Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 10
Boundary Control for Fast Transient Recovery
Lecture - 46
Sliding Mode Control Design in a Buck Converter

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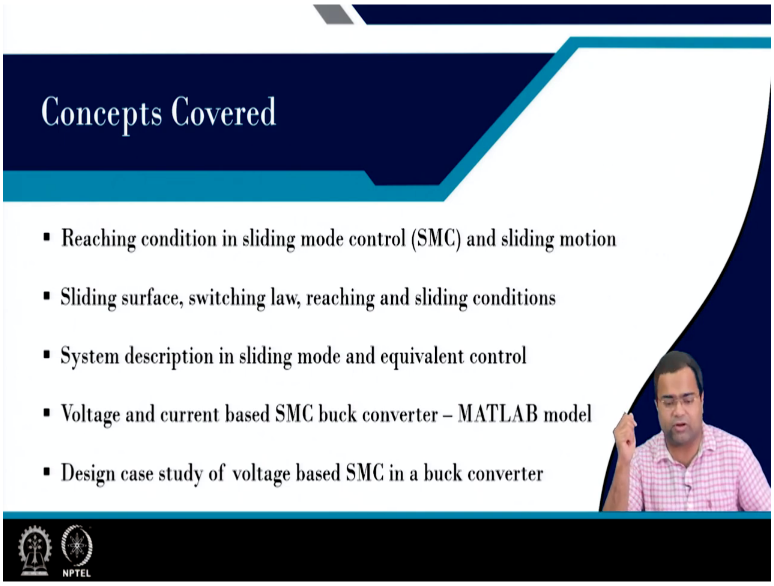

Welcome this is lecture number 46. In this lecture we are going to talk about Sliding Mode Control Design in a Buck Converter. In fact, this lecture is a continuation of our previous lecture; lecture number 45, where we have just introduced the concept of sliding mode control; that means, by means of switching, you know whether it is a linear system or a DC-DC converter we have shown vector field and where we have introduced the concept of switching.

And then in this lecture we will continue that how do you synthesize the switching law. So, that any arbitrary initial condition will reach towards the switching surface. So, in this lecture we are going to talk about the reaching a condition in sliding mode control and the sliding motion. Then we will talk about sliding surface, switching law, reaching and sliding conditions. Then we will talk about system description in sliding motion mode and equivalent control law.

Then voltage and current based sliding mode control in a buck converter that MATLAB model. And finally, we want to talk. We will want we want to consider one design case study of a voltage based sliding mode control in a back converter.

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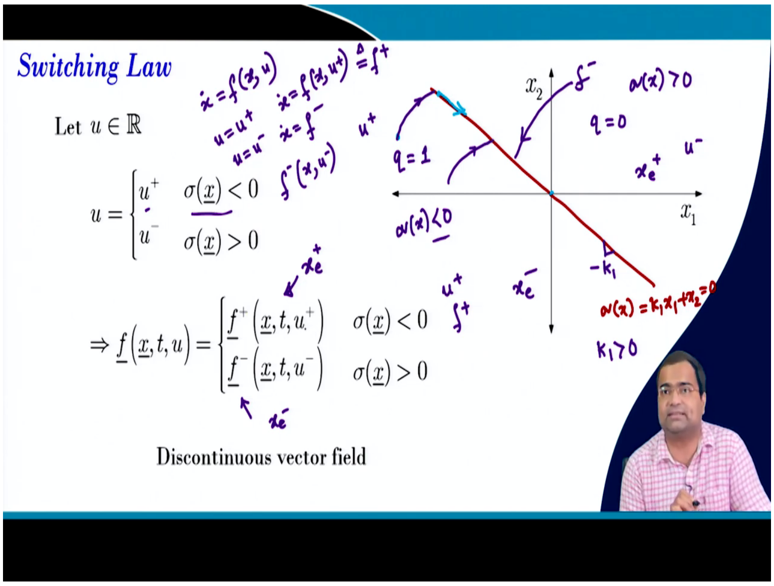

So, let us first consider a switching law. In fact, this we just talked at the end of the previous lecture. So, in this switching law suppose we take a second order system x 1 x 2 are the states and suppose we want to create an eigenvectors for example, equivalent to an eigenvector.

So, this is this side and if we extend to the other side and here it is our sigma x is equal to k 1 x 1 plus x 2 equal to 0. So, this line represents something similar to an eigenvector that we want to achieve it is our desired one ok and what we need to do? Suppose if you start any initial condition in on this line, then we need to ensure that motion will be either towards this.

If you can ensure the stability and it will asymptotically approach towards this origin that is our one of the target. So, and again another target is that if we take any initial condition here for example, then we should also ensure that it will actually attract towards the surface ok.

So, one of the condition is that any arbitrary condition either side of this line we should force them to take it to the line and once it reaches the line, then that line will satisfy the invariant principle; that means, it will move along the line.

So, we have to generate their switching logic, which will satisfy. In that case, it will look like we have synthesized one eigenvector and in a two dimensional plane it will look like a straight line. So, the whole motion will be constraint along the straight line if we can take it to the line and then the whole dynamics of the motion can be represented by a single order because it is a straight line motion ok.

So, that is one law. And what is what we are considering. So, let us consider we talked about a control law u; and the u can take either u plus and u minus. And in DC-DC converter example for buck both buck as well as boost converter in our previous example we saw the vector field we have considering in such a way that q equal to 1 that is our switch on in the left side of the surface.

And where the left side we have sigma x less than 0 in this case and sigma x greater than 0 in this case as in this case k 1 is greater than 0 ok. So, for k 1 greater than 0 ok and this slope is minus k 1 that is our design parameter ok. Then the switching law q equal to 1 in the left side and the right side we took q equal to 0. So, in general we can have discontinuous conduction control law left side with sigma less than 0 we will take this represent u plus, and this represents u minus.

Now, what was our original control law? Suppose we have x dot equal to f x any non-linear system f x u. Now, when you take u equal to u plus then we are getting x dot equal to f of x comma u plus and this we represent in terms of f plus ok. Similarly, when we take u equal to u minus then we represent x dot we represent equal to ok. So, we take x dot equal to f minus. And what is f minus? f minus is nothing, but x comma u minus ok.

So, and what we saw for since we have taken this left side u plus. So, naturally the vector field is f plus and for each case we have discussed that we can find out the equilibrium point ok. So, what is the equilibrium point corresponds to this vector field? Let us say x e plus.

And what is the equilibrium point corresponding to this vector field? Let us say it is x e minus. And what is our first requirement? That if I take f plus vector field to the left side of the surface, then x c plus; that means, the equilibrium point of this system must lie on the other side of the surface.

Similarly, f minus vector field that we have considered this side, this vector field should have its equilibrium point on this side. So, that it will try to come towards this and here any initial condition will try to go towards the equilibrium point. So, that is the first requirement. So, our vector field, which is considered in the left-hand side of the surface, must have the equilibrium point in the other side.

Similarly, the vector field on the right-hand side should have the equilibrium point on the left-hand side. The equilibrium points on either side will be attracted towards the switching surface. And that gives us what is called so; that means, the vector field is discontinuous f plus and f minus because we have control law u plus and u minus and this discontinuity happen either side of the surface ok.

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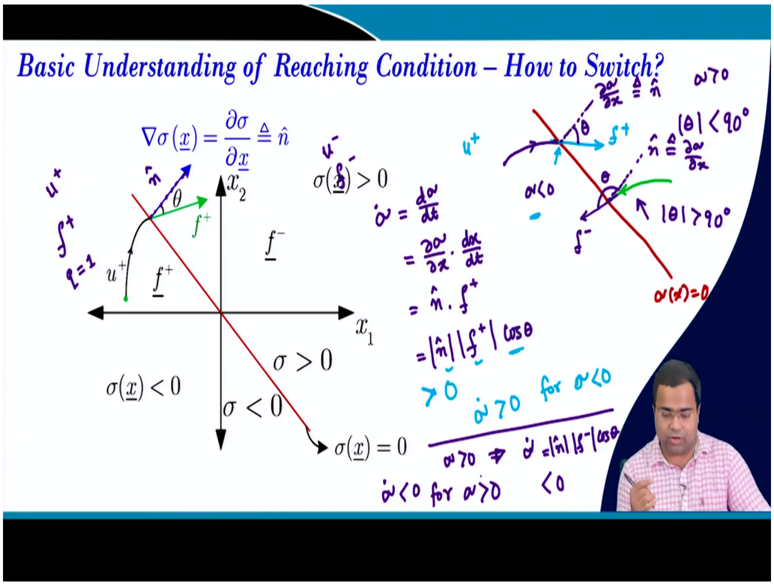

So, the basic understanding of a reaching law. So, what we need to ensure now? So, we have considered this side f plus right and we need to ensure that whenever we turn on and here; we have considered q equal to 1 for the DC-DC converter. In general, we write u plus right. On this side we wrote u minus and then we have vector field of f minus ok. And now, this is the surface sigma and this gradient vector of this sigma is our you know the surface vector which is the dou sigma dou x ok.

So, now imagine the trajectory, and we redraw this circuit diagram. Suppose this is our surface, this is our sigma x equal to 0. Now, let us say we have considered one trajectory or basically initial condition and here it is moving like this; this is the trajectory right. So, what is our interest? Whenever the trajectory intersect this surface or it is trying to intersect what we need to see, that we need to draw a tangent to this trajectory because that indicate.

So, if you have u plus the tangent, indicate f plus at this point. Because we know how to draw vector field at a given point. So, at this point, let us say that talking about this point where it is about to intersect. So, we draw the tangent on the vector field. And then what we draw? At the same time, we draw the normal to this vector field, which is dou sigma dou x.

And this dou sigma dou x is we represent like an equivalent sense it is a normal vector right. And we need to first identify what is the angle between these two. The normal vector and our vector field, which is nothing but the tangent at the point and tangent, which is drawn on the trajectory of the point that is a vector field.

So; that means, if we take sigma dot; that means, if we talk about sigma dot which is nothing but dou sigma dou d t d sigma d t and this can be obtained as dou sigma dou x into d x d t right. And if you do that, this is like our normal vector dot and this is like our f plus if you take in the left side. And this dot product, if you think just conceptually if we take the magnitude of this, then the magnitude of this into cos theta cosine theta angle.

So, if you see the diagram since we are going from left to right, and it is in the left side right and left side we have sigma less than 0. So, what we need to do the angle must be we must require the angle amplitude must be smaller than 90 degrees. Then only the left side trajectory will approach towards the surface.

If it is greater, the trajectory is going. I mean, it is coming out from the trajectory; that means, you imagine another scenario at trajectory you took you took the control law like this. Where if you draw a tangent here, suppose we draw a tangent at this point and this is a tangent, and this is our normal vector. So, this angle is greater than 90 degrees. This trajectory is moving out or going away from the surface.

So, first requirement from left side to right side to reach towards the surface your angle magnitude of the angle must be less than 90 degree and which tells that this must be greater than 0 this must be greater than 0 because this magnitude is greater than 0 and this angle is greater than 0.

So, which means we need to have sigma dot greater than 0 for sigma less than 0, if you take in the left side because left side with sigma less than 0. Similarly, if I can consider another scenario, for example, let us say another scenario. Let us rub this part and we are talking about another scenario. What is the scenario?

Suppose we took a trajectory coming towards it from this side, ok. We need to check whether it is coming towards it or not again. We take this point and we draw a tangent here and this is my f minus vector field. But, we have our tangent, the normal vector which is a surface and which we have represented, like a dou sigma dou x.

Now, in this case, you will find angle between this theta definitely is greater than 90 degrees. When the trajectory is coming from right to left, it has to be the angle between the tangent, which is the f minus and the angle; that means, angle between the tangent and the normal vector, that angle magnitude must be greater than 90 degrees.

So, for this case we need that angle must be greater than 90 degree. And if it is the case; that means we are talking about the second case when sigma greater than 0 because this side we have sigma greater than 0. Then we need that this sigma dot; sigma dot if we again write it will be again like a n f minus into cos theta, that must be less than 0 because cos theta greater than 90 degree will be a negative right.

Which means we are getting for sigma dot must be negative for sigma greater than positive. And this reaching condition we are going to discuss how it is going to translate into our generalized criteria of the reaching law.

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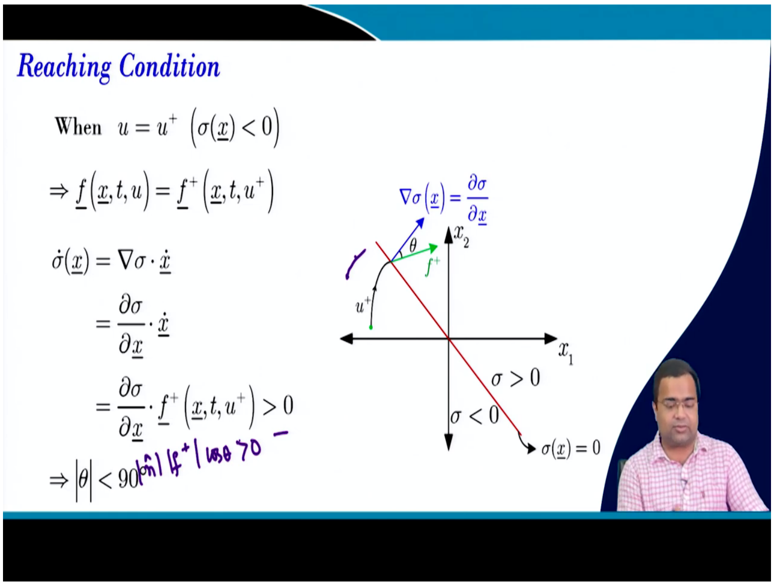

So, let us go to the reaching law, we told if we took u plus; that means, we have chosen left side it is going towards this. Then we have discussed that if we talk sigma dot we are getting that this sigma dot expression and it should be greater than 0 that we have just checked. That means the angle, which means that n dot that magnitude f dot magnitude into cos theta must be greater than 0. So, for this case, it is clear. So, angle should be less than 90 degree.

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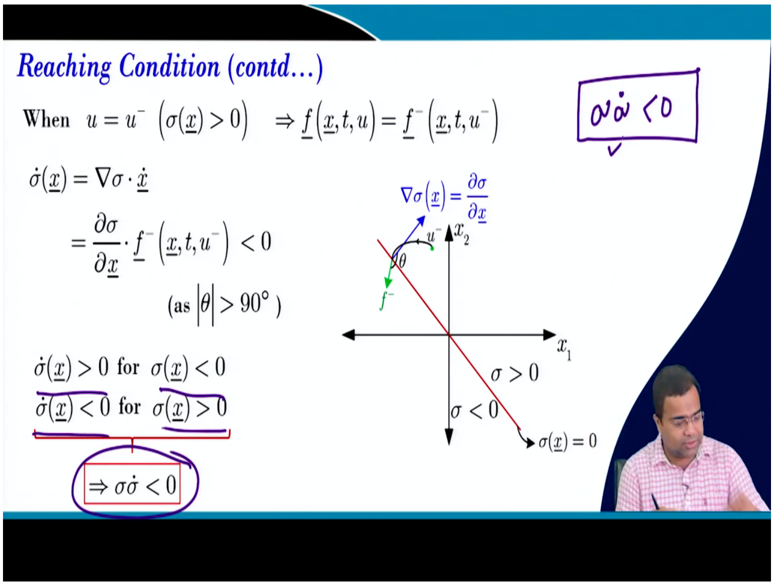

Similarly, if we take u minus from either side other side, it will be sigma dot and we check the angle should be greater than 90 degrees. So, for sigma left side you are getting sigma dot positive, for the right side we are getting sigma dot negative. So, if you combine, you will get sigma, sigma dot negative and that is the reaching law ok.

In fact, this can be also derived from the Lyapunov stability criteria, but we are not going to do that. Our ultimate aim is that sigma sigma dot should be negative and that will ensure and we need to choose the control law in such a way it satisfies this law in either side of the surface. Then it will ensure that trajectory will force to reach towards the surface that is the reaching law.

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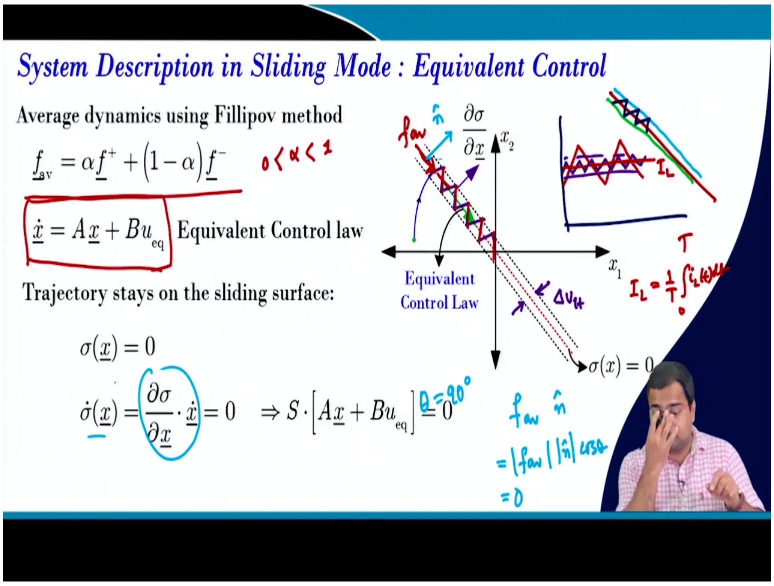

Then the system description, once it reaches the sliding surface, and we have discussed we need to insert one hysteresis band because we need to consider a practical sliding motion where otherwise suppose you know you consider once case. Suppose you know, for example, I am just giving one example. Suppose this is our switching surface ok.

And suppose we took a very narrow that means we took a very, very narrow band, very narrow band, very narrow band, very narrow band. So, you will find that this if you draw it will be very, very small. So, it will decrease. So, if we keep on decreasing this band then and the reflection will be in your inductor current in DC-DC converter.

So, if your average current is fixed for a larger band you will get like this. For a smaller band, you will get like this right. So, the smaller band your ripple will get reduced, but the switching frequency is increasing significantly for a larger band your ripple get is increasing switching frequency is reducing.

So, you cannot afford to have either very high switching frequency or very low switching frequency that we have discussed multiple time because this ripple comes from the design criteria. So, you need to specify a particular band which corresponds to a particular switching frequency.

And it is one of the difficulty or limitation of sliding mode control where your switching frequency will vary and in fact, there are many paper to address this and if you go to digital control in fact, you can regulate the switching frequency even more effectively. But, the bottom line is this: we need to regulate the switching frequency. And we need to decide certain delta v here some delta band some hysteresis band.

Otherwise, it is not practically possible to operate because your practical switching system or converter will have a finite rise time and fall time of the switch you cannot turn on and off arbitrarily fast. So, it has to be there has to be a limit. So, once you reach in the average sense, we can apply like because it is a switching right because this is a turn on. Then you know turn on these are the turn on dynamics right and then you can you have the turn off dynamics, this is the turn off dynamics.

And since they are fast. So, you can consider the average dynamics; that means, it is similar to a DC-DC converter. We are trying to talk about the average inductor current average inductor current right. And what we do for average inductor current? We generally 1 by T; that means, 0 to T we take i L t d t.

So, something similar can be achieved by Fillipov method, where you know it says that if you have a two vector field, you can get the average vector field which is moving towards this red actual surface will be alpha times f plus into 1 minus alpha time f minus.

And what will be alpha? Alpha is 0 to 1 because it is a fractional quantity. So, it will be the weighting factor so, it is an average dynamics. And once you substitute this average dynamic, then you can represent the dynamic of the average dynamics during the sliding motion using an equivalent control law, and this is something similar to our average model. Because we derive an average model from there, we derive average small signal model and so on.

So, this is something like an average model and we are interested to first derive the average model during the sliding motion and then we need to check whether the motion is asymptotically stable. What is the order of the dynamics during the sliding motion all we are going to discuss.

So, this is all equivalent control law. And during the sliding motion, you know we have sigma x equal to 0 that we have known and if you take the average trajectory which is there. So, this is our i average this trajectory is actually normal to this surface that is a normal to this surface.

That means we want the average trajectory. If you take it is if you take that i average magnitude then n average into cos theta, this theta this theta must be 90 degree because then only it will move along the line. Because if the theta is you know less than 90 degree then this trajectory will go towards this greater than 90 degrees.

So, in either cases it will move out of the line, but we want to ensure it will move along the line. So, it must be perpendicular to the surface normal like a normal vector. So; that means, the cos theta so, it will be 0; that means, sigma dot should also be 0 because these two will be a dot product, here it will be you know perpendicular.

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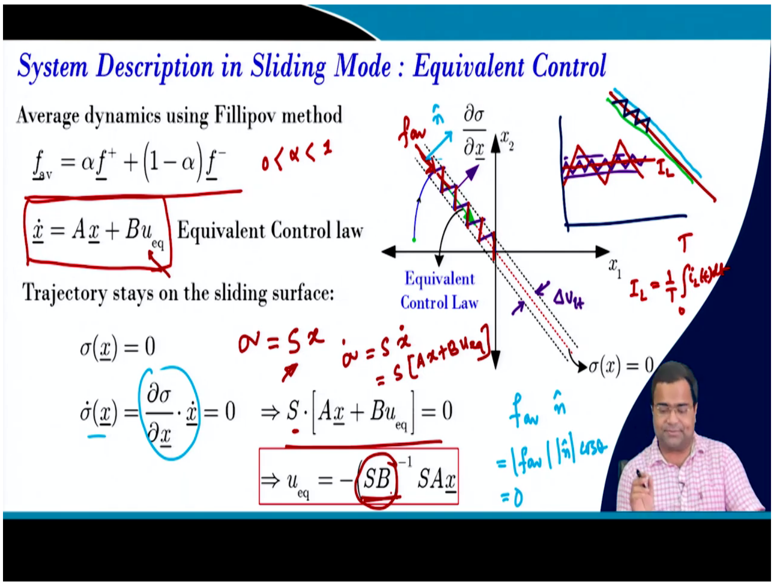

So, from here we can write that since this is 0 during the equivalent control law or sliding motion, we are getting. Now, whatever our switching surface we took in general expression, it was S into x right. And what was S? It was the hyper plane that we are going to discuss. I think we have discussed it.

And what is our so, if you take sigma dot it will be S into x dot and this is exactly and during the sliding motion we have S into x dot is A x plus B into equivalent control. So, during sliding motion we call it as an equivalent control because it is something like an average control right.

Because we have a switching control u plus and u minus in either side of the surface, but when you take the average, then it represented by the average control. And this if you solve the average control law is simply minus of S B inverse of S and this inverse must exist. So, we need to choose a this S particular, which is a mapping between the surface and the state vector in such a way with the input vector this you know this particular S B must exist, that there that must be full rank matrix.

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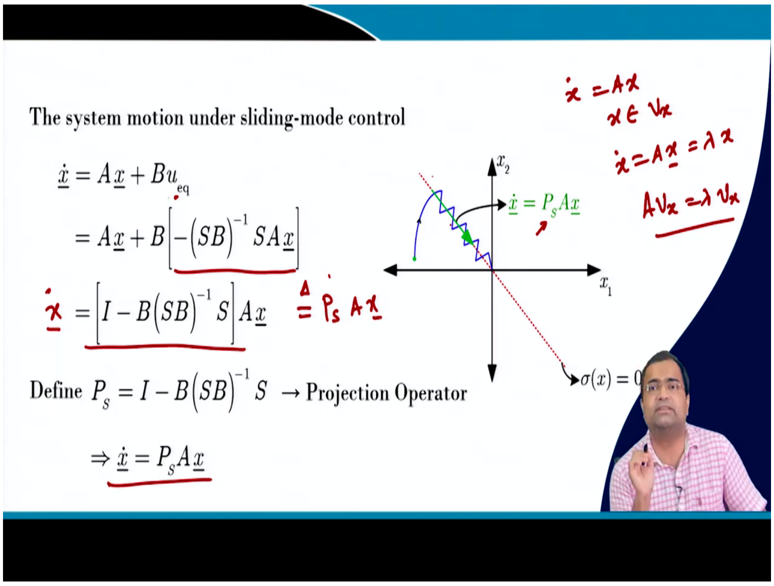

So, now in the sliding motion we obtain that if you substitute the equivalent control law from this expression, then during sliding motion, if you substitute here, you will get this equivalent control law. And then if you simplify so, this is the closed loop dynamics right because this is our closed loop dynamics and this we can represent something like some projection into A x and, if you recall, any state vector.

If you multiply with a with a linear transformation, then it will take a shape because we have transformed an orthogonal eigenvector to an inclined eigenvector by multiplying a transformation matrix. Similarly, here it is like a transformation matrix which takes the state and it will project in a way and the motion is along the surface that is the significance of this P s the mapping; that means, this projection will ensure that motion will be along the line ok.

So, we are defining this is a P s, and this is a projector operator. During the sliding motion, this is the equation. And this is the motion and during the motion as I told this will act like a projector operation which will, as if like you know in case of eigenvector. Also we can also multiply because we know during in the eigenvector x dot equal to A x.

So, if x belongs to any eigenvector that we have discussed. So, you can write x dot equal to A x and which is equal to sigma x. Because x is along the eigenvector and that we have learnt already right and from here we got A into v x equal to lambda into v x that we have learnt.

Similar to that here, it is like a projection operator which makes sure this projection operator ensures that motion is along the line ok. In the eigenvector case, it is moving along the eigenvector. So, it has an invariant property here. also it has an invariant property.

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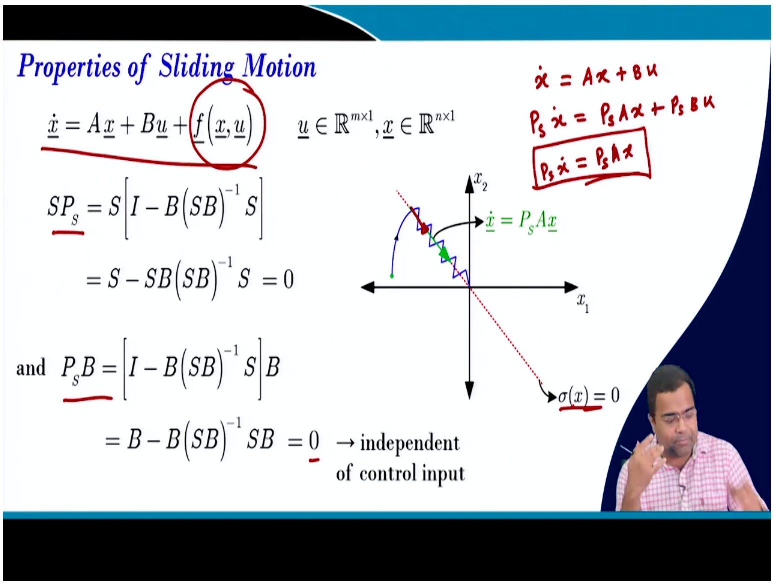

So, the property of the sliding motion. Now, if we recall our original system where we have uncertainty in the model matching right because we have a non-linearity which is due to the uncertainty. And now if we multiply S into P s because this is a projection operator. You can see it is 0; that means, if you take, if I x dot our original system let us say we time being we ignore the uncertainty, then if you multiply P s into x dot.

So, we will get P s A x plus P s B u. Since we are getting P s b u to be 0 0. So, this will be P s A x and this is exactly what during our sliding motion; that means, this is the equation during our sliding motion where the control input actually disappear; that means it is independent of control input.

That means when it is running, when it is going along the sliding surface, the control input has no function. So; that means, as if it is like a, but it is the equivalent control input. Control is there either side. You have a switching, but we are taking the average. And the switching law is making sure your average dynamics is moving toward the surface and during that time there is no control input.

So, your dynamics will be decided by the equation of motion and that will determine how fast you want to reach to origin right whether it is stable, what is that there are different properties.

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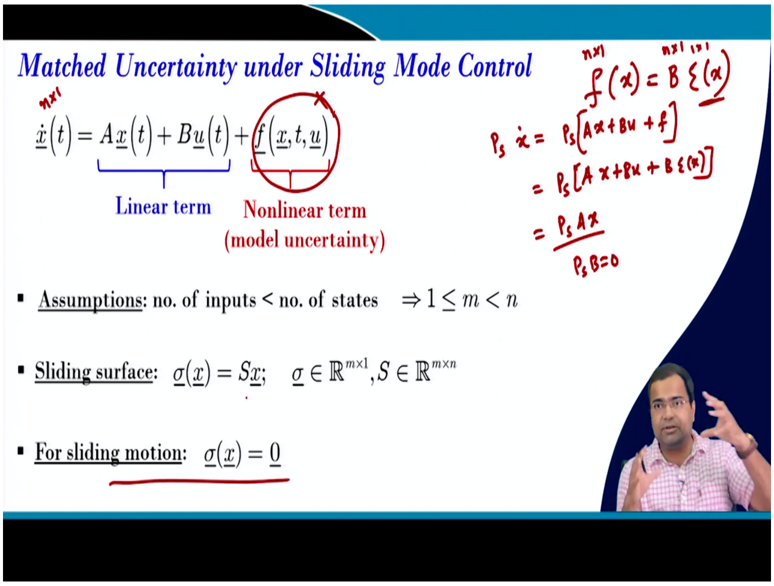

So, if you recall our nonlinearity. Now, suppose we talked about matched uncertainty. What is the matched uncertainty? If this uncertainty or the non-linearity can be written as something like a f of x it can be written as B time epsilon x right it is a non-linearity, but it can be represented by this.

Because see if this is n cross 1, this will be also n cross 1 and for a single input case it will be n cross 1 and this will be simply 1 cross 1 right. So; that means, if you can transform the system and in such a way the non-linearity is appearing along the input channel, then if you can do that, then during sliding motion we know that if we multiply P s.

So, P s into; that means, in our original equation, if you substitute A x plus B u plus; that means, f. And what is our f? A x plus B u plus B into epsilon x. So, it is P s into A x, since our P s into B is equal to 0. So, it will be independent of uncertainty; that means, during the sliding motion, the uncertainty that part the response that will be rejected; that means, first of all your assumption we took that number of input should be less than the number of state.

And we have defined hyper plane and a sliding motion. So, during this match uncertainty, you can ensure during sliding motion there will be no effect of the model uncertainty in the dynamics of the sliding motion.

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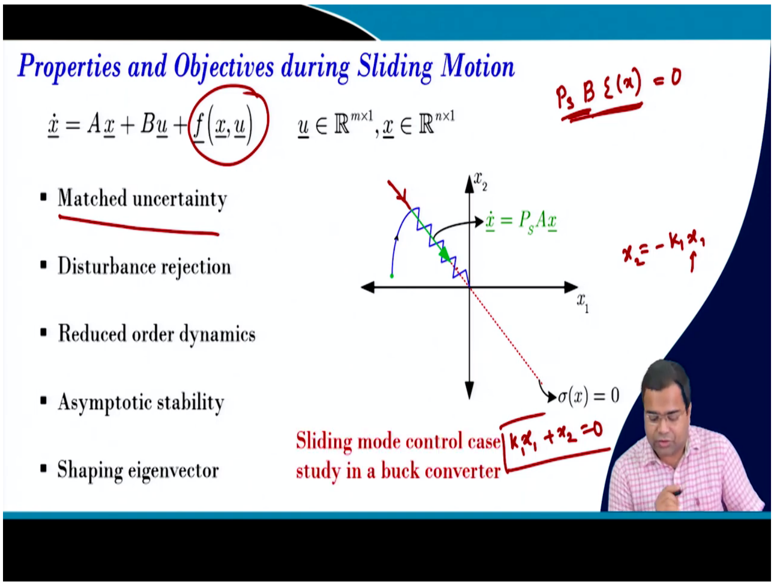

So, if we set the property, the matched uncertainty; that means the sliding motion will be independent of the uncertainty; that means, and if the disturbance also occurs in the input channel; that means, you know even also disturbance can occur. Then if you can write the disturbance in terms of some epsilon function x; that means, the disturbance occurs in the input channel. Then we know that P s into B this will be 0. So, this part will also vanish.

So, that means it will have a very good disturbance rejection. That is why sliding mode controls, you know people call about robust control because it is insensitive to the model uncertainty; it is insensitive it is very good; it has a very good disturbance rejection property provided that the matched uncertainty and you know the disturbance appear in the input channel. And also the matched uncertainty property is satisfied and another thing that motion will be reduced to the dynamics.

Because we know that k 1 x 1 plus x 2 equal to 0 and this equation; that means, we can write x 2 equal to minus k 1 x 1. And if we take a basis vector in x 1 coordinate. So, x 2 is simply a dependent function of x 1; that means, during this motion. So, if you can transform in that dimension. So, it can reduce that because it is very clear from the picture itself the whole motion is constrained along the line even though you have a two dimensional space.

So, it will have a first order dynamics because it is a one dimensional motion ok. And asymptotic stability and we need to shape the eigenvector ok. So, here now we are going to consider a buck converter case study.

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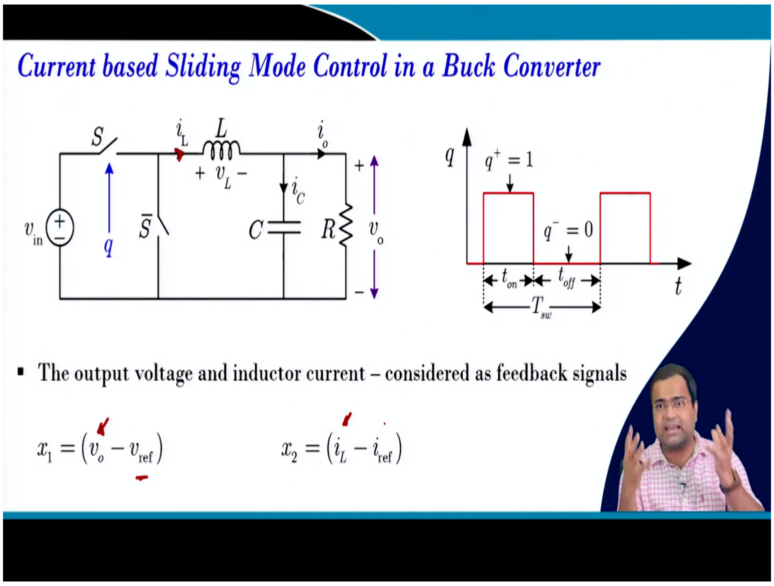

So, first we will take talk about current based control in a buck converter in which we want to take the inductor current sensing and output voltage as the sensor. So, we are sensing output voltage and, of course, there should be a reference voltage because we need to regulate the output voltage and also inductor current and we need to consider the reference current.

And in fact, we have discussed this reference current either can be a load current for a buck converter or you can generate this from you know something like you know from another variable you know or from the integrator of the error integration also, but here if you are considering load current; reference current is as if known.

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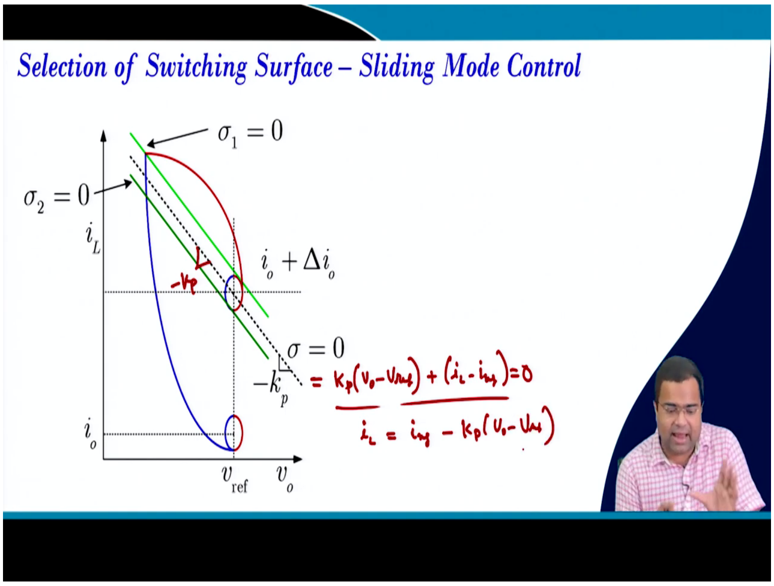

So, if we take the switching surface; this switching surface, we have discussed. So, this sigma is nothing, but our k 1 into v 0 minus v ref plus i L minus i ref that equal to 0 and that k 1 here it is k p. So, this k p is basically a negative slope negative slope because from here you can write, that your i L if you write it will be i ref minus k p into v 0 minus v ref. So, we will get a negative slope out of here and we have also considered that there has to be a hysteresis band, ok.

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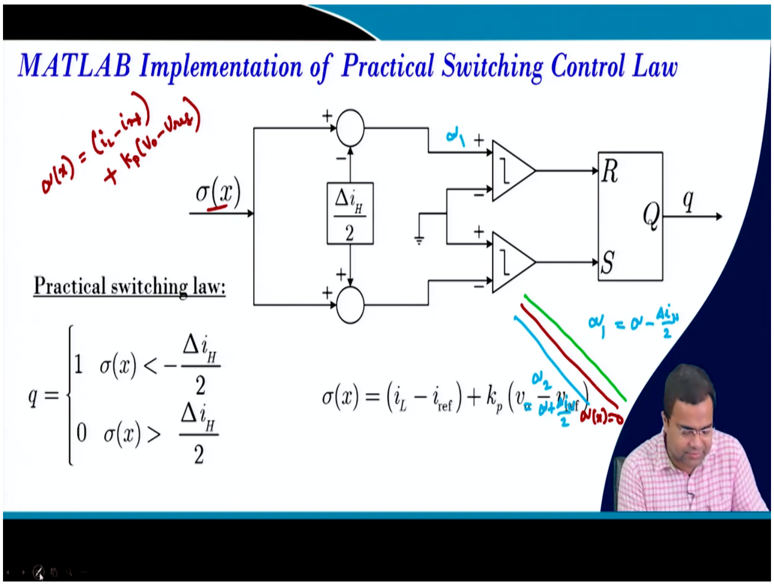

So, if you want to implement in a real implementation. So, this hysteresis; that means, this is our sigma x. What is our sigma x? We have chosen v 0 sorry what we have chosen that mean sigma x we have considered i L minus i ref plus we have considered k p into v 0 minus v ref. So, this is our sigma x and here we are putting a hysteresis band right because if we consider this is our sigma; sigma x equal to 0 and if we consider we have two surface one along this and the other along you know this path.

So, this we consider sigma 1 and this is sigma 2; that means, this is our sigma 1; sigma 1 is simply nothing, but sigma minus delta i H by 2 and sigma 2 is nothing, but sigma plus delta i H by 2 ok this is what is implemented and we are putting a hysteresis logic right.

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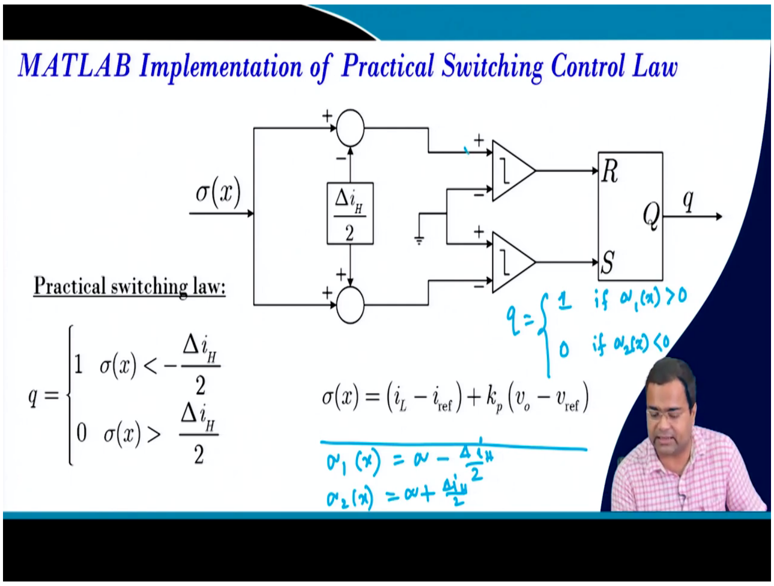

So, this is what we have considered you know this is our switching surface, and we told that sigma x is equal to sigma minus delta i H by 2 and sigma 2 x is nothing, but sigma plus delta i H by 2. So, whenever; that means, what is our switching logic q? q will be 1 if sigma 1 x is you know it is greater than 0, and it is 0 if sigma 2 x is less than 0 that is our switching logic right. So, you can see if it is greater than 0 ok.

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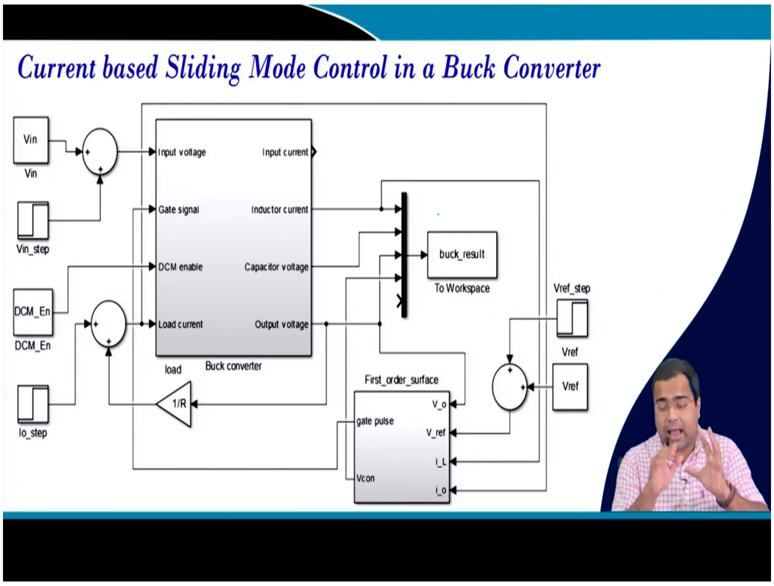

So, if we take the MATLAB implementation, I am not going to show MATLAB simulation here because we have a separate lecture for the representing this. And here we are talking about this is our sliding mode controller. We have considered voltage reference voltage, then inductor current, load current. Load current here we have considered at the reference.

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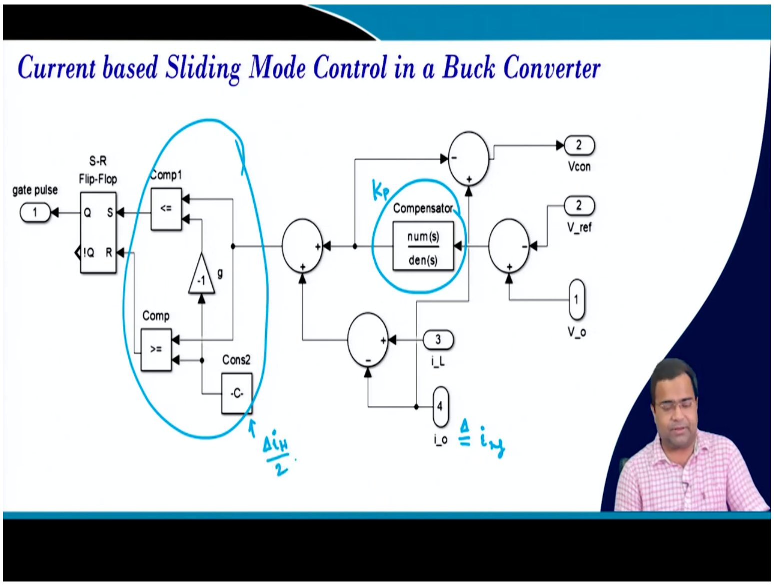

So, here reference I have chosen this is nothing but our i ref for this case. This is the i L and here we are going to set the controller. So, if you simply put a proportional controller, this can be k p. And we are tapping this signal because we can show how does the surface look like ok and this is what we have discussed. And this is nothing but our delta i H by 2 simply ok.

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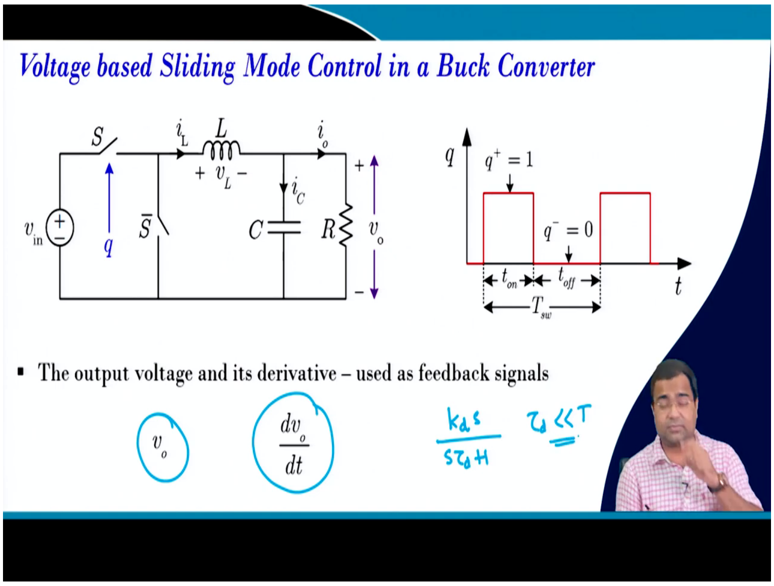

Then if you go for voltage based implementation, there we are taking two signals – one is the output voltage, and the other is the derivative of the output voltage at the feedback. But, remember we cannot implement a pure derivative. So, we need to consider something like a k d s into s tau d plus 1 where tau d has to be very small. If we talk about a kind of close to a if you want to operate close to a switching desired time period this t d should be very very smaller than this time period I means you know.

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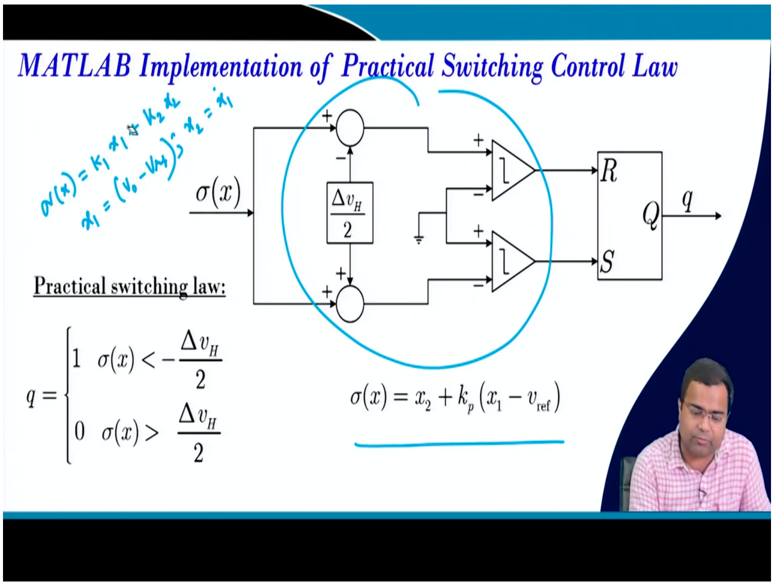

Again, here the same logic sigma x. So, what is our sigma x? We have chosen here. Sigma x is nothing but you know k 1 into your x 1 plus k 2 into x 2. And what is our x 1? We have chosen v 0 minus v ref. And what is our x 2? We have chosen x 1 dot that is our derivative ok other logics are same as the earlier. So, there is no difference and you can you know this is what exactly I told. So, here x 1 sorry x 1 I have taken ok.

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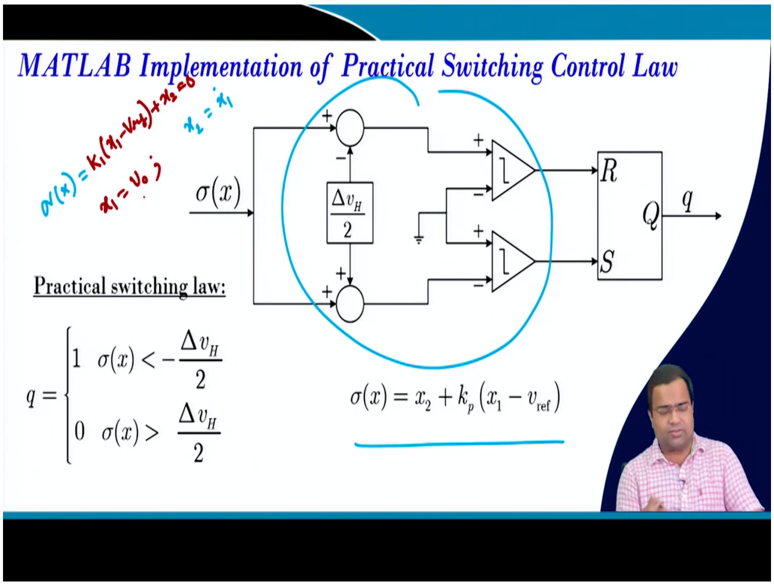

So, it is just a change in variable. So, you can you can modify this. So, what I have taken here in this case I have considered k 1 into x 1 minus v ref plus x 2 equal to 0. And where our x 1 is nothing, but simply v 0 and this is x 2 right.

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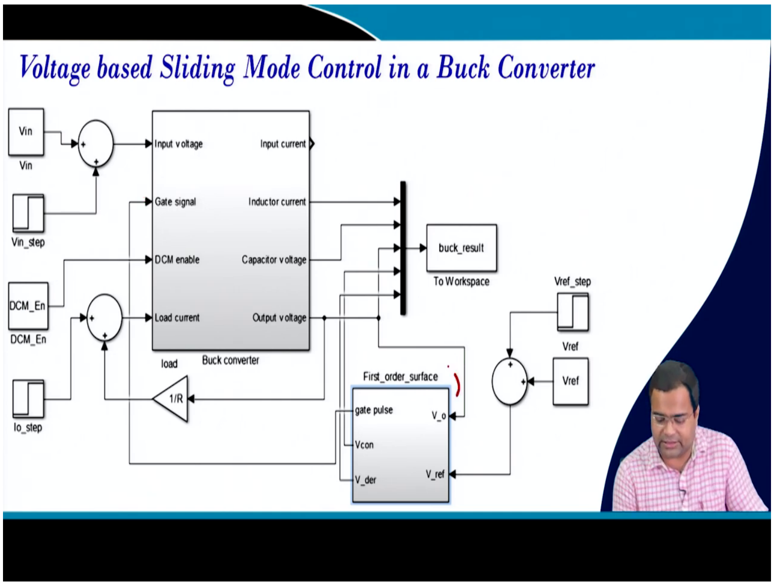

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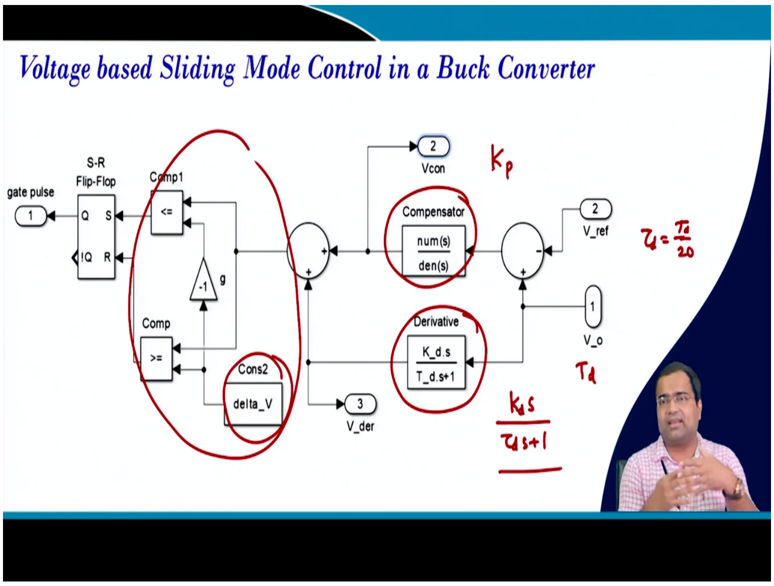

So, if you want to implement this is our voltage base where we are only taking output voltage and the reference voltage. So, here I told you the band limited derivative, which is something like, k d into s tau d s plus 1 and I want to achieve somewhat close to a desired you know time period.

So, I choose some T desired I mean somewhat nearby I want to achieve the switching frequency by setting this voltage band. And I choose that tau d here to be T by T d by 20 the smaller value right and this logic we have already discussed. And this we can set simply a proportional controller.

In future, you know and simulation, we will show that you can also incorporate integral action. So, that integral action will primarily, you know will not make a first order dynamic because during the sliding motion also integral will be there and that integral again brings back the other dynamics. So, in that case there will be no reduced order model, but you can make you know steady state error, you can eliminate the steady state error and so on.

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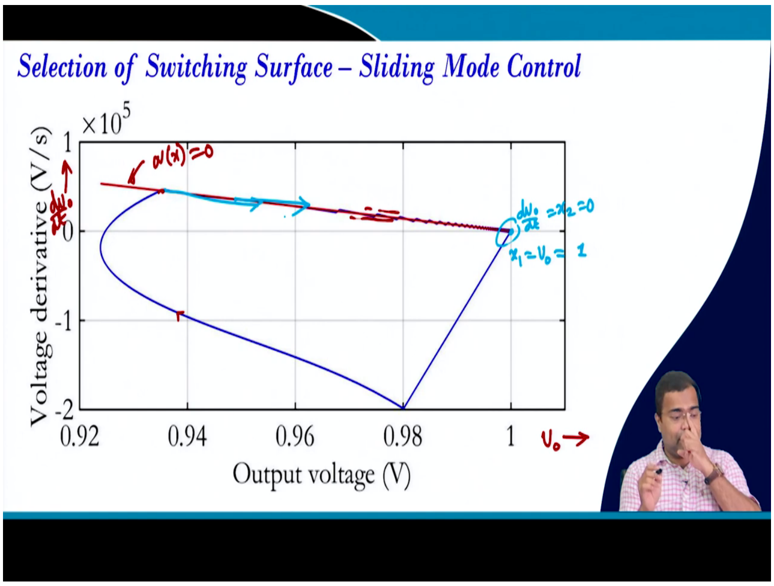

So, this is one of the simulation case study where this axis is our output voltage and this at axis is our d v 0 d t you can see. And this is our sigma x; sigma x equal to 0. And we have chosen a very narrow band. The band is very small. And you see along once this is the switching on state trajectory, once it switches here, it will be forced to move along this path.

So, it will like invariant vector field and this is our desired operating point where our x 1 which is nothing, but output voltage should be 1 and this is our derivative of the output voltage which is nothing, but x 2 that must be 0 at steady state. So, this is the point it will reach asymptotically. So, since it is coming asymptotically so, it is stable, but we want to consider this case study.

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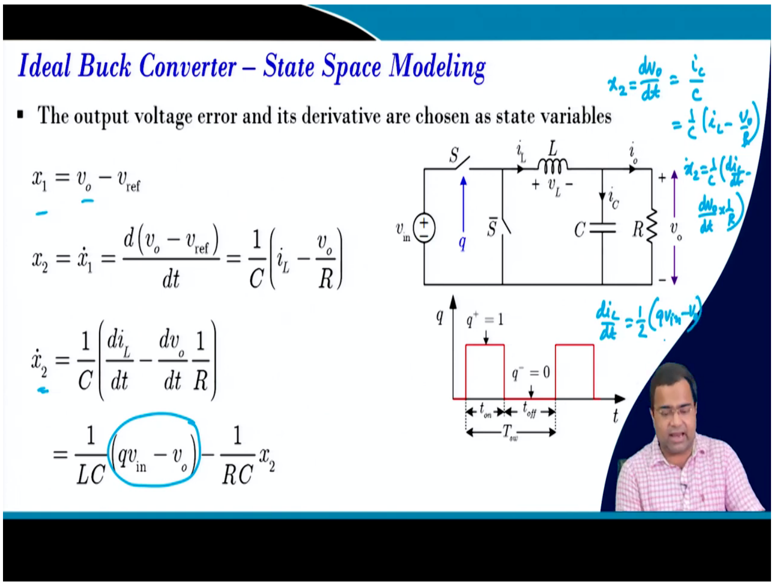

So, now we want to just show some stability condition and design criteria. So, if we take x 1 to be v 0 minus v ref then x 1 dot is equal to x 2. And if you represent x 2 dot because it is all coming from the state space model of the of the buck converter.

So, the voltage derivative of the buck converter you can write 1 by C; that means, this is the because we know what is x 1 is our output voltage. So, d v 0 d t is nothing, but for a buck converter, i c by C right and which is nothing but in this case it is our x 2 right; it is nothing but 1 by C i L minus v 0 by R, if you take a resistive load.

So, if you write x 2 dot it will be 1 by C d i L d t minus d v 0 d t into 1 by R. Now, you do further replacement you know you just write the equation of on time because d i L d t for a buck converter is nothing but 1 by L q v in minus v 0 which is our. So, then you rearrange.

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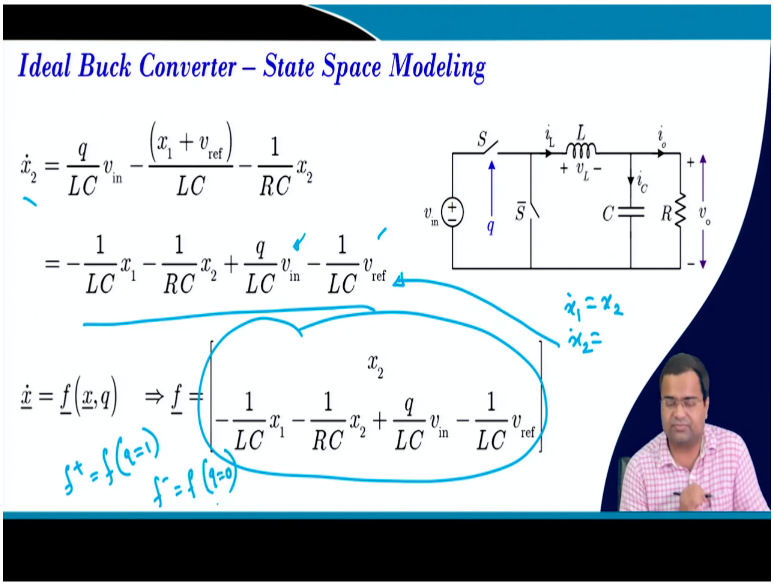

So, after rearrangement, you can obtain x 2 dot dynamics like this in terms of x 1 x 2 and it will also come v in and v ref. Then you can write x dot because you have x 1 dot. We know that x 1 dot is equal to x 2 and x 2 dot is equal to this particular equation. If you combine, you will get this vector field. Now, what is f plus? f plus is nothing, but f with q equal to 1 and f minus is nothing, but f with q equal to 0 right.

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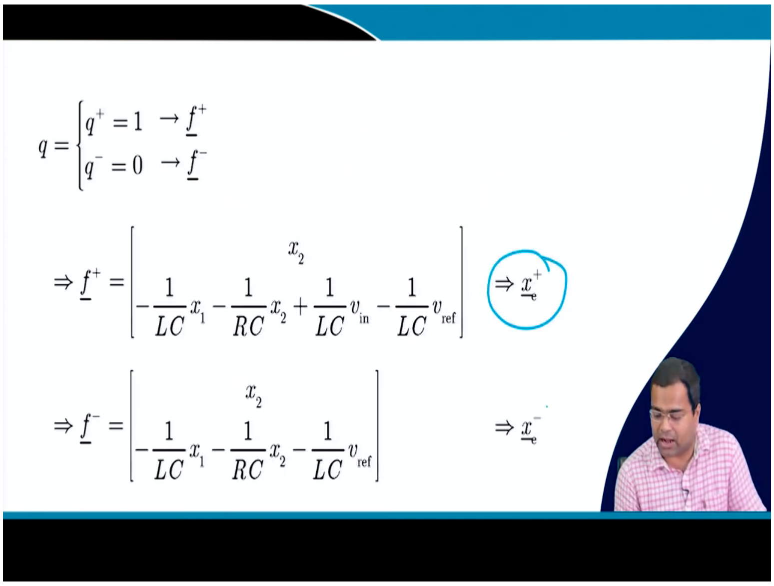

So, this also f plus for q equal to 1 f minus is q equal to 0. So, you can write f plus dynamics f minus dynamics. And if you find their equilibrium point; so, equilibrium point we want to plot how does it looks like.

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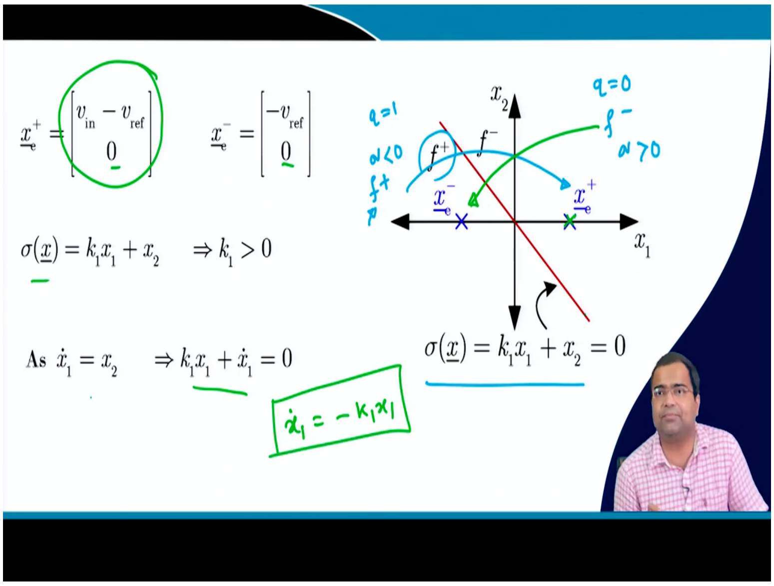

So, if you draw the plane this is our sigma x ok. For first we have considering the left side which is sigma less than 0, we are considering f plus that is; that means, q equal to 1 and this side we are taking q equal to 0 and which is f minus and here sigma greater than 0, here it is f plus right.

Now, we want that equilibrium corresponding to this vector field should lie in the other side and that is exactly is happening. Similarly, the equilibrium point corresponding to this particular vector field must lie in either side of this vector field.

So, that they will be forced to cross the switching surface and that we have shown right. And what is x e minus x e plus. So, it is just this point in both cases this is 0 because your derivative of the output voltage should be 0 at steady state, but in the first case it is v in minus v ref, in the second it is minus v ref right.

Then you can write sigma x and, if you substitute x 1 dot; that means, you will get the sliding motion is x 1 dot is equal to minus k 1 x 1. So, this is a first order reduced order dynamics during the sliding motion ok.

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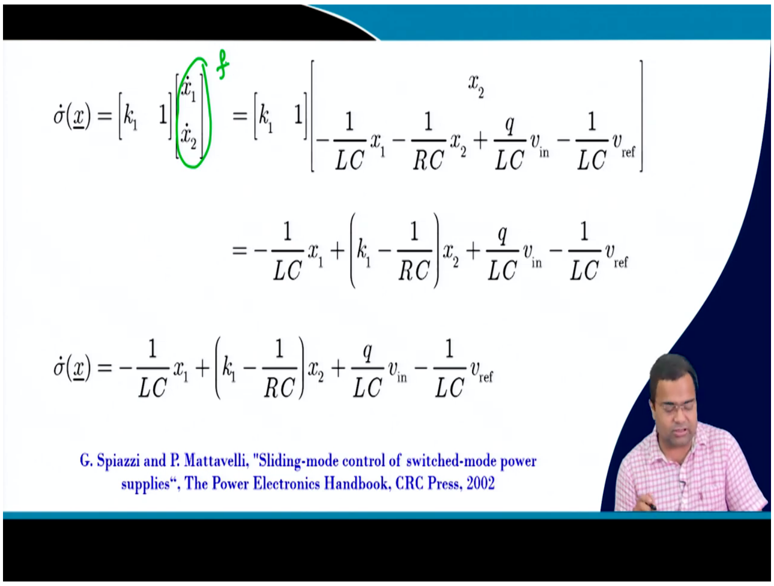

So, again, if you take sigma dot x then you can derive sigma dot x from the equation, ok because you already know what is my x 1 dot x 2 dot. But, again it is in terms of generic term because this quantity is nothing, but our vector field f right. Now, if you substitute, you will get this sigma dot x and this you will get detail in this book sliding mode control of a switched mode power supply.

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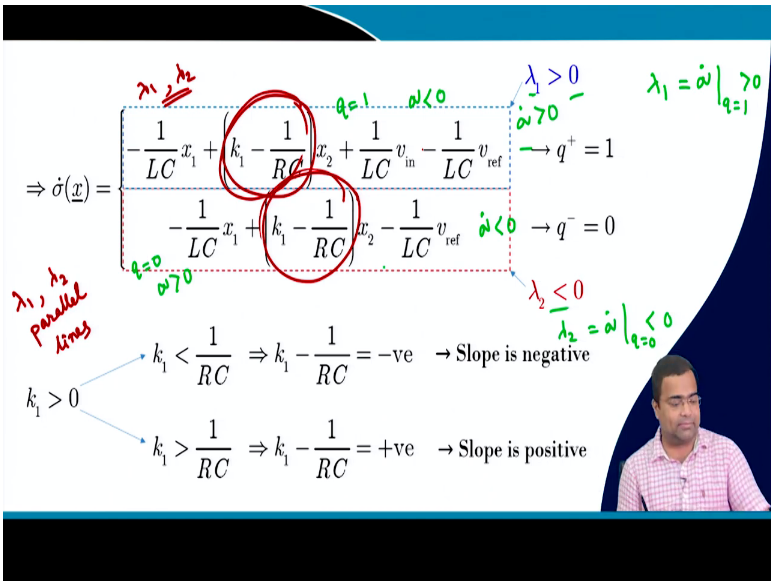

Then if you obtain sigma dot x and we know that our control law here it is q equal to 1 and here it is q equal to 0. And what we want this is nothing but sigma less than 0 and this is nothing, but sigma greater than 0; that means, we want this is sigma dot; sigma dot to be greater than 0 and we want that sigma dot to be less than 0. And this where it is for q plus q minus and this we are denoting at a lambda 1 s.

So, where lambda 1 is nothing, but sigma dot for q equal to 1 and lambda 2 is nothing, but sigma dot for q equal to 0 and which should be negative and this should be positive. This is the requirement. So, that it will be forced towards the surface. That is the requirement for the reaching condition.

Now, the k 1 greater than 0 there are two possibilities the gain must be positive. So, that if you look at here; that means, there are two possibilities either k 1 can be greater than 1 by R C or k 1 can be less than 1 by R C.

And depending upon that, you will get two condition; that means, the slope of the equation lambda 1 lambda 2 their slope, you see for lambda 1 lambda 2 the slopes are common because they are common; everything is common except for the offset point.

That means, I can say lambda 1 and lambda 2 they are always parallel line they are parallel lines because it is a straight line equation it will definitely not pass through origin. They are parallel line and their slope can be positive or negative right.

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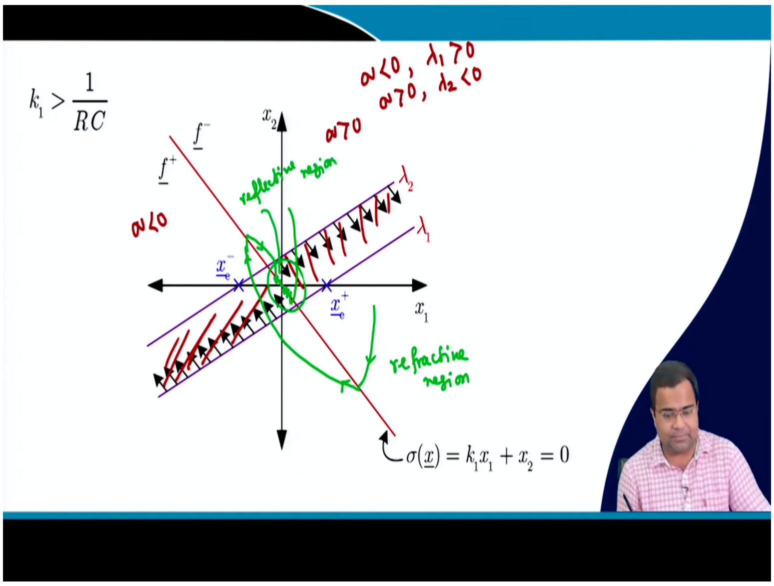

So, what was our requirement? Our requirement was that sigma for sigma less than 0 we are getting lambda s should be greater than 0. For sigma greater than 0 lambda 2 less than 0. And this side is sigma less than 0, this side is sigma greater than 0.

Now, if you see lambda; lambda greater than 0 is nothing but this region. So, anything in this region it will be under sliding motion right. Similarly, anything in this region will be under sliding motion. It is less than lambda lambda 2 right. So, this is our sliding motion condition right for greater than; that means, if your trajectory if your trajectory reaches here. Then it will be constrained to go along this line because either side it satisfies towards the surface ok.

But, it may not be that the case because here it may go here and it may go here and then it may come here and then again it will run like this because this side the trajectory will move away, this side coming in ok and we will discuss this. If one side trajectory goes away other side comes in, we call it as a refractive region and that we will discuss in the next lecture.

And when both sides come towards the surface, the region is called sliding region, or it is also called a reflective region, where both side trajectory are towards the switching surface ok.

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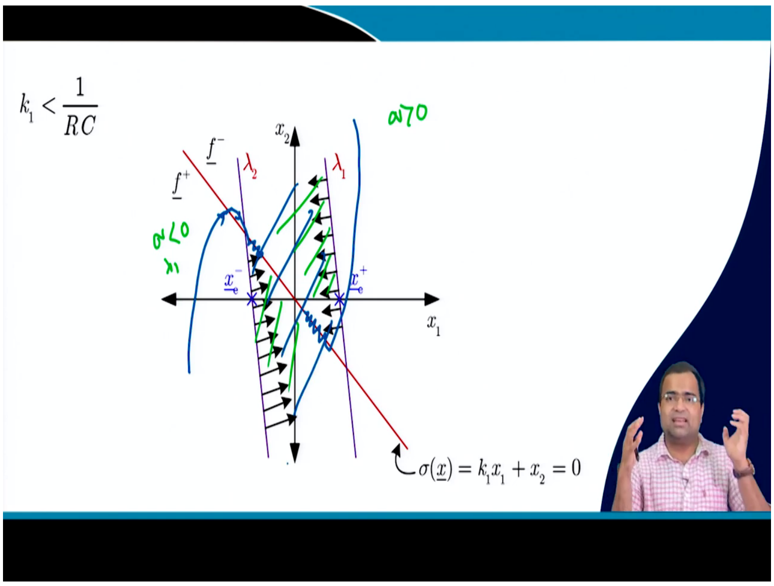

But, if you take k less than 1 by R F 1 by R C then sigma 1 sigma 2 slope will be negative and if the slope is negative and again we need to ensure that for sigma less than 0 sigma greater than 0. So, you can find out that this region is my sliding motion. So, you will get suppose if you have a trajectory like this, if the trajectory comes from any arbitrary point and reach here, then it will go along this path because both sides it is sliding motion.

Similarly, if the trajectory go towards this path, it will first move along this line because ok. So, because it will go it will come towards this and this side it will go either side it will go, because outside this region there may not be any sliding motion. So, in this region, you will have a sliding motion. So, you will get a more region of sliding motion and the system will be somewhat over damp.

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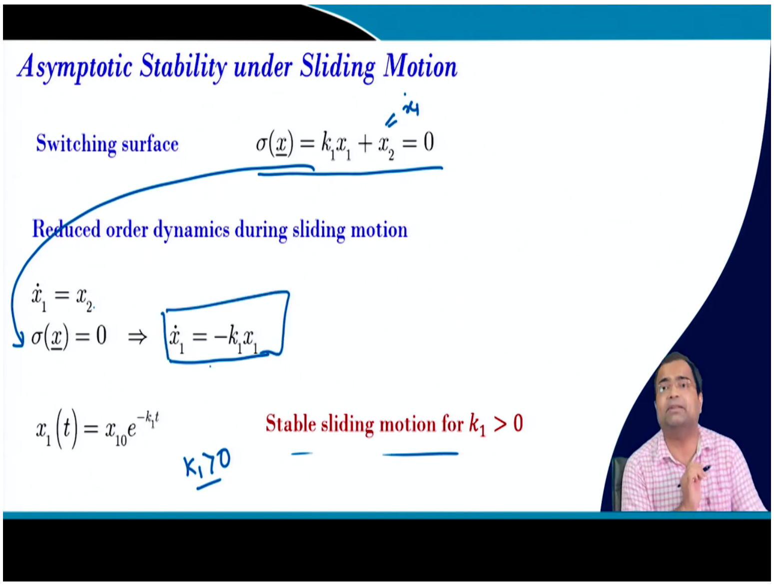

So, the asymptotic stability we saw that sigma x equal to 0 is the surface sliding surface and we know x 1 dot equal to x 2 because we have consider the output voltage is one of the state and it is derivative to the other state. So, and if you substitute x 1 x 2 equal to x 1 dot then this x 2 will become x 1 dot and then this equation from here we can substitute x 1 equal to minus k 1 x 1 it will be initial condition.

And since our k 1 is greater than 0. So, the motion is asymptotically stable; that means, it will be coming towards the origin and the stability condition that k 1 should be greater than 0 and that we have already discussed. So; that means, for the sliding motion you need to choose the gain, the k 1 must be positive because here our first requirement for positive we must have a negative feedback system for that k 1 is always positive. So, it will ensure our sliding motion is always stable asymptotically stable.

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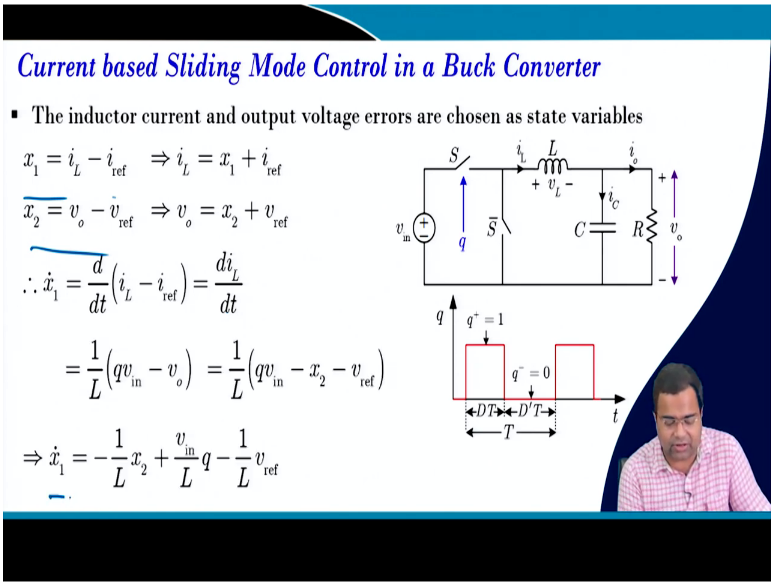

If you go to current based control, suppose you know we have discussed the voltage based control and we are going to show simulation case study in the subsequent lecture in the next week in MATLAB ok. In 12th week, we are going to show MATLAB case studies.

And the inductor current and the output voltage error are chosen at the state variable. here this is the error inductor current and the error output voltage because it is a current based control right. Here you can write x 1 dot in terms of this x 2 dot. So, you can represent x 1 dot in terms of x 2.

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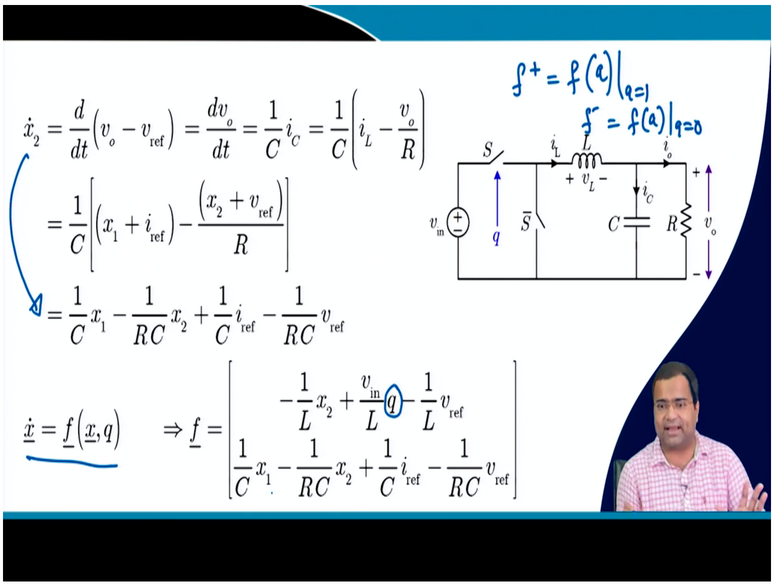

Similarly, you can write x 2 dot in terms of x 1. So that means we will have x 2 dot in terms of x 1 x 2 and you have already written that x 1 dot is in terms of x 2 there is no x 1 here. So, we will get overall vector f function of x and there is a q term if you go back to the previous slide.

So, there is a q term here right q so; that means, you will have f q, where you will have f plus which is nothing, but f of q for q equal to 1 and f minus is nothing, but f of q for q equal to 0. So, similarly you can again draw the vector field and you can get f you see there is a q and otherwise it is everything is fine. So, this q will make f plus and f minus; f plus and f minus whether q equal to 1 or q equal to 0.

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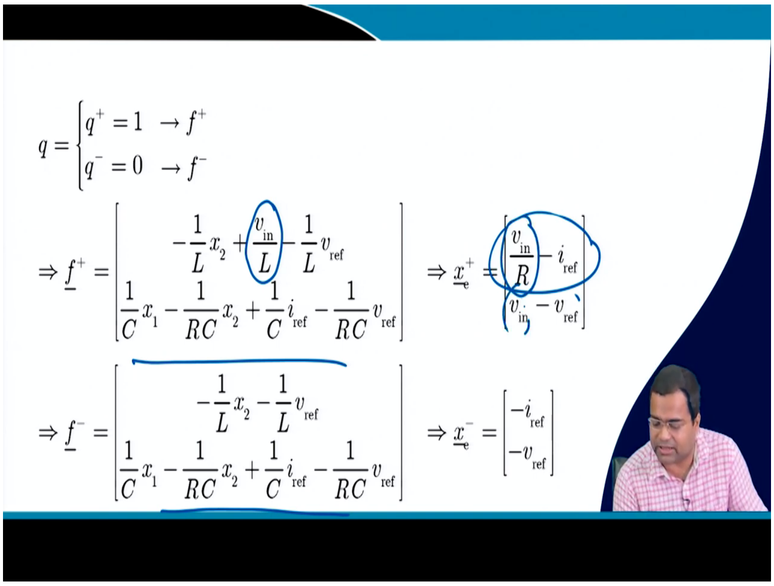

And you can get f plus where the input voltage term will be there and f minus everything is same, but input will not be there. So, these dynamics are common for both the cases right. So, you will find out the equilibrium point in this case will be this and this here. So; that means, you will have only this extra term will be there in the equilibrium point.

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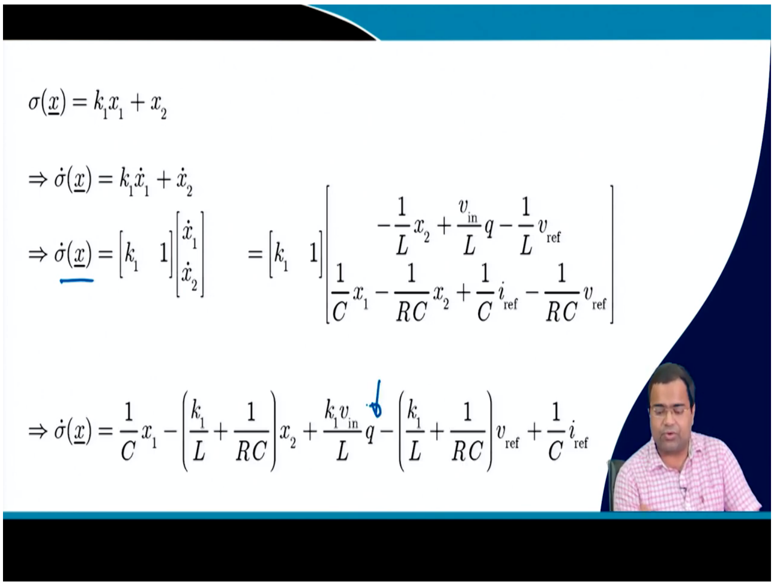

And you know, and the switching surface x 1 dot x 2; you can write down you know the equation. And we know that if we get sigma dot x right. So, again, if you draw the sigma x the curve and sigma dot x here, also we have a q.

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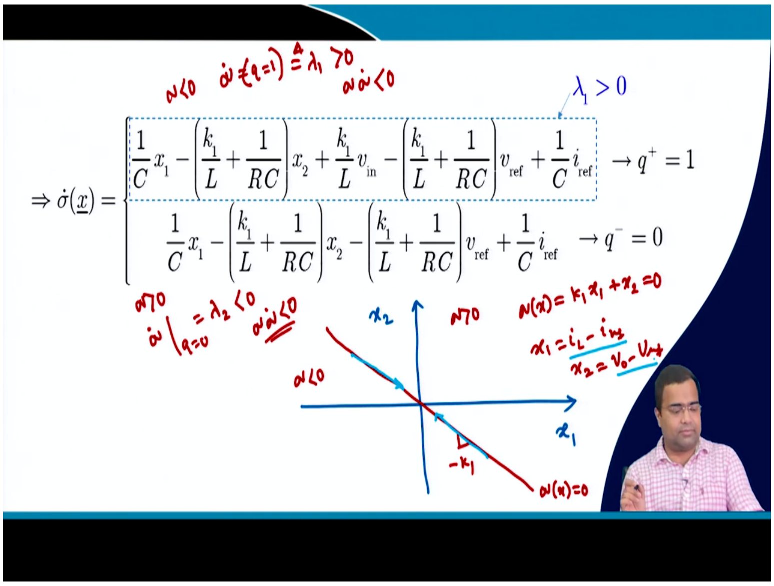

So, depending upon the q plus and q minus, you will get sigma 1 greater than 0 because suppose here we are talking about the current based implementation. So, if you draw this surface. So, here we are talking about x 1 and x 2. What was our x 1? We took x 1 if you go back. You know, if you recall, we took x 1 to be.

So, this is our state variable; error current and the error voltage right. So, if you go back, x 1 is the error current and the error voltage. So, I need to draw a surface which is something like this; this is my sigma x equal to 0. And what is my sigma x? It is nothing but k 1 into x 1 plus x 2 equal to 0.

Now, the question is what is x 1? x 1 is my inductor current minus a reference current and x 2 is minus output voltage minus v ref. And now, this slope is minus k 1 and again this side you have sigma less than 0, sigma greater than 0. So, for sigma less than 0 we are getting this to be lambda; that means, this is sigma less than 0. So, your sigma dot which is nothing, but for q equal to 1 is represented by our lambda 1 which should be greater than 0.

So, that it will satisfy sigma sigma dot less than 0. This is sigma greater than 0 and sigma dot what is nothing but for you know if you write for q equal to 0 it is nothing but lambda 2 and that should be less than 0. So, that you will get sigma sigma dot less than 0.

So, it will satisfy the reaching condition once it reaches then you can also do the similar carry out similar analysis and show that you will get you know asymptotically stable motion in either side ok. So, it will finally move towards the origin where equilibrium point is when i L will become i ref and v 0 will become v ref ok.

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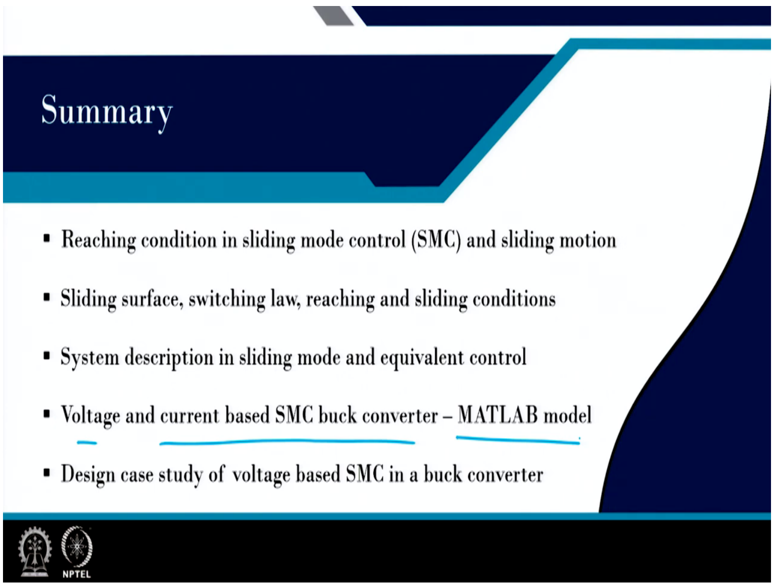

So, this I summarize that we have discussed the reaching a condition in sliding mode control. We have extended the concept in 45 lecture and we have elaborated different property of sliding motion ok under sliding motion. Then we have discussed first reaching law, then sliding mode control, then the concept of sliding motion. Then, in the sliding motion, what are the reduced order dynamics.

Then how the sliding mode control that motion actually offers like a match uncertainty it can eliminate robust disturbance rejection and then reduced order dynamics and then asymptotic stability. And finally, the shaping of eigenvector; that means, how to decide the slope and that will take case study in the last week in the simulation. But, in all cases it is stable. In one case, if the gain is large, you will find overshoot undershoot.

If the gain is somewhat smaller, there will be overdamp response the system will be sluggish ok. And we discussed sliding surface, switching law, reaching and sliding condition. Then we also discussed the system description under sliding motion their property. Then we have also discussed voltage based and current based control sliding mode control implementation using MATLAB model right.

And finally, we consider one design case study of voltage mode control, voltage based sliding mode control motion in a buck converter ok. So, with this I want to finish it here.

Thank you very much.