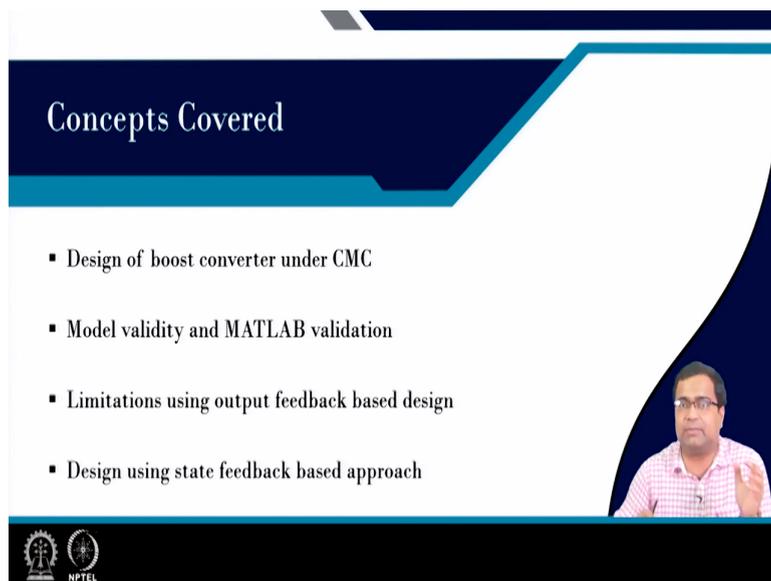


Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 08
Small-signal Design of Current Mode Control
Lecture - 39
Design of CMC Boost Converter – Output and State Feedback Approaches

Welcome this is lecture number 39. In this lecture, we are going to talk about the Design of Current Mode Control Boost Converter. Here we are going to consider the traditional approach of output feedback approach for design of current mode control, then we want to consider another approach, an alternative approach the state feedback design approach, but it is under current mode control.

(Refer Slide Time: 00:48)



Concepts Covered

- Design of boost converter under CMC
- Model validity and MATLAB validation
- Limitations using output feedback based design
- Design using state feedback based approach

NPTEL

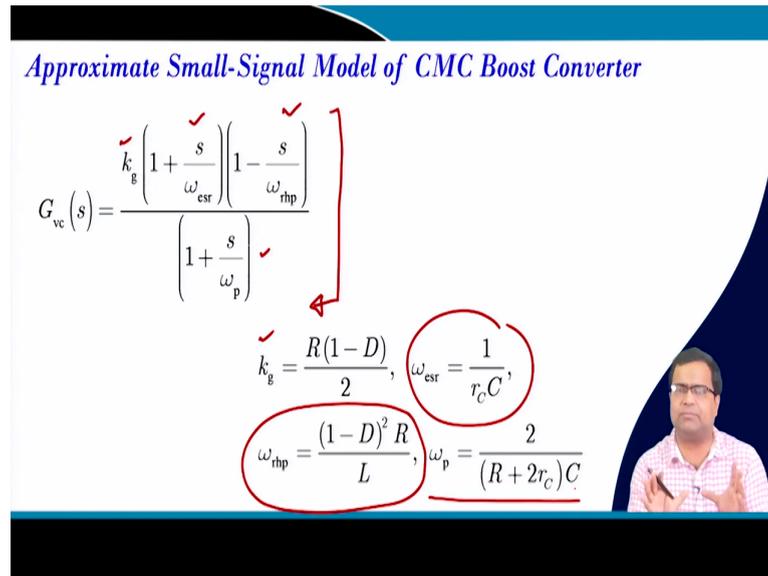
So, first we will design a boost converter under current mode control, then we will check the model validity using MATLAB AC transient simulation, then we want to find out what are the limitation using output feedback design approach and finally, also finally we are going to consider the design using state feedback based approach.

(Refer Slide Time: 01:10)

Approximate Small-Signal Model of CMC Boost Converter

$$G_{vc}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{esr}} \right) \left(1 - \frac{s}{\omega_{rhp}} \right)}{\left(1 + \frac{s}{\omega_p} \right)}$$

$$k_g = \frac{R(1-D)}{2}, \quad \omega_{esr} = \frac{1}{r_c C}$$

$$\omega_{rhp} = \frac{(1-D)^2 R}{L}, \quad \omega_p = \frac{2}{(R + 2r_c)C}$$


So, we will start with in fact in the previous lecture for a buck converter, we found that a first-order model is sufficient. If you do not consider any ramp compensation or if you know if your there is no sub-harmonic right, but in case of ramp compensation and sub-harmonic, then it is better to use discrete time model otherwise the design that I am going to consider it is much like a less than 0.5 duty ratio. So, where the first-order model will work perfectly fine up to the certain you know bandwidth. That means crossover frequency.

So, here first I will take the first-order transfer function of the boost converter, where this approximation is 1 esr 0 and then 1 rhp 0 and one pole because here again the inductor is replaced by a control current source and then using the first method of approximate small-signal approach, we can obtain the first-order transfer function.

So, 1 0 due to esr 1 0 due to rhp 0, but 1 a pole, but please remember generally we are familiar that this is an improper transfer function because it has 2 0 1 pole, but the actual system has two poles because there is a physical inductor. Since we are getting the approximate model that we are writing, otherwise this model I mean this is not a practically realizable system, but here we are taking the approximate version of an original boost converter where there is originally two poles, right. There are two poles.

So, here all this parameter like a DC gain of the plant, it is given esr 0 we know, then rhp 0, it is the same as voltage mode. What I mean, even if we take open loop converter voltage mode,

this rhp 0 remains same here and the pole actually we can obtain from this model. So, this we have discussed already earlier.

(Refer Slide Time: 03:09)

Open Loop Pole/Zeros: Boost Converter

$$G_{vc}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{esr}}\right) \left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

- **RHP zero** – Closer to Imaginary axis at high load and/or voltage gain, leading to severe bandwidth (BW) limitation

Now, in boost converter we have we want to design a closed loop control right. So, that means we have to shape the loop transfer function. So, that means our open loop control to output transfer function. Open loop means here the inner current loop is closed, but if we consider this one as if we disconnect the outer loop, then we have a control current reference and that is why we got G_{vc} .

So, this transfer function, we have discussed, has 1 RHP zero. We have discussed the RHP zero comes close to the imaginary axis under a high load and high voltage gain. This will lead to severe bandwidth problem because we will see when you want to design a compensator like a crossover frequency if you want to achieve certain desired crossover frequency, then that crossover frequency will be limited by the RHP zero. So, if the RHP zero comes close to the imaginary axis, the bandwidth of the boost converter will be very low.

(Refer Slide Time: 04:10)

Open Loop Pole/Zeros: Boost Converter (contd...)

$$G_{vc}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{esr}}\right) \left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

- **ESR zero** – Located at high frequency for lower ESR
- **Single pole** – Higher R leads to slower transient response
- **DC gain** – load resistance and duty ratio dependent, leading to poor load and line regulation



It also has 1 ESR zero which is located at very high frequency for low ESR and we will ignore this effect momentarily because this is located at very high frequency. Then, we have a single pole and that pole if the load resistance is high, then it will be a slower transient performance and the DC gain, the DC gain we will figure out it is basically a load dependent term as well as the duty ratio dependent term.

So, it leading to poor load regulation as well as line regulation here even though the current mode control has an excellent line regulation, but for boost converter it is not as you know excellent as like in a buck converter. It is better than voltage mode, but you know in terms of because there is a dependency in the closed loop. That means, our DC gain there is a duty ratio dependence. So, if there is a change in input, voltage duty will also change. As a result, there will be a slight shift in the DC gain of the control to output transfer function.

(Refer Slide Time: 05:06)

Primary Loop Shaping Objectives

$$G_{vc}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{esr}} \right) \left(1 - \frac{s}{\omega_{rhp}} \right)}{\left(1 + \frac{s}{\omega_p} \right)}$$

- To compensate RHP zero – to achieve flat gain (1P)
- ESR zero compensation – not needed for small ESR
- To cancel single pole – to arbitrary place CL pole (1Z)

Now, the primary loop shaping objective we need to compensate for RHP zero, ok. So, that means in order to achieve flat gain, so we will consider 1 pole of the controller which will be placed in coincidence with RHP zero. So, our controller should have a stable pole because we cannot we should not use an unstable pole to cancel unstable 0 because unstable pole 0 cancellation will make the system internally unstable, right.

So, the controller pole must be stable, but we will place the pole in coincidence with RHP zero, then ESR zero compensation it is not needed if the ESR is very small. So, in fact I will show you the case study where we took some ESR, but still the model matches accurately because the ESR is low and then we need to cancel the single pole here by means of a controller 0, ok. So, we want to place so that.

(Refer Slide Time: 06:05)

Primary Loop Shaping Objectives (contd...)

$$G_{vc}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{est}}\right) \left(1 - \frac{s}{\omega_{mp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

1 zero
2 poles including one at origin

- To consider an integrator – to eliminate SS error (IP)
- Compensator gain – to meet DC gain & crossover frequency
- A type-II compensator needed; PI is not enough here

That means now another we need to consider one integrator because the integrator is needed to achieve close to 0 steady state error, right or basically to achieve very high DC gain loop gain and then the compensator the DC gain of the compensator that we are going to consider that is required to meet certain crossover frequency.

So, that we will discuss in the design process. So, in this case we will find since we need 1 zero to cancel this pole of the system and 2 poles, including one at origin. So, this represents a type 2 compensator. So, here PI controller is not enough. So, in voltage mode in current mode control boost converter, we should not use a PI controller particularly for this approach because the PI controller I mean we have only 1 zero and 1 pole right and that was not sufficient to shape the loop.

So, here we need 2 poles and 1 zero because 1 pole additional pole is needed to anticipate the effect due to the ESR we want to place in coincidence to ESR, but once we go to state feedback design, then we will find approach design the PI controller should be sufficient. So, in this output feedback approach because we want to we do pole zero cancellation, we need a type 2 compensator.

(Refer Slide Time: 07:40)

Perfect Compensation – Boost Converter

$$K_{loop}(s) \approx \left(k_g \left(1 - \frac{s}{\omega_{rhp}} \right) \right) \times G_c$$

$$G_c = \frac{k_c \left(1 + \frac{s}{\omega_{cz}} \right)}{s \left(1 + \frac{s}{\omega_{cp}} \right)}$$

$\omega_{cz} = \omega_p, \omega_{cp} = \omega_{rhp}$

Now, for a perfect boost converter, that means if we consider if we ignore the effect due to esr because esr is located far away and we are talking about the control bandwidth which is limited by the right appearance zero. So, if the right appearance zero, that means your load is high esr 0 is negligible because it is far in the right-hand side.

So, its effect is negligible because your control bandwidth will see will be much lower than the rhp 0, right? So, the esr effect will be can be neglected. So, this is my approximate transfer function with 1 zero 1 pole and zero is the esr zero of the plant. Now the controller we talked about it is a type 2 compensator. So, in type 2 compensator, we need to place controller 0 to cancel the plant pole right and we need to place the controller pole to in coincidence with the rhp 0. That means, in coincidence with the rhp 0 right, but it is a stable pole remember right.

Now, with this the loop transfer function, that means if we place this, it gets canceled. That means the stable pole of the system is canceled by the stable zero of the controller, but we will have these two as well as one integrator DC gain of the controller and the planned DC gain and we are placing the controller pole in coincidence with the rhp 0.

(Refer Slide Time: 09:17)

Perfect Compensation – Boost Converter (contd...)

$s = \frac{\omega}{\omega_{rhp}} \times \omega_{rhp}$ $s = \sigma + j\omega$

$$K_{loop}(s) \approx k_g k_c \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)}{s \left(1 + \frac{s}{\omega_{rhp}}\right)}$$

$$\Rightarrow K_{loop}(s) \approx \frac{k_g k_c}{\omega_{rhp}} \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)}{\frac{s}{\omega_{rhp}} \left(1 + \frac{s}{\omega_{rhp}}\right)}$$

ω Frequency response, $s = j\omega$

$$K_{loop}(j\omega_n) = k_L \times \frac{(1 - j\omega_n)}{j\omega_n (1 + j\omega_n)}; \quad \omega_n = \frac{\omega}{\omega_{rhp}}$$

$\frac{s}{\omega_{rhp}} = \frac{j\omega}{\omega_{rhp}} = j\omega_n$

So, loop transfer function will become like this. Now, since you will find you have s by ω_{rhp} right. So, I want to write this as well. So if I write s , then j can write s by ω_{rhp} into ω_{rhp} right and this is exactly I have done. So, that is why this term is coming here and so, now it is s by ω_{rhp} and this term is my k_L .

Now, why I am writing s by ω_{rhp} ok? Now, we want to replace this. That means, if you go for frequency response, that means if you go for frequency response, we generally write s equal to $j\omega$ because if it is a stable system, generally we write s equal to $\sigma + j\omega$. But for a stable system, that means this σ term which is associated with the exponential decay it will decay and it will eventually you know come to steady state.

And then, the frequency response we simply take $j\omega$ because we will wait for long time, so that the system actually I mean all the transient effect is decay. I mean they are vanished, and it comes to the actual steady state behavior where we are exciting the system with some frequency ω . So, we are sweeping this frequency right. So, in this frequency response if we write s equal to $j\omega$, then s by ω_{rhp} will be $j\omega$ by ω_{rhp} and this we are writing ω_n where ω_n is nothing but this so that means we are writing $K_{loop} j\omega_n$. Is this ok?

(Refer Slide Time: 11:16)

Perfect Compensation – Boost Converter (contd...)

$$K_{\text{loop}}(j\omega_n) = k_L \times \frac{(1 - j\omega_n)}{j\omega_n(1 + j\omega_n)}; \quad \omega_n = \frac{\omega}{\omega_{rhp}}$$
$$\Rightarrow K_{\text{loop}}(j\omega_n) = r(\omega_n) \angle \theta(\omega_n)$$
$$\Rightarrow r(\omega_n) = \frac{k_L}{\omega_n} \quad \angle \theta(\omega_n) = -90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1 - \omega_n^2} \right)$$


So, now under perfect compensation in our K loop we have already written this expression $j\omega_n k_L$ by this term. So, if you obtain the in terms of polar form, that means this particular you know if you take the whole thing, that means if you take this whole thing you will see the magnitude of this and magnitude of this they will they are same.

So, they will cancel each other in terms of magnitude. So, the overall magnitude will be $r k_L$ by ω_n . This is clear, this can be derived, but if you take phase the phase due to this will be minus 90 degree and the phase due to the ratio of these two will be simply minus tan inverse of twice ω_n by $1 - \omega_n^2$.

(Refer Slide Time: 12:05)

Perfect Compensation – Boost Converter (contd...)

$$K_{\text{loop}}(j\omega_n) = r(\omega_n) \angle\theta(\omega_n)$$

$$\Rightarrow r(\omega_n) = \frac{k_L}{\omega_n}, \quad \angle\theta(\omega_n) = -90^\circ - \tan^{-1}\left(\frac{2\omega_n}{1-\omega_n^2}\right)$$

At gain crossover frequency ω_c

$$r(\omega_n)\Big|_{\omega=\omega_c} = \frac{k_L}{\omega_n}\Big|_{\omega=\omega_c} = 1 \Rightarrow k_L = \frac{\omega_c}{\omega_{\text{rhp}}}$$

$$\Rightarrow k_c = \frac{w_c}{k_g} = \frac{2w_c}{R(1-D)}$$

Handwritten notes on the right:

$$\omega_n = \frac{\omega}{\omega_{\text{rhp}}}\Big|_{\omega=\omega_c}$$

$$= \frac{\omega_c}{\omega_{\text{rhp}}}$$


So, that means we got the polar form the loop transfer function. Now, we need to get at gain crossover frequency. The amplitude, the magnitude should be unity. That means this is the unity at gain crossover frequency. That means here at gain crossover frequency, it is equal to 1 and what is my omega n my omega n is simply omega by omega rhp and at gain crossover frequency, this will be at gain crossover frequency, this will be simply omega c by omega rhp and this is equal to this is nothing but my k L.

So, we are getting this expression from here. So, you can write and if we substitute the k L expression because k L where is my k L. So, we have discussed in the previous slide, yeah this is my k L expression right k L expression k g that is the DC gain of the plant k c is the DC gain of the controller and the omega rhp 0. If we write all the expression, then we can get is here ok.

(Refer Slide Time: 13:22)

Perfect Compensation – Boost Converter (contd...)

At gain crossover frequency ω_c

$$r(\omega_n) \Big|_{\omega=\omega_c} = \frac{k_t}{\omega_n} \Big|_{\omega=\omega_c} = 1 \Rightarrow k_c = \frac{\omega_c}{k_g} = \frac{2\omega_c}{R(1-D)}$$

How to find gain crossover frequency ω_c ?

1. Set ω_c directly as $\omega_c = k \times \omega_{rhp}$ – what is k ?
2. Using phase margin (PM) criteria – find ω_c from desired PM

Handwritten notes:
 $G_c = k_c \times \frac{(1 + \frac{s}{\omega_{cp}})}{s(1 + \frac{s}{\omega_{cp}})}$
 $\omega_{ca} = \omega_p$
 $\omega_{cp} = (\omega_{rhp})$
 $k_c = ?$



Next, at gain crossover frequency, we have already computed the gain. Now how to find gain crossover frequency because if we can set the gain crossover frequency, then we can get the controller DC gain, right? If you recall what is there in our controller, our controller has a DC gain multiplied by controller has 1 0 and controller had 1 pole at origin and 1 pole and we have said that omega cz to cancel the pole of the open loop system. That means your control to output transformation.

So, this is already well known because we have cancelled what is omega cp? This is k same as rhp 0, sorry this is same as rhp 0. So, that means we know and we know the expression of rhp 0. So, that means this is also known. So, these two are known. Only unknown is the kc and that can be obtained from this expression.

Now, the question is for a given load current given duty ratio, what will be my crossover frequency, who will give me the crossover frequency right? So, there are two approaches. One approach, let us say the crossover frequency is a like at k times omega rhp zero. That means, either I can take k greater than 1 less than 1, but I will see what will be my k choice or the gain crossover. That means, omega c can be obtained from the phase margin criteria and we will discuss these two.

(Refer Slide Time: 15:04)

Case Study 1: Set $k=1$, i.e., $\omega_c = \omega_{rhp}$

$$K_{loop}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{csr}}\right) \left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)} \times G_c$$

At gain crossover frequency ω_c $w_n' = \frac{\omega_c}{\omega_{rhp}} = 1$

$k_c = \frac{2\omega_{rhp}}{R(1-D)}$

$\angle \theta(w_n') \Big|_{w_n'=1} = -90^\circ - \tan^{-1} \left(\frac{2w_n'}{1-w_n'^2} \right) \Big|_{w_n'=1}$

$\omega_c = k \omega_{rhp}$

$w_n' = \frac{\omega_c}{\omega_{rhp}} = 1$

$w_n' = \frac{\omega_c}{\omega_{rhp}} = 1$

So, in the first case at gain crossover frequency, which is ω_n dash, that means ω_c by ω_{rhp} let us consider a case study 1 where we are setting k equal to 1. That means we originally choose k_c equal to k times ω_{rhp} 0 and here we are taking k equal to 1. So, ω_c control crossover frequency is same as the ω_{rhp} 0. If we set it, then the k_c can be obtained from the previous. That means we have replaced ω_c by ω_{rhp} .

And what is my phase angle? Because we know that ω_n was originally ω_c by ω_{rhp} and that crossover frequency we replaced by this, that means dash replace. So, this dash symbol indicate that ω_n dash is nothing but ω_c by ω_{rhp} . Now, if we set this to be ω_c equal to ω_{rhp} , so this is 1 and if you consider this term will become infinity, ok.

(Refer Slide Time: 16:13)

Case Study: Set $k=1$, i.e., $\omega_c = \omega_{rhp}$

$$K_{loop}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{csr}}\right) \left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)} \times G_c$$

At gain crossover frequency ω_c $w_n' = \frac{\omega_c}{\omega_{rhp}} = 1$

$\angle\theta(1) = -90^\circ - \tan^{-1}(\infty) = -180^\circ$ \blacksquare PM = 0 degree

Not a stable design!! $\omega_c < \omega_{rhp}$ for stable design!!

Because there is a in the denominator 1 by in the denominator 1, you will have 1 minus 1. So, it is some constant by 0. So, it will be infinity and if infinity means tan inverse that is 90 degree. So, it is 180 degree. So, the phase margin is 0 degree. So, it is not a stable design. So, that means the omega c must be smaller than rhp 0.

(Refer Slide Time: 16:47)

Design using Phase Margin (PM) Criteria

At gain crossover frequency ω_c , find $w_n' = \frac{\omega_c}{\omega_{rhp}} = 1$

$PM_{desired} = 180^\circ + \angle\theta(w_n') = 90^\circ - \tan^{-1}\left(\frac{2w_n'}{1-w_n'^2}\right)$

$\left(\frac{2w_n'}{1-w_n'^2}\right) = \tan(90^\circ - PM_{desired}) \Rightarrow w_n' = k?$

Find controller DC gain $k_c = \frac{2k\omega_{rhp}}{R(1-D)}$

Handwritten notes: $w_n' = \frac{\omega_c}{\omega_{rhp}} = k$, w_c , $??$, 60°

And if we get anything better, so phase margin will be negative. So, that is an unstable system. So, our gain transfer frequency must be smaller than rhp 0, but how small that we will see. So, one approach, another approach, the design based on phase margin criteria. So,

at gain crossover frequency, we know the expression and the desired phase margin is nothing but 180 degree plus your phase angle at omega n dash and we are trying to find omega c. This is unknown and this expression, if we substitute, we will get this and if we rewrite this expression is nothing but tan of 90 degree minus phase margin desired.

So, if you set some phase margin lets us say 60 degree, then tan 30 degree. Then you can find out what is my omega n dash and what is my omega n dash it is nothing but omega c by omega rhp and this we have taken k because we have taken omega c equal to k times rhp, right? So, we can find out k from here and once I find out k, I can find out omega c and if I can find out omega c sorry you can k, then we can replace here.

(Refer Slide Time: 17:55)

Case Study: Design with 45 degree PM

$$K_{loop}(s) = \frac{k_g \left(1 + \frac{s}{\omega_{csr}}\right) \left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_p}\right)} \times G_c$$

At gain crossover frequency ω_c

$$PM = \angle \theta(\omega_n) \Big|_{\omega=\omega_c} - (-180^\circ) = 90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1 - \omega_n^2} \right) \Big|_{\omega=\omega_c}$$

Handwritten note: $\omega_n' = \omega_n \Big|_{\omega=\omega_c}$

$$PM_{desired} = 90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1 - \omega_n^2} \right) \Big|_{\omega=\omega_c} = 45^\circ \Rightarrow \omega_n' \approx 0.414$$

So, we will first design using 45 degree phase margin. If you set 45 degree phase margin, then for the same process that we have discussed the omega n dash omega n dash I will say dash that we have just discussed, which is omega n dash, sorry omega n at omega c that is omega n dash, ok. Now, here it is omega n because omega n dash is nothing but it is the omega n dash is nothing but it is the omega n at omega equal to omega c. That is it. So, it is nothing but 0.414.

(Refer Slide Time: 18:43)

Case Study 2: Design with 45 degree PM

$$PM_{\text{desired}} = 90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1 - \omega_n^2} \right) \Bigg|_{\omega_n = \omega_c} = 45^\circ \Rightarrow \omega_n \approx 0.414$$

$\omega_c = 0.414 \omega_{\text{rhp}}$

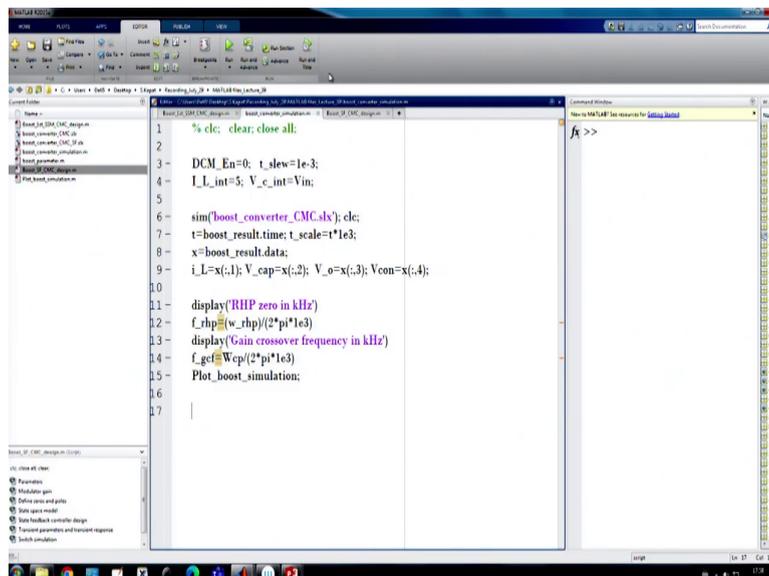
$$f_c \approx 0.414 \times f_{\text{rhp}}$$

$$r(\omega_n) \Bigg|_{\omega_n = \omega_c} = \frac{k_t}{\omega_n} \Bigg|_{\omega_n = 0.414} = 1$$

$$\Rightarrow k_c = 0.414 \times \frac{2\omega_c}{R(1-D)}$$


So, our omega that means our omega c will be 0.414 times omega rhp and we want to check. So, we want to check in this way if I set in my MATLAB code 0.414 is my crossover times rhp 0, then am I getting 45 degree. So, I will check in the reverse way, ok. So, let us check we want to check using MATLAB matching case study.

(Refer Slide Time: 19:14)



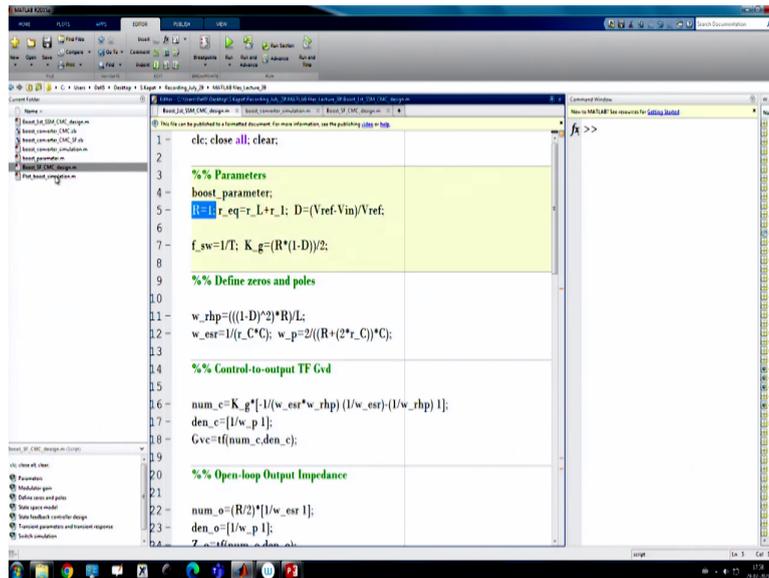
```

1 % clc; clear; close all;
2
3 DCM_En=0; t_slew=1e-3;
4 I_L_int=5; V_c_int=Vin;
5
6 sim('boost_converter_CMC.slx'); clc;
7 t=boost_result.time; t_scale=*1e3;
8 x=boost_result.data;
9 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3); Vcon=x(:,4);
10
11 display('RHP zero in kHz')
12 f_rhp=(w_rhp)/(2*pi*1e3)
13 display('Gain crossover frequency in kHz')
14 f_gc=(Wcp)/(2*pi*1e3)
15 Plot_boost_simulation;
16
17

```

So, let us go to the MATLAB here.

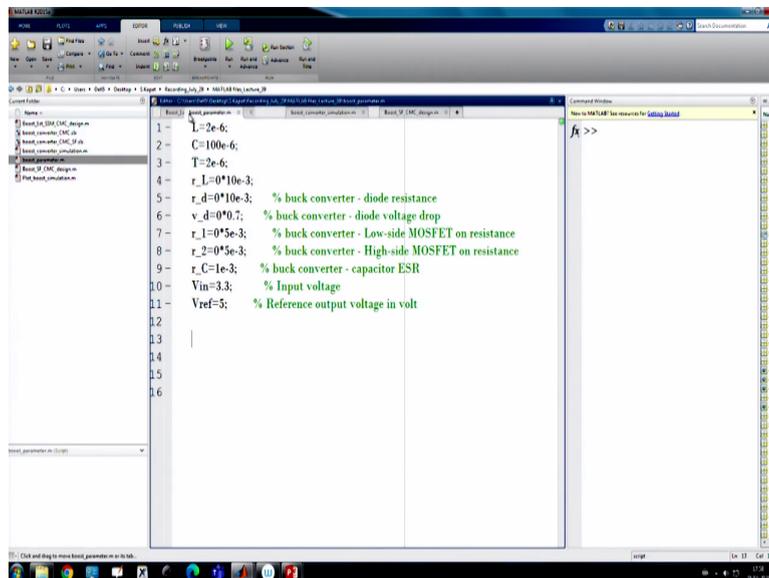
(Refer Slide Time: 19:17)



```
1 = clc; close all; clear;
2
3 %% Parameters
4 boost_parameter;
5 [r,eq=r_L+r_1; D=(Vref-Vin)/Vref;
6
7 f_sw=1/T; K_g=(R*(1-D))/2;
8
9 %% Define zeros and poles
10
11 w_rhp=((1-D)^2)*R/L;
12 w_esr=1/(r_c*C); w_p=2/((R+(2*r_c)*C));
13
14 %% Control-to-output TF Gvd
15
16 num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr) (1/w_rhp) 1];
17 den_c=[1/w_p 1];
18 Gvc=tf(num_c,den_c);
19
20 %% Open loop Output Impedance
21
22 num_o=(R/2)*[1/w_esr 1];
23 den_o=[1/w_p 1];
24 Z_o=tf(num_o,den_o);
```

We are talking about the boost converter design. So, all these model parameters load resistance is 1 ohm.

(Refer Slide Time: 19:26)



```
1 L=2e-6;
2 C=100e-6;
3 T=2e-6;
4 r_L=0*10e-3;
5 r_d=0*10e-3; % buck converter - diode resistance
6 v_d=0*0.7; % buck converter - diode voltage drop
7 r_1=0*5e-3; % buck converter - Low-side MOSFET on resistance
8 r_2=0*5e-3; % buck converter - High-side MOSFET on resistance
9 r_c=1e-3; % buck converter - capacitor ESR
10 Vin=3.3; % Input voltage
11 Vref=5; % Reference output voltage in volt
12
13
14
15
16
```

And if you go to the parameter file, I have taken you know the input voltage to 3.3 and output is 5 volt, ok. So, I can set it here.

(Refer Slide Time: 19:36)

```

1 clc; close all; clear;
2
3 %% Parameters
4 boost_parameter;
5 R=1; r_eq=r_L+r_1; D=(Vref-Vin)/Vref;
6
7 f_sw=1/T; K_g=(R*(1-D))/2;
8
9 %% Define zeros and poles
10
11 w_rhp=((1-D)^2)*R/L;
12 w_esr=1/(r_C*C); w_p=2/((R+(2*r_C)*C));
13
14 %% Control-to-output TF Gvd
15
16 num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr) (1/w_rhp) 1];
17 den_c=[1/w_p 1];
18 Gvc=tf(num_c,den_c);
19
20 %% Open-loop Output Impedance
21
22 num_o=(R/2)*[1/w_esr 1];
23 den_o=[1/w_p 1];
24 Z_o=tf(num_o,den_o);

```

Now, here the switching frequency is 500 kilohertz rhp 0 expression, everything is given.

(Refer Slide Time: 19:42)

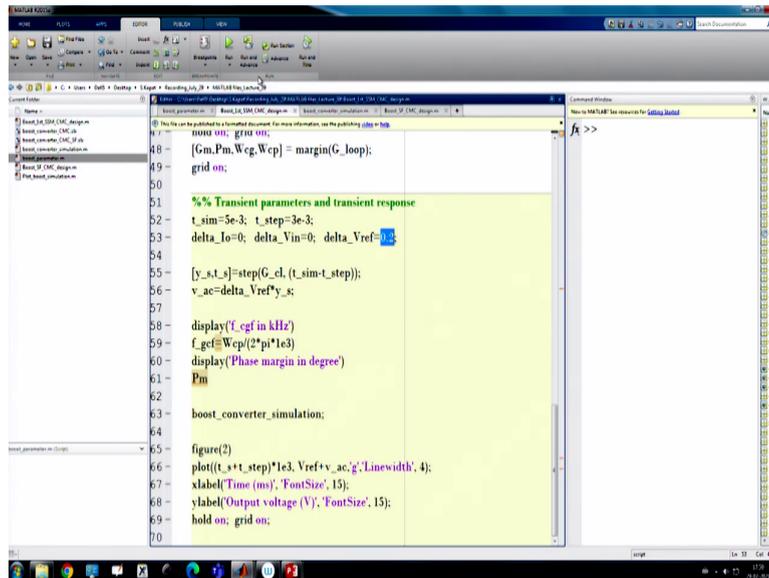
```

19
20 %% Open-loop Output Impedance
21
22 num_o=(R/2)*[1/w_esr 1];
23 den_o=[1/w_p 1];
24 Z_o=tf(num_o,den_o);
25
26 %% Design of Type-II Compensator based on Bandwidth
27
28 p=input('Select BW fraction of f_rhp ');
29 theta_rad=atan(2*p/(1-p^2));
30 theta=rad2deg(theta_rad);
31 PM=90-theta; w_c=p*w_rhp;
32 theta=(90-PM); theta_rad=deg2rad(theta);
33 K_c=(p*w_rhp)/(K_g); w_cp=w_rhp;
34 num_con=[K_c*den_c];
35 den_con=[1/w_cp 1 0];
36 Gc=tf(num_con,den_con);
37
38 %% Loop gain and closed-loop TFs
39 G_loop=Gvc*Gc; %% Loop gain
40
41 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
42 C_cl=C_loop/(1+C_loop); %% Closed-loop TF

```

Now, this will be the design, it will ask for all these you know what fraction of rhp 0 we are going to set and it will match.

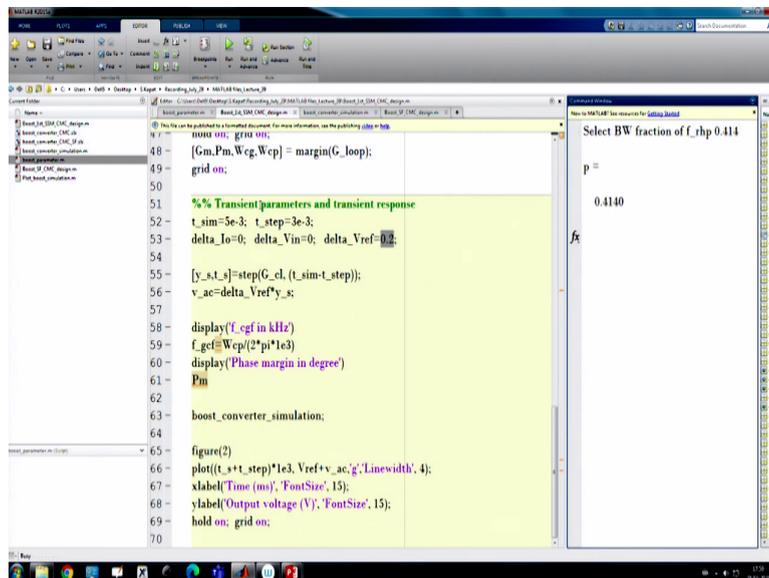
(Refer Slide Time: 19:50)



```
48- [Gm,Pm,Wcg,Wcp] = margin(G_loop);
49- grid on;
50-
51- %% Transient parameters and transient response
52- t_sim=5e-3; t_step=3e-3;
53- delta_Io=0; delta_Vin=0; delta_Vref=0.2;
54-
55- [y_s,t_s]=step(G_cl, (t_sim-t_step));
56- v_ac=delta_Vref*y_s;
57-
58- display(f_cg in kHz)
59- f_gc=(Wcp/(2*pi*1e3))
60- display('Phase margin in degree')
61- Pm
62-
63- boost_converter_simulation;
64-
65- figure(2)
66- plot((t_s+t_step)*1e3, Vref+v_ac,'g', 'LineWidth', 4);
67- xlabel('Time (ms)', 'FontSize', 15);
68- ylabel('Output voltage (V)', 'FontSize', 15);
69- hold on; grid on;
70-
```

The reference transient response I have applied a reference transient of 0.2 volt. So, it will change from 5 to point 5.2 at 3 millisecond. Let us run it.

(Refer Slide Time: 20:01)



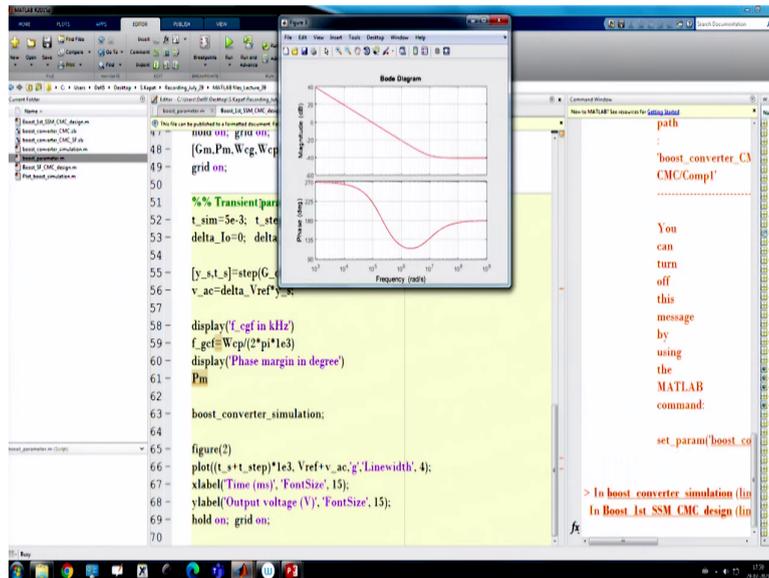
```
Select BW fraction of f_rhp 0.414
```

```
p =
0.4140
```

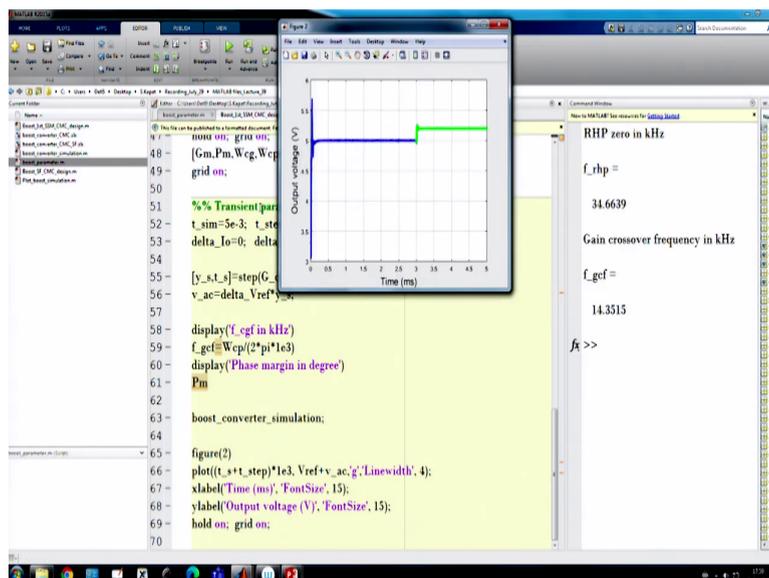
```
fz
```

So, I am select the fraction of rhp 0 that is my bandwidth. So, let us say 414, ok. If I do it, then my rhp 0.

(Refer Slide Time: 20:10)

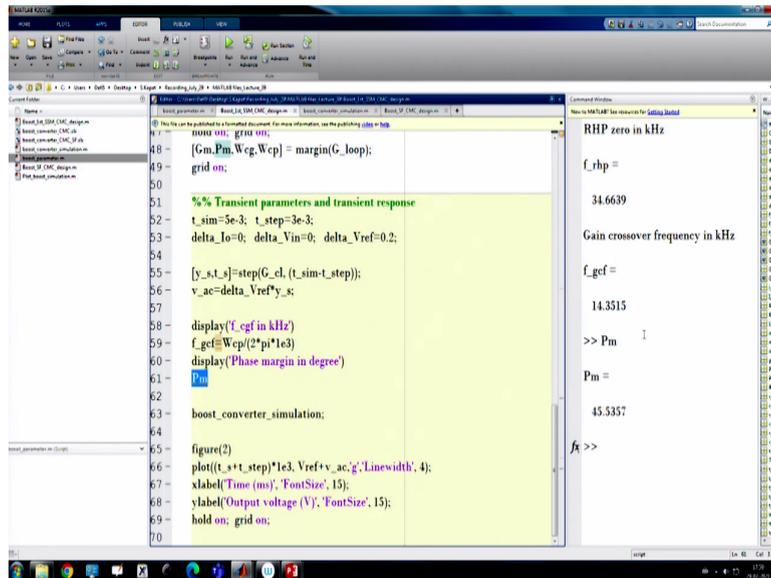


(Refer Slide Time: 20:11)



So, what is my phase margin? So, let us go to my phase margin. So, let us see what is my phase margin that I have obtained.

(Refer Slide Time: 20:23)



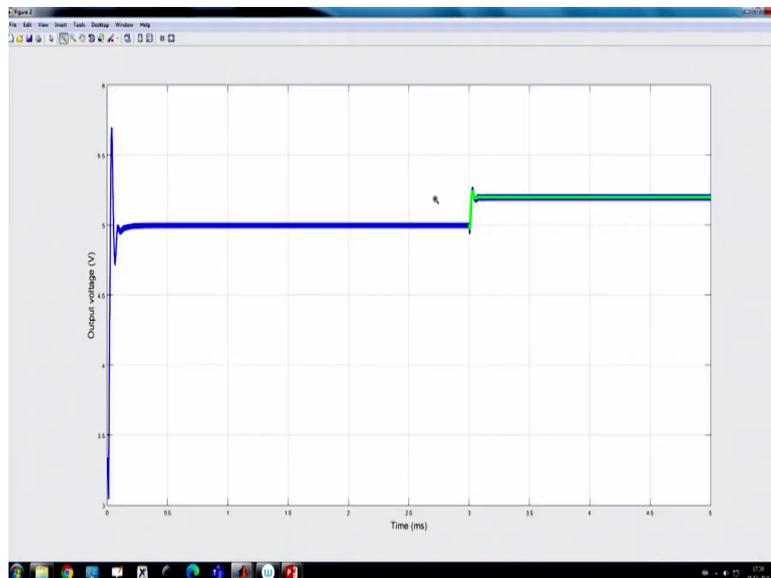
```
48 = [Gm,Pm,Wcg,Wcp] = margin(G_loop);
49 grid on;
50
51 %% Transient parameters and transient response
52 t_sim=5e-3; t_step=3e-3;
53 delta_Io=0; delta_Vin=0; delta_Vref=0.2;
54
55 [y_s,t_s]=step(G_cl, (t_sim-t_step));
56 v_ac=delta_Vref*y_s;
57
58 display(f_gcf in kHz)
59 f_gcf=Wcg/(2*pi*1e3)
60 display('Phase margin in degree')
61 Pm
62
63 boost_converter_simulation;
64
65 figure(2)
66 plot((t_s+t_step)*1e3, Vref+v_ac,'g','Linewidth', 4);
67 xlabel('Time (ms)', 'FontSize', 15);
68 ylabel('Output voltage (V)', 'FontSize', 15);
69 hold on; grid on;
70
```

Command Window Output:

```
RHP zero in kHz
f_rhp =
34.6639
Gain crossover frequency in kHz
f_gcf =
14.3515
>> Pm
Pm =
45.5357
fx >>
```

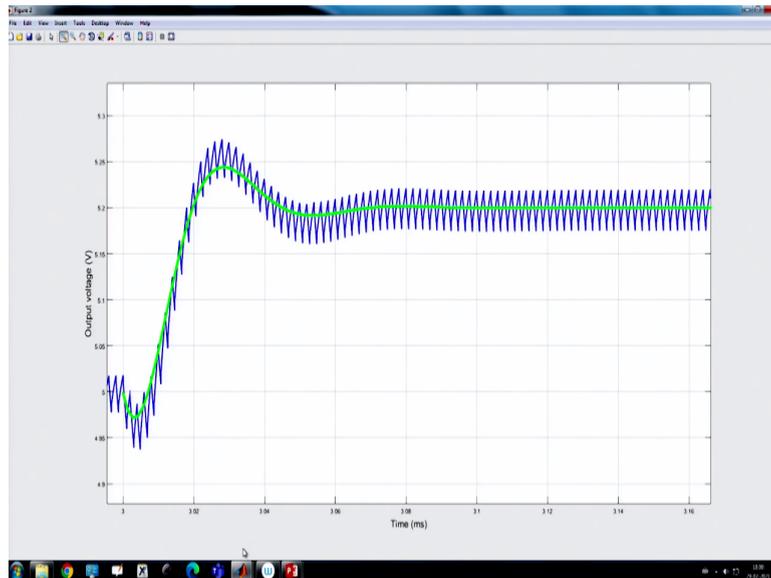
So, it is 45.5, and we set 45 degrees because we took an approximation 0.414, but actually it is an irrational number, right?

(Refer Slide Time: 20:36)



Now, if you go to the actual response and you will see that at 45 degree phase margin, this is my response of the converter. This is a reference tangent.

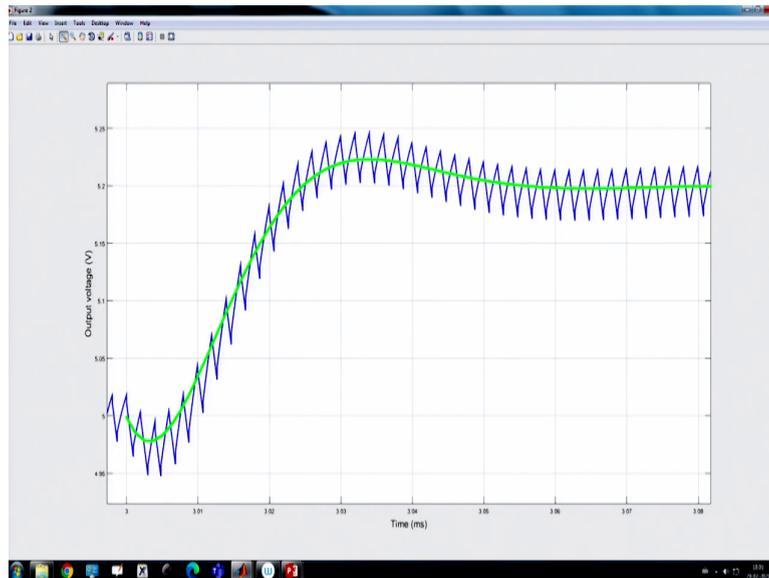
(Refer Slide Time: 20:40)



So, it is changing from 5 volt to 5.2 volt and you see the green one is coming from the model which is the from AC simulation. And then, we added the offset that is my operating point and then we match with the actual switch simulation. They are matching more or less closely, not very accurately, but that is reasonable now if we reduce. That means, if we know you make it one-third, it was.

So that means we are making one-third and we are finding what is my phase margin. Now, we are increasing phase margin. Earlier it was 45 degree now it is like a 53 degree. Then you will find the response is somewhat better because overshoot has reduced ok, but you see there is an undershoot behavior.

(Refer Slide Time: 21:34)



That means the initially instead of voltage going up, it is first going down and then going up. So, this undershoot behavior is due to the right appearance 0 because the non-minimum phase behavior it will result in some initial undershoot and then it will rise. And this non-minimum behavior will be more prominent once your load current increases right, ok. So, and your bandwidth will be really really very small ok. So, this is the design and we match the model and they are matching quite nicely.

(Refer Slide Time: 22:08)

Design Constraint – Phase Margin vs Crossover Frequency

- Higher gain crossover freq. (gcf) – poor phase margin (PM)
- Poor PM – higher voltage overshoot due to poor damping
- Higher phase margin – poor bandwidth – slower transient response
- Design thumb rule – $w_c = \min\left(\frac{w_{rhp}}{3}, \frac{2\pi f_{sw}}{10}\right)$

Can we further increase w_c with reduced overshoot/undershoot ?



So, that means we saw the model matching. Now here is the design constant. That means if I want to increase the crossover frequency.

(Refer Slide Time: 22:17)

```

1  clc; close all; clear;
2
3  %% Parameters
4  boost_parameter;
5  R=1; r_eq=r_L+r_l1; D=(Vref-Vin)/Vref;
6
7  f_sw=1/T; K_g=(R*(1-D))/2;
8
9  %% Define zeros and poles
10
11  w_rhp=((1-D)^2)*R/L;
12  w_esr=1/(r_C*C); w_p=2/((R+(2*r_C)*C));
13
14  %% Control-to-output TF Gvd
15
16  num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr)-(1/w_rhp) 1];
17  den_c=[1/w_p 1];
18  Gvc=tf(num_c,den_c);
19
20  %% Open-loop Output Impedance
21
22  num_o=(R/2)*[1/w_esr 1];
23  den_o=[1/w_p 1];

```

Let us consider again the MATLAB here. I want to first consider that if I use my phase margin to be, that means let us say I want to achieve one-third rhp 0 ok. I want to achieve one-third first and it is we can achieve this.

(Refer Slide Time: 22:34)

```

1  clc; close all; clear;
2
3  %% Parameters
4  boost_parameter;
5  R=1; r_eq=r_L+r_l1; D=(Vref-Vin)/Vref;
6
7  f_sw=1/T; K_g=(R*(1-D))/2;
8
9  %% Define zeros and poles
10
11  w_rhp=((1-D)^2)*R/L;
12  w_esr=1/(r_C*C); w_p=2/((R+(2*r_C)*C));
13
14  %% Control-to-output TF Gvd
15
16  num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr)-(1/w_rhp) 1];
17  den_c=[1/w_p 1];
18  Gvc=tf(num_c,den_c);
19
20  %% Open-loop Output Impedance
21
22  num_o=(R/2)*[1/w_esr 1];
23  den_o=[1/w_p 1];

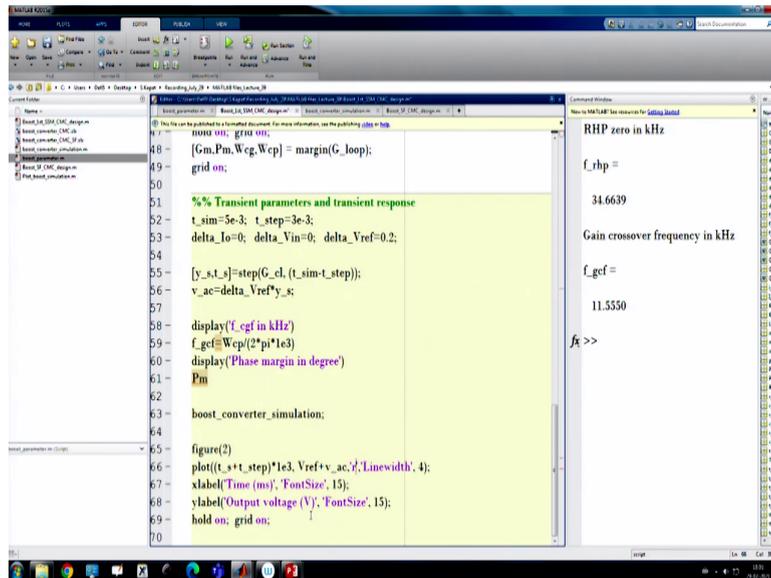
```

loop(s). To see more details about the loops use the command Simulink.BlockDiagram.getAlgebraic or the command line Simulink debugger by typing "aldebug boost_converter_CMC" in the MATLAB command window. To eliminate this message, set the Algebraic loop option in the Diagnostics page of the Simulation Parameters Dialog to "None"

> In boost_converter_simulation (lin In Boost_1st_SSM_CMC_design (lin Found algebraic loop containing: 'boost_converter_CMC/Boost Conv; 'boost_converter_CMC/Boost Conv; 'boost_converter_CMC/Boost Conv; 'boost_converter_CMC/load; 'boost_converter_CMC/Sum' (algeb

Now, I want to keep this and change the color. So, here I am changing the color.

(Refer Slide Time: 22:42)



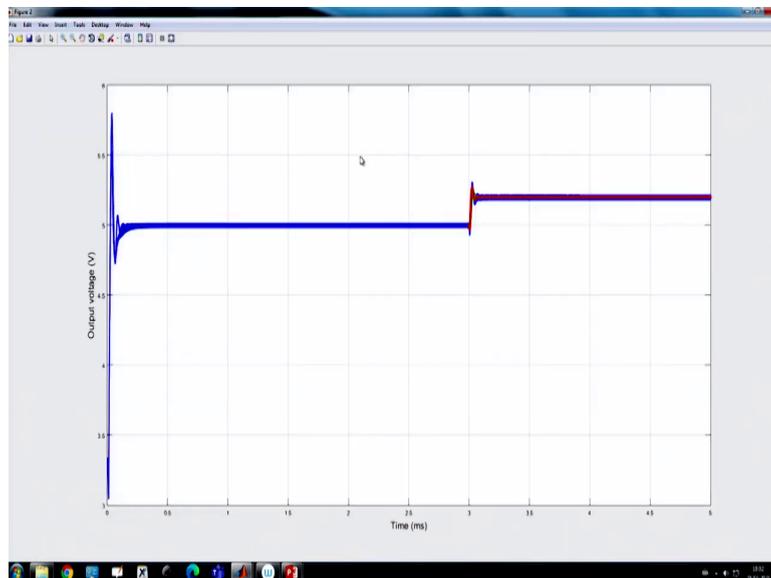
```
clear all; close all; clc;
% Boost converter simulation
% Transient parameters and transient response
t_sim=5e-3; t_step=3e-3;
delta_Io=0; delta_Vin=0; delta_Vref=0.2;
[y_s,t_s]=step(G_cl, (t_sim-t_step));
v_ac=delta_Vref*y_s;
display(f_gcf in kHz)
f_gcf=@Wcp/(2*pi*1e3)
display(Phase margin in degree)
Pm
boost_converter_simulation;
figure(2)
plot(t_s+t_step)*1e3, Vref+v_ac,'Linewidth', 4);
xlabel('Time (ms)', 'FontSize', 15);
ylabel('Output voltage (V)', 'FontSize', 15);
hold on; grid on;
```

Command Window Output:

```
RHP zero in kHz
f_rhp =
    34.6639
Gain crossover frequency in kHz
f_gcf =
    11.5550
fx >>
```

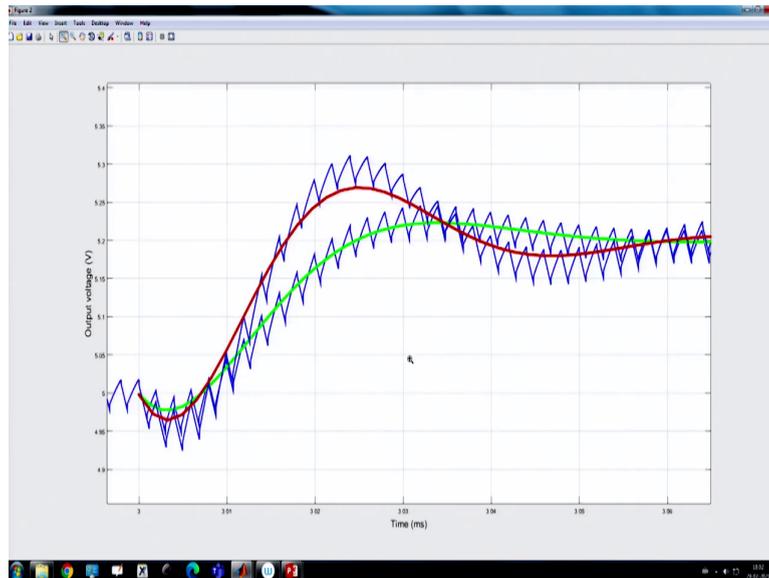
So, green I am using let us say red, ok and let us run it. So, here I want to increase to let us say one-half instead of one -third. I want to go for half of the switching frequency.

(Refer Slide Time: 23:03)



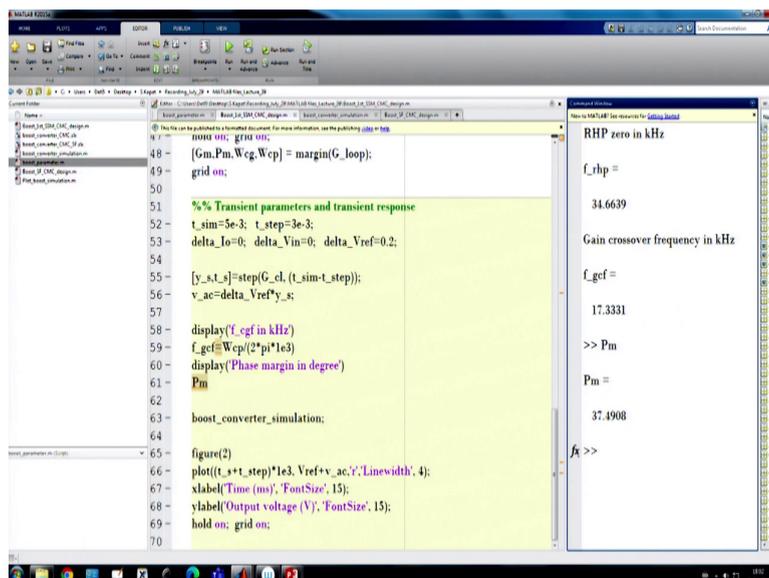
And let us see what happen.

(Refer Slide Time: 23:05)



So, if you see in the first case the green one, the matching is perfectly fine with the actual switch simulation because if you particularly look at this response, the green is following perfectly, but once you try to push the bandwidth high first of all there is a model mismatch is coming. Secondly, since the phase margin is poor because in earlier design we saw 53 degree phase margin.

(Refer Slide Time: 23:31)



Here we see the phase margin is 37.5 which has reduced significantly and lower phase margin will lead to higher overshoot right because the phase margin is inversely proportional

to or it is phase margin is you know some sense it is proportional to the damping ratio. So, if the poor phase margin means you have a smaller damping ratio, so this is a comparative result.

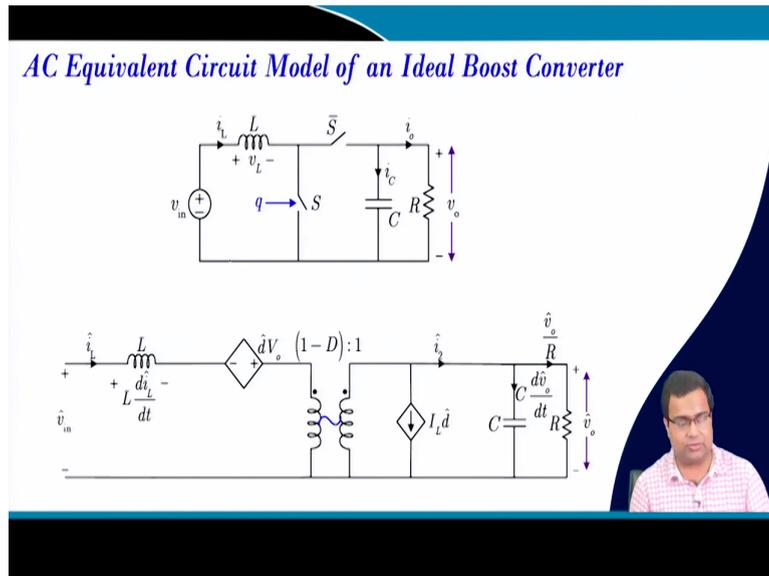
So, the red one is we want to increase the bandwidth, but the cost is the penalty is your response become overshoot become higher and it is something not acceptable, very high ok and maybe your settling time will be more or less same, but the overshoot is something which is unacceptable.

Another thing you may have some more additional right appearance 0 because you are trying to push the bandwidth ok. So, that means what we are trying to come there is a trade up higher gain crossover frequency. If you want to achieve poor phase margin, poor phase margin will lead to higher voltage overshoots because of poor damping.

And if your higher phase margin if you want to set high phase margin, then because you want to make the response like overshoot, you want to reduce, then it will lead to poor bandwidth and the slower transient response. And the design thumb rule is that generally the crossover frequency is set to the minimum of one-third of the ω_{hp} 0 frequency comma one-tenth of the switching frequency it is 2π . That means, it is in radian I am talking about.

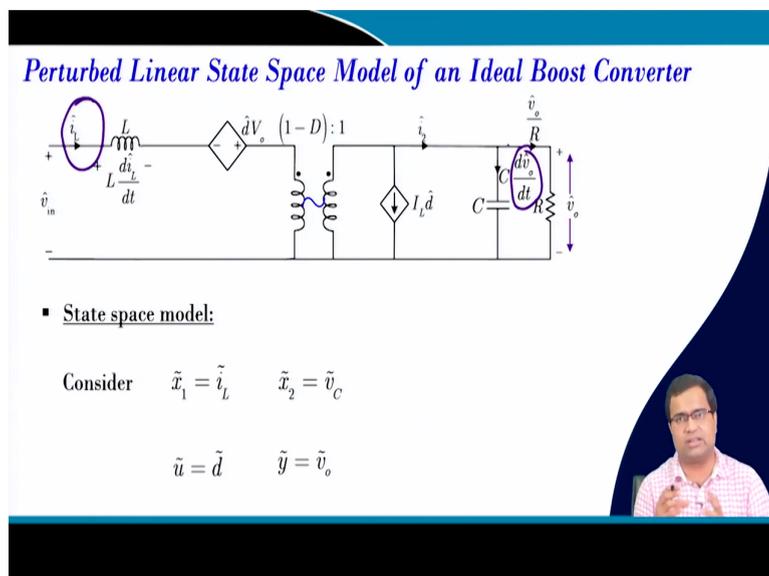
So, it is $2\pi f_{sw}$ by 10. That means, one-tenth if because if you take light load, then ω_{hp} 0 will be further right side then your model validity will come in terms of one-tenth of the switching frequency beyond which it will not work because that we have discussed earlier. But this tradeoff will not allow us to extend the bandwidth or response because if we want to go further up, then our overshoot is increasing right if you want to speed up. So, now, is there any if, is there any way to overcome this?

(Refer Slide Time: 25:41)



Let us go to the AC equivalent circuit of a boost converter, an ideal boost converter that we have already discussed. So, this is the AC equivalent circuit.

(Refer Slide Time: 25:49)



We want to achieve this because we want to design the current mode control using state feedback approach state feedback approach. So, for that we want to write the state space model. So, from this equivalent circuit I can take that x 1 perturbation is my inductor, current perturbation and x 2 perturbation is my capacitor voltage or the output voltage. In this case

because here if we take ideal the capacitor, voltage is same as the output voltage, but otherwise capacitor voltage and output voltage are different if there is an esr.

(Refer Slide Time: 26:21)

Perturbed Linear State Space Model of an Ideal Boost Converter

$$L \frac{d\tilde{i}_L}{dt} = \tilde{v}_{in} + V_o \tilde{d} - (1-D) \tilde{v}_o$$

$$\dot{\tilde{x}}_1 = \frac{[-(1-D)\tilde{x}_2 + V_o \tilde{u} + \tilde{v}_{in}]}{L}$$

Then a perturbed linear model: if you write the inductor current perturbed dynamics from this equation, from this circuit, then you can write x 1 perturb dot will be a function of this.

(Refer Slide Time: 26:33)

Perturbed Linear State Space Model of an Ideal Boost Converter

$$C \frac{d\tilde{v}_o}{dt} = (1-D)\tilde{i}_L - I_L \tilde{d} - \frac{\tilde{v}_o}{R}$$

$$\dot{\tilde{x}}_2 = \frac{[(1-D)\tilde{x}_1 - I_L \tilde{u} - \frac{\tilde{x}_2}{R}]}{C}$$

Similarly, if you write in the perturb output voltage dynamics, then you can obtain the perturbed dynamics of the x_2 perturb, which corresponds to the capacitor voltage. Here, it is the output voltage perturbation.

(Refer Slide Time: 26:44)

Perturbed Linear State Space Model of an Ideal Boost Converter

$$\dot{\tilde{x}}_1 = \frac{[-(1-D)\tilde{x}_2 + V_o\tilde{u} + \tilde{v}_{in}]}{L}$$

$$\dot{\tilde{x}}_2 = \frac{[(1-D)\tilde{x}_1 - I_L\tilde{u} - \frac{\tilde{x}_2}{R}]}{C}$$

Then we can write down the two state equation perturb because we are talking about the linear state space model and for the switching converter, after applying averaging and linearization, we are getting the perturb model which looks like a linear system.

(Refer Slide Time: 27:00)

Augmented State-space Model with Voltage Controller : Boost Converter

$$\tilde{x}_3 = \int_0^t \tilde{v}_e d\tau \quad \text{where } \tilde{v}_e = \tilde{v}_{ref} - \tilde{v}_o$$

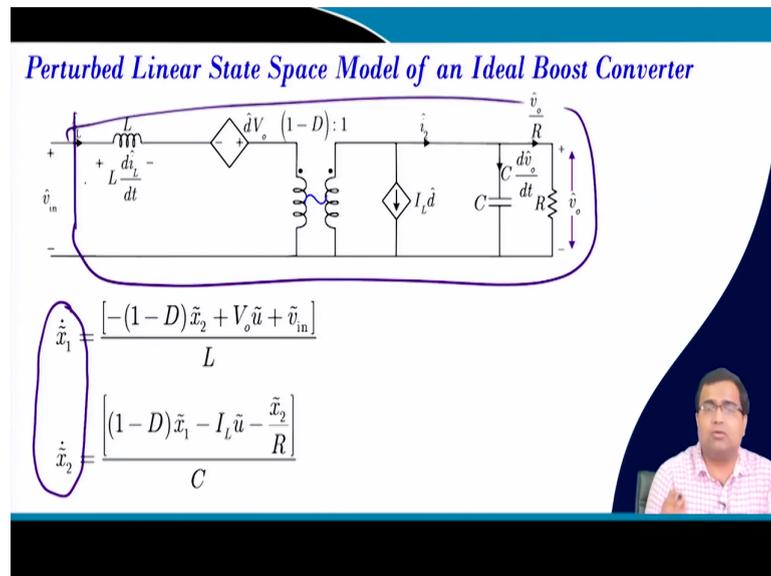
$$\dot{\tilde{x}}_1 = \frac{[-(1-D)\tilde{x}_2 + V_o\tilde{u} + \tilde{v}_{in}]}{L}$$

$$\dot{\tilde{x}}_2 = \frac{[(1-D)\tilde{x}_1 - I_L\tilde{u} - \frac{\tilde{x}_2}{R}]}{C}$$

$$\dot{\tilde{x}}_3 = \tilde{v}_e = \tilde{v}_{ref} - \tilde{v}_o = \tilde{v}_{ref} - \tilde{x}_2$$

Then we are now considering a PI controller.

(Refer Slide Time: 27:07)



So, the earlier two states or the state of the original system of the boost converter, but now in the output feedback of the current loop we are using a PI controller, right. So, the PI controller K_p into error voltage and K_i into integrator of the error voltage integration of the error voltage.

And the output of the integrator we are taking the x_3 that is another state. So, we can write now. The third state is this and we can write down first state we have already discussed, second state we have already discussed. And the third one is the augmented state, which is the derivative of the third state is nothing but the error voltage. And that is nothing but v minus x_2 perturb.

(Refer Slide Time: 27:49)

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} \quad \dot{\tilde{x}} = \begin{bmatrix} 0 & -(1-D) & 0 \\ (1-D) & -1 & 0 \\ C & RC & -1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} \frac{V_o}{L} \\ -\frac{I_v}{C} \\ 0 \end{bmatrix} \tilde{u} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tilde{v}_{in} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tilde{v}_{ref}$$

$$\Rightarrow \dot{\tilde{x}} = A \tilde{x} + B \tilde{u} + E_1 \tilde{v}_{in} + E_r \tilde{v}_{ref}$$

And if you write the perturb state space model, then you will obtain this perturb model and you know if you go back in the last equation, you also have a v ref perturbation term. So, if you want to write down that then you can also write down there is one more term 0 0 1 into V ref perturbation, ok. Then here you can write E you know some E r into V ref perturbation and this is my E r, this is my E 1, this is my B, and this is my A, ok.

(Refer Slide Time: 28:36)

$$\tilde{d} = F_m (-k_{cp} \tilde{x}_1 - k_{vp} \tilde{x}_2 - k_{vi} \tilde{x}_3) \triangleq -K \tilde{x} \quad F_m = \frac{1}{V_m}$$

$$\dot{\tilde{x}} = A \tilde{x} + B \tilde{u} + E \tilde{v}_{in} = A \tilde{x} - BK \tilde{x} + E \tilde{v}_{in} = (A - BK) \tilde{x} + E \tilde{v}_{in} + E_r \tilde{v}_{ref}$$

$$\Rightarrow \dot{\tilde{x}} = A_{cl} \tilde{x} + E \tilde{v}_{in} + E_r \tilde{v}_{ref}$$

And $\tilde{v}_o = C \tilde{x}$

Closed-loop Characteristic Equation

$$\Delta(s) = |(sI - A_{cl})|$$

So, now if we take the duty ratio, so it is the combination of modulator gain. Now, we are taking the current loop gain k c because you know if it if we consider the current loop gain in

current mode control also, we can consider a current loop gain that is this, and this is the proportional gain of the voltage controller. And this is an integral gain of the voltage controller.

And we can write in terms of state feedback form and what is my F m? F m I can simply take 1 by V m. That means I can use a PWM modulator. If you use a state feedback control, the whole error will actually go because if you go to lecture number 18, we have discussed in detail the implementation of state space and state feedback control. So, modulator gain is 1 by V m because we will consider a ramp ok. So, here we also have an additional term that we have discussed.

So, with this we will have E r into V ref perturbation. Here also we will consider the additional term E r into V ref perturbation and it is E r into V ref perturbation, ok. So, this is my closed loop equation, right and after closed loop again. You can add this term with this as well as this both. This is the output equation, then the closed-loop characteristic equation will be sI A closed loop. Now, coming back to how to place closed-loop poles. So, it is a third order system including the augmented state.

(Refer Slide Time: 30:18)

How to Place Closed Loop Poles?

The desired closed loop poles are selected by considering,

- (1) The crossover frequency $\omega_c = k \times \omega_{rhp}$ where $k < 1$
- (2) The pole due to output capacitor which is set to crossover ω_c
- (3) The pole due to inductor is kept 10 times faster for time scale separation
- (4) Zero of the PI controller – to be decided

$$\Delta_{cl}(s) = (s + a_1)(s + \omega_c)(s + 10\omega_c)$$

Handwritten notes:
 $k_i = a_1$
 $K_p + \frac{k_i}{s}$
 $= \frac{(K_p s + k_i)}{s}$
 $= K_i \times \frac{(s + \frac{s}{\omega_c})}{s}$

S. Kapat and P.T. Krein, "A Tutorial and Review Discussion ..." IEEE Open J. Power Electron. 2020.

So, the desired closed-loop pole can be considered one. First of all, we have to choose the crossover frequency smaller than rhp 0 because we discussed even in output feedback, we cannot go up to rhp 0. We have to reduce, but we want to increase this k factor compared to the output feedback response. So, the pole due to output capacitor, which is set to omega c

that is one pole. The other pole due to the inductor 10 times imagine in our output feedback approach. We have canceled the planned pole with a controller 0. That means, in the loop transfer function we lost the control over the voltage pole because we have cancelled the pole, right and that is one of the drawbacks of the output feedback approach because by because of pole zero cancellation, the system may not be controllable.

But here we are not cancelling anything, but we are considering the capacitor pole which is set to omega c and inductor pole we are taking 10 time faster because typically the thumb rule inner current loop has to be faster than the zero of the PI controller we are to be decided. That means, if I take a PI controller for regular current mode control, so we will generally take k p plus k i by s right. So, this will be simply k p into s plus k i by s.

So, I am talking about this 0. So, that mean this you can write if you take you know k i common I can write this like this 1 plus s by controller 0 by s. So, this approach was discussed because in this paper and which is this reference paper, we are using almost for the maximum part of this course.

(Refer Slide Time: 32:15)

Design Case Study using State Feedback Approach

The desired closed loop poles are selected by considering,

- (1) The crossover frequency $\omega_c = k \times \omega_{rhp}$ with $k < 0.5$ $k = 0.5$
- (2) The pole due to output capacitor which is set to crossover ω_c
- (3) The pole due to inductor is kept 10 times faster for time scale separation
- (4) Zero of the PI controller – ratio of k_{vi} and k_{vp}

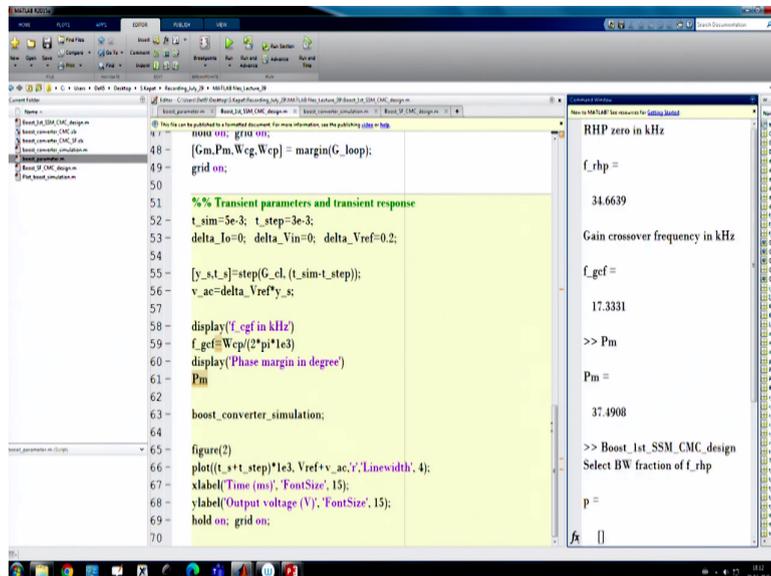
$$\Delta_{cl}(s) = (s + a_1)(s + \omega_c)(s + 10\omega_c); \quad a_1 = \frac{k_{vi}}{k_{vp}} = 10000$$

S. Kapat and P.T. Krein, "A Tutorial and Review Discussion ..." IEEE Open J. Power Electron. 2020.

Next in this design. So, first we are taking a case study. Let us say we want to take k to be 0.5 if we choose, ok or we can choose k to be 0.3 one-third or so or whatever. Now, we can set the capacitor pole inductor pole 10 times faster and the voltage controller of the PI let us say a 1 which is k integral gain of the voltage controller by proportional gain. So, here we are

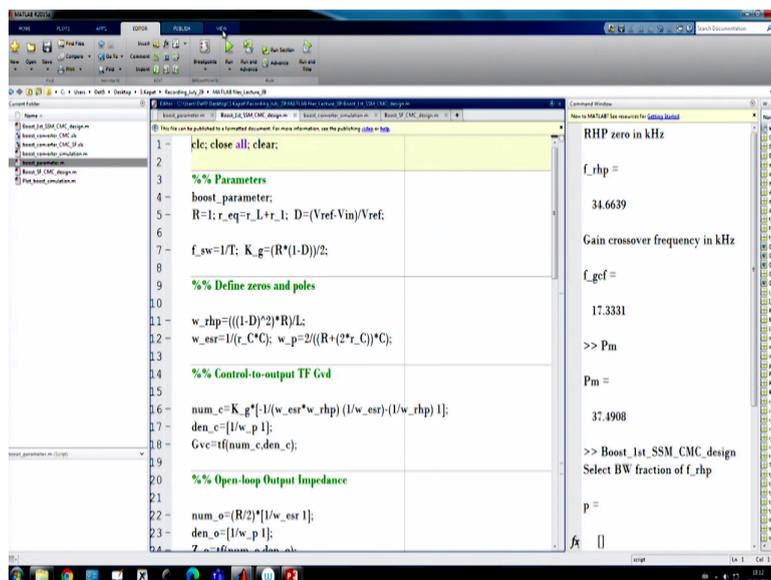
taking let us say some value. That means, the case study we are considering we are taking to be let us say some around 10000, ok. So, let us go to and take a case study.

(Refer Slide Time: 32:57)



So, here now we will first design using our state output feedback control and we want to first achieve ok, we want to first achieve here.

(Refer Slide Time: 33:16)



(Refer Slide Time: 33:20)

```

1 clc; close all; clear;
2
3 %% Parameters
4 boost_parameter;
5 R=1; r_eq=r_L+r_1; D=(Vref-Vin)/Vref;
6
7 f_sw=1/T; K_g=(R*(1-D))/2;
8
9 %% Define zeros and poles
10
11 w_rhp=((1-D)^2)*R/L;
12 w_esr=1/(r_c*C); w_p=2/((R+(2*r_c)*C));
13
14 %% Control-to-output TF Gvd
15
16 num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr)-(1/w_rhp) 1];
17 den_c=[1/w_p 1];
18 Gvc=tf(num_c,den_c);
19
20 %% Open-loop Output Impedance
21
22 num_o=(R/2)*[1/w_esr 1];
23 den_o=[1/w_p 1];

```

Command Window:

```

Select BW fraction of f_rhp 1/3
p =
    0.3333
f_cgf in kHz
    11.5550
Phase margin in degree
    53.5450
fx

```

(Refer Slide Time: 33:21)

```

1 clc; close all; clear;
2
3 %% Parameters
4 boost_parameter;
5 R=1; r_eq=r_L+r_1; D=(Vref-Vin)/Vref;
6
7 f_sw=1/T; K_g=(R*(1-D))/2;
8
9 %% Define zeros and poles
10
11 w_rhp=((1-D)^2)*R/L;
12 w_esr=1/(r_c*C); w_p=2/((R+(2*r_c)*C));
13
14 %% Control-to-output TF Gvd
15
16 num_c=K_g*[1/(w_esr*w_rhp) (1/w_esr)-(1/w_rhp) 1];
17 den_c=[1/w_p 1];
18 Gvc=tf(num_c,den_c);
19
20 %% Open-loop Output Impedance
21
22 num_o=(R/2)*[1/w_esr 1];
23 den_o=[1/w_p 1];

```

Command Window:

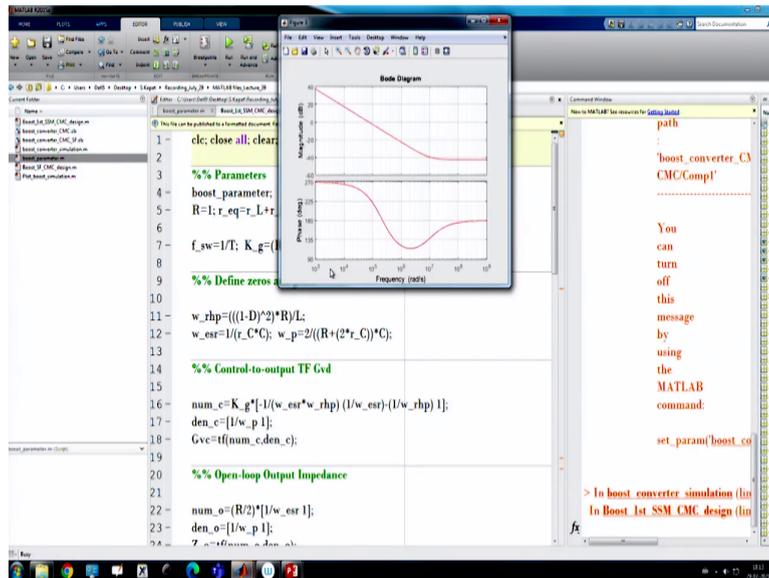
```

loop(s)). To see more
details about the loops
use the command
Simulink.BlockDiagram.getAlgebraic
or the command line
Simulink.debugger by
typing 'sdebug
boost_converter_CMC' in
the MATLAB command window.
To eliminate this message,
set the Algebraic loop
option in the Diagnostics
page of the Simulation
Parameters Dialog to
'None'
> In boost_converter_simulation (lin
In Boost_1st_SSM_CMC_design (lin
Found algebraic loop containing:
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/load'
'boost_converter_CMC/Sum1' (algeb
fx

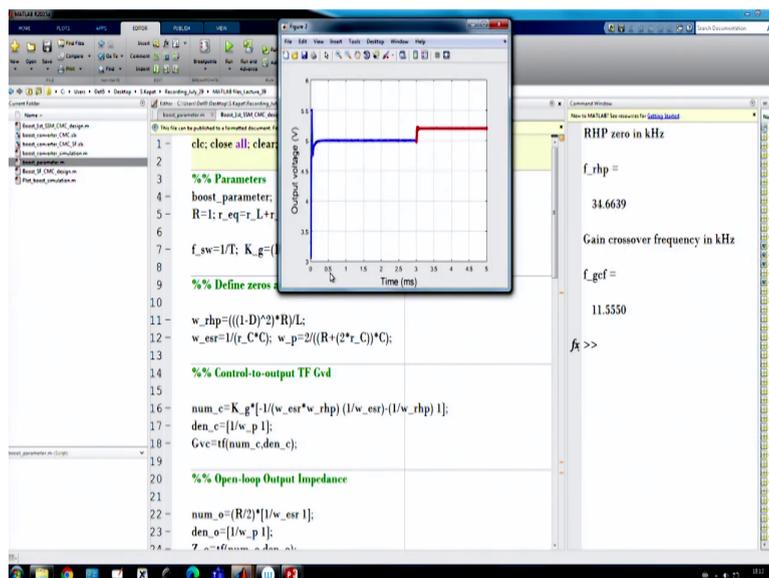
```

We want to achieve one-third of the rhp 0.

(Refer Slide Time: 33:23)



(Refer Slide Time: 33:24)



So, this is my response right.

(Refer Slide Time: 33:26)

```

%% Parameters
boost_parameter;
R=1; r_eq=r_L+r_1; D=(Vref-Vin)/Vref;

%% Modulator gain
V_m=1; Fm=1/V_m;

%% Define zeros and poles
w_rhp=((1-D)^2)*R/L; w_p=2*((R+(2*r_C))*C);
f_sw=1/T; w_sw=2*pi*f_sw;

%% State space model
a11_o=0; a12_o=-(1-D)/L; a13_o=0;
a21_o=(1-D)/C; a22_o=-1/(R*C); a23_o=0;
a31_o=0; a32_o=-1; a33_o=0;

Beq=[Vref*(Fm*L); Vref*((1-D)*Fm*R*C); 0];
Aol=[a11_o a12_o a13_o; a21_o a22_o a23_o; a31_o a32_o a33_o];
eig(Aol);
    
```

Command Window Output:

```

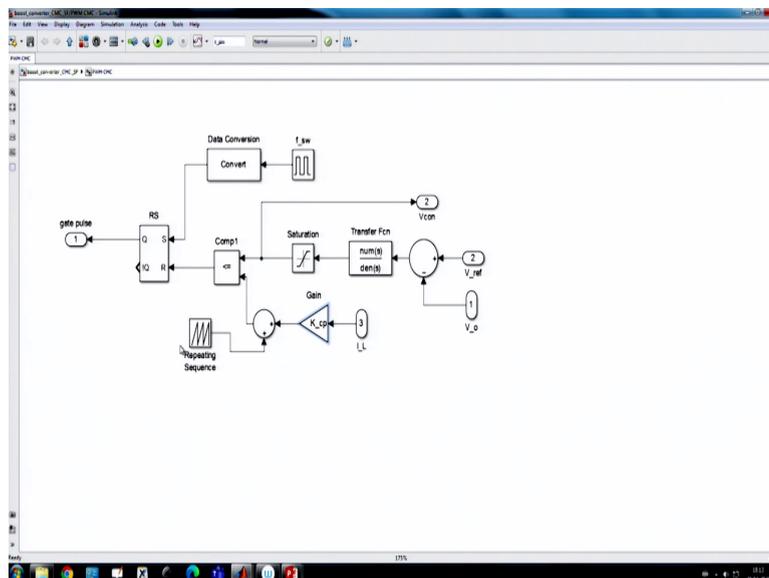
RHP zero in kHz
f_rhp =
    34.6639

Gain crossover frequency in kHz
f_gcf =
    11.5550

fx >>
    
```

Now, I am going to state feedback based design.

(Refer Slide Time: 33:30)



And if you go to the state feedback based design in the control, you know the here everything is same only I am adding a gain with the current loop and since I told that this has to be a compensating ramp because it will work like a modulator, but I am taking a very low ramp. It is just 1 voltage because your input voltage is very low. It is not very high, 1 volt, only 0 to 1 volt.

(Refer Slide Time: 34:00)

```

18 a11_o=0; a12_o=-(1-D)/L; a13_o=0;
19 a21_o=(1-D)/C; a22_o=-1/(R*C); a23_o=0;
20 a31_o=0; a32_o=-1; a33_o=0;
21
22 Beq=[Vref/(Fm*L); -Vref/((1-D)*Fm*R*C); 0];
23 Aol=[a11_o a12_o a13_o; a21_o a22_o a23_o; a31_o a32_o a33_o];
24 eig(Aol);
25
26 %% State feedback controller design
27
28 w_p1=10000; %% K_i/K_p
29 w_p2=min(0.33*w_rhp,w_sw/10);
30 w_p3=10*w_p2;
31 p_des = [-w_p1 -w_p2 -w_p3];
32
33 K_sf = place(Aol,Beq,p_des);
34 K_cp=K_sf(1);
35 K_vp=K_sf(2);
36 K_vi=K_sf(3);
37
38 num_con_SF=[K_cp K_vp K_vi];
39 den_con_SF=[1 0];
40 Ge=tf(num_con_SF,den_con_SF);

```

Command Window:

```

RHP zero in kHz
f_rhp =
34.6639
Gain crossover frequency in kHz
f_gcf =
11.5550
fx >>

```

Now, I am designing this first to obtain my first ratio. I told a 1 which is 10000. That means, K_i by K_p the other pole I am taking minimum of ω_{rhp} by 0.5. I can take let us say 0.33 because I want to compare the first or one-tenth of the switching frequency, right.

(Refer Slide Time: 34:24)

```

45
46 %% Switch simulation
47
48 DCM_En=0; t_slew=1e-3;
49 I_L_int=5; V_c_int=Vin;
50
51 sim('boost_converter_CMC_SF.slx'); clc;
52 t=boost_result.time; t_scale=*1e3;
53 x=boost_result.data;
54 i_L=x(:,1); V_o=x(:,2); V_o=x(:,3);
55
56 figure(1)
57 plot(t_scale,i_L,'LineWidth', 2); hold on; grid on;
58 xlabel('Time (ms)', 'FontSize', 15);
59 ylabel('Inductor current (A)', 'FontSize', 15);
60
61 figure(2)
62 plot(t_scale,V_o,'LineWidth', 2); hold on; grid on;
63 xlabel('Time (ms)', 'FontSize', 15);
64 ylabel('Output voltage (V)', 'FontSize', 15);
65
66
67

```

Command Window:

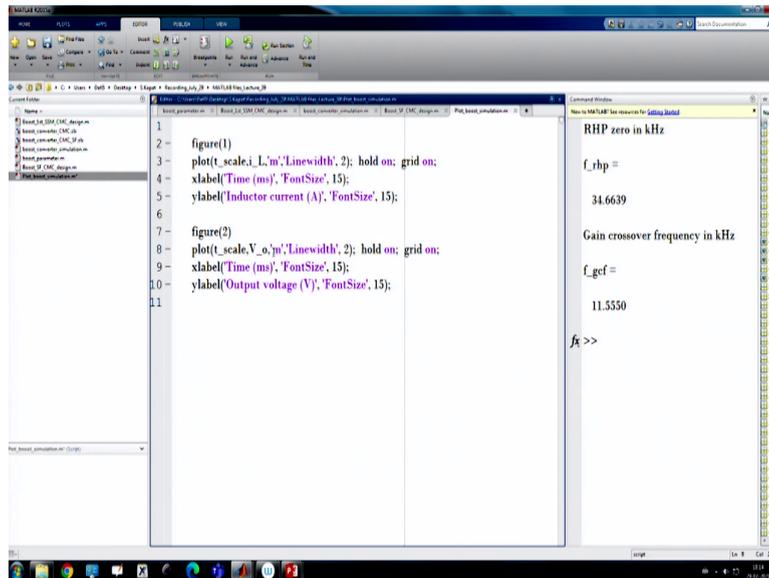
```

RHP zero in kHz
f_rhp =
34.6639
Gain crossover frequency in kHz
f_gcf =
11.5550
fx >>

```

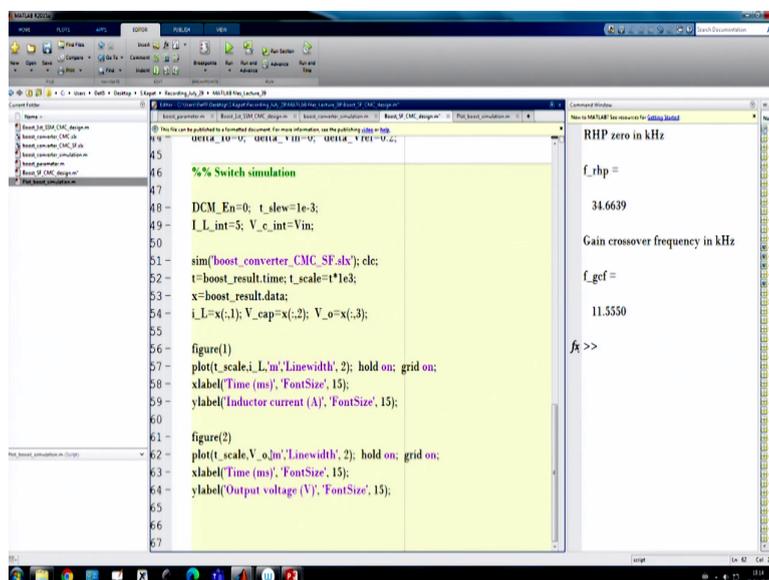
And then I want to match this response and for this case I want to use a different color.

(Refer Slide Time: 34:28)



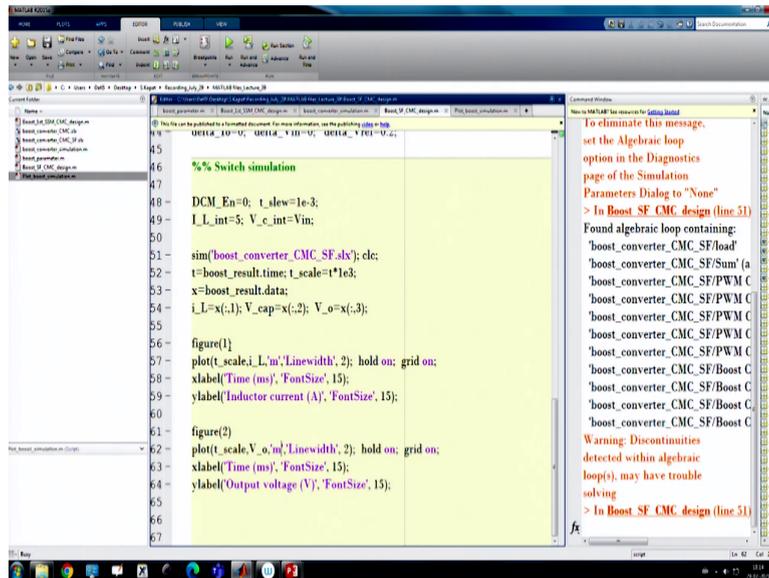
So, let us say I want to use green color, not green. I will say I will use magenta color, ok. So, because we have already drawn this, we have already obtained this. So magenta color and let us run this design. Sorry here, no need to change because we are already running it from here.

(Refer Slide Time: 34:57)



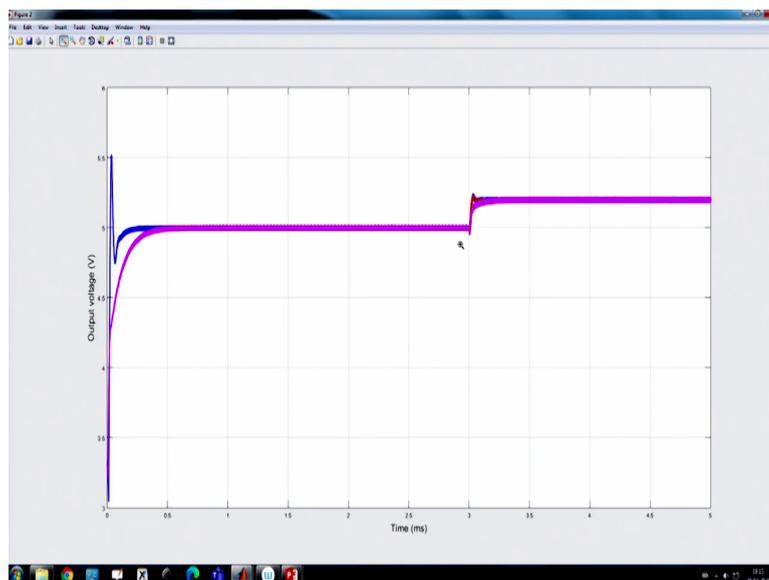
So, here I am using, I want to use magenta color, ok.

(Refer Slide Time: 35:04)



So, let us wait because we set here again one-third of them, so they are same as one-third here. In both cases, the crossover frequency is set to one-third. So, I have design by means of pole placement and then, it will give the value where in output feedback we set one-third of $\text{rhp } 0$ crossover frequency. Here also we set one-third of the $\text{rhp } 0$ and let us see what happens in the response of the converter, ok.

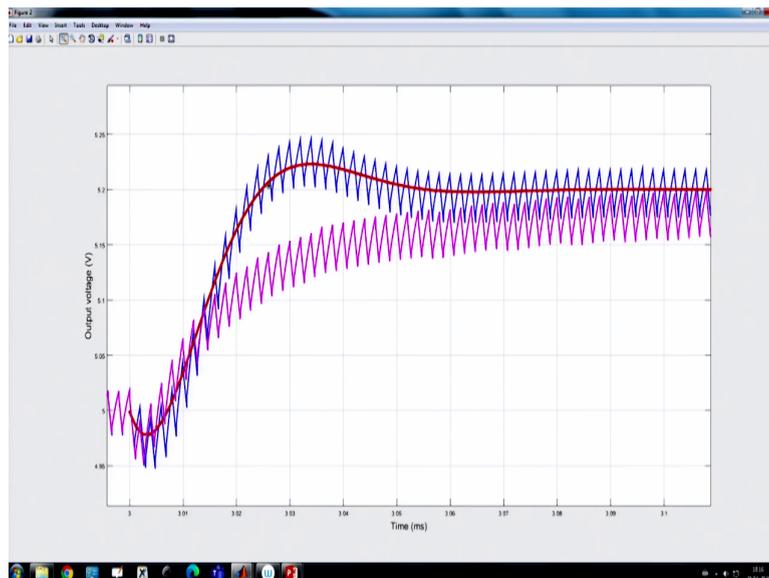
(Refer Slide Time: 35:32)



So, we will find if we, so this blue one, is the response coming from the output feedback design and that is using one-third of the $\text{rhp } 0$ and you have the red one is the stress coming

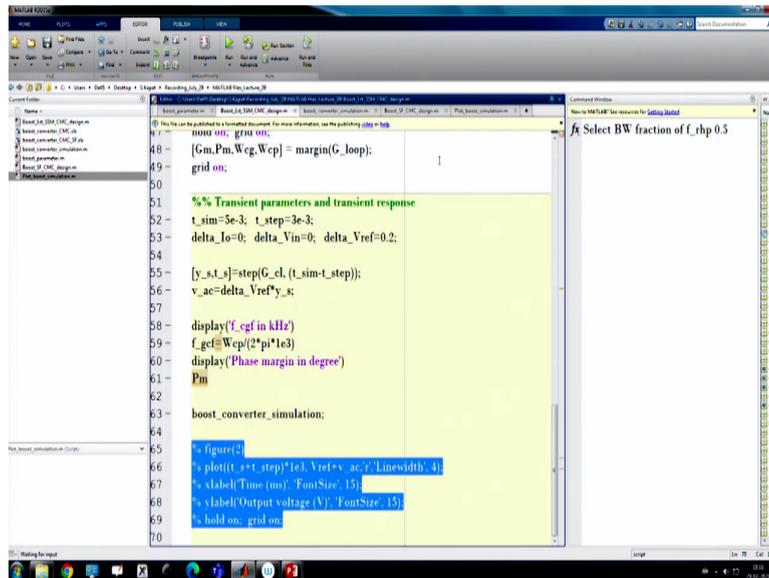
from you know model matching. So, I think so this is, that means if you go here, so the red one is coming from my small-signal model of the output feedback design and the blue one is coming from the output feedback switch simulation. For state feedback, I have not used model matching, but you can try it out. It is not difficult because you already have the closed loop equation, but here my objective is to compare.

(Refer Slide Time: 36:23)



So, if you compare these two, the result you will see. This goes to this value. You know this response is for one-third of the $\zeta = 0$. This is kind of overdamped response. I mean, it is overdamped. So, you have the flexibility to increase the gain ok, but this has overshoot. Now, I want to increase the bandwidth of the original design.

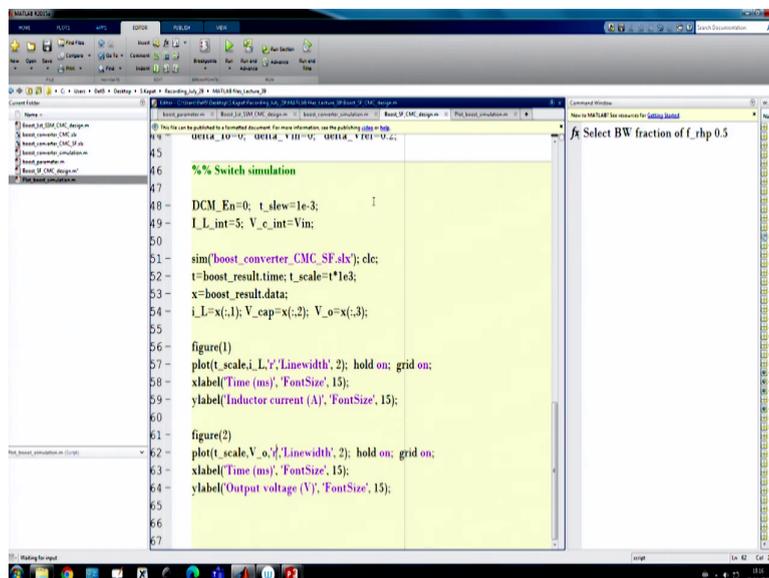
(Refer Slide Time: 36:47)



```
48 - [Gm,Pm,Wcg,Wcp] = margin(G_loop);
49 - grid on;
50
51 %% Transient parameters and transient response
52 - t_sim=5e-3; t_step=3e-3;
53 - delta_Io=0; delta_Vin=0; delta_Vref=0.2;
54
55 - [y_s,t_s]=step(G_cl,(t_sim-t_step));
56 - v_ac=delta_Vref*y_s;
57
58 - display(f_cg in kHz)
59 - f_gc=@(Wcg*(2*pi)*1e3)
60 - display(Phase margin in degree)
61 - Pm
62
63 - boost_converter_simulation;
64
65 % figure(2)
66 % plot(t_step*1e3,Vref+v_ac,'Linewidth',4)
67 % xlabel('Time (ms)', 'FontSize', 15);
68 % ylabel('Output voltage (V)', 'FontSize', 15);
69 % hold on; grid on;
```

That means I want to use let us say 0.5 and here I do not want to use this because model matching I do not want to go, but I want to first compare the response and the state feedback design I am using red color whereas, ok.

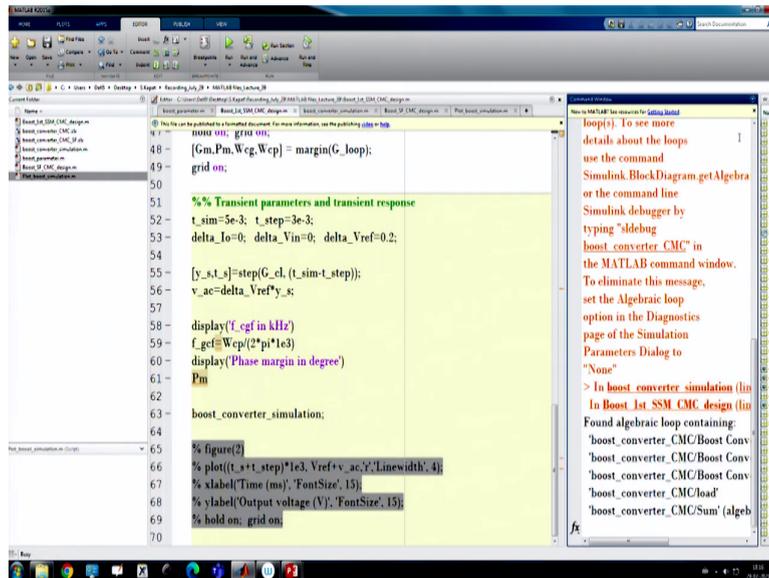
(Refer Slide Time: 37:03)



```
45 - delta_Io=0; delta_Vin=0; delta_Vref=0;
46
47 %% Switch simulation
48 - DCM_En=0; t_slew=1e-3;
49 - L_L_int=5; V_c_int=Vin;
50
51 - sim('boost_converter_CMC_SF.slx'); clc;
52 - t=boost_result.time; t_scale=t*1e3;
53 - x=boost_result.data;
54 - i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
55
56 - figure(1)
57 - plot(t_scale,i_L,'Linewidth', 2); hold on; grid on;
58 - xlabel('Time (ms)', 'FontSize', 15);
59 - ylabel('Inductor current (A)', 'FontSize', 15);
60
61 - figure(2)
62 - plot(t_scale,V_o,'Linewidth', 2); hold on; grid on;
63 - xlabel('Time (ms)', 'FontSize', 15);
64 - ylabel('Output voltage (V)', 'FontSize', 15);
65
66
67
```

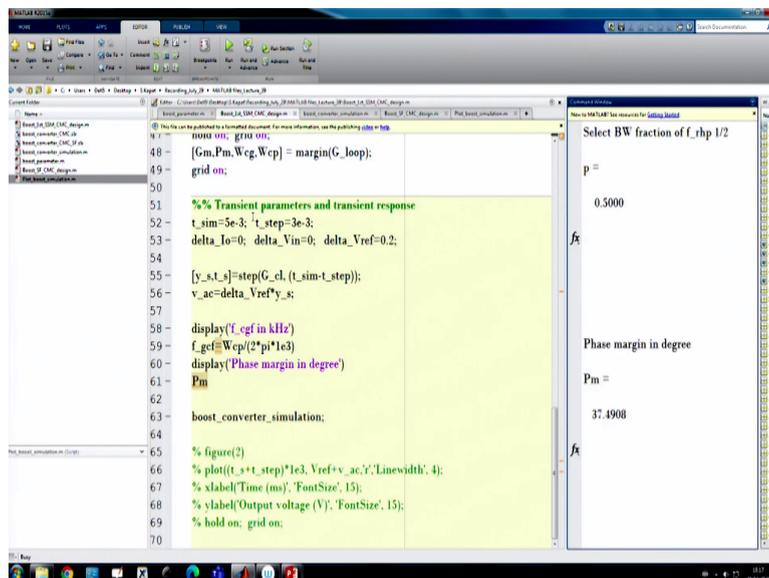
So, let us go and I am using half.

(Refer Slide Time: 37:11)



So, it will show one minute. So, what is happening?

(Refer Slide Time: 37:24)



So, it will show this it is like a, I am trying to achieve half.

(Refer Slide Time: 37:28)

```

Name:
  boost_converter_CMC_design
  boost_converter_CMC_1st
  boost_converter_CMC_2nd
  boost_converter_simulation
  boost_parameter
  boost_SFC_design
  boost_converter_simulation

Current folder:
  boost_converter_simulation

Name:
  boost_converter_simulation

48: [Gm,Pm,Wcg,Wcp] = margin(G_loop);
49: grid on;
50:
51: %% Transient parameters and transient response
52: t_sim=5e-3; t_step=3e-3;
53: delta_Io=0; delta_Vin=0; delta_Vref=0.2;
54:
55: [y_s,t_s]=step(G_cl,(t_sim-t_step));
56: v_ac=delta_Vref*y_s;
57:
58: display('f_cgf in kHz')
59: f_cgf=@Wcg/(2*pi*1e3)
60: display('Phase margin in degree')
61: Pm
62:
63: boost_converter_simulation;
64:
65: % figure(2)
66: % plot((t_s+t_step)*1e3, Vref+v_ac,'LineWidth',4);
67: % xlabel('Time (ms)', 'FontSize', 15);
68: % ylabel('Output voltage (V)', 'FontSize', 15);
69: % hold on; grid on;
70:
  
```

Command Window:

```

> In boost_converter_simulation (lin
In Boost_1st_SSM_CMC_design (lin
Found algebraic loop containing:
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/Boost Conv
'boost_converter_CMC/load
'boost_converter_CMC/Sum1' (algeb
  
```

(Refer Slide Time: 37:28)

```

Name:
  boost_converter_CMC_design
  boost_converter_CMC_1st
  boost_converter_CMC_2nd
  boost_converter_simulation
  boost_parameter
  boost_SFC_design
  boost_converter_simulation

Current folder:
  boost_converter_simulation

Name:
  boost_converter_simulation

48: [Gm,Pm,Wcg,Wcp] = margin(G_loop);
49: grid on;
50:
51: %% Transient parameters and transient response
52: t_sim=5e-3; t_step=3e-3;
53: delta_Io=0; delta_Vin=0; delta_Vref=0.2;
54:
55: [y_s,t_s]=step(G_cl,(t_sim-t_step));
56: v_ac=delta_Vref*y_s;
57:
58: display('f_cgf in kHz')
59: f_cgf=@Wcg/(2*pi*1e3)
60: display('Phase margin in degree')
61: Pm
62:
63: boost_converter_simulation;
64:
65: % figure(2)
66: % plot((t_s+t_step)*1e3, Vref+v_ac,'LineWidth',4);
67: % xlabel('Time (ms)', 'FontSize', 15);
68: % ylabel('Output voltage (V)', 'FontSize', 15);
69: % hold on; grid on;
70:
  
```

Bode Diagram:

Magnitude (dB) vs Frequency (rad/s)

Phase (deg) vs Frequency (rad/s)

Command Window:

```

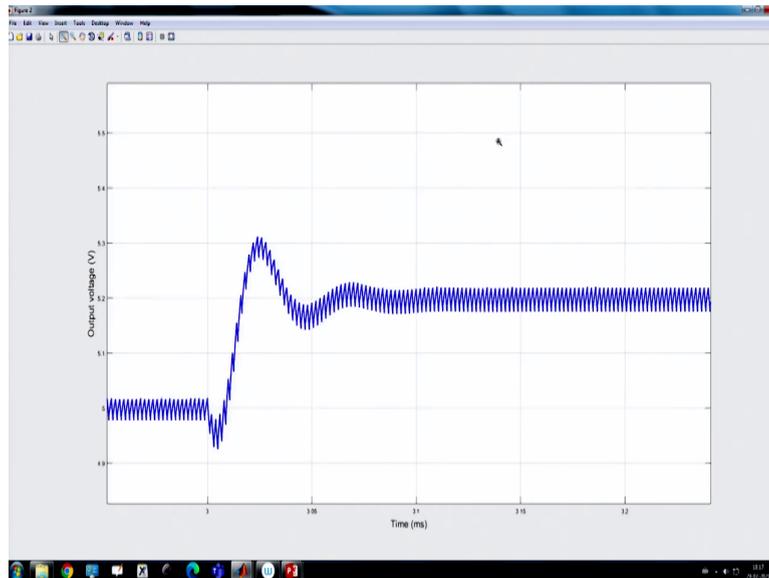
path
:
'boost_converter_CA
CMC/Comp1'

You can turn off this message by using the MATLAB command:

set_param('boost_co

> In boost_converter_simulation (lin
In Boost_1st_SSM_CMC_design (lin
  
```

(Refer Slide Time: 37:29)



These are responses because if I want to increase the bandwidth, so this is my half value. I want to increase the bandwidth now. That means, I am trying to achieve crossover frequency half of that is 0. I want to design the same thing using pole placement technique. That means, state feedback approach, but it is current mode control.

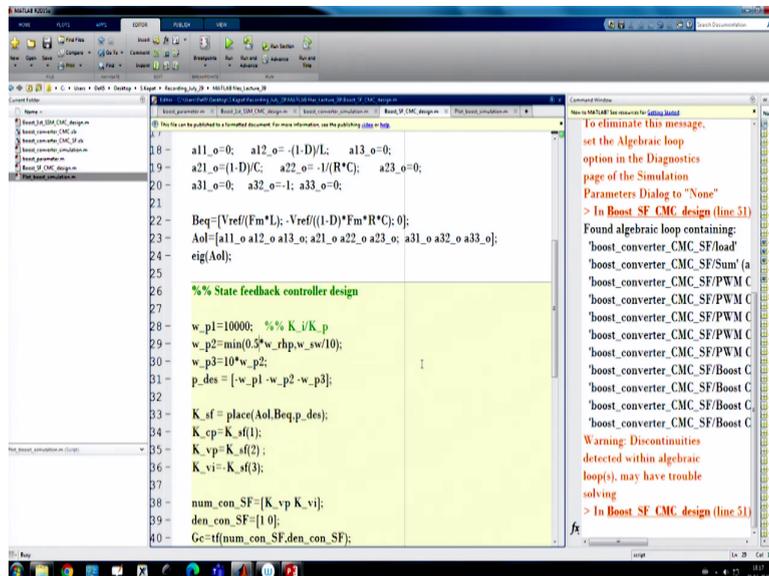
(Refer Slide Time: 37:55)

```
30 num_con_SF=[K_v*p N_v];
39 den_con_SF=[1 0];
40 Ge=tf(num_con_SF,den_con_SF);
41
42 %% Transient parameters and transient response
43 t_sim=5e-3; t_step=3e-3;
44 delta_Io=0; delta_Vin=0; delta_Vref=0.2;
45
46 %% Switch simulation
47 DCM_En=0; t_slew=1e-3;
48 I_L_int=5; V_c_int=Vin;
49
50 sim('boost_converter_CMC_SF.slx'); clc;
51 t=boost_result.time; t_scale=*1e3;
52 x=boost_result.data;
53 I_L=x(:,1); V_c=x(:,2); V_o=x(:,3);
54
55
56 figure(1)
57 plot(t_scale,I_L,'LineWidth',2); hold on; grid on;
58 xlabel('Time (ms)', 'FontSize', 15);
59 ylabel('Inductor current (A)', 'FontSize', 15);
60
61 figure(2)
```

So, then if I use this and let us see what is the response of the boost converter. When you design the compensator by means of a closed loop, you know by placing the pole, where the

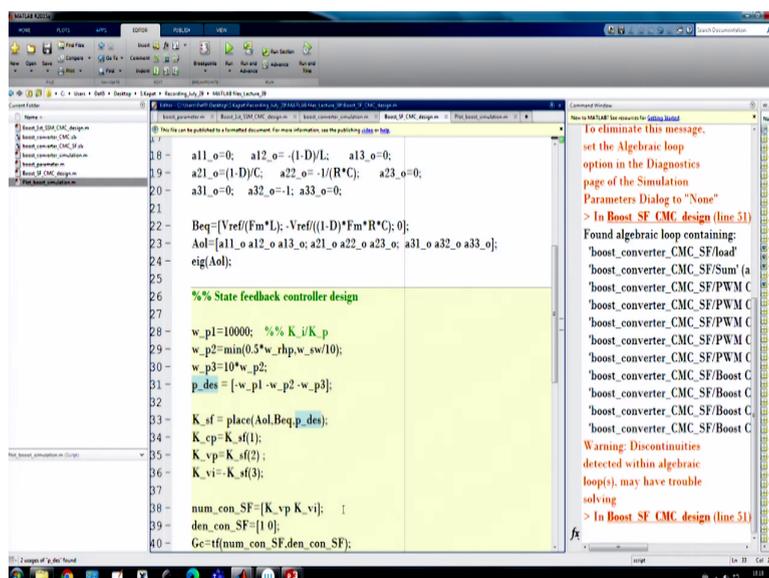
crossover frequency I have linked with the rhp 0. So, it is rhp 0 by 2, sorry here I have to change it. I am sorry.

(Refer Slide Time: 38:14)



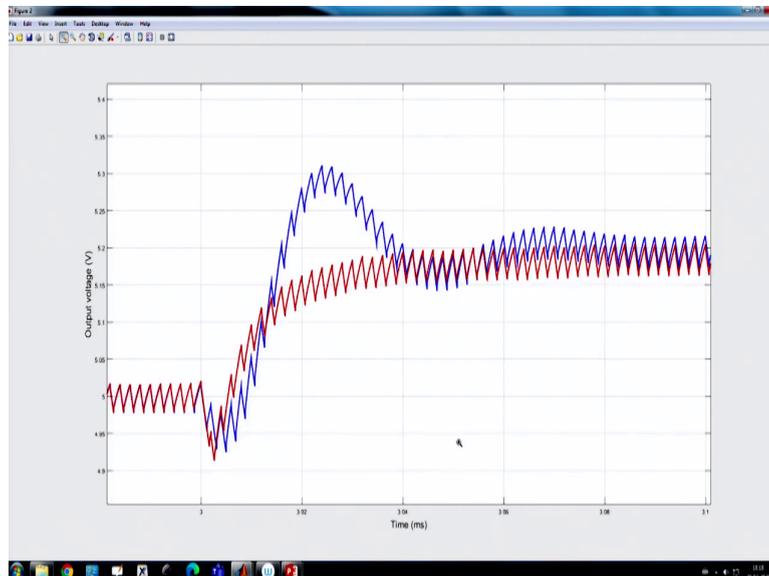
So, I have still kept this. Now, this response is not correct. So, I have to stop the simulation because we have used one-third. So, I have not changed it. So, now it is sluggish, yeah. So, let us change it once again. I want to achieve 0.5 times of rhp 0, ok and in this case I want to use the design which now I am setting half of the rhp 0, ok rhp 0 by 2 half of the rhp 0.

(Refer Slide Time: 38:35)



I want to compare whether using state feedback design approach can we really improve the performance or not? That is our objective ok.

(Refer Slide Time: 39:07)



So, now this is a response that I want to show. You can see the response due to the state feedback approach is much better because there is no overshoot. So, it reaches steady state almost same time. Both of them are reaching steady state, almost same time, but the first approach using output feedback has a high overshoot undershoot, and that is something not acceptable.

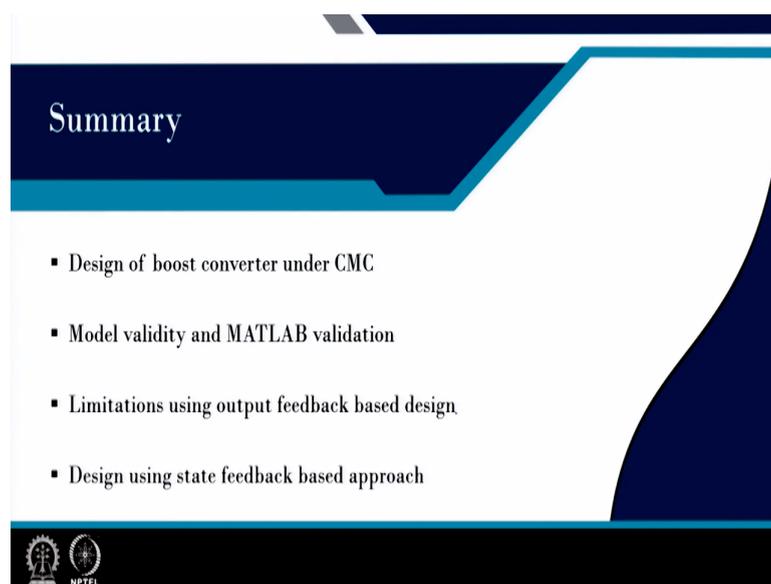
So, I think our original discussion, that means using the state feedback design approach we can leverage the more flexibility because we can increase the crossover frequency and we can improve the response without having overshoot. And of course, in this approach, we are taking all these three poles into consideration. That means, in earlier case there was a pole zero cancellation, but now it is not there.

So, we are we all this is it is the whole system is perfectly controllable. Now, we can use optimal criteria in order to not only for performance improvement, we can set some optimal criteria like a LQR criteria for certain cost function optimization that kind of criteria we can use, ok. So, that means the but only here we are considering a current loop gain which you may not need to change in analog control.

In fact, in analog control we use offline design because we do not do generally real time tuning. Then offline design this state feedback approach we can simply set a current loop gain and this approach can improve the performance much I mean much faster as well as much better because there is less overshoot compared to the traditional output feedback approach, ok.

So, that means this can be this method is very useful for design of current mode control even for buck you can try it out, but for boost you can very perfectly manage you know because of the $r_{hp} = 0$, we have a constraint we found in output feedback. If you want to increase the bandwidth, then your phase margin is all affected, but here there is almost no overshoot. So, it is nicely responding. So but if you go to digital control in real time tuning, you can easily incorporate the current loop gain ok and by that way, this approach can be very useful for digital control current mode control design.

(Refer Slide Time: 41:34)



So, in summary we have discussed design of boost converter under current mode control, we checked model validity using MATLAB, AC transient simulation, we have discussed limitation of output feedback based approach; we have identified what are the tradeoff and then we have discussed how to design state feedback current mode control using state feedback by its design approach. So, with this I want to finish it here.

Thank you very much.