**Control and Tuning Methods in Switched Mode Power Converters**
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**Module - 09**
**Large-signal Model and Nonlinear Control**
**Lecture - 44**
**Small – signal and Large-signal Model based Nonlinear Control**

Welcome, this is lecture number 44. In this lecture, we are going to talk about Small-signal and Large-signal Model based Non-linear Control.

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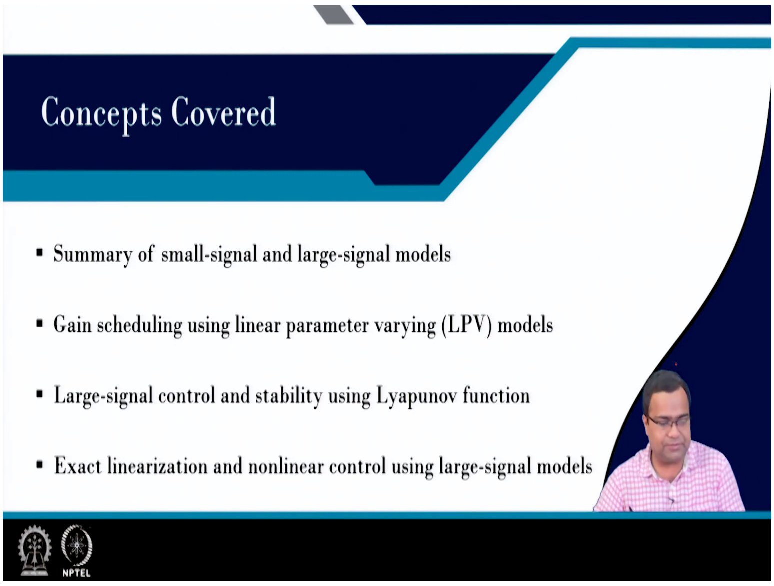


So, in this lecture first we will talk about you know summarize small-signal and large-signal models; then I will discuss gain scheduling method using linear parameter varying model. Then I will also talk about large-signal control and stability using Lyapunov function and then followed by exact linearization and non-linear control using large-signal models.

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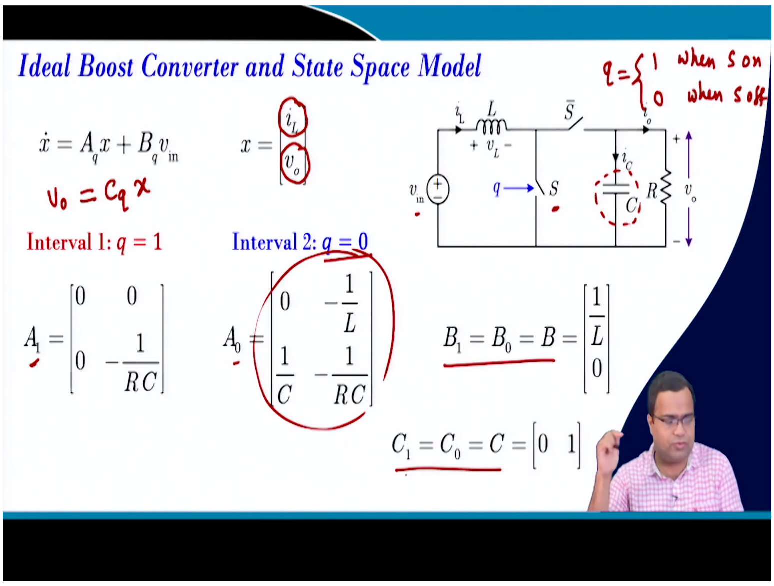


Recall the ideal boost converter and state space model. For an ideal boost converter, we can write the state space form. We are considering the state space model to be inductor current is one of the state; that means inductor current is one of the state inductor current and the other state we are taking output voltage. But actually it is the capacitor voltage we should take the state variable.

Since it is an ideal converter, so, the capacitor voltage and the output voltage are same. Then we can write x dot equal to A q x plus B q v in, where q is the gate signal and the we know that q equal to 1, when S is on right the main switch S is on and it is 0 when S is off. We know that ok. So, now, for q equal to 1 our A 1 matrix will be 0 0 0 minus 1 by RC and when q equal to 0, then we will get a 0 which is this matrix ok and this is standard thing we have already discussed multiple times.

Now, for a boost converter in continuous conduction mode, you know the input voltage is always connected. So, B matrix are common, they are all identical for two different configurations because the input voltage is common. It was the other case for the buck converter, where input voltage was connected and disconnected. So, we got different B matrices for a switch on and switch off whereas a matrix was common for the ideal buck converter, but it is just the opposite for a boost converter.

And, since we are talking about the ideal boost converter the capacitors are also common because what I have not written is the v 0 is equal to C q into x, where C q that means, q equal to 1 C 1, C 0 for an ideal boost converter they are same. But, for practical boost converter, if we incorporate the ESR 0; that means, the ESR of the capacitor then there will be discontinuity in the output voltage ripple and that will make two different C 1 and C 0 output matrix is different due to the ESR.

But, here we are talking about ideal converter. But we can we can do it for practical also, no problem.

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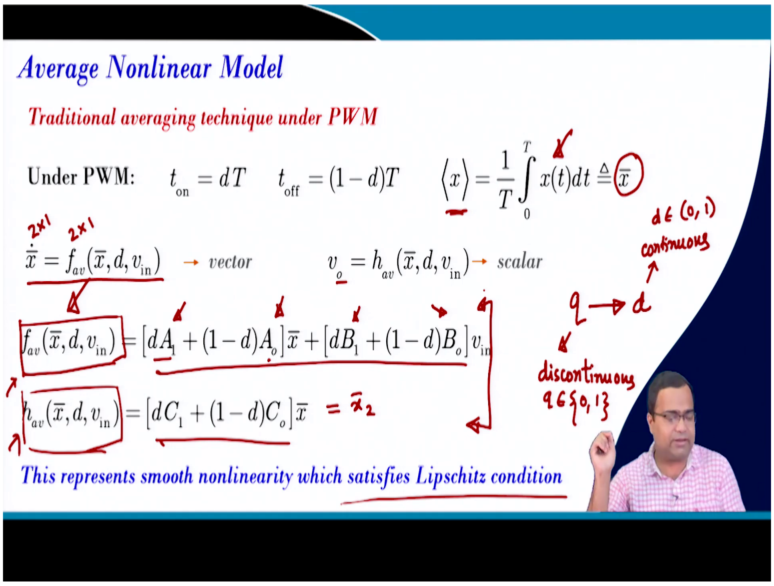


Next what we did? Next, we obtain the average small-signal model. We have discussed multiple time that traditional if we operate in pulse width modulation. We can apply traditional averaging technique where this average quantity represents the average over a complete switching cycle 0 to T, right. We are denoting as x bar, ok.

So, now, this x bar dot the average dynamics we will we will soon find and we have already discussed that it is basically a vector because we are taking two states. It will be 2 cross 1. Similarly, this vector function is also 2 cross 1, right; so, it is a vector. Whereas, the output voltage is a scalar because it is just the output voltage and this is a scalar quantity that we have discussed.

So, what is this f average? This is a standard averaging technique d into A 1 plus 1 minus d into A 0 where A 1 is the system matrix when the switch is on and A 0 is the system matrix when the switch is off. B 1 and B 2 are the system matrices input matrices when the switch are switch was on and off, respectively. So, by that way, we can obtain this average model.

And we can also obtain this average model and for the output voltage since our output voltage is constant and for this ideal case it will be simply x 2 bar because we have taken x 2 bar; x 2 is the instantaneous capacitor voltage for the ideal case it is the output voltage. So, we have to take bar means because you are talking about the average model ok.

Now, if we see this f average; that means, this vector function as well as this output scalar function they are basically smooth function because we have replaced actual gate signal q by d. You see with the original switch model, the q was discontinuous because discontinuous because it is a switching either it is on or off. When we take the average, then this discontinuous function is replaced by a continuous function.

Continuous function in this case q can take the value discrete value 0 and 1 whereas, the d belongs to the open interval of 0 to 1. Because we are not considering the extreme cases that mean d equal to 0 and d equal to 1, we can take then it will not behave like a DC-DC converter because if we operate either fully on or fully off state, then we will not get the desired output voltage ok.

So, we need a switching operation. As a result or duty ratio should be in between 0 to 1. It should not be equal to 1 or 0 equal to 0. But this can happen when we go for switching surface based control where we allow the duty ratio to saturate because in that case the I mean if we allow duty ratio to saturate we can obtain something called time optimal or the fastest transient response, that we will be discussing in the next week.

But, in this lecture we want to limit the duty ratio between 0 to 1. Since d is not d is a continuous function so, both the vector and scalar functions are continuous and we can take the partial derivative of this as well as this you know as many times as we want and that is why and there will be you know continuous; that means, there will be different derivative will exist for various higher order derivative will exist, so, satisfy the Lipschitz continuity.

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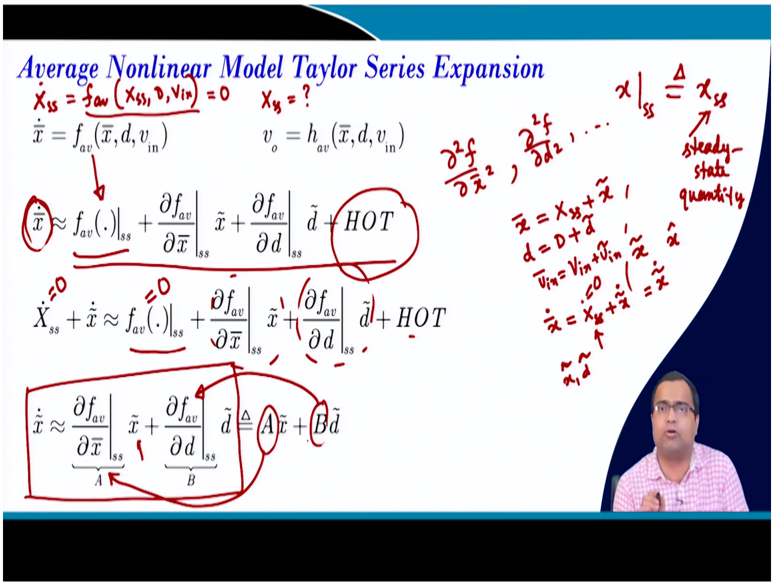


So, if satisfy the Lipschitz continuity and, in fact, you know it satisfy that you can take differentiation at any point any number of derivative, but the derivative will exist because there is no discontinuity. So, this is a perfect example that this nonlinearity for this nonlinearity we can apply Taylor series approximation. If we cannot if we have a function with a discontinuous function, then we cannot apply Taylor series. So, it is a continuously differentiable function.

So, next this average quantity, we can write in Taylor series because we have average dynamics that can be replaced by approximately this function at a certain point. So, here I am writing when I am writing this any x ss this represents actually x at steady state; that means, these are the steady state quantity steady state quantity, ok. So, that means, when you are talking about steady state means we are trying to obtain the approximate Taylor series approximation around a steady state point.

But, you can also approximate the using the Taylor series for any arbitrary point which need not to be steady state which need not to be operating point. But, whether you are going to get the perturb linear model or not that we are going to discuss, but the Taylor series approximation can be applied for any arbitrary point. It need not to be an operating point or it need not to a steady state.

But, in this case we are considering steady state because we want to obtain linear model under a around a steady state operating point. So, what is the higher order term? If you write down, the higher order term the higher order term means it consists of dou 2 f dou x 2, then it consists of dou 2 f dou d 2 and other higher order term and that we are writing clubbing together to write higher order term.

So, now, if I write all the instantaneous average quantity; that means x to be it is steady state value plus perturbation. So, sometime we write perturbation to be tilde, sometime we write hat. So, you can assume that these are perturb quantity. So, when you write any state same similarly, we can write duty ratio to be capital D and then d perturbation; similarly, we can write input voltage average V in plus small v in perturbation and so on, right? So, you can write everything.

Now, left side, once you take x average dot; that means you can write X x dot plus x tilde dot interestingly since it is a steady state point. This part will be 0, this part will be 0. So, it will become x tilde dot, right? So, write this since this part is 0, similarly if you solve; that means, if you solve this equation; that means, here if I simply take steady state dot will be equal to f average x steady state. So, I am writing x steady state comma d capital D comma capital V in.

So, this term must be 0, because this is the operating point steady state. So, from this we can solve for X x in terms of duty ratio input voltage and so on. So, this quantity will also be 0 at steady state that we have discussed. So, this is 0. So, we will have this term, we will have this term, but higher order term we are neglecting because we are trying to obtain linear model and we already have discussed earlier.

When you are we carried out circuit averaging technique, state space averaging technique where we ignore the product of perturbation that also ignored, right. Product of x 1 dash or tilde you can say these terms we have neglect; that means, higher order terms we have neglected earlier. So, they are not coming to picture. So, the approximate model here is simply this model where we can write A into x tilde plus B into d d tilde where A is nothing, but my this particular function and b is nothing but our this particular operation ok.

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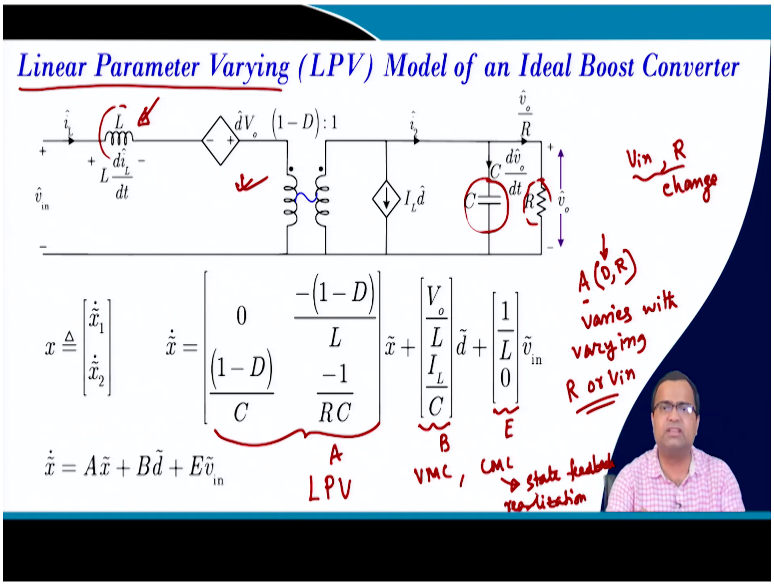


So, we can obtain now linear. Why we call linear parameter varying model? First of all, this represents a linear model perturb linear model that we have discussed because we will see that because we have linearized around the steady state point and there is no product of perturbation term. So, it is a perturb quantity or the deviation variable; that means, we have obtained x dot and this was our A matrix, this was our B matrix and this is a disturbance matrix because supply disturbance.

Now, you will see in the a matrix. If you take this a matrix, a matrix has information of the duty ratio, a matrix has the information of load resistance. L and C are the system parameters, which are generally fixed during the design process. We generally do not vary inductor or capacitor, but in a practical circuit there can be variation of inductance and capacitance because the inductance is nothing, but it is coming from the magnetic BH curve.

So, this inductance may not behave like a constant value if we try to operate towards the saturation of the non-linear region of the magnetic curve, but if we operate them in the linear region, then inductor will be more or less constant and the variation is not very significant same thing is for the capacitor. So, we are assuming the L and C variations are insignificant, but and those are fixed at the design stage. So, they are not going to vary during the run time except for small variation inductance that I have discussed.

But, what here going to vary? Suppose if there is a change in input voltage, input voltage can change load resistance can change. So, this quantity can change right because the load resistance is the external quantity which actually because this load because you are supplying the boost converter is supplying to a load it may be a processor load or sorry, it can be you know if you talk about electric vehicle, auxiliary power. So, there are many applications varying boost converters. So, the load can vary.

When the load varies, supply can vary, then the duty ratio will vary at steady state resistance. So, A matrix depends on the parameter. That is why this model is called linear parameter varying model linear parameter varying model where the A matrix consists of parameter and those parameters will be decided and you will see this A matrix will vary it varies with varying R or V in which is the duty ratio.

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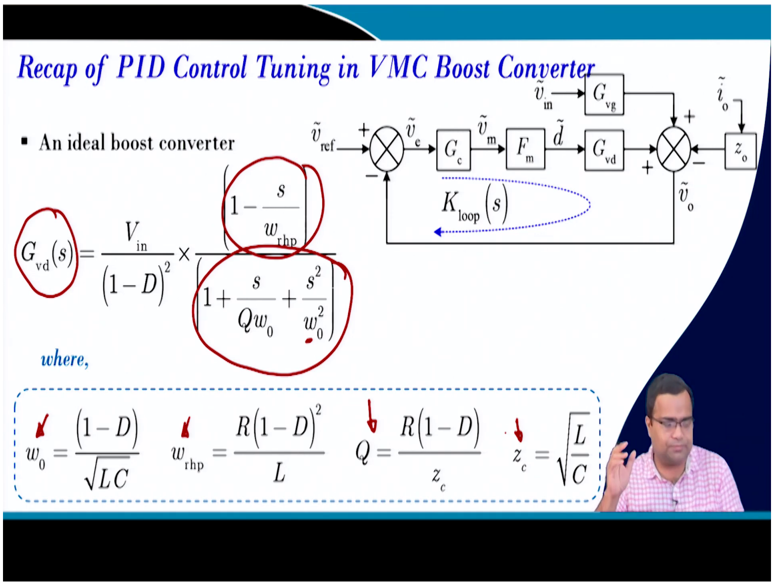


Now, if we recap, you have this linear model; that means, this is our linear model this particular circuit, we can obtain various transfer functions. So, we can design a closed loop control of this boost converter either by you know voltage mode control that we have discussed. We can design using current mode control and we have also learned the current mode control has an alternative state feedback realization.

So, state feedback realization right that also we have learned. So, state feedback realization. So, we can implement different type of control using this perturb or linear parameter varying model. For example, if we recall our boost converter under voltage mode control, we have used a PID controller where the control to output transfer function of the boost converter consists of one right half plane 0 and of course, there are two stable complex conjugate pole in general.

And, we have already discussed that what are these. Omega 0 the natural frequency, then what is the rhp 0 right half plane 0 frequency, then the Q factor, and this is also a function of characteristic impedance. So, all this thing we have discussed I think in lecture number 36 where we discuss the design of a boost converter using voltage mode control.

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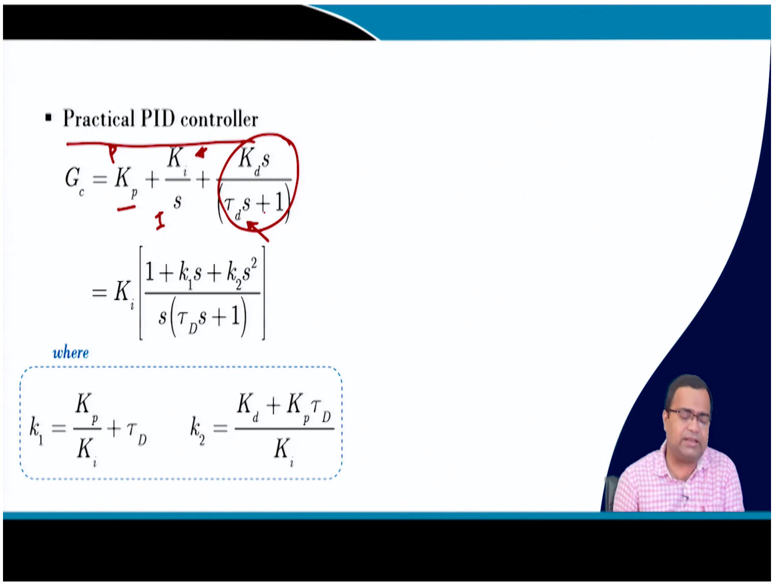


And, we have also discussed in that lecture that we can use a practical PID controller for example, where the structure of the practical PID control is K p that proportional term this is my p control, this is my i control and this is my band limited derivative. So, it is a band limited derivative. This is my derivative control ok where we have a derivative filter. So, this is a derivative filter.

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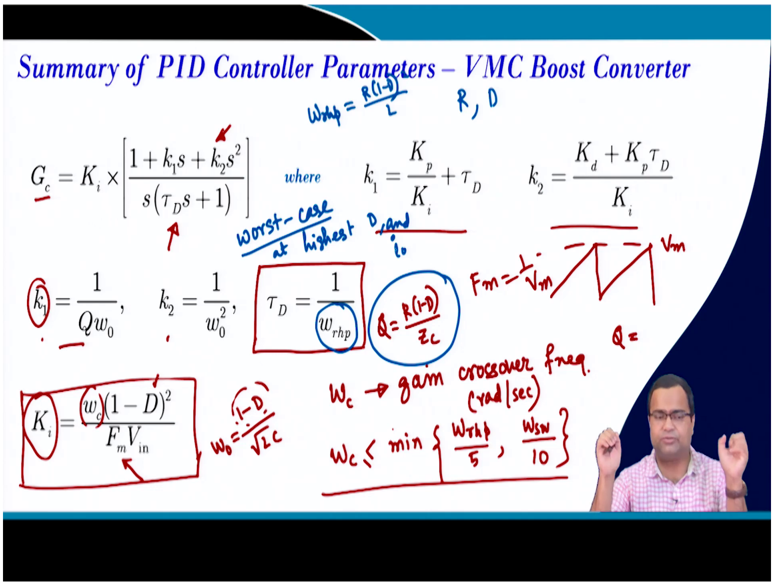


Now, in summary, we have a controller, which is a practical PID controller that structure looks like this. We can set this PID controller parameter because we have discussed the analytical method of design with stable pole zero cancellation. We have cancelled the stable pole by means of the 0s of this PID controller and if you do that then, but what we did for the right half plane 0 in we cannot cancel because we unstable pole 0 cancellation is not allowed otherwise it will make the system internally unstable.

So, we placed a right half plane 0, we placed the controller one pole in coincidence with the rhp 0; that means, we want to coincide the controller pole with the rhp 0 and that is why I have chosen. And, this part and after that we have analyzed you know by computing the gain crossover frequency.

And, we saw this the bandwidth; that means, which is the gain crossover frequency that we have discussed. We have discussed the gain crossover frequency; it is in radian per second radian per second. So, this gain crossover frequency we found for a for a voltage mode control it should be less than equal to what minimum of rhp 0 by 5 comma switching frequency by 10 all are in radian per second. So, it should be the maximum that we have discussed, ok.

So, this once we select this omega C and, since our duty ratio is known and the modulator gain for voltage mode control, we are assuming that your controller that saw-tooth waveform, which is already decided. And if this is our V m then we know that F m equal to 1 by V m that we have already discussed. So, you can select the integral gain and the other gain parameter can be already decided.

Now, since you see the controller gain k 1, k 2, k i tau D k 2, so, what is what was omega 0? So, for a boost converter we know omega 0 was 1 minus D square root of L C; that means this quantity is duty ratio dependent and what we learn about Q? We learn about Q is equal to you know if you go back, yeah Q equal to R into 1 minus D by z c, ok; that means, our Q was what was our Q R? 1 minus D by z c that is the characteristic impedance and those things we have learned; that means, we have learned this.

So, this is also function and we have also discussed rhp 0 is also a function of. So, what was our rhp 0? It was R 1 minus D whole square by L because if you go back we can say R 1 minus D whole square by L. So, that; that means, we have discussed. So, all these parameters of the controller depend on load resistance and duty ratio and duty ratio is a function of input voltage.

If we want to so, if we design by traditional a way where we keep we design this compensator or a PID controller or type III compensator at the very beginning stage. We use of the chief component and we select it and then we leave it, then throughout the process when there is a load variation supply variation we are not going to change in the steady state analog control.

So, as a result, you have to design this controller based on the worst case. So, we have to design based on the worst case and the worst case means it happens generally at highest you know at highest duty ratio and highest load current. But, your converter may not operate in this worst case condition all the time. In fact, most of the time it will operate in other operating point where because of this worst case design we lose the performance benefit. So, it will be very slow.

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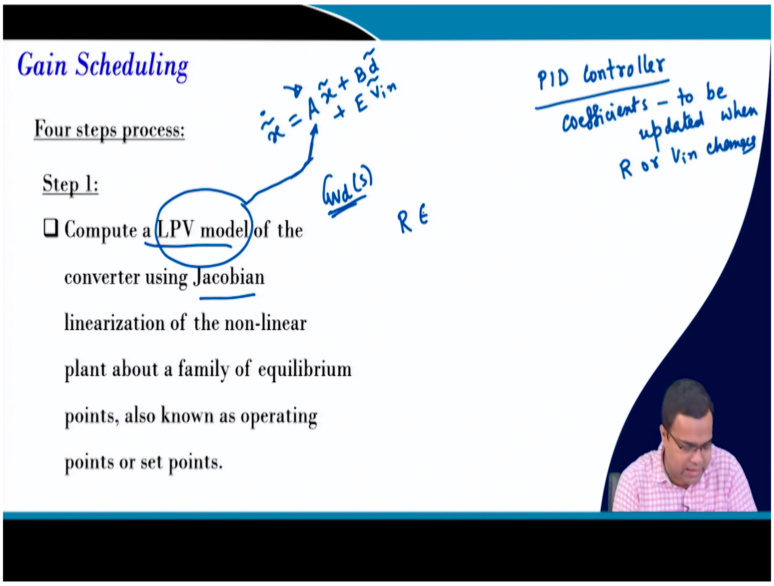


So, one of the way is the gain scheduling technique; that means, we need to update the controller parameter. So, there are two ways. We can still use a PID controller throughout, but this PID controller coefficient; that means, if we take the coefficient of the PID controller of or its coefficient, I can say. So, the coefficient should vary; that means, this coefficient to be updated when R or V in changes and that is kind of scheduling when to change, how to change.

So, this gain scheduling has four steps. Now, step 1: we first compute a linear parameter by model. In fact, we have it. In fact, we have this model which is A into x tilde plus B into d tilde plus E into V in tilde and this is something like a function of load resistance and duty ratio. So, we this linear parameter varying model which is arising due to this A matrix this A matrix we can obtain by Jacobian. So, in our case we do not need to obtain the linearization matrix all the time because we know the linearized model.

But, what we need to obtain? We need to obtain the coefficient or basically a matrix and we need to obtain control to output transfer function for various load resistance and the you know duty ratio or input voltage. So, we need to get a family of equilibrium point; that means, what is. So, that require what is my range of load.

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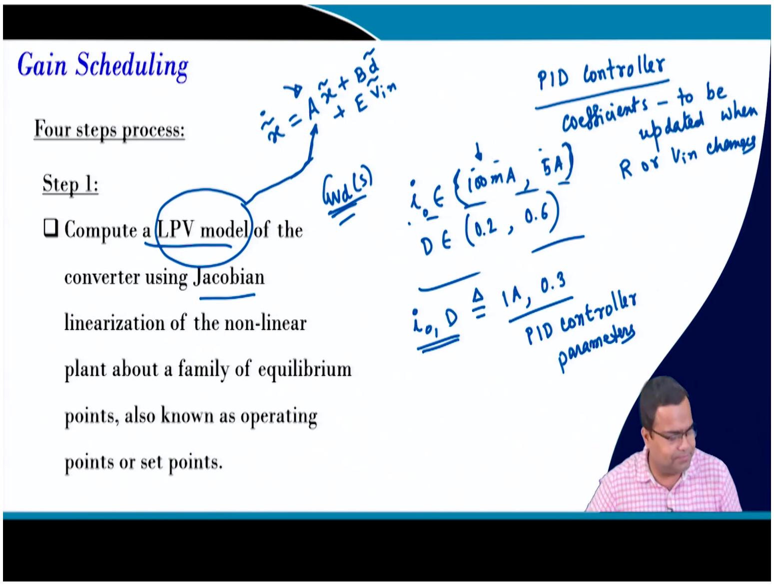


Or in other word, if I talk in terms of load current so, this requires the information of my load current. What is my load current? So, let us say it varies from 100 milliampere to you know to let us say 5 ampere. Similarly, if your duty ratio varies from let us say 0.2 to let us say 0.6. So, this, that means, within this load range and duty ratio, we need to compute this G vd for a certain range because you know we need to take some combination; that means, we need to take certain combination of D 0.

For example, I can take load current to be 1 ampere and duty ratio to be 0.3 like that. So, this combination we can take this combination and we have to design the controller; that means you get the PID controller parameters; that means we need to obtain the different operating point. So, we will get multiple operating point because how many in a current you will you will take and how many duty ratio will take?

So, you may get infinite number of set, but we are not interested in so many you know operating point. So, you have to choose some operating point where we have more interest and particularly; we need to consider the highest and lowest load current condition because if you unnecessarily use you know very you know this highest load current you have a real restriction in terms of right half plane 0.

But, the lowest load current you have real restriction in terms of Q peeping because at light load your damping can be really poor and so, you have to take special care in either PID controller or type III compensator, whatever it is.

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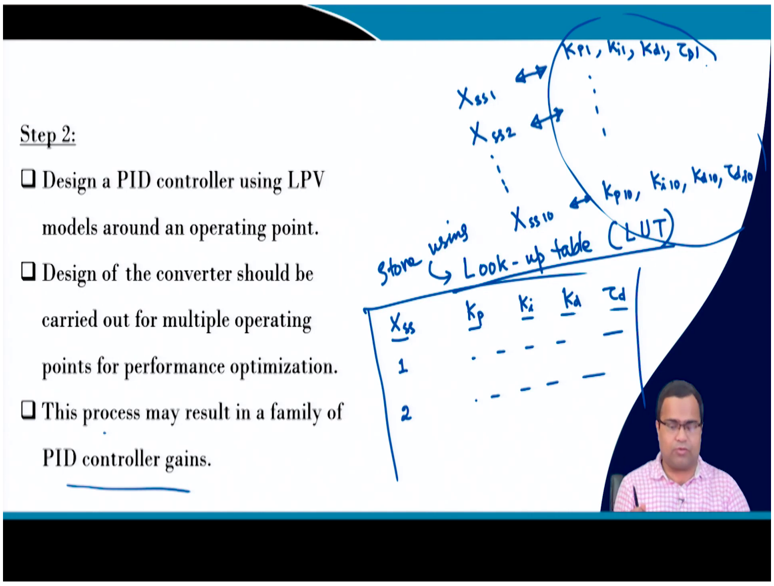


So, step 2, as I said we need to design PID controller for various operating points. We can say that operating point may be X s1 then we can say operating point X s 2 and so on. Maybe we can have ten such operating points. For each operating point, I can say I have assigned some PID controller gain; that means, I can write here K p 1, K i 1, K d 1, tau d 1.

So, by that way we can write K p 10, K i 10, K d 10 and tau d 10. So, these parameters once we obtain offline, we can simply plug into the look-up table. We write look-up table, right? So, here lookup table is my x-axis and here I can say K p, K i, K d, tau d. So, for number 1 point, you will get a set of controller. Number 2 point, you will get a setup controller and these are stored in a lookup table.

So, store using lookup table ok. So, should be multiple operating point we can compute and then this may result in a family of PID controller gain. That means you will get multiple set of PID controller gain and that will create a family of controller gain.

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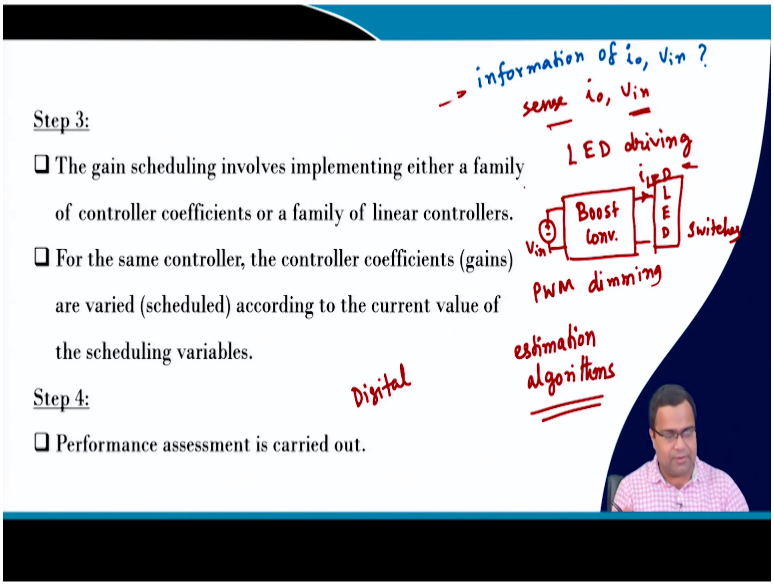


Then, what we will do? For each coefficient, I can use a lookup table the scheduling variable that means, but the question is how can we get the information of information of load current and input voltage, how can we get? So, one way we can do direct sensing; that means, you sense I 0 and V in.

Generally, this V in sensing is not a difficult task because if you go to commercial I C we can keep that provision that input voltage sensing. But the load current sensing is something is very difficult in fact. But, if I take let us say LED driving application, LED driving application a boost converter, let us say let us say you have a boost converter. It has a supply V in and it is driving an LED load.

Now, this is a dimmable LED; that means, it is PWM dimming that means, the LED current which is going out LED current that will either high-low, high-low, high-low, but it will change between it is nominal value and 0 value, nominal, 0, nominal, 0 and depending upon the number of string this value can change. So, you know the value of the current and if there are multiple string, we will know what are the different nominal current rating and we are only controlling the switches of the dimming switches right of the boost converter.

So, as a result, by changing this dimming switches whenever we are changing, we are essentially getting the information of the nominal load current and that can be used to tune the controller or get the parameters. So, this is another possibility. Similarly, if you go for battery, it is easy because you know battery charging anywhere sensing current.

If you are going for other applications, you are going for electric vehicle, where the boost converter. There can be multiple auxiliary power supply or sometime this boost converter is used to in step up the DC link voltage for the inverter where this boost converter output current can be measured because in high power application, it may not be very difficult to sense current where because the switching frequency is not very high.

If you are going for low voltage high current applications, this might be difficult. This is because the sensing current in the high current path is really a tough, difficult job, but we can also use some kind of estimation algorithm which can which may not require you know to sense the current, but then we can get the information and can schedule it.

And, finally, once we set or we change the controller parameter using lookup table, then we will get we can try to get the maximum benefit using this algorithm and by that way we can improve the performance. So, your design will not be limited by the worst-case scenario anymore, but all these are very effective when you go for a digital control because this lookup table arrangement the updating parameter that makes sense when you implement a digital control, but this may be really difficult in analog control.

So, whatever gain scheduling technique we are talking about non-linear control, these are the motivating case to go for digital control for future, ok.

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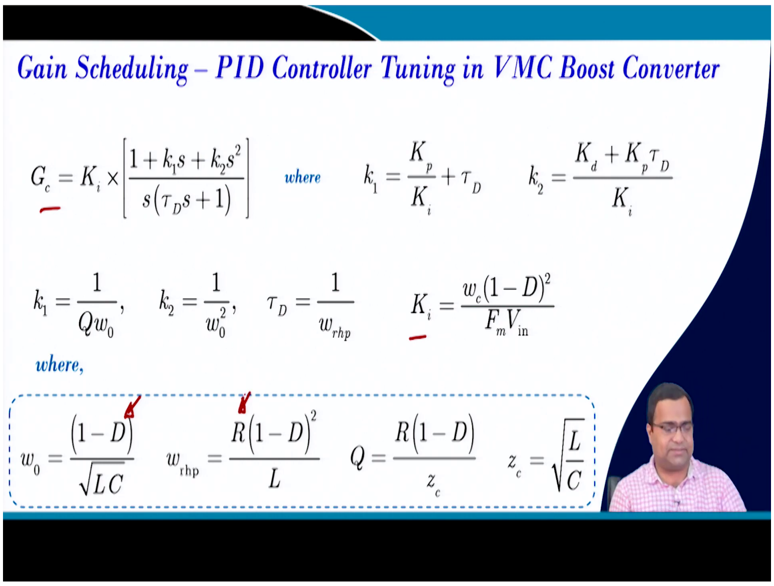

So, in the gain scheduling, we have a controller, and the parameters we have already discussed. The integral gain can be obtained. All the parameters information are given. So, we need to update the controller gain while when the duty ratio and the load resistance or rather load current changes and that we have discussed.

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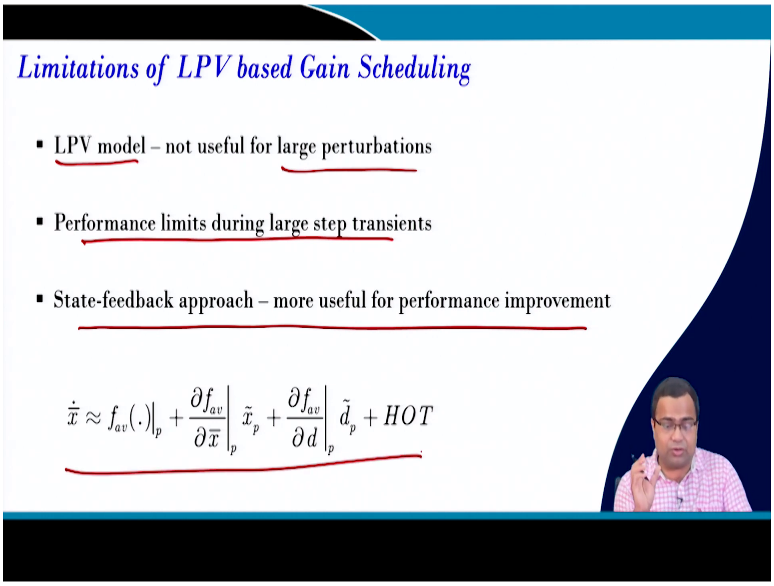


So, this way we can get a better response than you know fixed compensator based design; that means, where you are fixing the compensator gain at the very beginning in analog control, where we do not have flexibility to change the controller value abruptly because they are implemented in the off chip or outside using parasitic. That means, you know actual I would say not parasitic actual component ok where changing resistance outside component, capacitance, all these values are really difficult.

But, once you plug into digital where the coefficient is just a number that can be updated by this method. So, this method can improve performance over the worst case design where the controller gains are fixed in analog control. So, this will you know because we are going to take the case study in the next week I mean in the week 12, when we talk about analog control using fixed compensator worst-case design.

Then we will talk about the design based on the gain scheduling or basically updating the controller parameter and then we will also consider some other non-linear control to compare ok. So, limitation, still the linear parameter for varying model they rely on the small-signal model because if we have a large perturbation because of the validity of the model if the perturbation of the duty ratio is large then the model is not valid.

So, due to this validity of the model, we need to slow down the controller even if you update the parameter. It is not sufficient because the duty ratio perturbation is considered to be small. So, the performance limit is due to the model limit because of using small-signal model, right?

And, we have also seen the state feedback approach can be a better alternative compared to you know traditional current mode control as well as voltage mode control because voltage mode control is not sufficient for a boost converter. And, there we can use again this Taylor series model and so and we can get the linear model and design based on state space.

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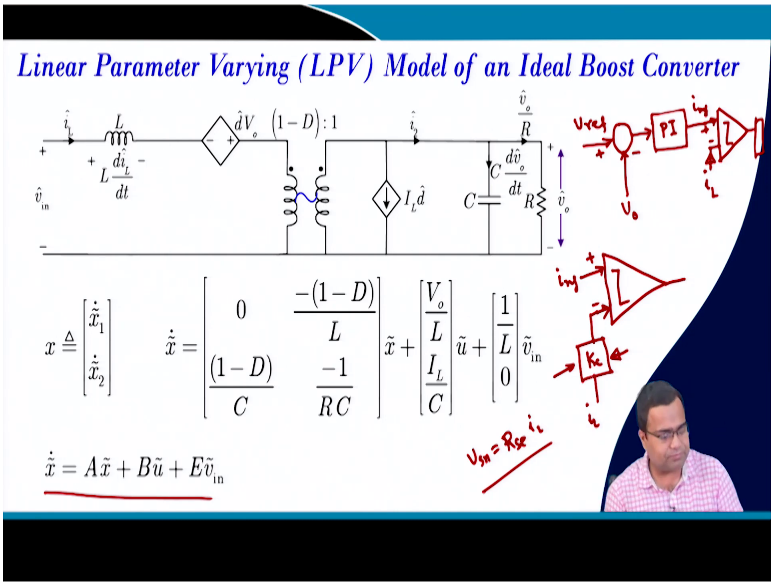


So, again, the linear parameter varying model state space model where what we did here we have implemented like you know we are taking this output voltage feedback sorry, this is my reference voltage, this is my reference voltage. So, this is my reference voltage, this is plus this. Let us say the output voltage.

So, I am not drawing the complete circuit, then we have in case of state feedback we have a PI controller and this output is going to some. It is like a reference current. And what we are doing? We are directly comparing with inductor current and that is going to the latch circuit or latch circuit, right. I am not going to discuss because this we have discussed multiple time.

So, in traditional current you know, I would say state traditional output feedback current mode control. This is the way.

But, in case of state feedback control, we can also add one gain block here; that means, you know after this i ref; i ref is coming in which is compared with our inductor current, now we have a current loop gain current loop gain i L and that is to compare and again it goes to the latch. So, this current loop gain it can be already placed because we told that the sense voltage of the inductor current we have written it is the R s equivalent to i L, right.

So, it is coming from the sense resistance. Sometime we need to put intentionally change the gain to do some optimization, right and that is very much possible in digital control as well as if we want to change this kent in analog also although it is difficult, but it is possible.

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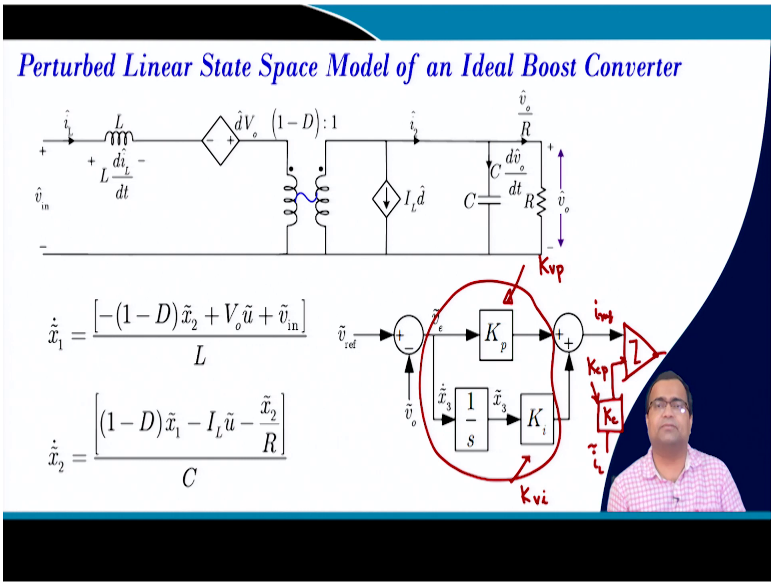


So, in state feedback control, what we did? We have considered this PI controller separately; we have just modelled it and this is a very well known augmented model. And, this reference, as if this is my reference current, is compared with the I would say there is gain K c and that is my actual inductor current and if we take tilde perturbation. So, this is compared right, and this is going to the modulator.

So, this state feedback control we are taking both current state and the voltage error where we have a proportional gain of the voltage controller we write is a K v p voltage controller proportional gain, this is like a current voltage controller integral gain and this is nothing but your current controller proportional gain, ok.

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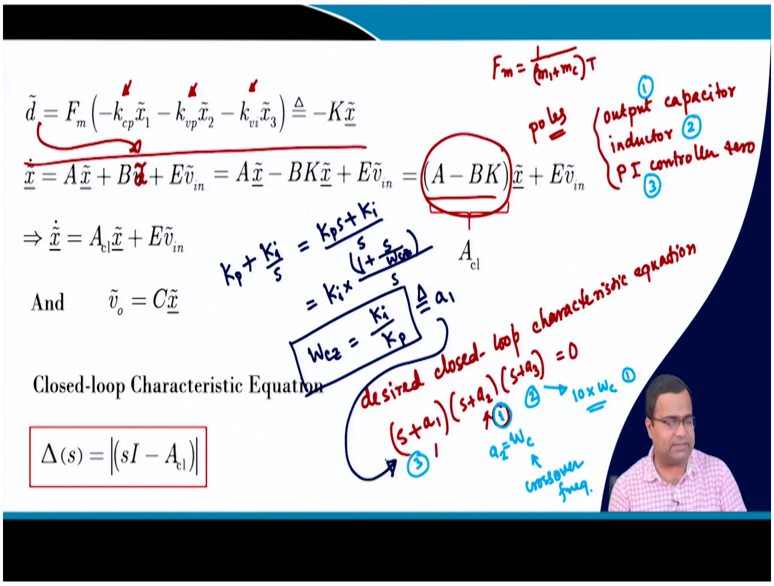


So, we can take this gain voltage controller integral gain, voltage controller proportional gain, and current control proportional gain. We can frame it into something like a feedback control where the duty ratio is generated by their combination.

And, what is the modulator gain? We know the modulator gain we have chosen 1 by m 1 plus m C into T in case of current mode control, where m 1 is the rising slope of the inductor current m C is the ramps. So, if you do not use a ramp, then it is 1 by m 1 t rising slope of the inductor current.

So, then if we substitute this d expression; so, this should be d expression duty ratio, if you substitute here, then you will get this and this is my closed loop matrix and then and then my output matrix. So, the closed-loop characteristic equation, which is the closed loop matrix, is this and we can shape this closed-loop characteristic impedance by placing the poles?

And, since it is a third order system so, we will have a polynomial consisting of third order. So, our desired our desired I would say our desired closed-loop characteristic equation. Our characteristics equation can be written as s plus a 1 s plus a 2 s plus a 3 and this whole thing to be 0.

So, we generally tend to prefer real pole, but I will tell what is this. So, in this converter case, we have discussed that this a 1 I mean a 2 will choose because there are three poles. So, one is due to the output capacitor output cap, one is due to the inductor and other is due to the PI controller 0, PI controller. So, these are the poles, no poles of the closed loop system.

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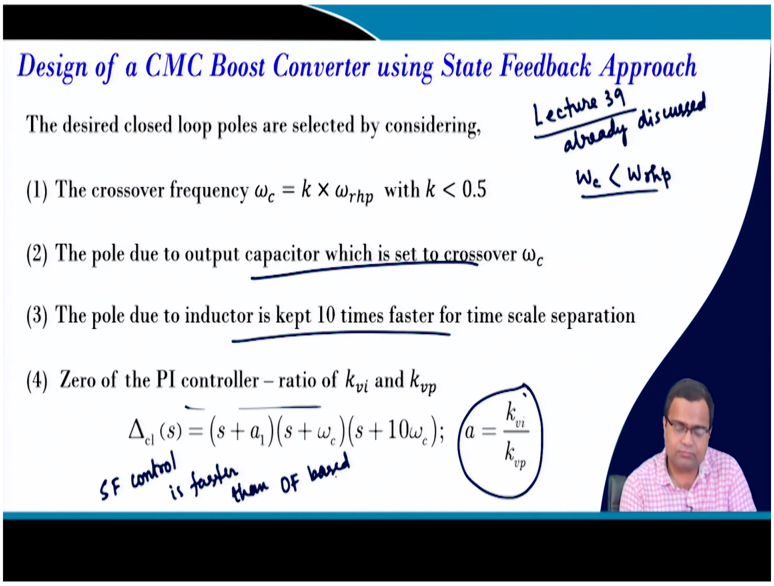


So, for the first case, what we can do? So, what we are going to discuss one of the pole let us say this is number 1. So, if I use different colour. So, this is number 1 this I will place at omega c; that means, I will set a 2 equal to omega c where omega c is the crossover frequency. Why? Because in boost converting output feedback, we have discussed the crossover frequency has to be some fraction of rhp 0. So, one pole I am placing, which is due to the capacitor, is the omega c.

The other pole is due to the inductor that is due to the inductor. So, this is 1, now talking about the second pole. This should be roughly 10 times omega c or basically the pole due to 1; that means, the pole due to 1 10 time because inductor should be faster than inductor dynamics should be faster than the capacitor. And, the third pole which is corresponding to this is nothing, but you know if we take k p plus k i S by S. So, we can write by taking k i we can write you know 1 plus; that means, let me write down again.

Here if we take the k p pole so, that means, we are talking about k p plus k i by s which is nothing, but k p s plus k i by s i can write it k i common into 1 plus s by omega controller z 0 by s. And, what is omega controller 0 because we are taking k i by k p and which I have called here is my a 1. This is here.

So, if we choose say k p by k i something, then we can place it and this case study we have already discussed in lecture number 39. So, lecture 39 already discussed we have already discussed. So, I am repeating and generally our crossover frequency should be smaller than rhp 0, but, here if we choose, let us say 5 nearly that.

So, pole due to the output capacitor is the 10 time faster and the 0 of the; that means k vi by k p of the voltage controller. And we have seen this can achieve faster response than output feedback. So, we have seen the state feedback control state feedback control or state feedback design is faster than or output feedback based. Here it is a state feedback based design state feedback or let me erase this part.

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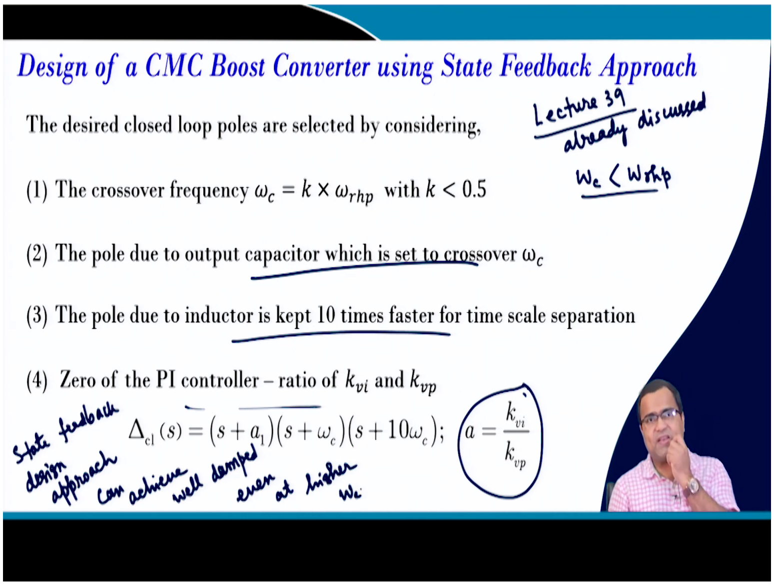


So, what we discussed? State feedback design approach can achieve can achieve well damped as well as faster. Well damped response even sorry, even at higher crossover frequency because that is the problem in output feedback design. So, we can achieve faster.

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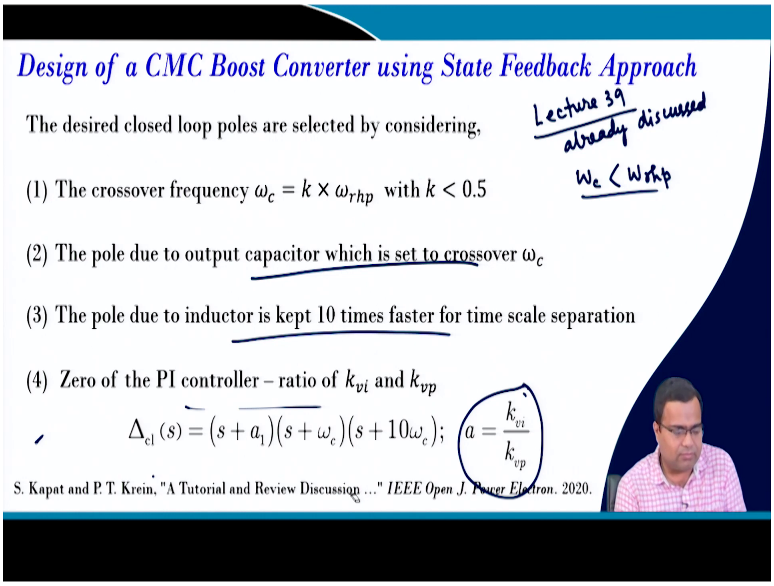


But, because of the limitation and that part we have discussed in this review paper. So, you can check it out that it can respond much faster that we have already discussed here, ok.

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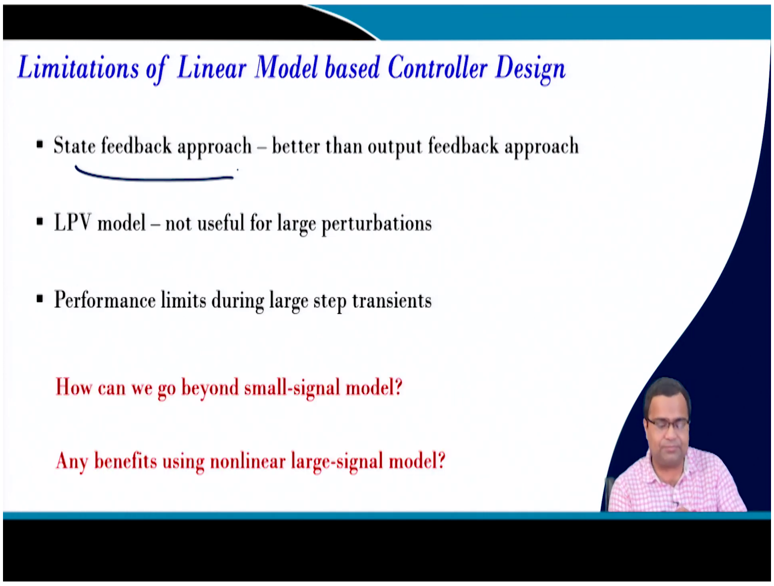


But, here still we are using linear control though state feedback approach is better, but due to the linear approach it cannot handle large perturbation. So, performance is limited. So, can we go beyond small-signal model? Any benefit using non-linear or large-signal model?

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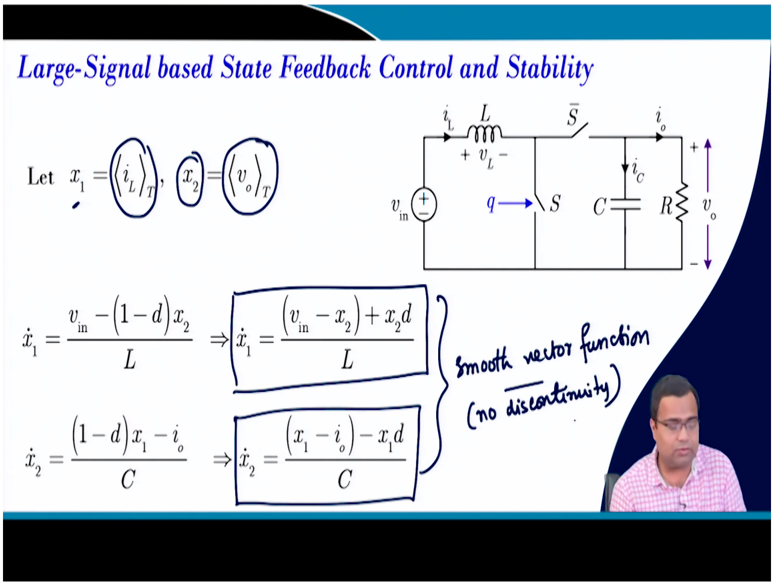


So, again we can rewrite the equation and if we write suppose the x 1 state to be the average inductor current and x 2 state to be the average output voltage, then we know that average dynamics x 1 dot is this, this is well known and x 2 dot also this part we know. So, we are not spending time on this and here we are writing these are all smooth function smooth vector function. Because there is no discontinuous control input and there is no discontinuity.

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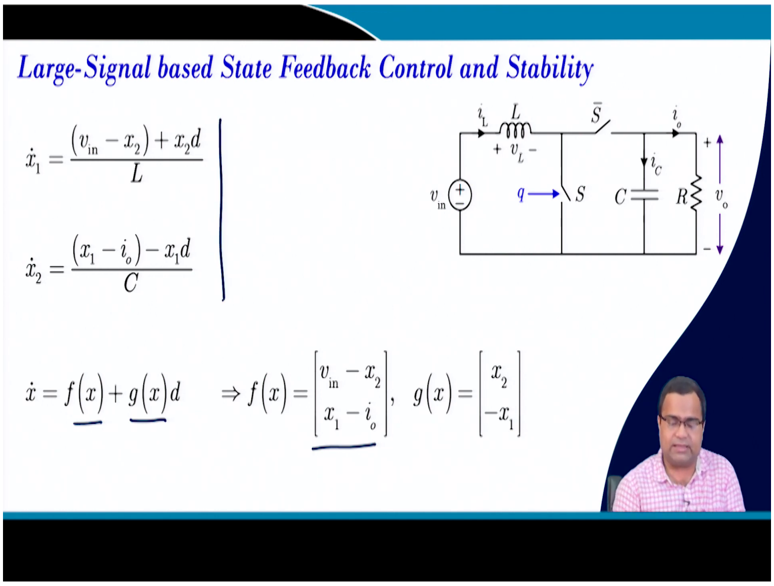


So, this we can discuss. Now, we can represent this something like x dot equal to f x plus g x into d format, where we can write f x and g x in this format by rearranging.

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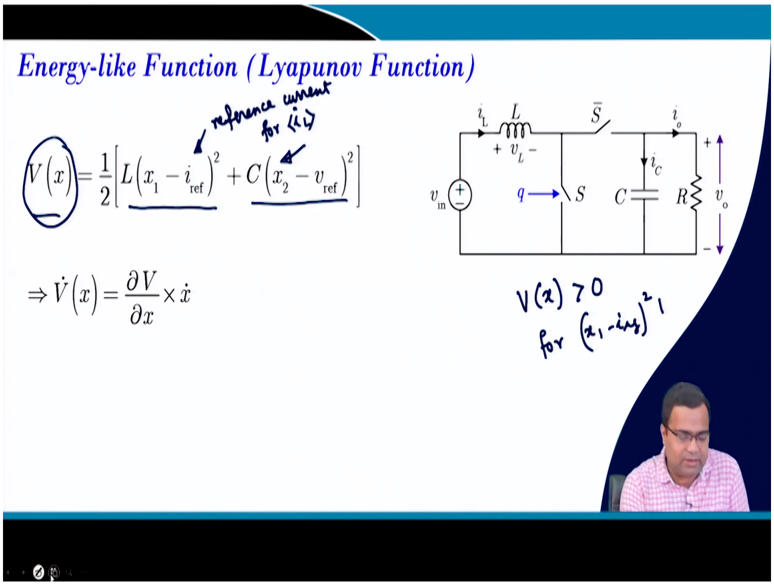


What is the next task? Suppose we create an energy function kind of energy like function where half of if I take the reference i ref to be the reference current for average inductor current. That means we want the inductor current average value to track the reference current or they should be equal at steady state.

Similarly, we want the average value of the output voltage should be equal to the reference voltage that is our desired, and that is why we have constructed the energy like function is like look like half L i square, but this is an error current and half C v square and this is error voltage.

Now, if you differentiate this energy like function because this energy function can be shown to be greater than 0 for as long as this error; that means, x i minus i ref for if I take this square plus.

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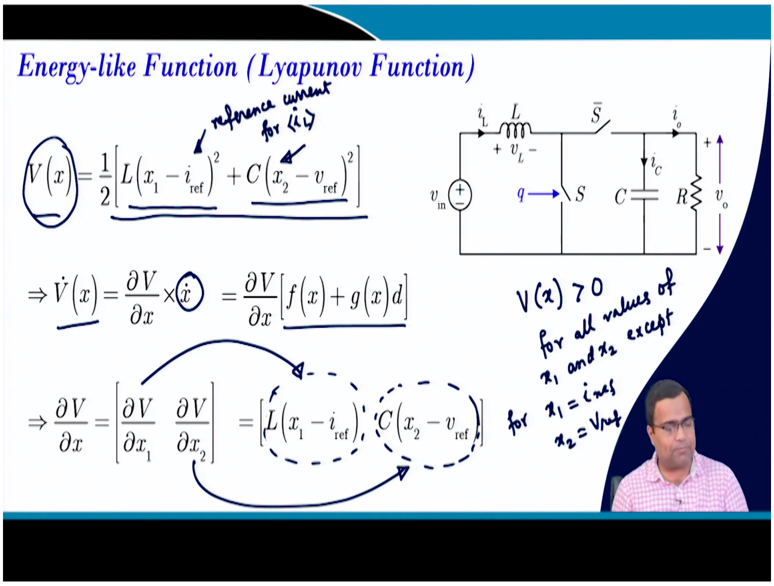


Or I think If x it can be shown to be x 1 and x 2. Except for x 1 equal to 0 x, x 1 equal to i ref x 1 equal to i ref and x 2 equal to v ref except for that point it is always 0 because the energy like function. So, if you take the differentiation of this; that means, rate of change of energy we can write dou V dou x into x dot and x dot is nothing, but our f x plus g x into d and that we have discussed in the previous slide.

Now, what is dou V dou x? It is dou V dou x 1 dou V dou x 2 and what is my dou V dou x 1? If I differentiate this function with respect to x 1, it will be simply L times this. So, this is nothing, but our dou V dou x 1 and this one is simply nothing, but this one, ok. So, you obtain this.

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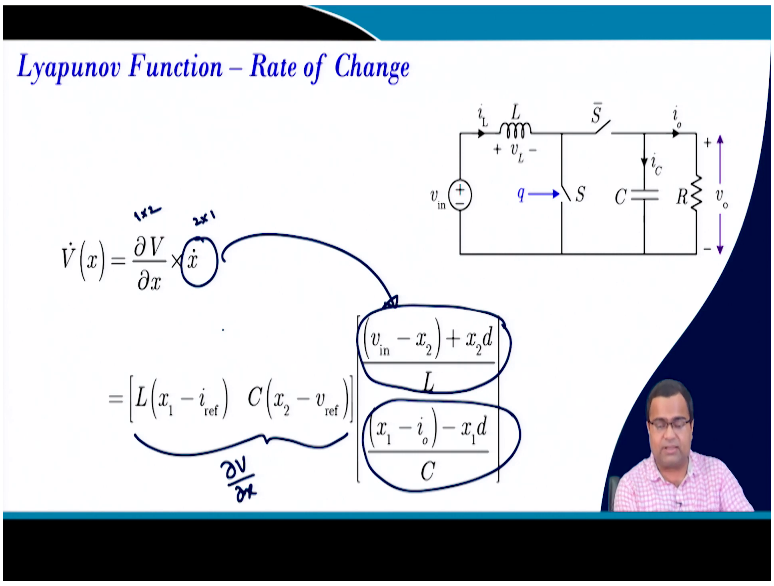


Then we already know this whole thing is nothing, but my dou V dou x then what about x dot? It is nothing, but this because it is a 2 cross 1, this is a 2 cross 1 and this is 1 cross 2 right and this is coming from state space representation to the dynamical equation, correct? This we have already discussed.

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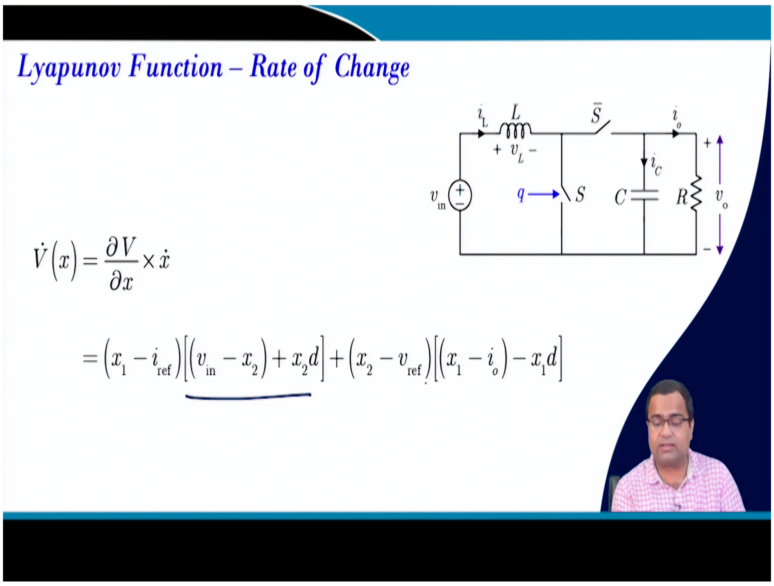


Next if you substitute; that means, if we simplify this multiplication, after simplification we are getting this x 1 minus i ref into this x 2 minus v ref into this.

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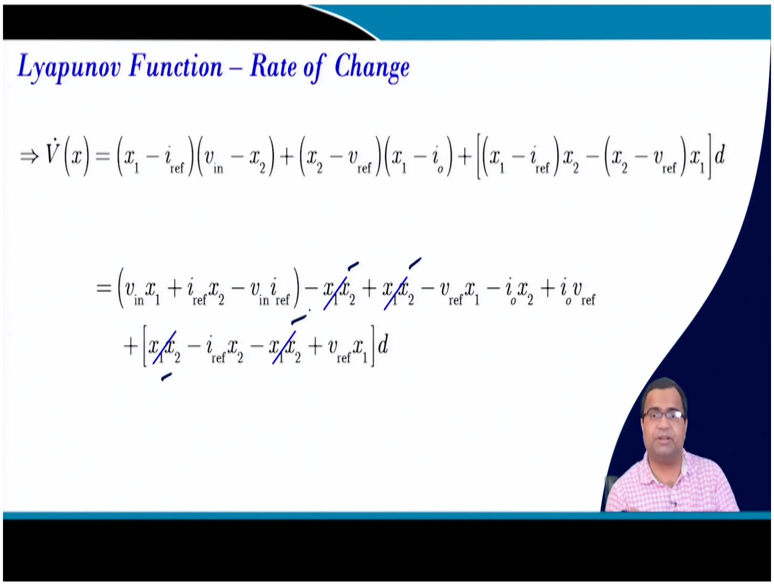


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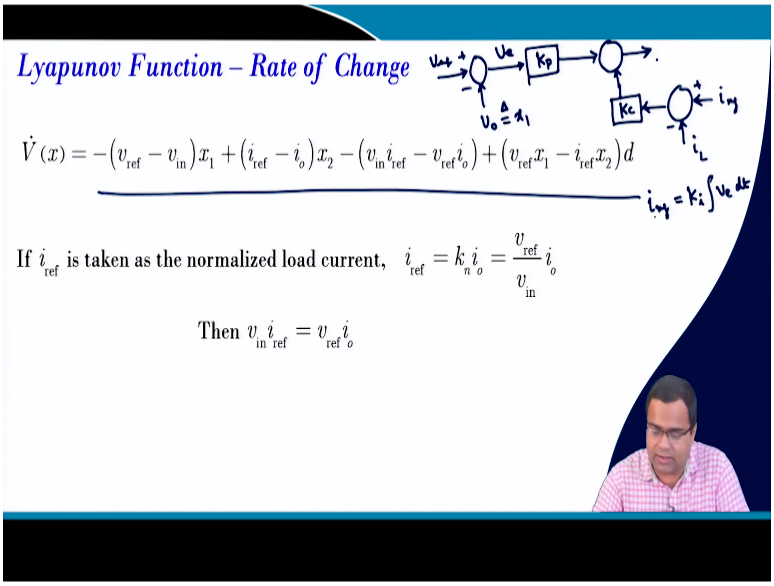


And, if we rearrange this function then, we will get this function. This all will be eliminated and will get this kind of structure; that means, this x 1, x 2 cross terms are gone; x 1, x 2 is gone. These cross terms are eliminated. Now, we got this form; that means, this kind of form.

Now, what is my reference current? In the earlier case, we consider reference current; that means, I am talking about a state feedback realization where we have a reference voltage and this is my output voltage. So, this is my error voltage ok error voltage this into k p ok and here we have denoted this term to be x 1, we have denoted.

Then, this is added with what? This is added with two things. I mean, one thing is this is the voltage loop gain, this is a K c the current loop gain, and that is coming with one is our i ref which is plus and another is i L which is minus. So, in i ref and this whole thing is my you know the control current, and that is compared with the saw-tooth waveform.

So, the i ref can be because we have discussed in lecture number 18 in state feedback, we have an alternative realization this can be taken simply our voltage loop integral. So, this is my voltage loop because you know. So, this is something like integral gain into our error voltage, d t. This we have considered for small-signal based design. So, here we are considering this quantity to be the normalized load current.

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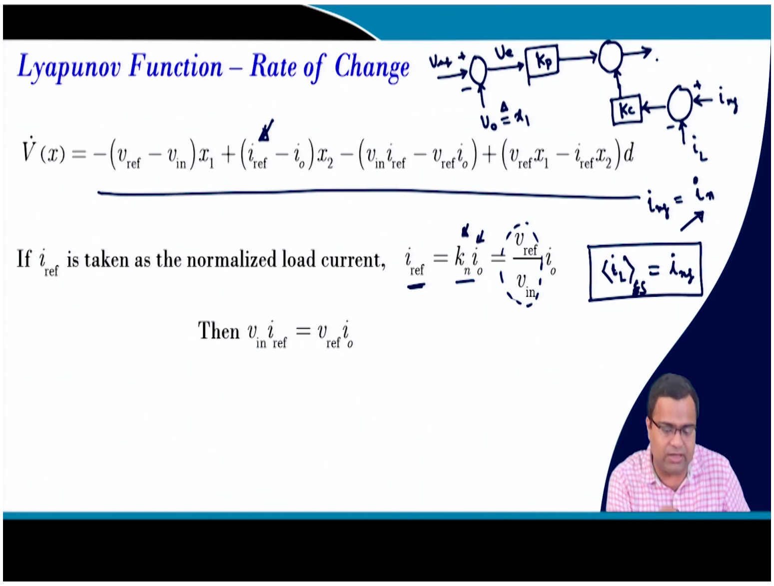


So, let us say we are considering the normalized load current. So, this is my normalized load current i reference which is nothing, but the normalization factor into the load current and for a boost converter normalization factor is v ref. So, this normalization factor is carried out in order to achieve the steady state value average value at steady state should be equal to this i ref. That is my objective.

And, you know, the average inductor current is equal to v ref by v in times the average load current. That is the boost converter. So, we have considered this. Now, if you substitute this i ref with the normalized load current, then what we will get?

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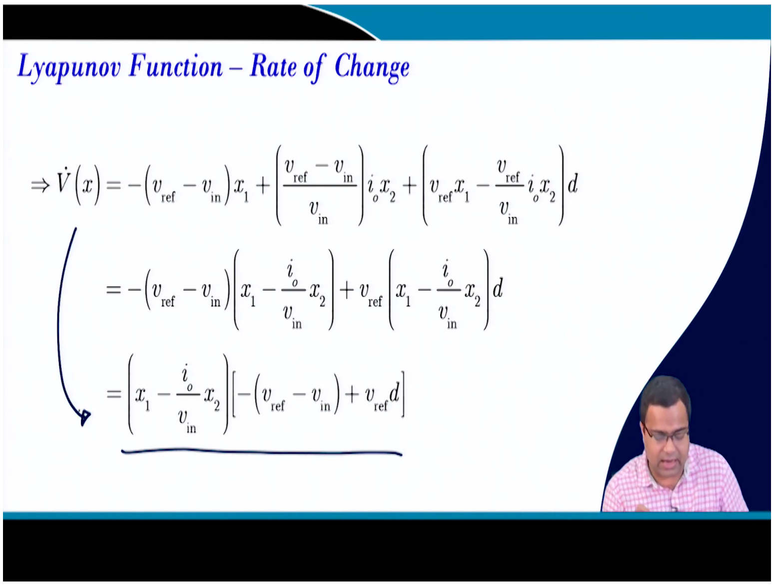


We have substituted will get this equation V dot x this equation.

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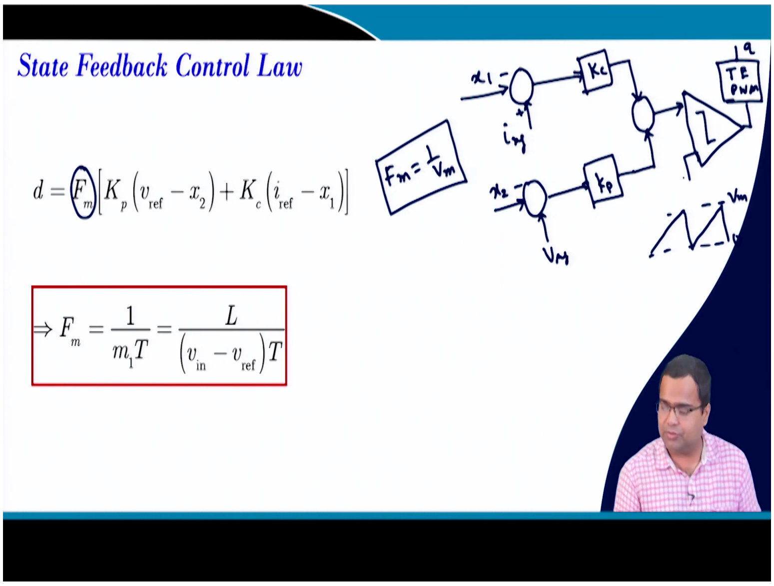


And, next our control duty ratio is the modulator gain because I told you that this is our x one that is my inductor current subtraction this is my i ref plus there is one more is my x 2 which is my output voltage, it is minus reference voltage. Here we have K p here we have K c and they are added up, and that is my that is directly compared with and that is generating my gate signal. I mean there will be latch circuit of course, there will be a latch circuit. So, we will have a latch circuit after that. So, this will go to your I will say trailing edge PW and that will generate the due to gate circuit.

So, what is my modulator gain? So, if this is my V m and this is my 0 voltage. So, the modulator gain F m is nothing, but 1 by V m, but if you take because if you do not take the ramp; that means, the other possibility you can also make it 0. Suppose instead of taking ramp if you make it 0 because there is no ramp compensation.

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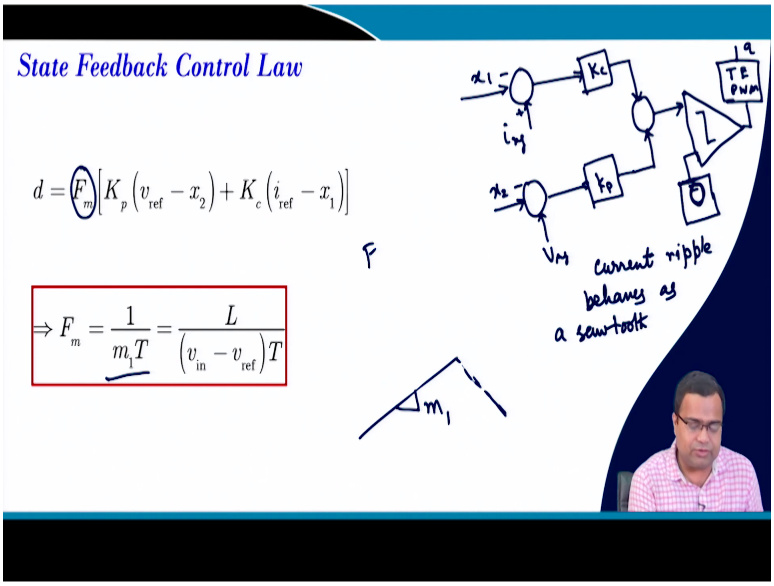


Then this whole quantity will be this will be 0 that is 0 grounded or you can simply write this is 0 quantity, that is 0. In that case our as if the ripple due to this current the current ripple there is a current ripple behaves as a sawtooth. So, the modulator gain is nothing but a rising slope of the current because the rising slope of the inductor current will act like a saw-tooth waveform. So, this slope is m 1 and this we are using for m 1 using for the modulator gain.

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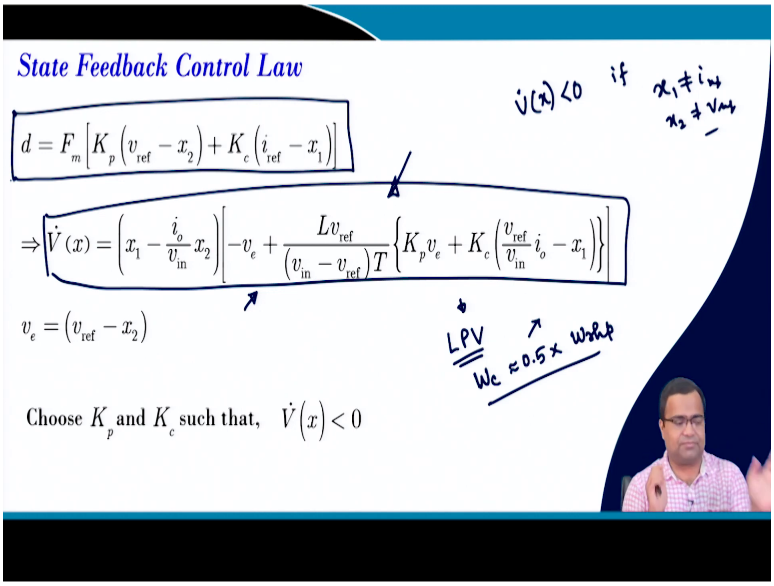


If you substitute, then this expression V dot x will look like this and what we need for stability we have to ensure that V dot x is strictly negative if x is not equal to 0 except for origin it should be; that means, sorry if I would say because it should come to the reference value.

So, if x not equal to i ref x 1 and x 2 not equal to v ref I mean as long as they come here. It should be negative. It should decrease. That will ensure the large-signal stability and we will see when you design state feedback control we have a constraint in the linear LPV model, where we have considered the K factor. The crossover frequency we choose a maximum sort of 0.5 times your rhp 0, but if we go for large-signal base control in state feedback, we can even increase slightly above.

So, that we can and we can ensure the large-signal stability under what condition you know it will basically the top switch will collapse because sorry, the top switch will turn on and your induct voltage will collapse and induct will keep on rising. So, the linear model the only difficulty. We cannot push further the model validity, but here, since you are using a Lyapunov function, we can ensure the large-signal stability using this non-linear model.

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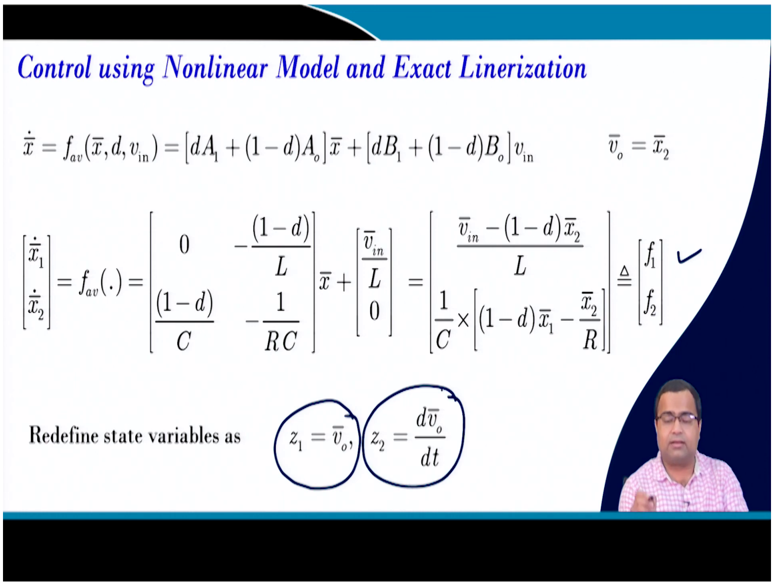


Another control we can use because this control we have used a linear state feedback control; that means, if you see the duty ratio is a linear function of state. But your closed loop dynamics is still non-linear because your average model is non-linear and that way we try to improve the performance. And if you go to linear control, we have linearized using Taylor series.

So, the perturb model was valid only for small day variation in the perturb quantity around their steady state value; otherwise, it is not valid. So, we have a validity issue in the small-signal model because of the perturbation size. But if we go to non-linear control where we can get do exactly relation. Even the perturbation is large. The model still remains valid, but we want to linearize by using by replacing duty ratio as some function of states in such a way that we can cancel the nonlinearity.

So, again, if you take the average model that we have discussed we can write this average quantity in this is well known and this just we have discussed now. Now, we are redefining the variable. One variable we can take the output voltage and the other variable we can take the derivative of the output voltage.

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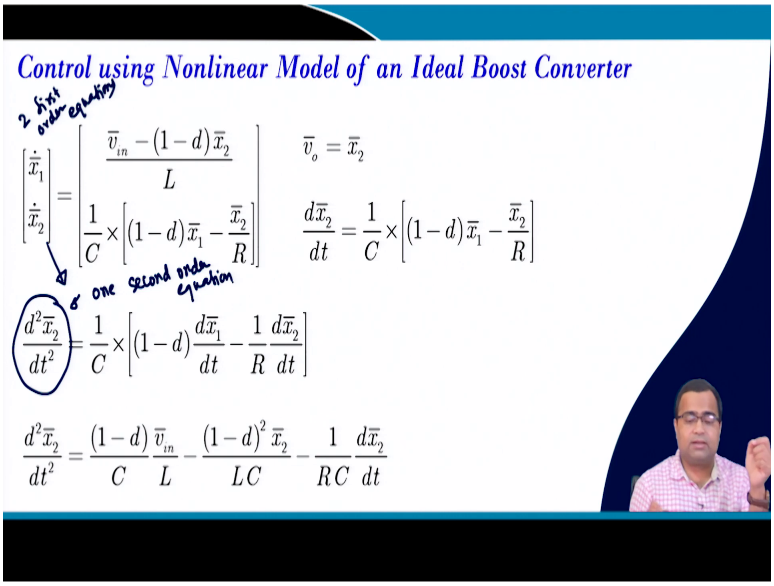


Then, we can take x 1 dot x 2 dot. We can write it down in terms of if we take the double derivative of the output voltage, it will be like this and so, this can be represented from this model, where here it is a state. It is a state space model where we are writing. It is a second order system. We are writing two first order. We are writing two first-order equations. Here we are writing one second order equation, right.

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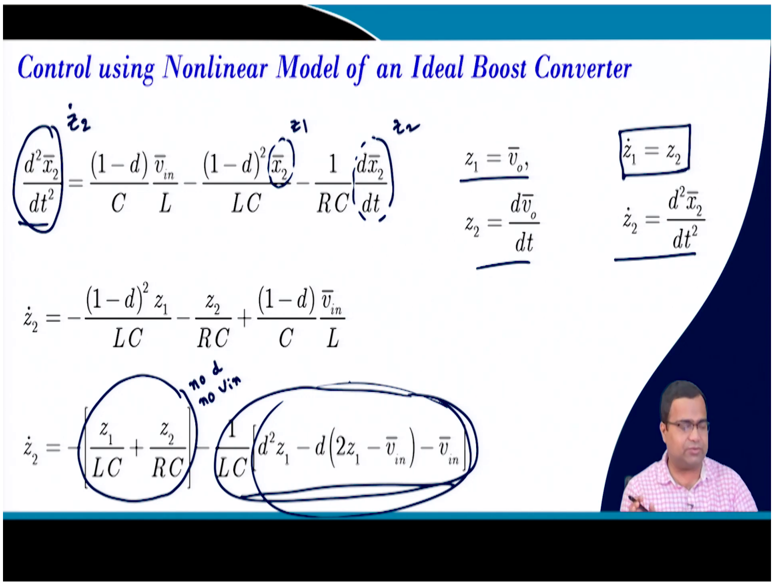


And, then, since we have redefined that v 0 to be z 1 and derivative to be d v 0 dt. So, that means, z 1 dot is z 2. This is straightforward, but what is z 2 dot? It is nothing but the double derivative of the output voltage and this we have just written, yeah, here. It is here, it is already here. So, this quantity will be replaced by my z 2 dot z 2 dot; this quantity will be replaced by z 1 and this quantity will be replaced by z 2 and this exactly we wrote ok; after rearrangement.

Then, we can further write to separate the duty ratio term; that means, this quantity there is no duty ratio, no duty ratio, no d even no v in. So, we have separated because this term we have to select the duty ratio in such a way that we can eliminate this whole term and replace with another control variable which will be a linear function of z 1, z 2.

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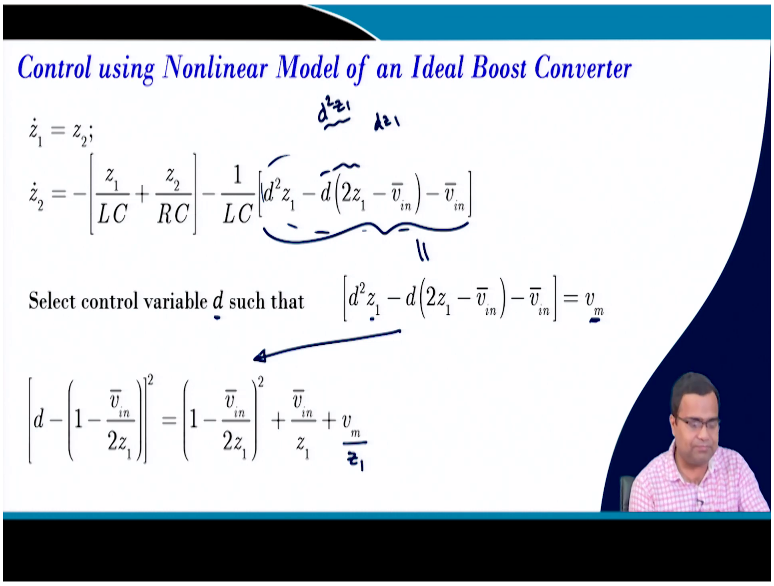


Then what we did? z 1 dot equal to z 2. This is the model. So, select the control variable d such that this quantity can be replaced by v m so that there is no non-linear function because there is a product of d square z 1 is a non-linear term right d into z 1 which will come from here it is also non-linear and input voltage in offset we want to eliminate. So, we want to make a linear combination.

So, this that means, this requires if I set this quantity. So, this should be v m by z 1 sorry, z 1. Then because we are dividing z 1, so, that means, this twice d and then we are making into a square form.

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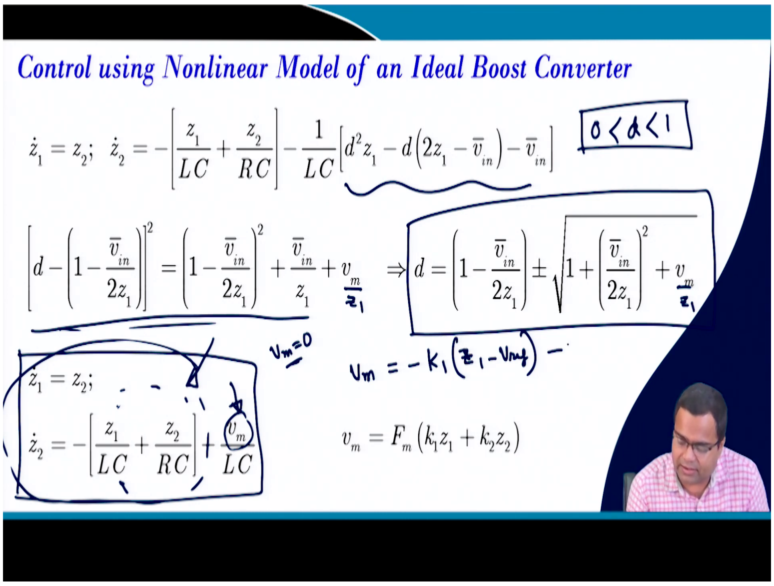


Then what we will get? This function that we have to replace this equal to z 1 and this turns out to be so, this should be z 1 z 1. That means, if we replace the duty ratio in this way, then my whole term here it will look like this frame. So, now, it is a linear. There is no non-linearity, and this is another control variable remember which has to be which should be derived as a function of z 1, z 2.

This represents v m. Let us say v m equal to 0; no other control variable. This is the equation, where it is a linear equation. But the location of the poles will be decided by the system parameter LC, but that poles may not give sufficient transient performance. That means we want to achieve some desired transient performance that cannot be made.

So, this extra variable that we placed in order to create a linear function of z 1 z 2 in such a way the closed-loop poles can be placed in a way to achieve faster response, but interestingly the whole control problem solved to a linear control problem because we have eliminated the non-linear control by means of this.

So, there is no Taylor series linearization. Remember, there is no duty ratio perturbation limit, but here the duty ratio must be greater than 0 and less than 1. It should be it should not take the extreme value 0 and 1. And, where we can make z 1 in fact, in more generic term we can take a v m to be minus k 1, what is z 1? z 1 is the output voltage.

So, you should take something like z 1 minus the reference voltage because we need to achieve we need to regulate the output voltage to the desired value.

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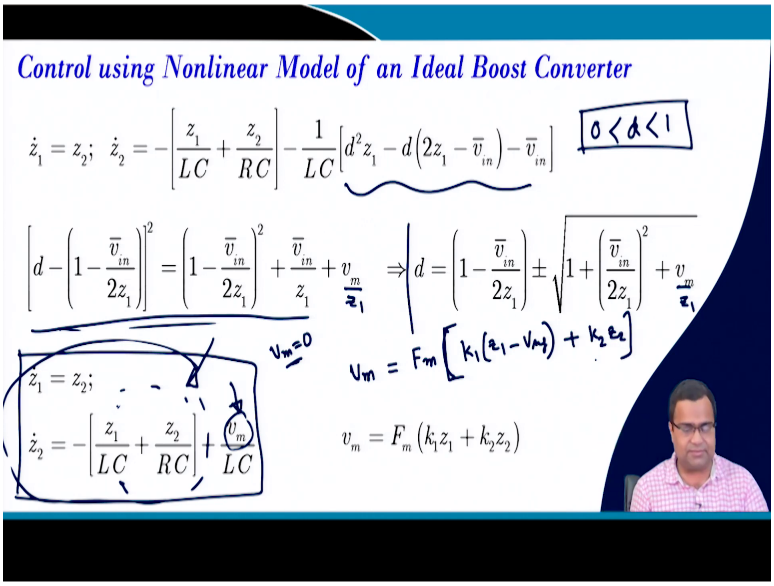


Another one will be and if we write ok just a minute if we write in terms of v m. So, there is a modulator gain into here we should write k 1 z 1 minus v ref because z 1 is output voltage plus k 2 into z 2. Why z 2 because the derivative output voltage should be 0 at steady state in average sense and what is the modulator gain?

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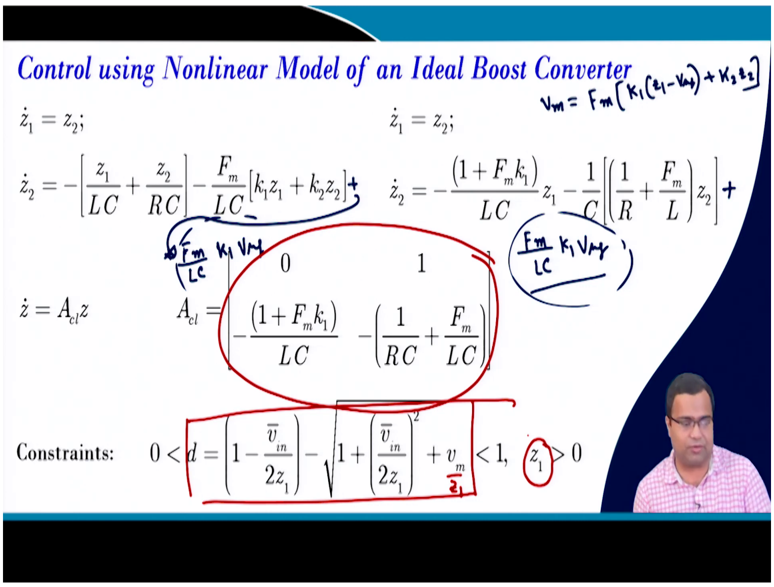

Because as if we are implementing this control logic and there should be a reference voltage. So, since we have replaced, v m to be v m to be what? v m to be F m k 1 z 1 minus v ref plus k 2 into z 2 ok. So, that means, there will be an additional term which will be plus you know if you take here it should be F m. So, this should be plus F m by LC into k 1 by into v ref. So, this term offset term will also be there along with this; that means, this term should come here, ok.

Similarly, we will have another term, which will be F m LC k 1 v ref, but we want to analyze stability for this system. So, this offset term is just you know as if it is if there is no offset term we are trying to get you know the stabilization at origin because we want to take any arbitrary initial condition should be driven towards origin, but here we want to drive towards the v ref.

So, only adding this offset term will only change the operating point, but the stability will be determined by this particular you know term; that means, this particular term will decide the stability. So, our stability will be decided by this particular matrix. So, if you solve by suitably, but there is a constant because when we take the duty ratio expression; the duty ratio has this.

So, this will put a limit and here we have z 1 tau and the limit also the output voltage must be positive because output voltage cannot be negative. So, it should be positive ok. So, in this constant we need to find the control law.

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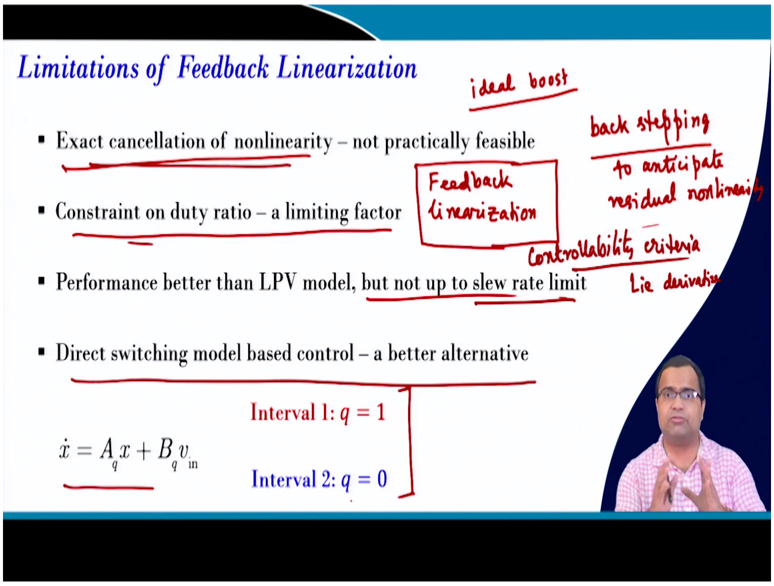

But, what is the limitation here? The exact cancellation is not possible because we have considered only ideal boost converter remember ideal boost converter, but in reality the boost converter is not ideal and you will get more complex you know model with name including parasitic and which can create real problem in exact cancellation that may not be possible. So, practically feasible that is why most of the feedback linearization comes with something called back stepping.

And, which will give you some kind of robust performance because there will be some residual to anticipate residual nonlinearity because we are relying on the exact cancellation and that is not possible. If that is not possible, if there is some residual nonlinearity how to make sure it is stable, that is the first criteria.

Secondly, we got a constraint on the which is a limiting factor and the performance better than LPV model but not up to the slew rate limit. Another thing I can say the non-linearization cancelling of the nonlinearity may not be a good idea, rather keeping the non-linear model and taking the advantage may be the better idea. And, this in order to you know apply this method comes from the concept of feedback linearization feedback linearization.

And, this require I mean if you want to achieve this linearized form it also need to satisfy something like a similar to you know controllability criteria controllability criteria and that require Lie derivative Lie derivative, but we are not going to discuss because this is beyond this course and it is a non-linear control course. But, one thing I can say we can use a large-signal model where we do not need to limit a small duty ratio perturbation.

But, even with that we really face difficulty exact cancellation is real difficult and we still have a limit on the duty ratio even though the variation is large, but it cannot achieve up to slew rate limit and it is very sensitive to cancellation as well as the parameter. So, then direct switching based control is the better alternative where we can go up to the slew rate limit. That means we can get the fastest response and that we can use a switching model that we have started with.

So, that means, where we discuss different non-linear control technique and one of the case studies using Lyapunov function based approach we will take in the next week I know in last week in the MATLAB case study, that, we want to show that if we incorporate the large-signal model with Lyapunov stability criteria. Then we can ensure the stability when the duty ratio variation happens beyond the small-signal limit. And, then we will show the switching control base approach is the best one to get the fastest response and to get up to the slew rate limit.

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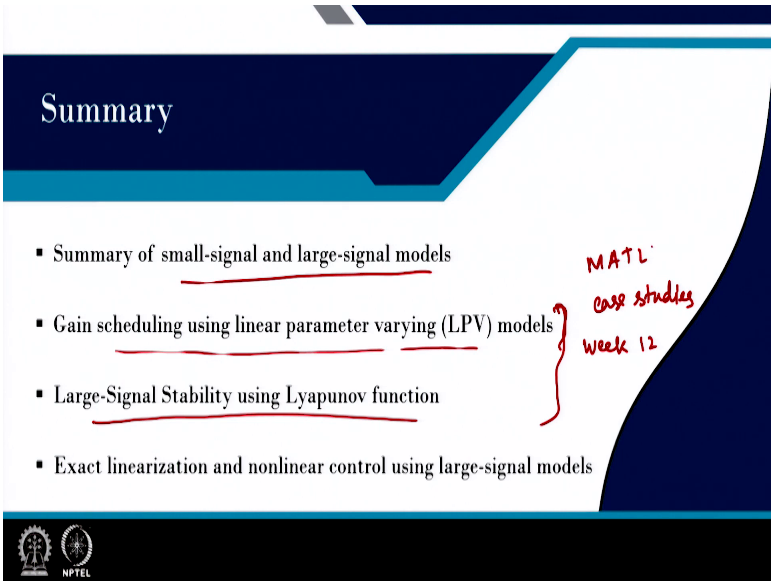


So, with this I summarize that we discuss small-signal model, large-signal model, we discussed gain scheduling and linear parameter varying model. We also discussed large-signal stability using Lyapunov function. We also discussed exact linearization and non-linear control using large-signal model.

So, and some of the case study that you know gain scheduling and then Lyapunov function based approach we will consider in week 12, we will take case studies using MATLAB case studies. So, with this I want to finish it here.

Thank you very much.