Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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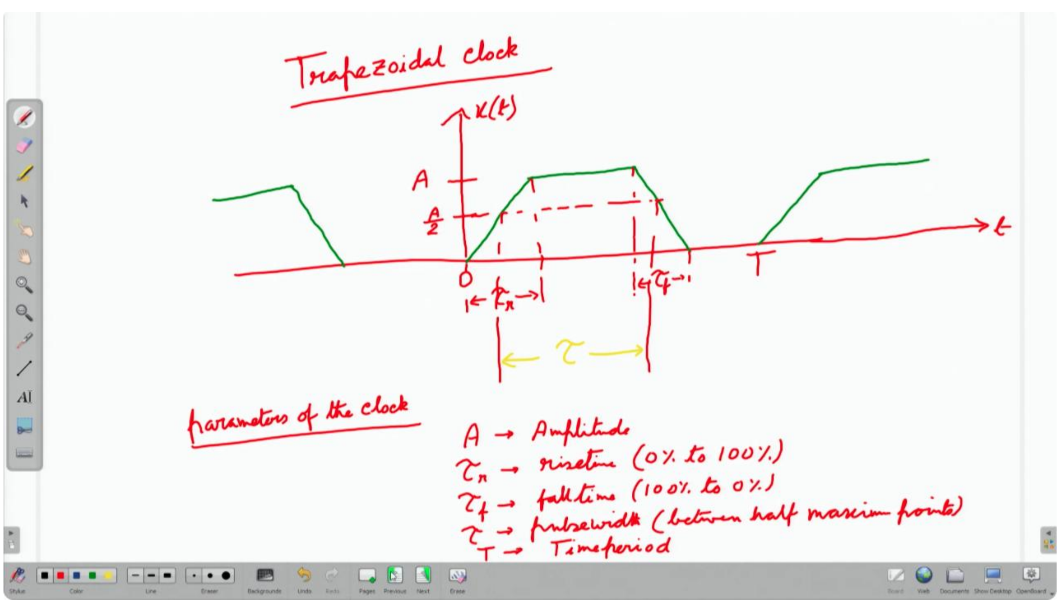
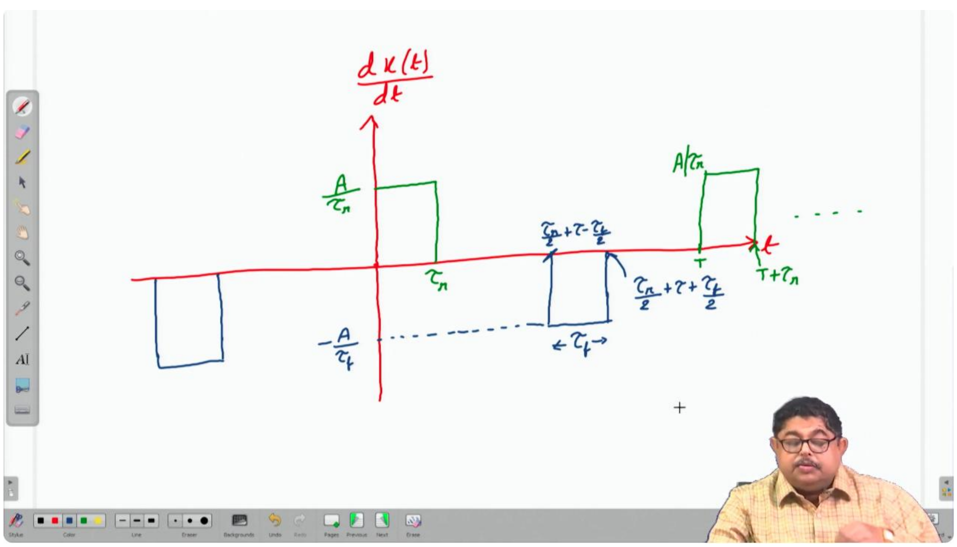
Week :02
Lecture 10: Trapezoidal Clock

Welcome to this 10th lecture of the course on EMIMC and Signal Integrity Principles Techniques and Applications. In the previous lecture, we have seen the algorithm for getting the Fourier coefficient in very elegant way. Now, today we will apply all that to the trapezoidal clock. Now, with this knowledge we will try to find the what is the Fourier coefficient for a clock which has finite rise time and fall time that is a trapezoidal clock. So, let us see a trapezoidal clock, trapezoidal clock or trapezoidal pulse tran . So, let me first draw there are some parameters of the clock that we need to understand.

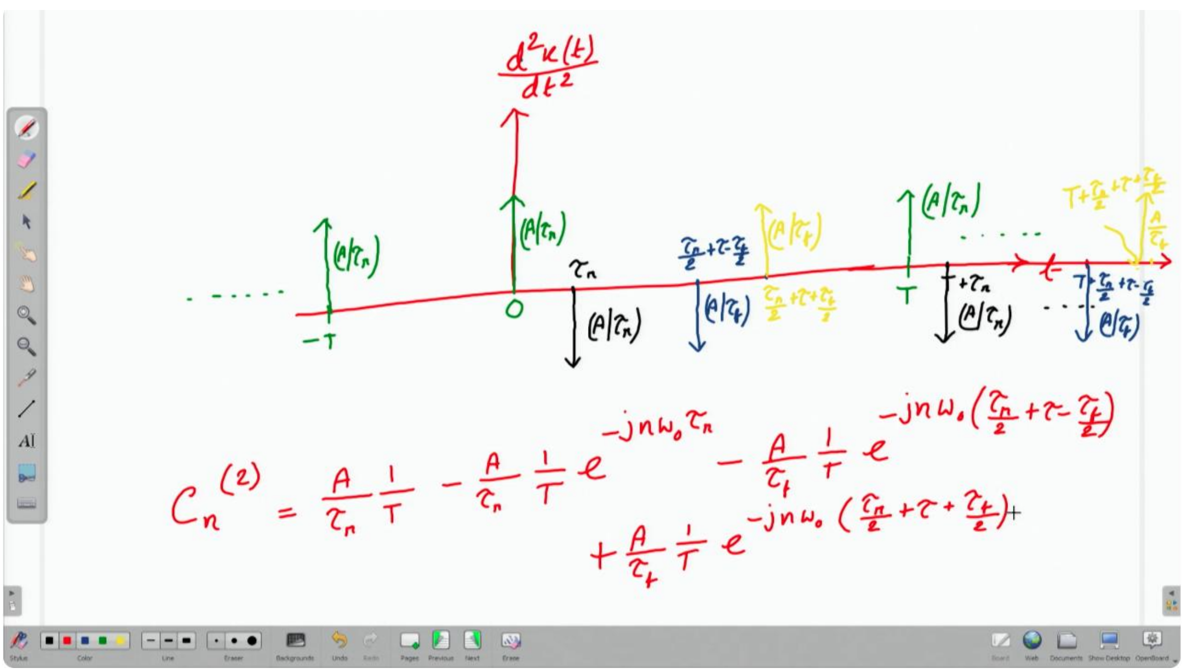
So, first let me draw the clock. So, let us draw the clock. For our analysis we will take that the time the clock takes to go from 0 to the peak value A that is tau r. You know, but in industry they sometimes take 10 percent to 90 percent of the waveform the time it takes to go from 10 percent of the peak to 90 percent of the peak sometimes they take as 20 percent to 80 percent of the peak different industry uses different things, but here we will take this definition that tau r is this similarly our fall time is this and for this one what is our pulse width that will take that it is between two half amplitudes amplitude is A, A by 2 is half amplitude. So, if I take this this is our tau and this is our time period when it is. So, what are the parameters of the clock? Parameters of the clock one is A is the amplitude, tau r is rise time 0 percent to 100 percent, tau f is fall time 100 percent to 0 percent, tau pulse width between half maximum points and T is the time period. So, with this we will try to find the spectrum that is the c n values. So, let us first differentiate it once.



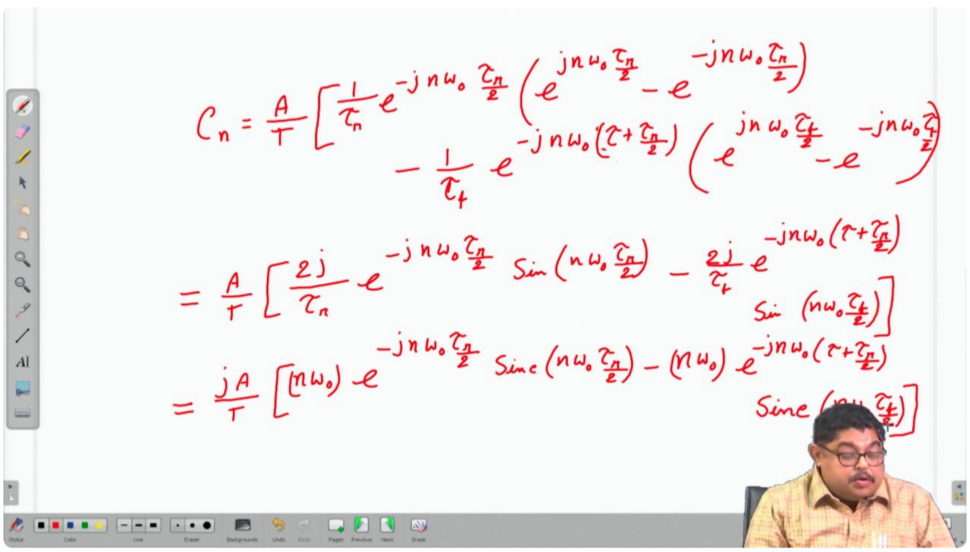
So, what will happen after first differentiation d x t d t. So, this portion will give me a rectangular portion again you know that amplitude will be A by tau r of that rectangular portion. So, let me use green colour. So, it is A by tau r and it will go up to tau r. Then there is a after differentiation the portion from this tau r to this line the flat line that will be 0 then again at tau f there will be something. So, let us say this is the falling portion. Now, what is the value of the falling portion that we will have to find? That means what is the value when the from the maximum it starts falling. If you see that if I look at that it is the half maximum of the rising portion that is I can say occurring at tau r by 2. So, from tau r by 2 if I give tau it is coming at the second half maximum point and from that if I subtract tau f by 2 and coming to the point where the clock is starting falling. So, this value of this will be let me. So, at time tau r by 2 plus tau minus tau f by 2 I will see that there is again a tau r falling the means falling. So, after differentiation it becomes this. So, this width is tau f or you can find out. So, this point is tau r 2 by a this point will be tau r by 2 plus tau plus tau f by 2 that means this width is tau f. Then at T there will be this repetition that it will be a by tau r this point will be T plus tau r that is it. So, this will go. Similarly, here again this value is what that means this value will be minus a by tau f this side. So, this is rising edge let us see what will happen here this is again a falling portion. So, I can say that there will be. So, like this it will go.



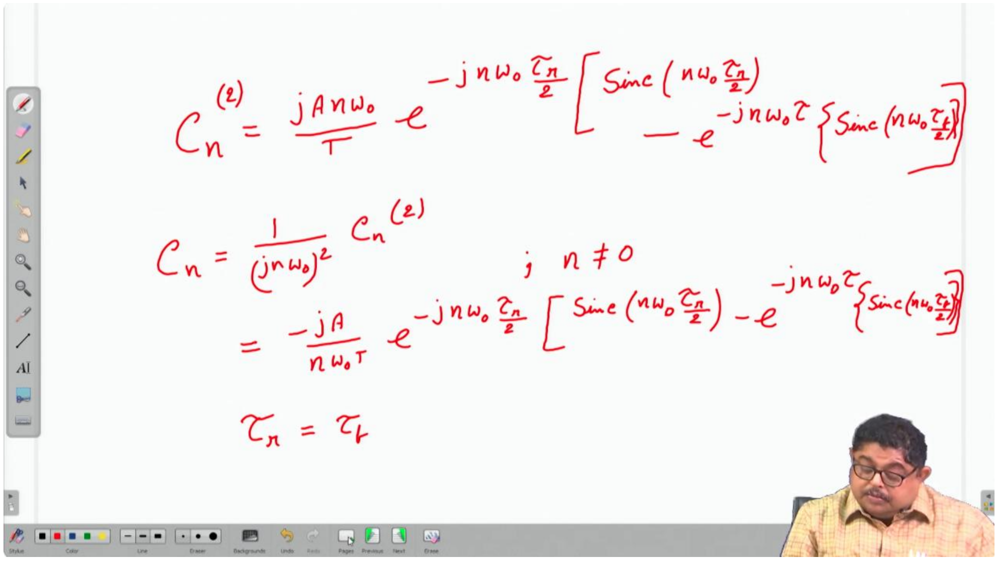
So, still we have not got any impulse train. So, we will have to make another differentiation. So, we will have this one. If I differentiate you see this this one should be green. So, this one will give me an impulse this one will give me a rising impulse. Now, this one that means at tau r what it that will give me a falling impulse this one will give me a falling impulse. Similarly, here you see this will give me a falling impulse this will give me a rising impulse. So, there will be 4 types of things. So, let us do that carefully that dx 2 dt. So, let me take the first rising impulse as this one. So, this is a by tau r it will be repeated at T there will be another a by tau r. Similarly, this is at 0 this is at minus T there will be a by tau r. Then there will be at tau r there will be a falling one. So, let me choose this color that at tau r there will be a falling one with value a by tau r. Similarly, it will be at T plus tau r T plus tau r there will be again its next occurrence. So, like this it will also go this will go like this this will go like this. Now, let us see the other ones. So, at tau plus that you see previous one tau r by 2 plus tau minus tau a by 2 there will be a falling impulse train. So, tau r by 2 plus tau minus tau a by 2 there will be a falling impulse strain again that value will be a by tau f and since I do not have much space I am writing it here. So, what color is still remain in this one. So, can write at tau r by 2 plus tau plus tau a by 2 there will be a rising impulse strain a by tau f this whole things will be again repeated. So, we can say that here if I look at this look at at at T plus tau r by 2 plus tau minus tau f by 2 there will be a falling impulse strain a by tau f at T plus. Tplus tau r by 2 plus tau plus tau f by 2 like this. So, there are 4 different pulse trains. So, all of them were obtained after second differentiation. So, can I write c n 2 is equal to a by tau r the green one 1 by T minus a by tau r 1 by T it is shifted by tau r.So, e to the power minus j in omega naught tau r minus a by tau f 1 by T e to the power minus j in this has been shifted by j in omega naught tau r by 2 plus tau minus tau a by 2 plus a by tau f 1 by T e to the power minus j in omega naught tau r by 2 plus tau plus tau f by 2 this is my c n 2.



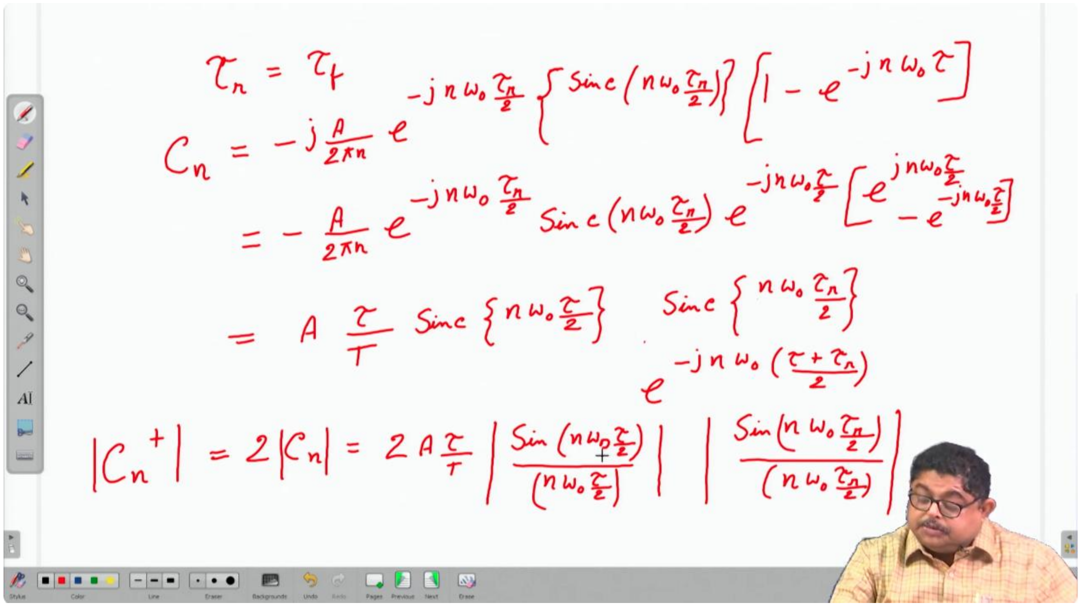
So, I can simplify this I take a by t out then 1 by tau r e to the power minus j in omega naught tau r by 2 e to the power j in omega naught tau r by 2 minus e to the power minus j in omega naught tau r by 2 minus 1 by tau f e to the power minus j in omega naught tau f by 2 omega naught sorry omega naught tau a by 2 plus tau r by 2 into e to the power j n omega naught tau a by 2 minus e to the power minus j in omega naught tau by 2. So, a by t you take common here this will give you a sign. So, can I write directly that 2 j by tau r e to the power minus j n omega naught tau r by 2 sign of n omega naught tau r by 2. Similarly, minus 2 j by tau f e to the power minus j n omega naught tau plus tau r by 2 into sign of n omega naught tau f by 2 j a by t n omega naught e to the I am trying to now make sinc out of this from this sign. So, e to the power minus j in omega naught tau r by 2 sinc of n omega naught tau r by 2 minus n omega naught e to the power minus j n omega naught tau plus tau r by 2 into sinc n omega naught tau a by 2.



So, e to the power minus j a in omega naught by t e to the power minus j in omega naught tau r by 2 all I have taken out. So, sinc of n omega naught tau r by 2 minus e to the power minus j n omega naught tau sinc in omega naught tau f by 2. So, this is sorry this is my C n 2 check previous this is also C n 2 this is C n 2 C n 2 is still going on. So, what is C n? C n will be 1 by double differentiation j n omega naught whole square C n 2 n naught is equal to 0 because for DC this is not the case. So, is equal to if you put that then minus j a by n omega naught t e to the power minus j n omega naught tau r by 2 all those things that sinc n omega naught omega naught tau r by 2 minus e to the power minus j n omega naught what is happening omega naught tau f by 2. So, this is the result for trapezoidal clock. Now, this thing can be or in a in most of the cases the in trapezoidal clock the rise time and fall time they are same. So, if we make that that tau r is equal to tau f then this can be further simplified.



So, that we will see now that for tau r is equal to tau f that means, rise time and fall time are equal which is most of the cases that happens. So, C n becomes minus j a by 2 pi n e to the power minus j n omega naught tau r by 2 then you can now take this sinc is out. So, sinc n omega naught tau r by 2 if you take it out what you are left with is 1 minus e to the power minus j n omega naught tau you see from previous this e to the power e to the power minus j n omega naught tau that will be there. So, again you can take e to the power minus j n omega naught tau by 2 common that will give you another sinc term. So, minus if you do that minus j a by 2 pi n e to the power minus j n omega naught tau r by 2 sinc of n omega naught tau r by 2 e to the power minus j n omega naught tau by 2 then you will be left with e to the power j n omega naught tau by 2 minus e to the power minus j n omega naught tau by 2. So, you can do this manipulation and you will get finally, this value this you can easily do I have done enough. So, sinc n omega naught tau by 2 then sinc n omega naught tau by 2 then e to the power minus j n omega naught tau plus tau n by 2. So, you see this one and this one these two exponential terms they are here and there are two sink terms one is dependent on the pulse width tau another is dependent on the rise time or fall time because now rise time or fall time are similar. So, this is the expression there are two sinc terms product in a normal pulse train that means, where rise time fall time is not there you have one sink function here you have product of two sinc function one is like the pulse train another is due to the rise time. Now, this is the double sided spectrum. So, what is the single sided spectrum that is C n plus that will be equal to 2 C n is equal to 2 a tau by T if we take the magnitude part only then this e to the power term will go and so we will have sin n omega naught tau by 2 by n omega naught tau by 2.Then sin n omega naught tau r by 2 n omega naught tau r by 2 this is the magnitude spectrum of a trapezoidal clock. Now, here again as before we can remove omega naught by noting that omega naught is 2 pi by t that will bring our t because t is in instead of in terms of frequency this we can write.



So, if we do that then our C n one sided spectrum magnitude that will be 2 a tau by t sin n pi tau by t by n pi tau by t sin n pi tau by t sin n pi tau r by t by n pi tau r by t. What will be our angle or phase spectrum? We have already seen that how to for sin what happens? So, that will be plus minus n pi tau plus tau r by t from this that is phase spectrum will be addition of these two. So, that will be sin space and this sin space you know that sin may be plus minus that way is the spectrum and what is the DC value C0? C0 is a tau by t. So X t can be written as C0 plus one sided spectrum n is equal to 1 to infinity C n plus magnitude cos n omega naught t plus angle of C n. . . . . Now, you see that one of the popular choice is or most of the cases people try to make the duty cycle 50 percent. So, if I do that that mean then tau by t will be half. So, the first sinc term that will be the first sinc term is sin n pi by 2 by n pi by 2. So, this will go to 0 for n even that means, the even harmonics are absent if the clock has 50 percent duty cycle you only have odd harmonics of the clock. However in practice clock duty cycle fluctuates it is the designer makes it 50 percent, but today it may be something 49 percent tomorrow it may fluctuate it is very rare that it will tend at same 50 percent. Now, the problem is near 0 the sinc term varies widely you know a sin theta that you see that near 0 that has a very fast variation whereas, near peak it is a quite stable. So, slight variation in duty cycle from 50 percent to some other value can cause even harmonics to fluctuate rapidly whereas, odd harmonics are relatively stable this have is this variation. So, in a radiated emission test if the offending frequency is an even harmonics of the clock then emission level measured at one day may fluctuate widely on another day due to variation on clock duty cycle. So, you should always check whether even harmonic is the culprit or not. So, remember these in a field test you should remember that even harmonics try to find out and if from your one day it is often not meeting the limit in another day it is meeting the limit sometimes the manufacturer may report you that that he has done it, but at your end you are finding that it is not there check the even harmonic. If the even harmonic is there try to find that if you can bring the duty cycle proper then that may go away. So, we have now got the spectrum of the trapezoidal clock a practical clock waveform. Now, we will have to estimate that what are the bounds of this clock that means, if I tell you that at 5 harmonic what may be the highest value of this spectrum. Now, in field you cannot always calculate everything. So, we will try to get some upper bound of this harmonic spectrum the upper bound of the spectrum. So, that we can find out that if our those harmonics are meeting that bound then we are safe that they cannot cross the limit emission limits. So, that will take up next day.