Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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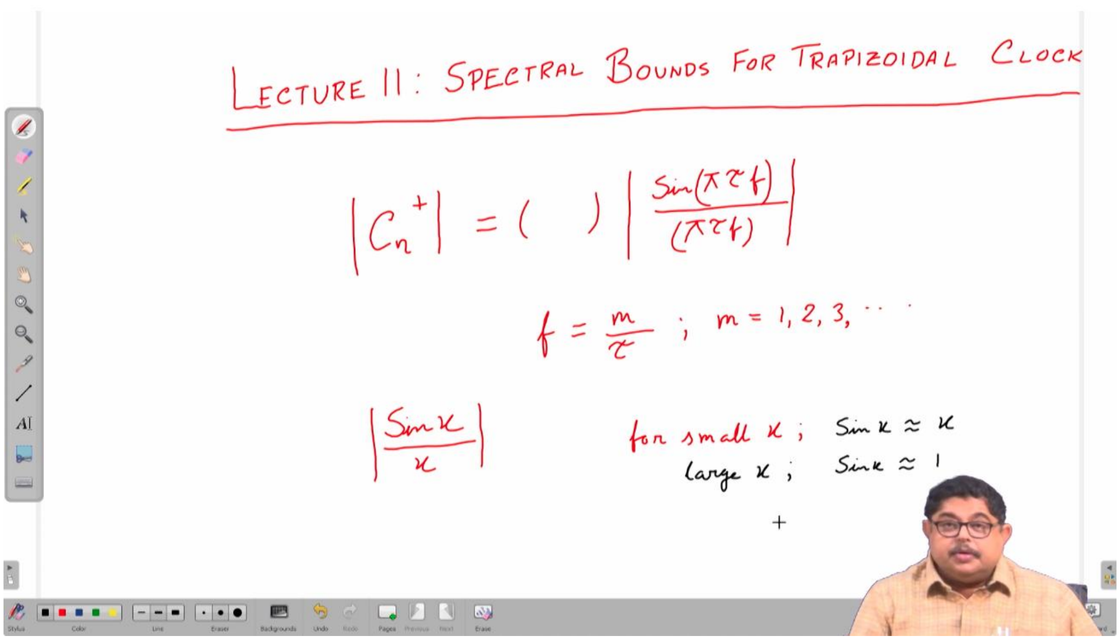
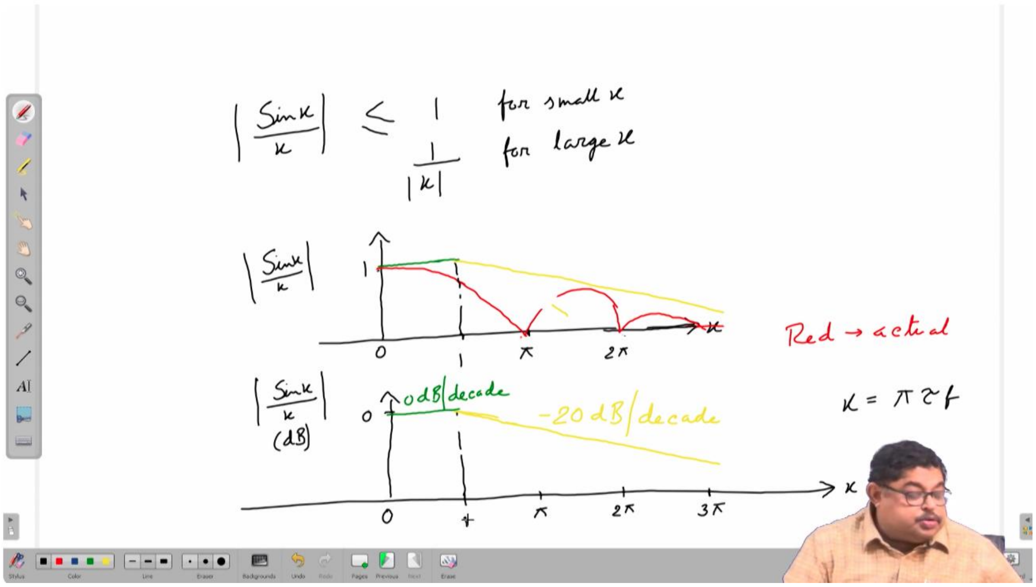
Week :03
Lecture 11: Spectral Bounds for Trapezoidal Clock

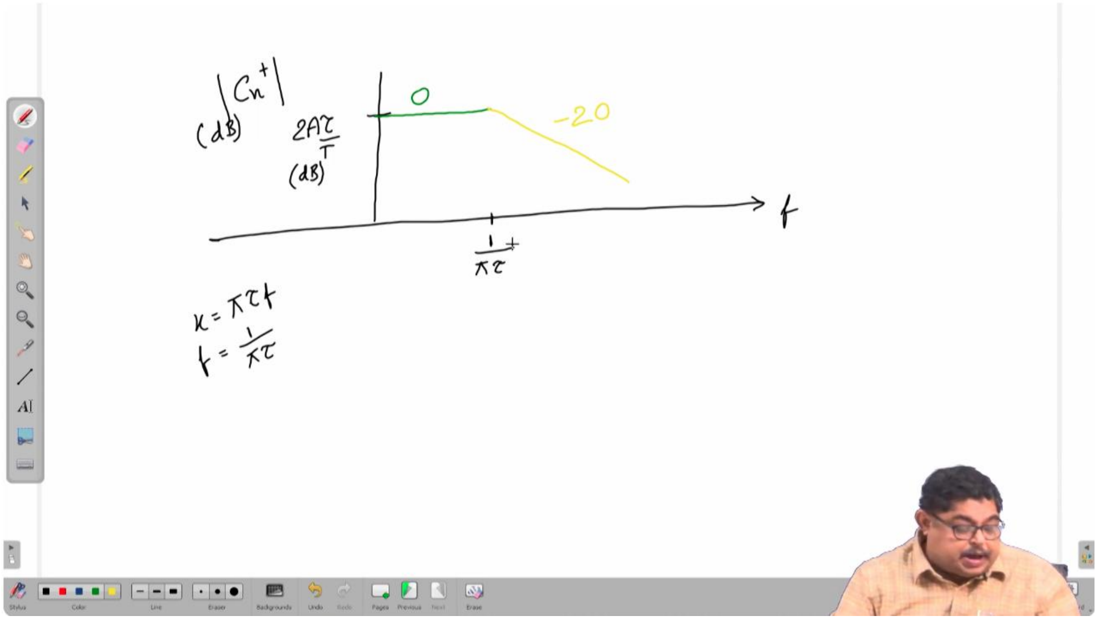
Welcome to the eleventh lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. Now, in the previous class we have seen the spectrum of a trapezoidal clock. Now, today we will see that how an EMC engineer will estimate the upper bound of the spectrum, so that he can make judicious choices whether the clock that the he is testing is satisfying or complying with the regulatory requirements. So, we recall that a square wave or a rectangular pulse train both are same, I sometimes I say square sometimes I say rectangular, but the thing is it is a pulse train and with zero rise and fall time. So, we have seen already that it is spectral magnitude that means c n plus that is with some constant it is of the form sin pi tau f by pi that means, a sinc function the magnitude of a sinc function. So, now we know that the spectrum is discrete and this spectrum or spectral coefficients are 0 when we have f is equal to m by tau where m is an integer, this is one sided so no plus minus required. Now, what is the upper bound of this, what is the maximum value of this at any frequency if I say that at any frequency I specify what is the upper bound of that can I simply say that or I will have to calculate it from here. So, to answer that question consider the upper bounds of the function sin x by x. So, we know that for small x for small x this sorry the sin x is equal to x and for large x sin x is almost 1 this we know from our elementary classes.



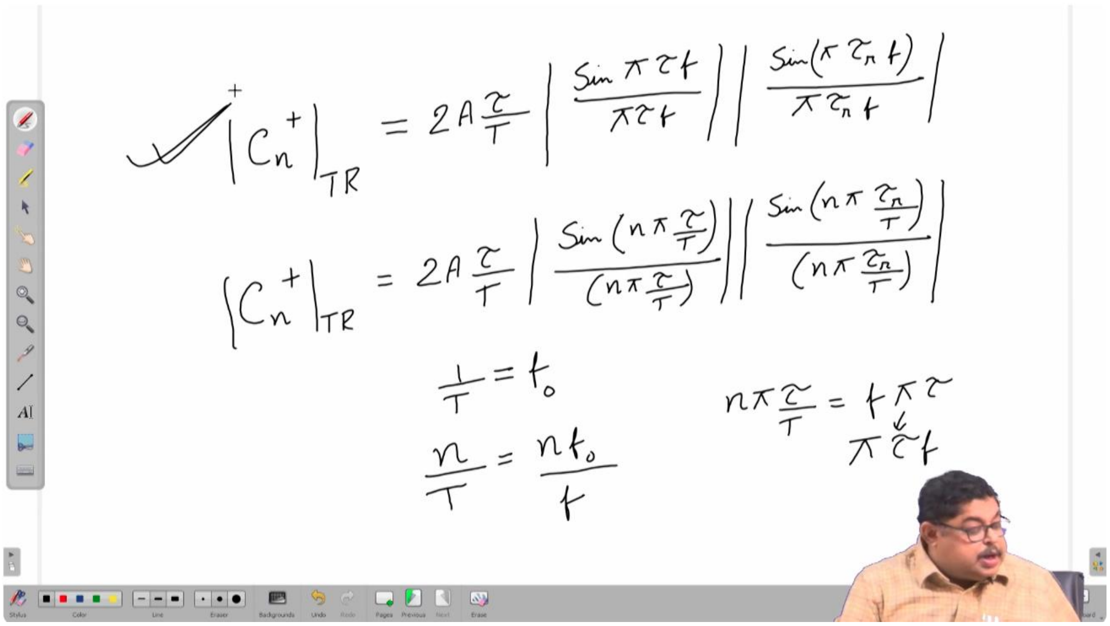
So, now we can say that if that is so then what is these two values. So, can I say that the sin x by x is less than 1 for small x and 1 by x magnitude for large x. So, you see that instead of calculating the sin x by x I can now say that basically the all this information can be drawn as an as two asymptotes one is this and another is something like this 1 by x. So, it will be a line. So, I can say that x and sin x by x is this is the upper bound and where they are meeting they are meeting at a value x is equal to 1. So, with this now if I draw the sin x function you see that we know sin x will be it has its first null at pi then its second null is at 2 pi the third null is will be at 3 pi etcetera and so its. So, this magnitude will be smaller. So, this is the actual value this red one. So, I can say that red is actual, but its upper bound is the green one is for x less than 1 it is a constant value and then the yellow one is the always you see actually my drawing is wrong let me remove this. So, something like this and so in a dB scale now if I put it generally I say that we always put it in dB scale. So, if I now put this thing in dB scale that means sin x by x thing. So, the first sorry there I have not shown you this value one thing here this value is 1 in the y axis which is sin x by x. So, at x is equal to 0 we know that it is 1. Now if I put it convert it to dB, dB means actually here we are talking of spectral amplitudes. So, it will be 20 log as I said that any primary quantity is 20 log that we discussed before. So, this value in d b will become 0 and the asymptote what will be it is a constant value up to 1. So, here the x side let me keep it. So, let me put those values 1 then somewhere here pi somewhere here 2 pi I have now extended the axis. So, somewhere here there will be 3 pi and in dB scale I can say that the magnitude at 0 that will be 20 log 10 1 log to the power base 10 of 1. So, that is that is why I have called it 0 dB and up to 1 it will go with a it will be flat and it will go with a 0 dB per decade slope. I think you know this from those bode plots you have learnt in your undergraduate courses that 0 dB per decade means if I increase the frequency by 1 decade then the change will be 0 dB that means it will be flat going, but then there is a break point here. So, this is the break point and there I will get a line which will start from 0 dB and it will go like this. So, asymptote this is the asymptote upper bound. So, this slope slope of this line a following line in bode plot you know it is minus 20 dB per decade that means if you change the frequency by 1 decade that means if it this frequency let us say if it is corresponds this x corresponds to the frequency let us say 1 megahertz if I make it 10 megahertz I will get at 10 megahertz minus 20 dB. So, that is the meaning and so this is the asymptotic thing or this is the upper bound actually this we were seeking. So, you see that I do not need to calculate this I always know that is sinc of x graph will look like this. So, for x now I can put that x is equal to pi tau f. So, the first asymptote will have a slope of 0 dB per decade this is the first asymptote the green one its value is 0 and it will go up to what is the value.



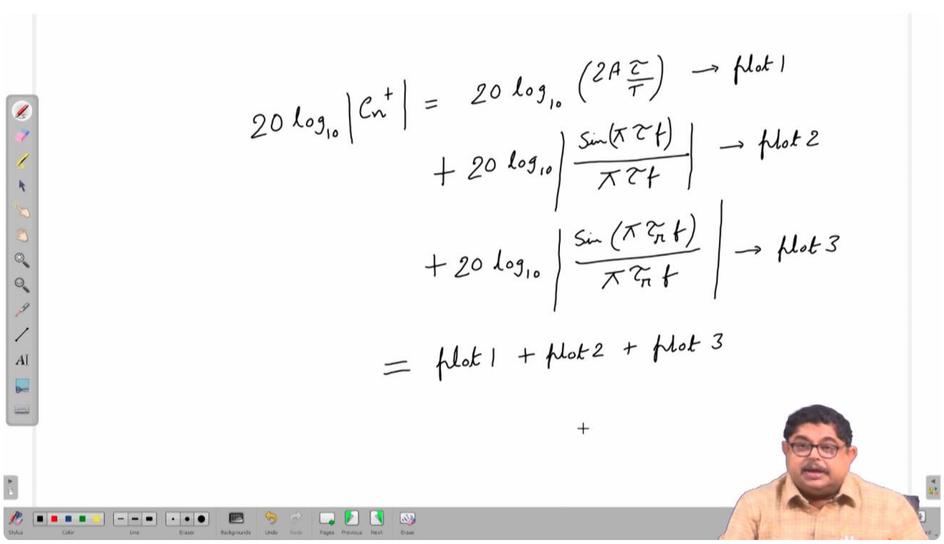
So, f is equal to so or let me draw it in a new one that let us now convert the same thing to frequency plot. So, can I say that this side I have c n plus magnitude my actual that rectangular pulse train its one sided spectrum that is in dB. So, if you look at the expression I will get some value here that is 2 a tau by t, but in dB that means, in 20 log form and then it will the first at in asymptote will go with a and I will now just write 0 here that will indicate 0 dB per decade slope. So, over the line if I write 0 it is 0 dB per decade and how long it will go it will go you see your x is equal to pi tau f. So, then the 0 will come that will come when f is equal to 1 by pi tau 0 now. So, this will come at 1. So, if we put x is equal to 1 f is equal to 1 by pi tau that means, this value is 1 by pi tau then. So, up to that it will continue with this 0 dB then so that means, the break point in first break point in frequency that is 1 by pi tau here there is only one break point. So, now, it will go with a minus 20 dB per decade slope. So, now, you can easily determine that what is the upper bound. So, if I tell that ok I have a clock of 10 megahertz at 100 megahertz that means, at 10th harmonic what will be the upper bound the actual value will be lower than that, but to be safe side that if I know the upper bound I will check whether that is complying with the standards. So, the second that in asymptote starts from 1 by pi tau frequency and with a minus 20 dB per decade slope. Now, let us go to trapezoidal clock this was for 0 rise time and fall time.



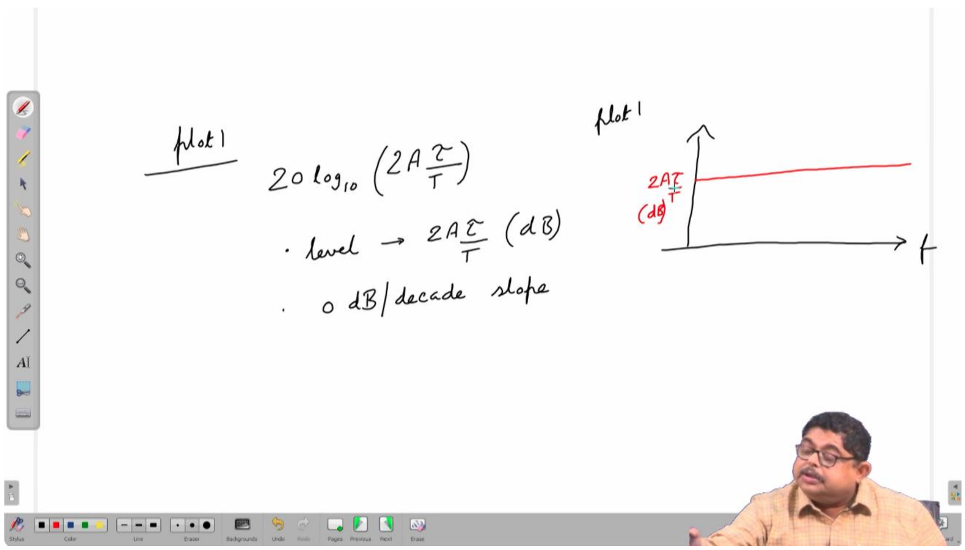
Now, with trapezoidal clock we know that sorry this color is not conducive always in the white background. So, let me take a black color. So, I am again writing we know that for trapezoidal clock trapezoidal it is we have seen it is 2 a in the previous class we have seen that 2 a tau by t these are always amplitude is always positive tau by t is duty cycle is always positive then sin pi tau f by pi tau f to sin pi tau r f by pi tau r f. Now, here you can ask me that how you got this we have seen it in terms of a n omega t. So, let us see that that we have seen it I think in last class we have seen c n plus t r was 2 a tau by t then where was that expression I think we have ended here last the sin n pi tau by t by n pi tau by t sin n pi tau r by t by n p tau r by t. Obviously, for rise and fall time equal that was our thing I think up to this we have seen last day. So, now, 1 by tau 1 by tau is what 1 by tau is your f naught fundamental frequency 1 by tau 1 by t is f naught fundamental frequency that means, the clock frequency and n by t is n f naught. So, these are the discrete spectral points. So, this one I am calling f if I do that you see n pi tau by t becomes f pi tau that is what I have written that pi tau f pi tau f same thing. Similarly, here also if you put that n by t n by capital T is nothing, but f. So, pi tau r f that is why I have written like this. So, this is a valid expression. So, this one now we will have to determine what is it spectral bounds.



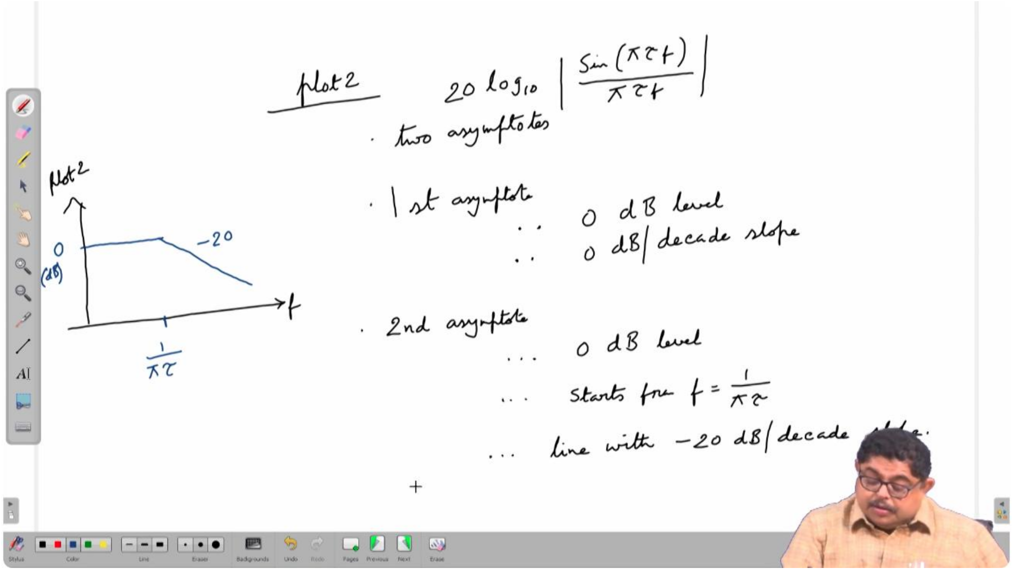
So, this is product of 2 sinc functions you see we have seen that a single sinc function how it is spectral bounds are that means, what are the it produces 2 asymptotes one is a flat line horizontal line another is a falling line. Now, if I make product so, how it will look like you see this is product of 3 things one is this some dc value this one then a one sinc function this is another sinc function. So, let us take 20 log of this side that means, 20 log c n plus magnitude of the trapezoidal block. So, I can write that 20 log 10 of c n plus will be 20 log of 2 a log of 2 a tau by T plus 20 log 10 sin pi tau f by pi tau f plus 20 log 10 sin pi tau f by pi tau t log 10 sin pi tau r f. So, can I say that I will add this as an individual one so, I will break this that there will be plot 1 for this one. So, this will give me plot 1 only this part will give me plot 2 and this part will give me plot 3. So, resultant plot will be plot 1 plus plot 2 plus plot 3. So, I will first describe each of these plots and then I will sum them up ok. You understand that if I take log of product of 3 things then it will be basically a summation that is the advantage of taking logs.



So, let us discuss about plot 1. What is plot 1? It is 20 log 10 2 a tau by T. So, what is its value? So, value or we call it level. So, what is its level? Its level is 2 a tau by T in dB and it is constant you see there is no frequency things. So, I can say that it is any constant thing means 0 dB per decade slope. So, that means, if I draw it that plot 1 versus f let me take some colour let us take red. So, it will be this and what is this value? 2 a tau by T in dB whatever value will come I will take 20 log of that. So, with that value it is going this is plot 1.



Then let us come to plot 2. Plot 2 we know it will be it will consist of 2 asymptotes. So, that is my first point that it is 2 asymptotes will form this plot. Now, what is plot 2? Let us take this value is 20 log 10 then sin pi tau f by pi tau f. So, it will consist of 2 asymptotes what is the first asymptote? For first asymptote for first asymptote what is this level it does not have any multiplier because multiplier we have taken in plot 1. So, I can say that it is level will be 0 dB level 1. So, 20 log 1 is 0 dB level and what is it slope? A slope is also 0 dB per decade slope. What is the second asymptote? It is value is again 0 dB level and it will start from starts from So, f is equal to 1 by pi tau it is a line with minus 20 dB per decade slope. So, if I draw it again f versus plot 2 let me take green colour here. So, we know that first asymptote will go like this then it is like this. So, this is minus 20 this is 0 dB and this break point is 1 by pi tau.



Now let us go to plot 3. So, what is plot 3? 20 log 10 sin pi f tau r by pi tau r f sorry let me write as I was writing pi tau r f. So, again we know that there will be 2 asymptotes for this what is first asymptote that will have 0 dB level level 1. So, 1 by pi tau r and when it starts from or sorry goes up to f is equal to 1 by pi tau r and goes up to 1 by tau r what is the slope? 0 dB per decade slope and second asymptote what is its value? Again it is 0 dB level starts from f is equal to 1 by pi tau r and slope is minus 20 dB per decade slope. So, if I draw it plot 3 versus f then again it is sorry I should have going here then is calling. So, this is 0 minus 20 and this point is 1 by pi tau r. So, we have seen that all 3 are known just we will have to sum them that will be doing them doing in the next class. Thank you.