)} \right]$$

$$\theta = 0$$

$$p = jn\omega_0$$

$$e^{-\alpha \frac{T}{2}} \ll 1 \quad \alpha \text{ large}$$



So, with this we will do the approximation that I is can be written as minus j by 2 1 by alpha plus p minus j omega r minus 1 by alpha plus p plus j omega R. So, we can write that as minus j by 2 alpha plus p plus j omega R minus alpha minus p plus j omega R divided by alpha plus p whole square plus omega R whole square. So, this becomes our omega R by alpha plus p whole square plus omega R square. So, we can write it now breaking the things that means, putting the value of p we can say that this is alpha square plus j 2 alpha n omega naught minus n square omega naught square omega naught square plus omega R square ok. So, we have we could evaluate I the integral.

$$I \approx -\frac{j}{2} \left[ \frac{1}{(\alpha + p - j\omega_n)} - \frac{1}{(\alpha + p + j\omega_n)} \right]$$

$$= -\frac{j}{2} \left[ \frac{\alpha + p + j\omega_n - \alpha - p + j\omega_n}{(\alpha + p)^2 + \omega_n^2} \right]$$

$$= \frac{\omega_n}{(\alpha + p)^2 + \omega_n^2} = \frac{\omega_n}{\alpha^2 + j2\alpha n\omega_0 - n^2\omega_0^2 + \omega_n^2}$$



So, now let us see what happens to our  $C_n$ . So,  $C_n$  is  $C_n$  square wave plus  $e$  to the power minus  $j n \omega_0 t$  by 4. Before this before this so, this actually if I take this common there will be a if I take  $k$  common here there will be a  $1$  minus  $e$  to the power minus  $j n \omega_0 t$  by 2. So, from there I have told you the trick that you take  $e$  to the power minus  $j n \omega_0 t$  by 4 common that will give you a  $\sin$  term. So, that will do now and that is the if you take this common then you get  $2 j \sin n \omega_0 t$  by 4 by this is my addition  $\omega_0 t$  by 4 to make it  $C$ . So, I will have to multiply by  $n \omega_0 t$  by 4. So, this is and then that I part or  $k$  is there. So,  $k$  by  $t$  then I part is this  $\omega_0 R$  by  $\alpha$  plus  $p$  whole square plus  $\omega_0 R$  square. So, this is the total thing. So, now, we can just make one step for simplification  $C_n$  square wave plus  $k$  by 2  $\text{sinc } n \omega_0 t$  by 4  $e$  to the power minus  $j n \omega_0 t$  by 4 then the last part is that  $p \omega_0 R$  by  $\alpha$  plus  $p$  whole square plus  $\omega_0 R$  square.

$$C_n = C_{n, \text{sq wave}} + e^{-j n \omega_0 \frac{T}{4}} 2j \frac{\sin(n \omega_0 \frac{T}{4})}{n \omega_0 \frac{T}{4}} n \omega_0 \frac{T}{4}$$

$$= C_{n, \text{sq wave}} + \frac{K}{2} \text{sinc}(n \omega_0 \frac{T}{4}) e^{-j n \omega_0 \frac{T}{4}} \frac{K \omega_n}{(\alpha + p)^2 + \omega_n^2}$$

$$= C_{n, \text{sq wave}} + \frac{K}{2} \text{sinc}(n \omega_0 \frac{T}{4}) e^{-j n \omega_0 \frac{T}{4}} \frac{p \omega_n}{(\alpha + p)^2 + \omega_n^2}$$



So, what is  $C_n$  square wave we know? So, we can now write  $C_n$  is equal to  $C_n$  square wave means  $V$  by 2 this is without any rise time fall time. So, it will be  $\text{sinc } n \omega_0 t$  by 4  $e$  to the power minus  $j n \omega_0 t$  by 4 this is the  $C_n$  square wave plus  $k$  by 2 again  $\text{sinc } n \omega_0 t$  by 4  $e$  to the power minus  $j n \omega_0 t$  by 4  $p \omega_0 R$  by  $\alpha$  plus  $p$  whole square plus  $\omega_0 R$  square. Actually we are not so used to write in this board that is why many times the pen slips. So, you see that here this there is the first part is the square wave part the next part is the whole effect of the ringing. So, we can say that if we take this  $V$  by 2 common  $V$  by 2  $\text{sinc } n \omega_0 t$  by 4  $e$  to the power minus  $j n \omega_0 t$  by 4 then we get  $1$  plus  $k$  by  $V$   $p \omega_0 R$  by  $\alpha$  square plus  $2 \alpha p$  plus  $p$  square plus  $\omega_0 R$  square. So, this is the whole

effect of the ringing. This is I can say the  $C_n$  of a square wave. So, this part is the effect of ringing so, that means, we can say that Fourier coefficients of a ringing clock is the Fourier coefficient of a non-ringing clock and then it is multiplied by a band pass transfer function. You can easily see that this is a band pass transfer function and that transfer function is band pass that filter that it is centered about I can say an angular frequency of this part I think you understand that this is a band pass filters function. So, its center frequency will be if I call it  $\omega'$  that is  $\sqrt{\alpha^2 + \omega_n^2}$  plus it is headed for me  $\alpha^2 + \omega_n^2$ . And, compared to  $\omega_n$  the ringing frequency  $\alpha$  is much smaller. You see  $\alpha$  value is large as I said that time with respect to  $1 - e^{-\alpha t}$  by 2 or  $t$  by 4 that was very small, but if you compare  $\alpha$  with  $\omega_n$   $\alpha$  is much smaller  $\omega_n$  is the ringing frequency. So, ringing frequency is higher than damping constant. So,  $\alpha$  is much less than  $\omega_n$ . So, this  $\omega'$  that means, the band pass filters center frequency is almost you can say is  $\omega_n$ . That means, at ringing frequency you see the spectrum has an additional term. So, there will be a jump in the spectrum at ringing frequency.

$$C_n = \frac{\sqrt{V}}{2} \text{sinc}\left(n\omega_0 \frac{T}{4}\right) e^{-jn\omega_0 \frac{T}{4}} + \frac{K}{2} \text{sinc}\left(n\omega_0 \frac{T}{4}\right) e^{-jn\omega_0 \frac{T}{4}} \frac{b\omega_n}{\{(\alpha + b)^2 + \omega_n^2\}}$$

$$= \frac{\sqrt{V}}{2} \text{sinc}\left(n\omega_0 \frac{T}{4}\right) e^{-jn\omega_0 \frac{T}{4}} \left[ 1 + \frac{K}{V} \frac{b\omega_n}{\alpha^2 + 2\alpha b + b^2 + \omega_n^2} \right]$$

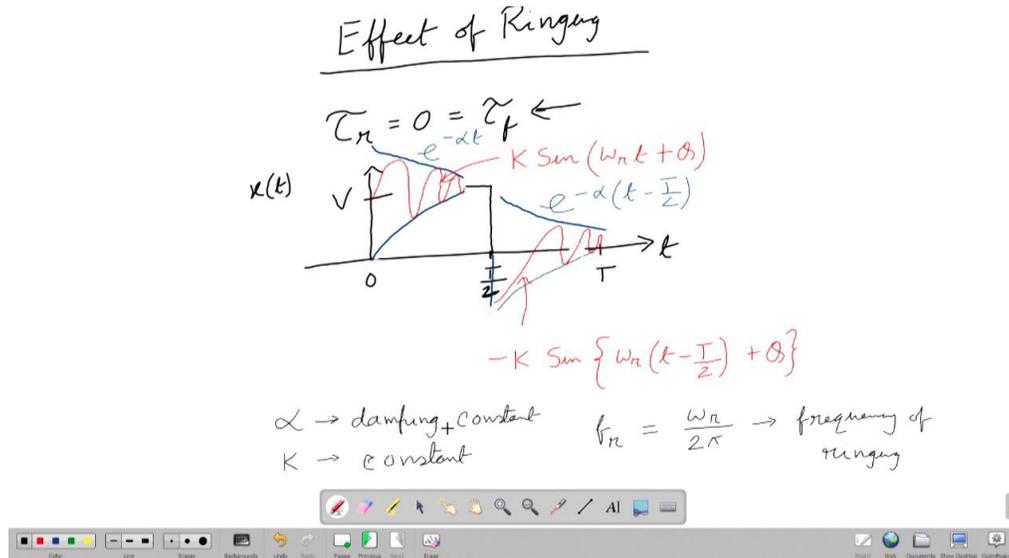
*$C_n$  sq. wave*

$\omega' = \sqrt{\alpha^2 + \omega_n^2}$   
 $\alpha \ll \omega_n$   
 $\omega' \approx \omega_n +$

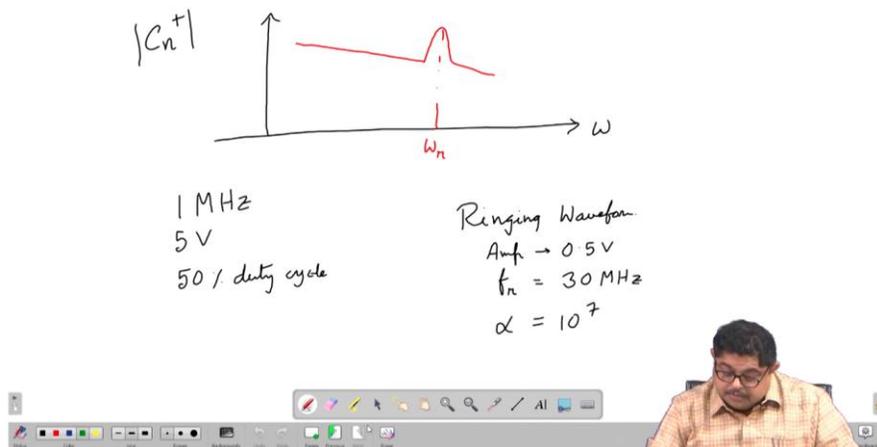


So, this jump an EMC engineer should understand that if at some high frequency you see that there is a jump that means, suppose you are having a spectrum like this  $C_n$  plus versus  $F$  or  $\omega$  same thing. Now, there suppose at high frequency you are getting something then suddenly there is a jump. So, this one is  $\omega_n$ . So, if you see this you should be able to recognize that this is due to ringing so that means, you will have to do something for checking the parasitic inductance capacitance you will have to nullify them and then this will go. Because at certain high frequency this may exceed the regulatory limits. So, let us calculate let us do a practical problem calculation that what is this spectral jump at the ringing frequency. So, let us take again a clock again that same

1 mega charge clock 5 volt instead of 1 volt let us make it a 5 volt clock 50 percent duty cycle 50 percent duty cycle and let us say that ringing waveform have. So, what were the parameters of the ringing waveform? Let us go back and see when we discuss the ringing waveform. So, it has its own amplitude. So, ringing waveform should have an amplitude ringing waveform should have what is its frequency of ringing that means, what is omega R or F R and what is the damping constant alpha F R and this.



So, ringing waveform having amplitude let us say 0.5 volt that means, over a 5 volt you are having a ringing of 0.5 volt then its F R let us say 30 megahertz you see it is much higher than 1 megahertz ring and alpha as I said what is a typical alpha value 10 to the power 7 alpha is a damping constant without any unit. So, 10 to the power 7 quite high value. So, for a 1 megahertz clock e to the power minus alpha t by 2 etcetera that will be quite small than 1. So, that have assumption was justified. Now, we will have to calculate this what is the jump how much is this jump better to calculate it in dB. So, this is the problem at hand. So, let us see this part that this part is the jump in spectrum it occurs at omega R. So, let us take this whole thing something.



So, let me call that delta is K by V naught by V we were calling it V isn't it this V is the ringing clocks amplitude p omega R by alpha plus p whole square plus omega R increase in spectral magnitude will be in dB it will be it is a spectral quantity primary quantity because spectrum is in terms of the primary thing. So, 20 log to the base 10 what is the jump. So, 1 plus delta by sorry by 1 previously it was 1 now it is 1 plus delta. So, increase in spectral magnitude will be 20 log 10 this. So, basically I will have to calculate delta once I can calculate delta I can do that. What is K? K is the amplitude you see K is the amplitude of the ringing waveform what is F r is given and damping constant alpha is also given. So, K is given 0.5 volt already V for the wave we have taken 5 volt p is J n omega naught now it is said that F r is 30 mega hertz whereas, F naught is 1 mega hertz. So, that means, we can say that n is 30. So, value of p is how much omega naught we know the angular frequency due to 1 mega hertz.

So, p is equal to J 30 into omega naught means 2 pi into 1 mega hertz that is 10 to the power 6. So, if you do this you will get J 1.885 into 10^8. So, if you do the way to get the points etcetera always are creating problem 1.885 into 10 to the power 8 and so, if that is there also it is said that ringing frequency omega r is 30 mega hertz. So, omega r will be 2 pi into 30 mega hertz that is 1.885 into 10 to the power 8 and omega r is 2 pi into 30 into 10 to the power 6, alpha is already said.

$$\Delta = \frac{K}{V} \frac{p \omega_n}{(\alpha + p)^2 + \omega_n^2}$$

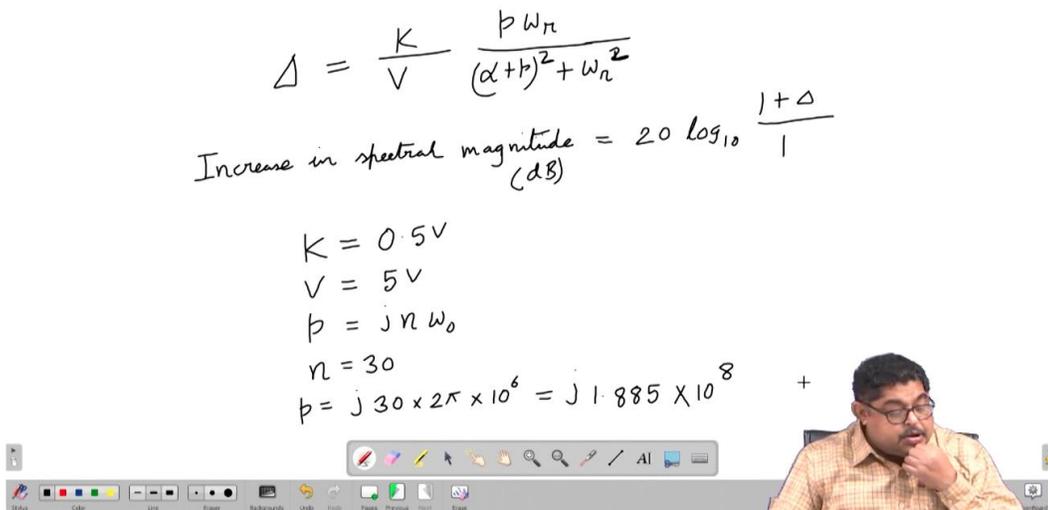
$$\text{Increase in spectral magnitude (dB)} = 20 \log_{10} \frac{1 + \Delta}{1}$$

$$K = 0.5 \text{ V}$$

$$V = 5 \text{ V}$$

$$p = j n \omega_0$$

$$n = 30$$

$$p = j 30 \times 2\pi \times 10^6 = j 1.885 \times 10^8$$


So, we can calculate delta if you put all these values in the delta it will come to be 0.9446 I think I will have to take this much width then only the points will come properly. Increase of spectral magnitude is will be 20 log 1.9446 by 1 that is a. So, that will be something like 5.78 dB. So, you see that at the ringing frequency that means, at 30 mega hertz you are having a almost 6 dB jump that means, whatever you have almost

double of that is coming at that frequency. So, that is an EMC engineer cannot neglect it. So, though we require some time to analyze it, but it is an important parameter of signal integrity that you if you have parasitic effects, parasitic inductance capacitance resistance present in the clock circuit then you will have ringing and you should be aware because this may suppose otherwise at that high frequency the thing was ok. You have designed a clock you it was passing the regulatory limits, but in practical case when there will be ringing you will get that there is a double the value at that high frequency 30 mega hertz. So, that may fail your whole thing. So, that time as an EMC engineer as a consultant you will have to say that there may be some ringing effect you just look down this thing that whether the ringing frequency is like this and then you try to nullify those parasitic effects to avoid this spectral jump. So, we have seen spectral bounds for our signal. Now, that signal or that clock that is passing through a system. So, in the next lecture we will see that when it passes through the system what is the upper bound here because ultimately that system. So, that system output we will see we cannot see what is the output of the clock because clock is driving the system. So, that thing we will see in the next class that what is a bound for that system. Thank you.

$$\omega_n = 2\pi \times 30 \times 10^6 = 1.885 \times 10^8$$

$$\Delta = 0.9446$$

$$\text{Increase of spectral magnitude} = 5.78 \text{ dB}$$

