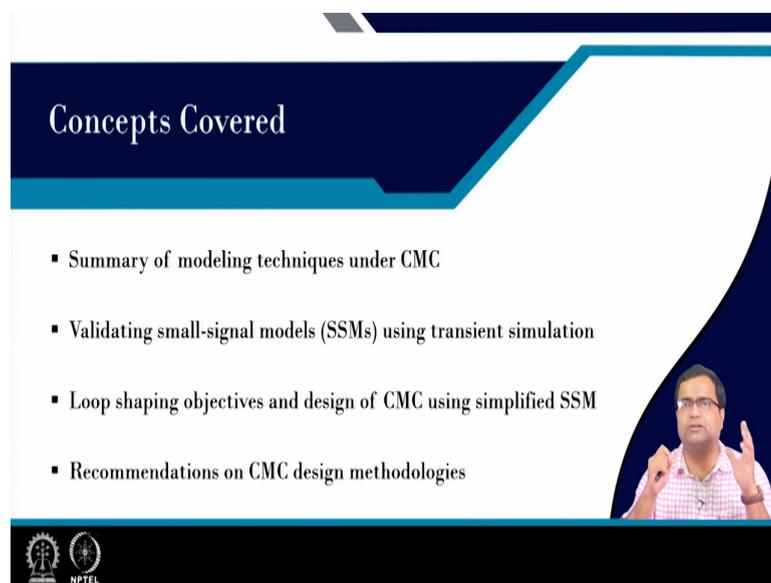


Control and Tuning Methods in Switched Mode Power Converters
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Module - 08
Small-signal Design of Current Mode Control
Lecture - 38
Design CMC in a Buck Converter and MATLAB based Model Validation

Welcome, this is lecture number 38. In this lecture we are going to talk about Design of Current Mode Control in a Buck Converter and MATLAB based Model Validation.

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Concepts Covered

- Summary of modeling techniques under CMC
- Validating small-signal models (SSMs) using transient simulation
- Loop shaping objectives and design of CMC using simplified SSM
- Recommendations on CMC design methodologies

NPTEL

So, in this lecture we are going to first summarize different modeling techniques under current mode control. Then we need to validate small-signal model using transient simulation MATLAB AC transient simulation. Here we want to you know simulate, validate the closed loop converter.

Then you know after that we will make assessment like what are the design technique what are the recommendation which model to be used for design of current mode control, Then we want to identify loop shaping objective and design of current mode control using simplified small-signal model. And finally, the recommendation of design of current mode control design methodology.

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Output Impedance of a Practical Buck Converter

$\langle i_L \rangle \approx \langle i_c \rangle$

$$z_o(s) = \frac{R \times \left(1 + \frac{s}{w_{ESR}}\right)}{\left(1 + \frac{s}{w_p}\right)}$$

$w_p = \frac{1}{(R+r_c)C}$

$w_p = \frac{1}{RC}$, $w_{ESR} = \frac{1}{r_c C}$

$z_o = -\frac{\tilde{v}_o}{\tilde{i}_o} \Big|_{\tilde{i}_c = 0}$

So, if we look at the very first you know I think in week 6 in the model development we have considered, I think it is yeah week 6. We talked about a simplified model of under current mode control with the closed current loop where we have considered the approximation. We have considered that our average inductor current is approximately equal to control current.

That was our approximation and with this approximation we got very simplified model and the output impedance looks like this which consisting because your for a buck converter inductor is in the output side. So, inductor can be replaced by a control current source and then if we want to find out the output impedance, then we need to output impedance.

If we want to find out output impedance, then it is nothing but minus v_0 tilde by i_0 tilde where we want to take i_c tilde to be 0 right. We got the output impedance to be it has one ESR 0 which is 1 by RC into C that is the ESR then one pole. In fact, this pole should be more accurate pole it should be. In fact, it should be 1 in a more accurate sense R into r_c comma C because we have ignored the r_c .

So, that means, let me write again ω_p is equal to 1 by R plus r_c into C , but if load resistance is higher than ESR which is generally the case, then it can be approximated as 1 by RC . And one observation; that means, this is an output impedance and which you know looks like a first-order system like unlike in voltage mode control where we have second order denominator polynomial, but here it is only first-order and with an ESR 0.

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Control-to-Output TF of a Practical Buck Converter

$$G_{vc}(s) = \frac{R \times \left(1 + \frac{s}{w_{ESR}}\right)}{\left(1 + \frac{s}{w_p}\right)} = z_o(s)$$

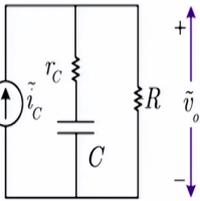
Handwritten note: $G_{vc} = \frac{\tilde{v}_o}{\tilde{i}_c} \Big|_{\tilde{i}_o = 0}$



And if we take the control to output transfer function in that case in this case what we are considering G_{vc} which is nothing but \tilde{v}_o by \tilde{i}_c by setting the external load perturbation to be 0 and again this is same as the output impedance.

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Control-to-Output TF of a Practical Buck Converter

$$G_{vc}(s) = \frac{R \times \left(1 + \frac{s}{w_{ESR}}\right)}{\left(1 + \frac{s}{w_p}\right)} = z_o(s)$$


Handwritten note: $G_{vc} = 0$

DC gain of the control-to-output TF $G_{vc}(0) = R$

Load dependent DC gain – poor load regulation!!

Approximate first-order model – robust compensation



Under first-order model and so first-order model this, these two are same and the DC gain if we take the control to output transfer function. The DC gain means if we put S equal to $j\omega$ and ω equal to 0, then it is simply the load resistance. Here, the DC gain is load dependent. If we recall in voltage mode control, the control to output transfer function has an

input voltage term and the DC gain was input voltage dependent. We can reduce or it can in fact, make the DC loop gain independent of input voltage by incorporating input voltage feed forward.

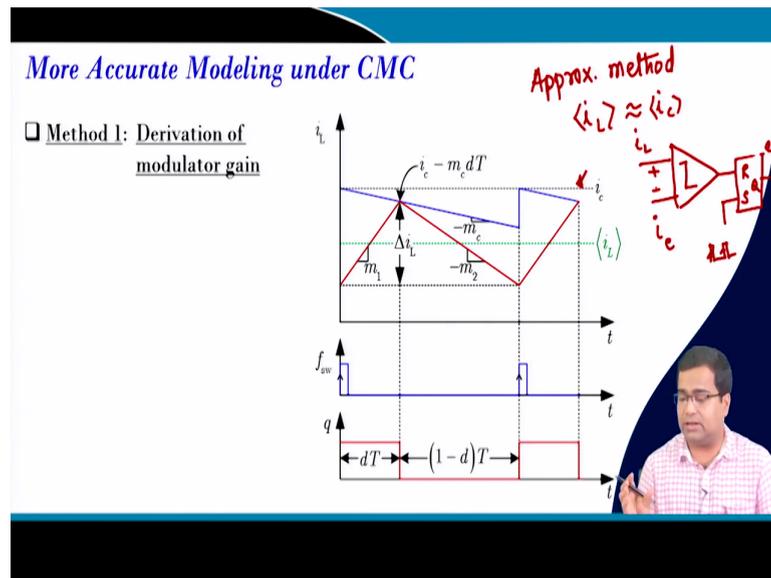
Here our DC gain is load resistance dependent and this will lead to poor load regulation. In case of voltage mode control, it has poor load regulation. And this can be anticipated by considering load current feed forward and that also we are going to discuss. Now, one of the major advantage in this current mode control that your double pole, in case of voltage mode control or direct duty ratio control, is transformed into a first-order pole.

But this is an approximate transfer function because here we have ignored the dynamics of the inductor because we are assuming the inductor dynamics to be much faster. So, that can be approximated like just like a capacitor r_c the pole due to the r_c , r_c network. So, that means, this is a first-order pole, and this can give us very robust compensation.

Because earlier we face difficulty when we have a double pole it was even very difficult to compensate. Even the exact pole 0 cancellation was not sufficient when we are operating under light load or maybe higher load resistance value. Then actual circuit we saw oscillatory behavior where our model response was perfectly fine.

So, because that was due to the poor damping into the actual switching converter. But since we are closed, we have closed the current loop. So, it virtually looks like a first-order system. So, it can be the compensation become much more robust. So, there is no double pole effect neither there is any queue peaking.

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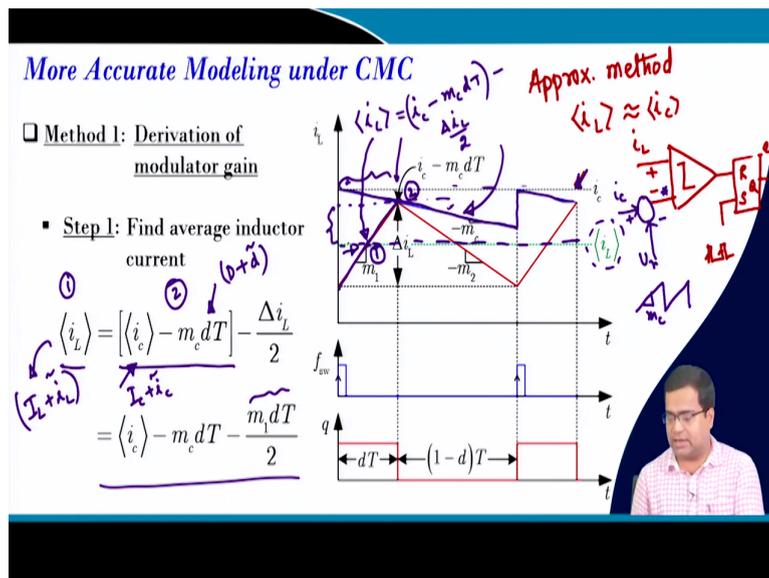
Now, this is the first-order model and this first-order model we also saw G_{vg} that means audio susceptibility was 0 because in this model there is no input voltage perturbation term is not there, it is not there in this model. But in actual converter we know in under current mode control the audio susceptibility is excellent that means, even if you make a large change in supply voltage there will be almost no change in the output voltage is very excellent line regulation as well as excellent supply transient response.

But in the approximate model it shows 0, but there will be a slight effect very you know almost insignificant effect. But in order to improve the accuracy of the model then we have discussed that earlier we have considered in method 1 in the approximate method, what we have considered approximate method? We have considered that i_L average was approximately equal to i_c average right, but now we are not considering i_L average equal to i_c average. So, we are considering some information of you know duty ratio as well as ripple.

But not exactly ripple information, but slope due to the ripple also we want to consider. So, there are two methods to do that if you look at. So, this is a middle brook method where you know if we consider this is my actual current control current, which is coming out of the voltage controller. Then we are considering a compensating ramp that means, if you recall that our original current loop this was our control current.

And this is our inductor current, and this is a comparator, then it goes to our R S latch and this is my Q. So, here we took clock because it is trailing edge modulation. So, in this case, we have considered that inductor current is directly compared with the peak current and then if you want to reset that if the inductor current goes above peak current, then the comparator will reset the latch circuit. Now, in this case, what we are doing instead of adding the controller voltage directly.

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We are considering an additional block; that means, i_c is there which is my output of the voltage controller, but in addition to that we are considering a ramp compensation and this ramp either that means, it is like a sawtooth waveform and this is with a slope of m_c . So, this is my ramp.

Ramp compensation is generally subtracted from the peak current or it is added with the inductor current. So, the both are identical. So, in this case, the waveform shows that you have a dotted line which is a pure current; that means, this is I am talking about this part. Here is the control current. Now, we have subtracted the ramp current. So, the actual current reference that means, if I take this particular term, this waveform is going down.

Because your ramp is subtracted, so its slope will be falling, but actual ramp is rising slope and this is compared to the inductor current. So, you can see the inductor current is rising when it hit the peak current then switch turns off. Now, we want to obtain the average value

of this inductor current; that means, this is my average value of the inductor current and this average value of the inductor current.

We can simply take this average itself. That means this point ok. So, one approximation we can directly compare this approximation. If we do that then what is this point; that means, let us take this point is 1 what is this point? This point nothing but if I take this peak value because value is something which is not in our hand, but we want to find out average current in terms of the reference current which is like actual control current coming minus the ramp.

So, this is my actual reference current, which is compared with the inductor current. So, I want to obtain the average current in terms of the reference current as well as the current ripple. So, this can be obtained that this i_c this point that means, this point i_c minus $m_c dT$ which is the value here minus this half of this ripple minus this half of this ripple. And this is exactly find the average inductor current. So, it is nothing but this value and this is nothing but the point which is number 2 point. So, this is my number 2 point coming, and this is a half of the ripple right.

So, the number 1 the average inductor current I am getting from here, I am taking the number 1 point, is my average inductor current equal to number 2 minus half of the ripple. And then what is the ripple expression. During this particular duration, it is m_1 into dT that is my current ripple and that divided by 2, so this is the expression. Now, once we obtain this current average expression in terms of this.

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More Accurate Modeling under CMC

$$\frac{m_1 dT}{2} = \frac{(m_1 + \tilde{m}_1)(D + \tilde{d})T}{2} = \frac{m_1 DT}{2} + \frac{M_1 T}{2} \tilde{d} + \frac{DT}{2} \tilde{m}_1 + \frac{\tilde{m}_1 \tilde{d} T}{2}$$

negligible

- Step 2: Replace $\langle i_L \rangle = I_L + \tilde{i}_L, \langle i_c \rangle = I_c + \tilde{i}_c, d = D + \tilde{d}, m_1 = M_1 + \tilde{m}_1$
- Step 3: Obtain the perturbed linearized equation

$$\tilde{i}_L = \tilde{i}_c - \left(M_c + \frac{M_1}{2} \right) T \tilde{d} - \frac{DT}{2} \tilde{m}_1$$

$$\tilde{d} = f(\tilde{i}_c, \tilde{i}_L, \tilde{m}_1)$$

If we rewrite, then what we have to do? This is my average dynamics and in order to get perturbed linear model we have to replace this average quantity by the steady state quantity plus perturbation. That means I need to replace my average inductor current by steady state inductor current plus perturbation. Then average control current which is coming out of the controller or it is a current fixed current reference or it can be the output of the controller or it is just a control that we are sending from the outside that means, it is there if there is no ramp compensator this is a peak reference.

But, now along with this current control current, we are subtracting the reference ramp slope that means compensating ramp. So, it is the DC value and the perturbed value then the duty ratio can be replaced by capital D plus d perturbation and the rising slope we are only considering rising slope can be written as the capital value that is the steady state value and its perturbation. Then we want to obtain perturbed value.

So, if we go back. So, we will consider only perturbation; that means, here it is I_L plus i_L perturbation. So, this is my quantity. Then what about this? quantity it is my I_c plus i_c perturbation right then, for m_c it is a slope of the ramp which is fixed. So, we are not taking any perturbation in the ramp slope, then we can write d to be D plus d perturbation right. And then we can write this term. We can write $m_1 dT/2$.

That means, if we consider $m_1 dT/2$ we can write M_1 plus m_1 perturbation then capital D plus d perturbation $T/2$ and this can be separated like $M_1 DT/2$ plus. Now there are two term one is m_1 ok. So, that will find that means, what is this? $M_1 d$ perturbation $T M_1 T/2 d$ perturbation plus $DT/2 m_1$ perturbation. However, we are ignoring that means, there is one more term which is m_1 perturbation d perturbation $T/2$, but this we are ignoring we are you know this is negligible because for small perturbation this is negligible otherwise this will introduce a non-linearity.

So, we have to obtain a perturbed linear model. Then you separate out the DC term. We ignore the non-linear term, then you will end up with only the linear term. And then that means, this is the equation which is very important that should keep in mind. Now, what you have to do? From this equation, our objective is to write d as a function of i_c tilde i_L tilde and m_1 tilde. There are other constant parameters.

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More Accurate Modeling under CMC

- Step 3: Obtain the perturbed linearized equation

$$\tilde{i}_L = \tilde{i}_c - \left(M_c + \frac{M_1}{2} \right) T \tilde{d} - \frac{DT}{2} \tilde{m}_1$$

$$\Rightarrow \tilde{d} = F_m \left(\tilde{i}_c - \tilde{i}_L - \frac{DT}{2} \tilde{m}_1 \right)$$

where $F_m = \frac{1}{\left(M_c + \frac{M_1}{2} \right) T}$



So, perturbed current after that so, this is the expression that we are looking for and this is a modulator gain and what is this modulator gain is nothing, but this term 1 by this term right, 1 by this particular term.

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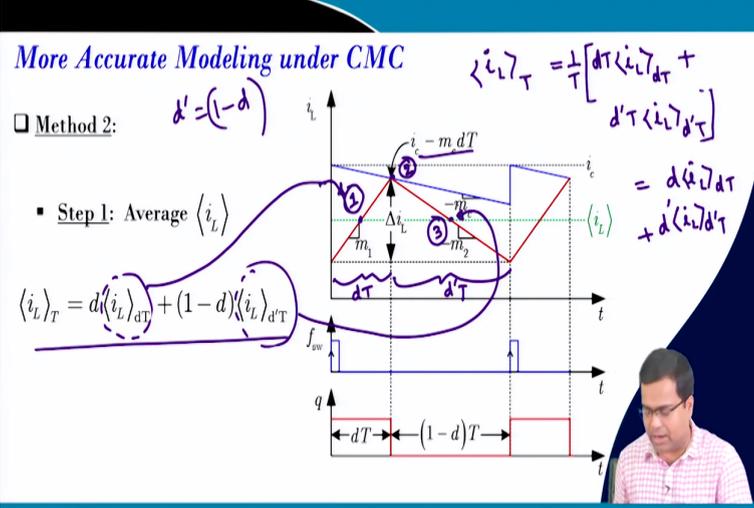
More Accurate Modeling under CMC

Method 2: $d' = (1-d)$

- Step 1: Average $\langle i_L \rangle$

$$\langle i_L \rangle_T = d \langle i_L \rangle_{dT} + (1-d) \langle i_L \rangle_{dT}$$

$$\langle i_L \rangle_T = \frac{1}{T} \left[dT \langle i_L \rangle_{dT} + d'T \langle i_L \rangle_{dT} \right]$$

$$= d \langle i_L \rangle_{dT} + d' \langle i_L \rangle_{dT}$$


Another approach method 2: So, here in the first approach we took the average current if we go back we took the average current as point a. So, the point we have taken as the average current, but now in another scenario we can take the average current. So, this was my point 1,

now, and this was our point 2 in the earlier case and now, we are considering an additional point to be 3.

So, what I am considering now here my average inductor current average inductor current over cycle T is my average inductor current for this duration dT. So, I want to take the average inductor current during the dT plus the average inductor current due to dT dash T that is 1 minus dT, but now we are taking two points and their weightage is this is my dT and this is my d dash T or 1 minus dT. So, in this case we will take simply d that means, if we take 1 by T.

So, this will be my dT and this will be d dash T and the bracket close and that means, it will be d into i L dT plus 1 minus d into i L d dash T ok. So, I think it is not visible. So, let us take it or you can write in terms of d dash into i L d dash T and in this case we are writing d dash to be 1 minus d and this is exactly writing on. Now, here the point to remain same, which is nothing but i c minus m c into dT.

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More Accurate Modeling under CMC

□ Method 2:

- Step 1: Average $\langle i_L \rangle$

$$\langle i_L \rangle_T = d \langle i_L \rangle_{dT} + (1-d) \langle i_L \rangle_{dT} \dots (1)$$

$$\langle i_L \rangle_{dT} = \left[\langle i_c \rangle - m_c dT \right] - \frac{m_1 dT}{2} \dots (2)$$

$$\langle i_L \rangle_{dT} = \left[\langle i_c \rangle - m_c dT \right] - \frac{m_2 (1-d)T}{2} \dots (3)$$

So, if we write again i L T is that average and what is i L dT that means? If I take go back i L dT is nothing but this point 1 the current at point 1 and what is that means, I will say my i L dT this particular term is nothing but this, this point and if I take this, this point is nothing but my point 3 ok.

So, average if we take then we can get that $i_L dT$ which was my our earlier equation when we are taking an extra term, then what we are doing? That means, extra term means this is my at point 3 it is my point 1 and this quantity in all cases is my point 2 corresponds to point 2 because this point is same.

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More Accurate Modeling under CMC

Substituting (2) and (3) in (1), $\frac{\partial f}{\partial i_c} = 1$

$$\langle i_L \rangle = \left[\langle i_c \rangle - m_c dT \right] - \frac{m_1 T d^2}{2} - \frac{m_2 T (1-d)^2}{2}$$

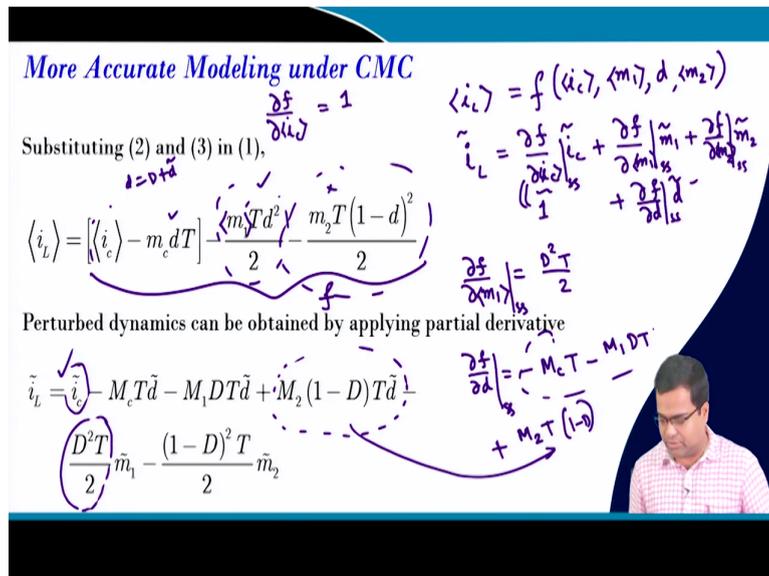
Perturbed dynamics can be obtained by applying partial derivative

$$\tilde{i}_L = \tilde{i}_c - M_c T \tilde{d} - M_1 D T \tilde{d} + M_2 (1-D) T \tilde{d}$$

$$\frac{D^2 T}{2} \tilde{m}_1 - \frac{(1-D)^2 T}{2} \tilde{m}_2$$

Handwritten notes on the slide include:

- $\langle i_c \rangle = f(\langle i_c \rangle, \langle m_1 \rangle, d, \langle m_2 \rangle)$
- $\tilde{i}_L = \frac{\partial f}{\partial i_c} \tilde{i}_c + \frac{\partial f}{\partial m_1} \tilde{m}_1 + \frac{\partial f}{\partial m_2} \tilde{m}_2 + \frac{\partial f}{\partial d} \tilde{d}$
- $\frac{\partial f}{\partial m_1} = \frac{D^2 T}{2}$
- $\frac{\partial f}{\partial d} = -M_c T - M_1 D T + M_2 T (1-D)$



So, then average inductor current we can write after this substitution that means, if we substitute this into this and this into this then we will get this complete equation then again we apply perturbation. We can simply apply Taylor series. That means, you know in this expression suppose if I get it is very easy to get suppose if I get i_L perturbation is a function of i_c perturbation sorry m_1 average.

So, I can consider this to be average then d then m_2 then I can obtain i_L tilde to be $\frac{\partial f}{\partial i_c} \tilde{i}_c + \frac{\partial f}{\partial m_1} \tilde{m}_1 + \frac{\partial f}{\partial m_2} \tilde{m}_2 + \frac{\partial f}{\partial d} \tilde{d}$. So, what is this term? This is simply nothing but i_c you see this term is coming nothing but i_c because it is 1 this term is simply 1 equal to 1 because I am taking this whole term to be f . So, f if you differentiate with respect to i_c average, I am talking about this is average quantity, then you have only term here, and coefficient is 1. So, it is 1, so it is 1.

Now, if we differentiate with respect, to suppose if with respect to m_1 . So, if you differentiate equal to m_1 then you will get d^2 then we have to substitute steady state. All substitution steady state at steady state we are finding right. So, from this equation, if we

write what is my double dot average. So, this is simply 1 right, then what is my double dot m 1 average. So, you will find only this term has this.

So, it will be and calculated at steady state. So, it will be $D^2 T$ by 2 $D^2 T$ by 2 and this is the time the coefficient $D^2 T$ by 2 ok. So, this into m 1 perturbed similarly this all average quantity. We have to differentiate otherwise you can simply substitute d equal to let us say capital D plus \tilde{d} you can solve that, but this is a much easier way to do that. Now, if you do, what is my double dot d .

So, you will first get this term so, minus at steady state minus M_c although M_c small M_c capital they are same because we are not perturbing it into T because we are differentiating with respect to d . Then for the second term like this term what will get $M_1 dT$ because there is a d^2 term if you differentiate $2d$ and D should be capital because you are finding at steady state $2D^2$ get cancel.

Then if you take this term what you will get, you will get minus. So, I think there should be plus no minus fine. So, minus m_1 that means, we are talking about now minus because we are talking about this term. This term we are talking about minus $M_2 T$ and there is a square term.

So, it will be $1 - D$ and square means 2 will come out. So, 2^2 get cancel, now we have to differentiate with respect to $1 - D$. So, there will be plus sign into T . That means it will come as this term. So, it will be $M_2 - dT$ that into delta this term. So, this whole term will come, all terms will get it right.

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More Accurate Modeling under CMC

Perturbed dynamics can be obtained by applying partial derivative

$$\tilde{i}_L = \tilde{i}_c - M_c T \tilde{d} - M_1 D T \tilde{d} + M_2 (1-D) T \tilde{d} - \frac{D^2 T}{2} \tilde{m}_1 - \frac{(1-D)^2 T}{2} \tilde{m}_2$$

$$\tilde{d} = F_m \left(\tilde{i}_c - \tilde{i}_L - \frac{D^2 T}{2} \tilde{m}_1 - \frac{(1-D)^2 T}{2} \tilde{m}_2 \right)$$

where $F_m = \frac{1}{M_c T}$ since $M_1 D = M_2 (1-D)$

So, the perturbed dynamics can be obtained by applying partial derivative as I discussed then you can get all this delta d term F_m is the modulator gain, what is the F_m ? So, it turns out to be $\frac{1}{M_c T}$ because at steady state $m_1 D T$ that means, if you take the steady state inductor current this is $m_1 D T$.

And this is $m_2 (1-D) T$ right. So, if I take the steady state rising slope $m_1 D T$ and falling slope $m_2 (1-D) T$ then the inductor current ripple at steady state is equal to $m_1 D T$. It is also equal to $m_2 (1-D) T$ so; that means, these two $m_1 D T$ is equal to $m_2 (1-D) T$ and if you that then this term actually will be 0 this term will have no contribution because these two will cancel each other. So, you will get m_c .

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More Accurate Modeling under CMC

In a Buck Converter

$$m_1 = \frac{v_{in} - v_o}{L} \quad \text{and} \quad m_2 = \frac{v_o}{L}$$

Thus $\tilde{m}_1 = \frac{1}{L}(\tilde{v}_{in} - \tilde{v}_o)$ and $\tilde{m}_2 = \frac{1}{L}\tilde{v}_o$

Overall small signal model becomes

$$\tilde{d} = F_m (\tilde{i}_c - \tilde{i}_L - k_1 \tilde{v}_{in} - k_2 \tilde{v}_o)$$


Now, in a buck converter it is a buck converter only I mean if you take a buck converter m_1 is $\frac{v_{in} - v_o}{L}$ m_2 is $\frac{v_o}{L}$. So, I can take m_1 perturbation is nothing but $\frac{1}{L}$ into input voltage perturbation minus output voltage perturbation m_2 should be output voltage perturbation by L . So, the overall small-signal becomes F_m into i_c perturbation i_L perturbation minus i_L perturbation minus k_1 into v_{in} minus k_2 into v_o and all this we are writing from here, ok.

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More Accurate Modeling under CMC

$$\tilde{d} = F_m (\tilde{i}_c - \tilde{i}_L - k_1 \tilde{v}_{in} - k_2 \tilde{v}_o)$$

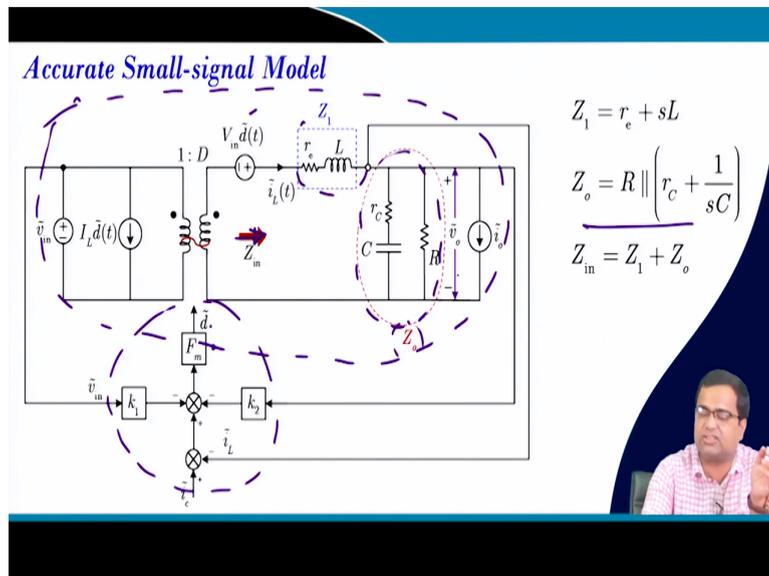
Method	F_m	k_1	k_2
Method 1	$\frac{1}{\left(M_c + \frac{M_1}{2}\right)T}$	$\frac{DT}{2L}$	$-\frac{DT}{2L}$
Method 2	$\frac{1}{M_c T}$	$\frac{D^2 T}{2L}$	$\frac{(0.5 - D)T}{L}$



So, then if we tabulate, so under the first method your F m is this and under the second method your F m is this then k 1 can be obtained by the first method this k 2 is this and for the second method your k 1 is this and k 2 is this. So, this can be obtained whatever we have discussed so far.

And this is the basic equation now we can plug in this equation to our original ac equivalent circuit model that means, if we recall that when we derive by ac equivalent circuit using either average switch model or circuit averaging technique, then the ac equivalent circuit there was open loop deterioration perturbation was there right.

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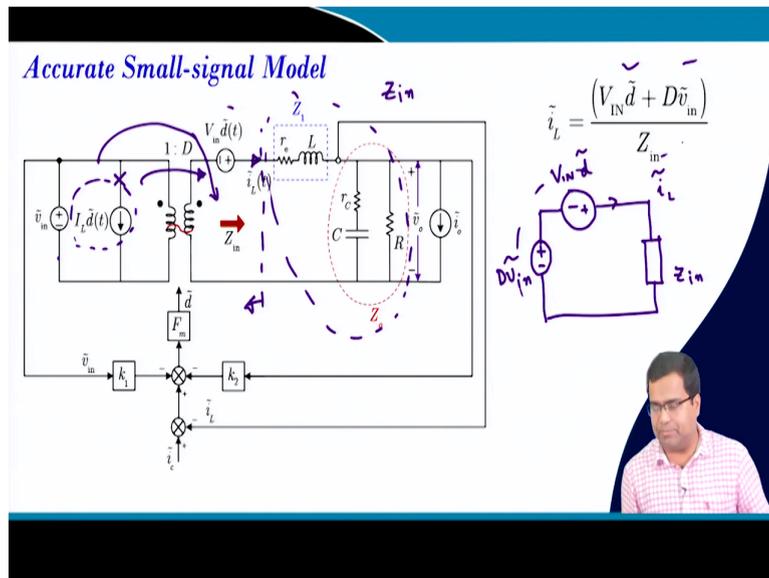


So, now if I going back this is my ac equivalent circuit that we have discussed earlier they have already discussed earlier. So, in this circuit the perturbation, the duty ratio is the one which is coming from the closed loop and this expression we have got from our this from this expression ok.

So, this we have just plugged in that means, d perturbation is F m into k 1 in bracket minus k 1 into v in perturbation minus k 2 into v 0 perturbation minus i L perturbation plus i c perturbation and if you see yes only i c perturbation and all are minus. Here we are considering Z 1 which is the impedance of the inductor plus is equivalent resistance. An equivalent resistance consists of inductor dcr plus switch rds on all are included.

And then Z_0 which is the output impedance because we know for approximate small-signal model, this branch becomes output impedance right. So, that we are writing output impedance and if we look from this side of the converter, then the input impedance is nothing but the R L circuit that is in series with the parallel combination of this Z_1 Z_2 .

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Now, if we want to get the accurate small-signal model if we obtain i_L perturbation that means, we want to obtain this current i_L perturbation and let us consider this whole branch as an impedance and this impedance is nothing but my Z_{in} impedance. That means, and if we do not consider any load perturbation, that means and if I draw this circuit; that means, if you draw this side here we know that if we this current source; that means, the it can be vanished and we can remove because it is across the voltage source it has no meaning.

And this voltage source, if you map to this secondary side and with a step-down ratio of 1 is to a D then you can draw this circuit like this. So, this is our so we have one source here that means, this is our $v_{in} D$ into v_{in} which is the mapped one in this side right. Then it is series with V_{IN} into d perturbation. This has an overall impedance of Z_1 Z_{in} and this is my current expression. So, my inductor current will be the sum of these two voltages which is here divided by the impedance.

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Accurate Small-signal Model

$$\tilde{i}_L = \frac{(V_{IN} \tilde{d} + D \tilde{v}_{in})}{Z_{in}}$$

$$\tilde{i}_L = \frac{D}{Z_{in}} \left(\tilde{v}_{in} + \frac{V_{IN}}{D} \tilde{d} \right)$$

Now, if you rearrange, we take d by Z in out because we want to arrange in a way. So, input voltage plus this. So, this is just a rearrangement from this stage to this stage, but this stage was important. Now, we will see we can write the inductor current if I write that means it is like this. So, you have an input voltage perturbation which is summed up and this side we are getting something like v in by d and this is our duty ratio and this multiplied by D by Z in is my i_L perturbation ok.

(Refer Slide Time: 30:51)

Control-to-Output TF with Closed Current Loop

$$\tilde{i}_L = \frac{D}{Z_{in}} \left(\tilde{v}_{in} + \frac{V_{IN}}{D} \tilde{d} \right)$$

$$T_v = \frac{V_{IN} F_m Z_o k_2}{Z_{in}}$$

Handwritten notes: $\tilde{d} = F_m(\tilde{i}_L - \tilde{i}_L - k_2 \tilde{v}_o)$

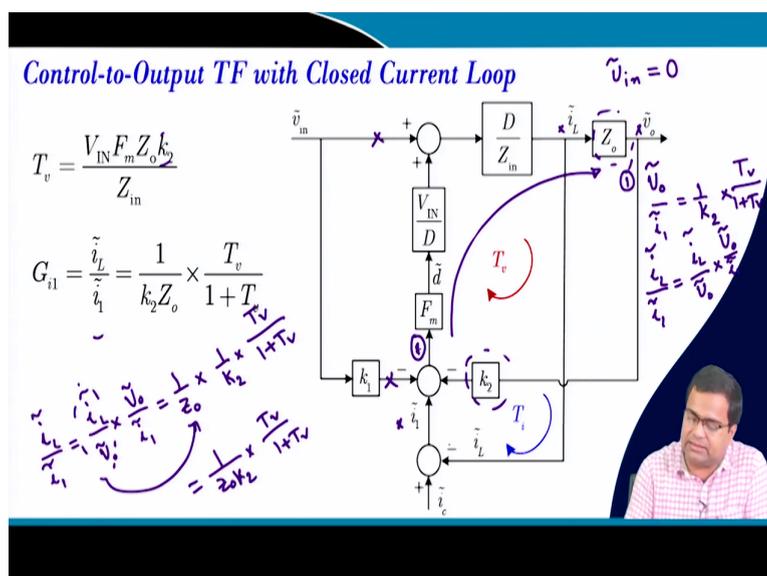
So, this is exactly that. So, we talked about this branch. We talked about this branch. Then how are you generating d perturbation? It is F_m into minus k_1 into F_m in bracket. That means we know that d perturbation was F_m in bracket i c perturbation minus i L perturbation. So, it is added here this is subtracted then minus k_1 into v in perturbation minus k_2 into v 0 perturbation.

So, this is $k_1 k_2$ that we have discussed and then i L to v 0 if you go back. So, this is my i L that means, this current is my i L perturbation, and this branch is my Z_0 . So, then what is the voltage across this branch this branch voltage is nothing but current multiplied by this Z_0 , Z_0 impedance and this is the Z_0 impedance ok. So, you can write i L perturbation in terms of this.

Now, we want to find out there here you can see two loop one is the inside like a voltage loop right. In that case, let us forget that current loop for the time being. We are only talking about the voltage loop. If you remove the current loop, then what is there in the voltage loop? That means, if you start from the number 1 and move along this path right. Then what is there in the voltage loop.

So, it starts with F_m is there V_{IN} by D is there, then V_{IN} by D into d into z. So, $D D$ gets cancelled. So, Z_{in} is there V_{IN} is there, then k_2 is there and. So, here Z_0 is there. It is a voltage loop right. So, this is clear. So, voltage loop gain is clear once you obtain the voltage loop gain now.

(Refer Slide Time: 33:00)



Then you have to obtain the current loop gain ok. So, what are you going to achieve? Now, it looks like we have started from this point right that means; we have started from here we are ignoring that means we are ignoring let us say equal to 0. So, that means, we can simply remove this path we are not considering the perturbation in the input voltage for the time being. Now, if we look at this point to this point. So, this to this point is nothing but what. So, if we look from i_L that means, i_L by i_1 .

So, that means, we start from journey here we are going like this and then ok. So, first I want to obtain this v_0 that means, I want to obtain what is my i_1 that means, what is my this point forget about that this is my number 1 point. So, that means, what i_1 we have already obtained the voltage loop gain T_v . Now, once you obtain voltage loop gain then for the voltage loop point that means, this is the current loop.

So, what is my ok first let us find what is my v_0 to in that means, this is my v_0 and this is my i_1 that means, $1 \cdot 2 \cdot 1$ dash what is there. So, you see this path contain everything forward path expect for k_2 that means I can write it is 1 by k_2 into my voltage loop by 1 plus voltage loop right, because it is nothing but the expression between this point. This point is nothing, but I want to get a forward path divided by 1 plus loop voltage loop.

And the forward path has the voltage loop gain. I mean transfer function, but excluding this term k_2 that is there, but it is now it is in the feedback path. So, we have to write 1 by k_2 into T_v by 1 plus T_v right. Now, we want to obtain what is my i_L to i_1 it is nothing but i_L to v_0 into v_0 to i_1 right, ok.

So, let me write here that means we want to obtain i_L to i_1 which can be written as i_L to v_0 into v_0 to i_1 what is i_L to v_0 this term. That means, it only this impedance will come that means, it is 1 by Z_0 . So, this is the term then multiplied by what is v_0 to i_1 . So, this we already saw it is 1 by k_2 into T_v plus 1 plus T_v right. So, it will become 1 by $Z_0 k_2 T_v$ plus 1 by T_v . So, this is what the expression next once we get that means, this is now my current loop, now what is my current loop?

(Refer Slide Time: 37:07)

Control-to-Output TF with Closed Current Loop

$$T_v = \frac{V_{IN} F_m Z_o k_2}{Z_{in}}$$

$$G_{i1} = \frac{\tilde{i}_L}{\tilde{i}_1} = \frac{1}{k_2 Z_o} \times \frac{T_v}{1 + T_v}$$

$$T_i = G_{i1}$$

if $k_2 = 0$
 $T_i \approx \frac{F_m V_{in}}{Z_{in}}$
 $F_m = \frac{1}{V_m}$

Now, my current loop that means, I will start from the loop that means, I will start from this point number 2 point and I want to come back sorry. I start from this point number 2 and I want to come back to this part. This is my current loop right. So, this is my 2 dash, so that means, what is my current loop transfer function it is nothing but the one which is obtained because we have not considered any feedback gain in the current loop if we consider then they will be different otherwise this is the one which we obtain i_L tilde by i_1 tilde.

(Refer Slide Time: 37:48)

Control-to-Output TF with Closed Current Loop

$$T_i = G_{i1}$$

$$G_{ic} = \frac{\tilde{i}_L}{\tilde{i}_c} = \frac{T_i}{1 + T_i}$$

So, that means, we got the current loop gain. Now what is my i_L by i_c . So, in i_L by i_c that means now we are talking about the number 3 point and number 3 point dash. So, we are getting but in 3 to 3 dash and this is nothing but the forward path has everything, which is the current loop transfer function divided by 1 plus current loop transfer function. So, this is the transfer function ok.

(Refer Slide Time: 38:16)

Control-to-Output TF with Closed Current Loop

$$G_{ic} = \frac{\tilde{i}_L}{\tilde{i}_c} = \frac{T_i}{1 + T_i}$$

$$G_{vc} = \frac{\tilde{v}_o}{\tilde{i}_c} = Z_o \times G_{ic}$$

$$= \frac{\tilde{v}_o}{\tilde{i}_L} \times \frac{\tilde{i}_L}{\tilde{i}_c} = Z_o \times \frac{T_i}{1 + T_i}$$

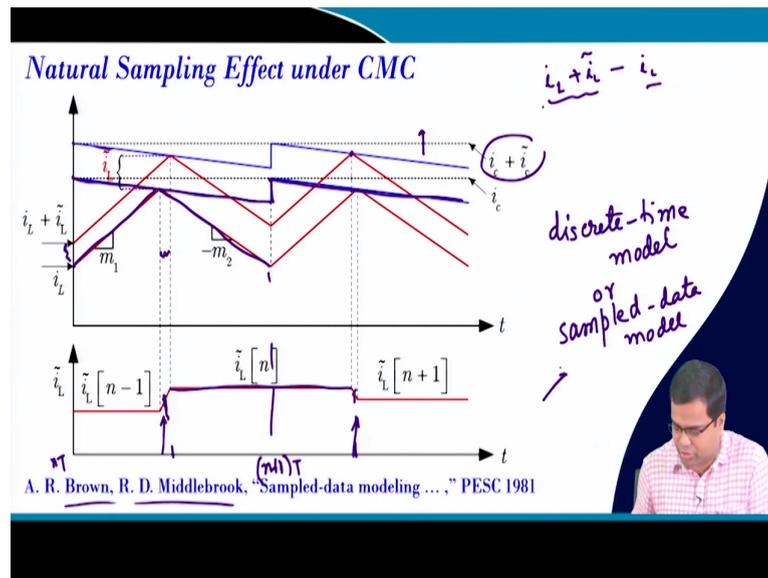
$\omega_c = Z_o \times \frac{T_i}{1 + T_i}$

$|T_i| > 1$
 $\omega_c \approx Z_o$

So, once we get current loop transfer function then we have to get what is my control, because our ultimate control to output transformation is v_o by i_c ok. This can be obtained as v_o by i_L into i_L by i_c right. What is v_o by i_L it is Z_o into and what is i_L by i_c it is T_i by $1 + T_i$ right and that means, my G_{vc} is equal to Z_o into T_i by $1 + T_i$.

So, this is my current loop gain. If my current loop gain is very very larger than 1. Then G_{vc} will approximately become Z_o and this is nothing but our control to output transfer function in case of the approximate model. So, both the model will match once the current loop gain is very high, but if the current loop gain start decreasing, then there will be a significant mismatch in the model and that we will see.

(Refer Slide Time: 39:35)



Another thing the natural sampling effect you know where in the Brown and Middlebrook thesis though info I mean it was reported that if you take any perturbation in the inductor current that means, if you consider a nominal inductor current. This is my normal inductor current, and this is my nominal current reference minus the ramp slope that we have discussed right, which will be compared with the inductor current.

Now, the current reference is perturbed. So, it is shifted up, similarly your ramp slope the whole thing will get shifted up and inductor current is also perturb there is a perturbation shifted up. Now, due to this perturbation, you see there is a change in the sample inductor perturb; that means, if we obtain the i_L plus i_L perturbation minus i_L that means, this is the perturbed inductor current the original.

So, there is an initial perturbation. So, this i_L perturbation during the n th cycle that means, we can say this is my nT this is my $n+1T$ because switching cycle. So, during the n th switching cycle, this perturbation will be reflected because we have perturbed the initial current of the inductor current.

We have also perturbed the current reference. So, there will be a perturbed current and that perturbed current will remain same till it hit the next time when it hit the reference current that means, that reference current minus the ramp like a sawtooth waveform or basically ramp compensation. So, this is the overall current reference and this indicate that it only changes when, as if it is almost after one cycle, it is changing. So, this is like sampling and

hold behaviour sampling effect, in this is naturally present in even analogue current mode control.

(Refer Slide Time: 41:35)

More Accurate Small-Signal Model with Sampling Effect under CMC

$$H_e(s) = 1 + \frac{s}{\omega_n Q_z} + \frac{s^2}{\omega_n^2}$$

$$Q_z = \frac{1}{\pi [\alpha_c (1-D) - 0.5]}$$

$$\omega_n = \frac{\pi}{T}$$

R. B. Ridley, "A new, continuous-time model for current-mode ...", TPEL 1991.

And then this current mode control if you want to investigate you need to include discrete time model generally it is discrete time model is used or you can say or you can say sample data model. So, sample data model means, we are modelling between the two sampling instant whereas if these two are the sampling instead, but the discrete time one is more generic one.

That means, discrete time your time period the sampling time can be smaller than the switching period and so on. Here we are making like a there is no actual sampling, it is like a stroboscopic or the virtual sampling just to create a model of the system right sampled data model.

But then Ridley actually incorporated that means. This is the sampled data model, if we incorporate. If you obtain the continuous time model, then in our earlier model, which was there, you can get a current loop transfer function. In the current loop, there is a sampling transfer function will come and this effect is coming due to the sampling effect and we are not going to derive or discuss this in this course because you know this is left as an advanced topic. But this sampling effect can be included.

(Refer Slide Time: 43:03)

More Accurate Small-Signal Model with Sampling Effect under CMC

$$G_{vc}(s) = \frac{\tilde{v}_o}{\tilde{z}_c} = \frac{R}{k_c} \times \frac{1}{1 + \frac{RT}{L} [\alpha_c(1-D) - 0.5]} \times F_p(s) F_h(s)$$

$\alpha_c = 1 + \frac{m_c}{m_1}$

$k_c \rightarrow$ current sensor gain

Handwritten notes:
 $F_m = \frac{1}{m_1 T}$
 $F_m = \frac{1}{(m_c + m_1) T}$

And if you include the sampling effect then the control to output transfer function will be written like this. This is similar to our first-order pole with one ESR 0 1 pole that means, this one and this is due to the sampling effect ok and this model is use almost most of the industry they use this model, but still this hold effect has a second order pole like a 2 pole the 1 pole. So, we want to see if we can we use our first-order model, which is very easy to design, then we want to compare what happened with the Ridley model ok.

(Refer Slide Time: 43:46)

Small-Signal Modeling Techniques under CMC – Comparative Study

- Methods considered – (i) approximate first-order model, (ii) more accurate model and (iii) Ridley model of control-to-output TF
- Validating step reference transient – link with closed-loop bandwidth
- To consider all cases with and without ramp compensation

So, we want to do a comparative study using approximate first-order model more accurate model as well as a Ridley model for getting control to output transfer function. We want to validate using reference transient which is linked with the closed-loop bandwidth and then we want to consider with and without ramp compensation using MATLAB case study.

(Refer Slide Time: 44:09)

```

44 - num_on; grid on;
45 - (Gm,Pm,Wcg,Wcp) = margin(G_loop);
46 - %% Transient parameters and transient response
47 - t_sim=5e-3; t_step=3e-3;
48 - delta_Io=20; delta_Vin=0; delta_Vref=0;
49 -
50 - [y_s,t_s]=step(Z_oc,(t_sim-t_step));
51 - v_ac=-delta_Io*y_s;
52 -
53 - buck_converter_simulation;
54 -
55 - figure(2)
56 - plot(t_s+t_step)*1e3,Vref+v_ac,'LineWidth',4);
57 - hold on; grid on;
58 - xlabel('Time (ms)', 'FontSize', 15);
59 - ylabel('Output voltage (V)', 'FontSize', 15);
60 -
61 -
62 - % display('f_cgf in kHz')
63 - % f_cgf=Wcg/(2*pi*1e3)
64 - % display('Phase margin in degree')
65 - % Pm
66 -

```

(Refer Slide Time: 44:13)

```

1 - close all; clear; clc;
2 -
3 - %% Parameters
4 - buck_parameter; Vin=12; Vref=1;
5 - D=Vref/Vin; M1=(Vin-Vref)/L;
6 - R=1; f_sw=1/T;
7 - r_eq=r_L+r_1; alpha=(R+r_eq)/R;
8 - k2=(0.5-D)*(T/L);
9 -
10 - %% Modulator Gain
11 - V_m=input('Set ramp voltage in V ');
12 - Fm=1/V_m; Mc=V_m/T;
13 -
14 - %% Define zeros and poles
15 - w_z=1/(r_C*C); w_p=1/((R+r_C)*C);
16 -
17 - %% Open-loop Output Impedance
18 - num_o=R*(1/w_z);
19 - den_o=[1/w_p];
20 - Z_o=tf(num_o,den_o);
21 -
22 - %% Control-to-output TF 1st-order model
23 - num_c=R*(1/w_z);
24 - den_c=[1/w_p];

```

Let us go to the MATLAB. So, in this MATLAB we are talking about the model comparison that means, here all these codes like parameters. So, initially we are talking about 12 volt

input, then we obtain you know whatever transfer function we have written so far. This is a MATLAB code.

That means, you know all duty ratio M1 rising slope of the inductor current I am operating initially let us say 1 ohm resistance then switching frequency is 1 by T the time period is 2 microsecond then r equivalent is r L plus r 1, r 1 is the on state resistance of the inductor ok. And the k2 I talked about k2 corresponds to the v 0 feedback loop that you know if we go back to the transfer function. So, we have talked about k 2. So, k 2 is this particular term this particular we are talking about this particular term ok.

So, now, let us go back k 2. Then this is my V m is the amplitude of the ramp. If we use a ramp, it is varying from 0 to V m maximum voltage and the modulator gain is simply 1 by V m ok. If you take the slope of the compensating ramp, it is nothing but V m by T because it is varying from 0 to V m for a timing periodic interval of T time period T and this is my ESR 0 and this is my pole that we have discussed, ok.

(Refer Slide Time: 45:37)

```

19 - den_o=[1/w_p 1];
20 - Z_o=tf(num_o,den_o);
21
22 %% Control-to-output TF 1st-order model
23 - num_c=R*[1/w_z 1];
24 - den_c=[1/w_p 1];
25 - Gvc_1=tf(num_c,den_c);
26
27 %% Control-to-output TF Accurate
28 - z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
29 - w_o=w_o_ideal*sqrt((R+r_eq)/(R+r_C));
30 - Q=alpha*((r_C+r_eq)/z_c)+(z_c/R);
31
32 - num_imp=((alpha*R)/(D^2))*[1/(w_o^2) 1/(Q*w_o) 1];
33 - den_imp=[1/w_p 1];
34 - Z_in=tf(num_imp,den_imp);
35 - T_v=(Fm*V_in*k2*Z_o)/Z_in;
36 - Gv1=(1/k2)*(T_v/(1+T_v));
37 - T_cl=Gv1/Z_c;
38 - G_i_cl=T_cl/(1+T_cl);
39 - Gvc_2=G_i_cl*Z_c;
40
41 %% Control-to-output TF Ridley
42 - alpha_c=1/(M*beta);
43 - w_o=1/(sqrt(L*C));
44 - Q=alpha*((r_C+r_eq)/z_c)+(z_c/R);
45 - num_ridley=((alpha_c*R)/(D^2))*[1/(w_o^2) 1/(Q*w_o) 1];
46 - den_ridley=[1/w_p 1];
47 - Z_in=tf(num_ridley,den_ridley);
48 - T_v=(Fm*V_in*k2*Z_o)/Z_in;
49 - Gv1=(1/k2)*(T_v/(1+T_v));
50 - T_cl=Gv1/Z_c;
51 - G_i_cl=T_cl/(1+T_cl);
52 - Gvc_2=G_i_cl*Z_c;
53
54 %% Control-to-output TF Approximate
55 - num_app=[1/w_z 1];
56 - den_app=[1/w_p 1];
57 - Gvc_3=tf(num_app,den_app);
58
59 %% Control-to-output TF Final
60 - Gvc=Gvc_1*Gvc_2*Gvc_3;
61
62 %% Control-to-output TF Final
63 - Gvc=Gvc_1*Gvc_2*Gvc_3;
64
65 %% Control-to-output TF Final
66 - Gvc=Gvc_1*Gvc_2*Gvc_3;
67
68 %% Control-to-output TF Final
69 - Gvc=Gvc_1*Gvc_2*Gvc_3;
70
71 %% Control-to-output TF Final
72 - Gvc=Gvc_1*Gvc_2*Gvc_3;
73
74 %% Control-to-output TF Final
75 - Gvc=Gvc_1*Gvc_2*Gvc_3;
76
77 %% Control-to-output TF Final
78 - Gvc=Gvc_1*Gvc_2*Gvc_3;
79
80 %% Control-to-output TF Final
81 - Gvc=Gvc_1*Gvc_2*Gvc_3;
82
83 %% Control-to-output TF Final
84 - Gvc=Gvc_1*Gvc_2*Gvc_3;
85
86 %% Control-to-output TF Final
87 - Gvc=Gvc_1*Gvc_2*Gvc_3;
88
89 %% Control-to-output TF Final
90 - Gvc=Gvc_1*Gvc_2*Gvc_3;
91
92 %% Control-to-output TF Final
93 - Gvc=Gvc_1*Gvc_2*Gvc_3;
94
95 %% Control-to-output TF Final
96 - Gvc=Gvc_1*Gvc_2*Gvc_3;
97
98 %% Control-to-output TF Final
99 - Gvc=Gvc_1*Gvc_2*Gvc_3;
100

```

Then open loop output impedance we have discussed for using first-order model. We have discussed the control to output transformation, this is nothing but same as the output impedance. So, I have written Gvc 1 it is the control to output transfer function using method 1 which is the approximate model.

Now, for the accurate model, then we obtain $z_c Q_0 Q$ we obtain numerical you know because whatever model we have discussed using accurate model means I am talking about this all this derivation we have considered input impedance and all this thing. So, with this we have discussed that we have ultimately obtained this control to output function over here.

So, this is our expression we are talking about we are we want to obtain the control to output transfer function using accurate model here. So, for that we need to derive the current loop transfer function voltage loop transfer function first, then current loop transfer function, then G_{vc2} that is the control to output transformation using accurate model.

(Refer Slide Time: 46:42)

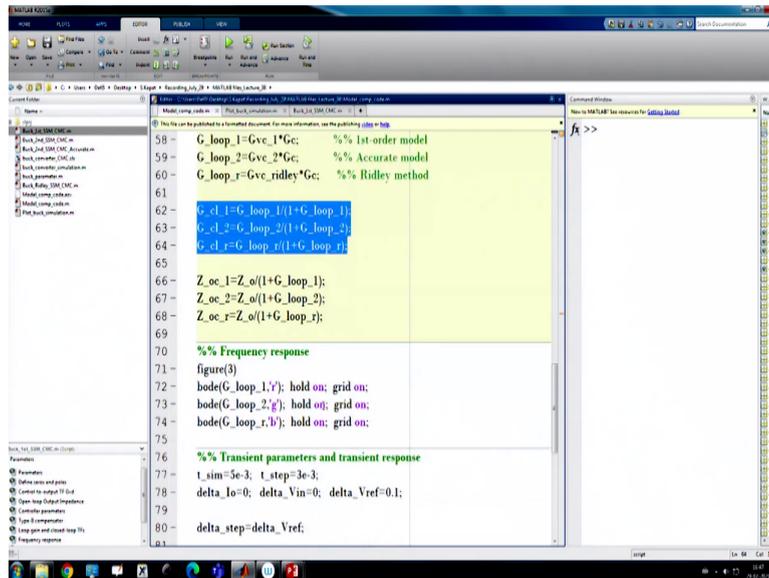
```

37 - T_cl=G_s1/Z_o;
38 - G_i_cl=T_cl/(1+T_cl);
39 - Gvc_2=G_i_cl*Z_o;
40
41 %% Control-to-output TF Ridley
42 alpha_c=1+(M_e/M1); w_n=pi/T;
43 w_pr=(1/(R*C))+((T/(L*C))*(alpha_c*(1-D))+0.5);
44 Q_n=1/(pi*((alpha_c*(1-D))+0.5));
45 k_den=1+(((R*T)/L)*(alpha_c*(1-D))+0.5);
46 F_p=tf([1/w_z 1],1)/w_pr 1);
47 H_e=tf([1/(w_n^2) 1],(w_n^2 1),1,1);
48 Gvc_ridley=R*(1/k_den)*F_p*H_e;
49
50 %% Type-II compensator
51 f_c=input('Select BW in kHz ... ');
52 K_c=(2*pi*f_c)^3/R;
53 num_con=K_c*e^den_c;
54 den_con=[1/w_z 1 0];
55 Gc=tf(num_con,den_con);
56
57 %% Loop gain and closed-loop TFs
58 G_loop_1=Gvc_1*Gc; %% 1st-order model
59 G_loop_2=Gvc_2*Gc; %% Accurate model
60 G_loop_ac=Gvc_ac*Gc; %% Bessel method
  
```

Then we are talking about the Ridley model, where alpha everything is whatever, as per the equation that we have discussed, like you know if we have discussed this equation we have considered. So, it is a Ridley model ok, now we are designing the type 2 comp. So, design will discuss in this course I mean in this lecture later, but suppose I have designed the compensator using just a first-order model.

And then I have certain objectives. I want to achieve a desired bandwidth. Let us say one tenth of the switching frequency. So, 1 10 means? So, if it will ask for what is my crossover frequency, I will set 1 10. That means, 500 kilohertz is my switching frequency I will set 50 kilohertz then it will automatically generate I am using a type two compensator. So, what will be my compensation?

(Refer Slide Time: 47:31)



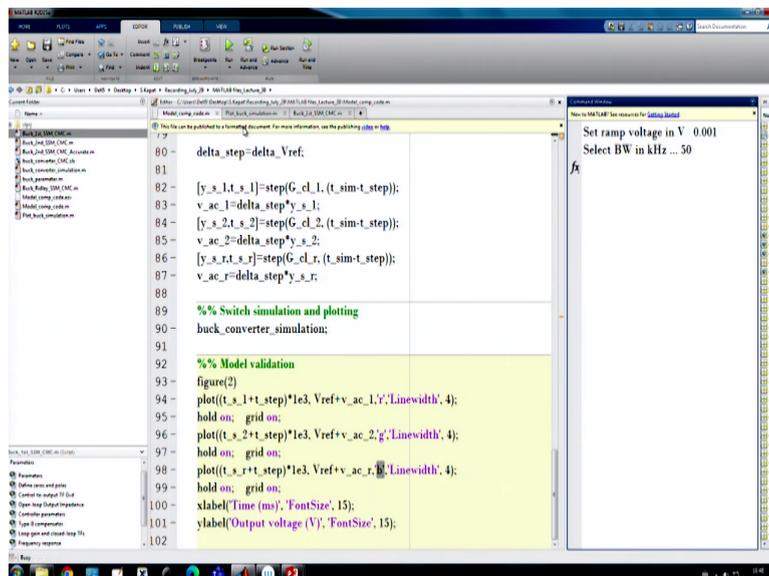
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58 - G_loop_1=Gvc_1*Gc;      %% 1st-order model
59 - G_loop_2=Gvc_2*Gc;      %% Accurate model
60 - G_loop_r=Gvc_ridley*Gc;  %% Ridley method
61
62 - G_cl_1=G_loop_1/(1+G_loop_1);
63 - G_cl_2=G_loop_2/(1+G_loop_2);
64 - G_cl_r=G_loop_r/(1+G_loop_r);
65
66 - Z_oc_1=Z_o/(1+G_loop_1);
67 - Z_oc_2=Z_o/(1+G_loop_2);
68 - Z_oc_r=Z_o/(1+G_loop_r);
69
70 %% Frequency response
71 - figure(3)
72 - bode(G_loop_1,''); hold on; grid on;
73 - bode(G_loop_2,'g'); hold on; grid on;
74 - bode(G_loop_r,'b'); hold on; grid on;
75
76 %% Transient parameters and transient response
77 - t_sim=5e-3; t_step=3e-3;
78 - delta_io=0; delta_vin=0; delta_vref=0.1;
79
80 - delta_step=delta_vref;

```

I will discuss this part, then it will draw the loop transfer function. So, same compensator is used for all 3 transfer function that means, the approximate one then accurate one and the Ridley model, then it will draw the Bode plot of the loop transfer function then we will obtain the closed loop transfer function using three methods.

(Refer Slide Time: 47:49)



```

80 - delta_step=delta_vref;
81
82 - [y_s_1,t_s_1]=step(G_cl_1,(t_sim-t_step));
83 - v_ac_1=delta_step*y_s_1;
84 - [y_s_2,t_s_2]=step(G_cl_2,(t_sim-t_step));
85 - v_ac_2=delta_step*y_s_2;
86 - [y_s_r,t_s_r]=step(G_cl_r,(t_sim-t_step));
87 - v_ac_r=delta_step*y_s_r;
88
89 %% Switch simulation and plotting
90 - buck_converter_simulation;
91
92 %% Model validation
93 - figure(2)
94 - plot(t_s_1+t_step)*1e3, Vref+v_ac_1,'LineWidth',4);
95 - hold on; grid on;
96 - plot(t_s_2+t_step)*1e3, Vref+v_ac_2,'g','LineWidth',4);
97 - hold on; grid on;
98 - plot(t_s_r+t_step)*1e3, Vref+v_ac_r,'b','LineWidth',4);
99 - hold on; grid on;
100 - xlabel('Time (ms)', 'FontSize', 15);
101 - ylabel('Output voltage (V)', 'FontSize', 15);
102

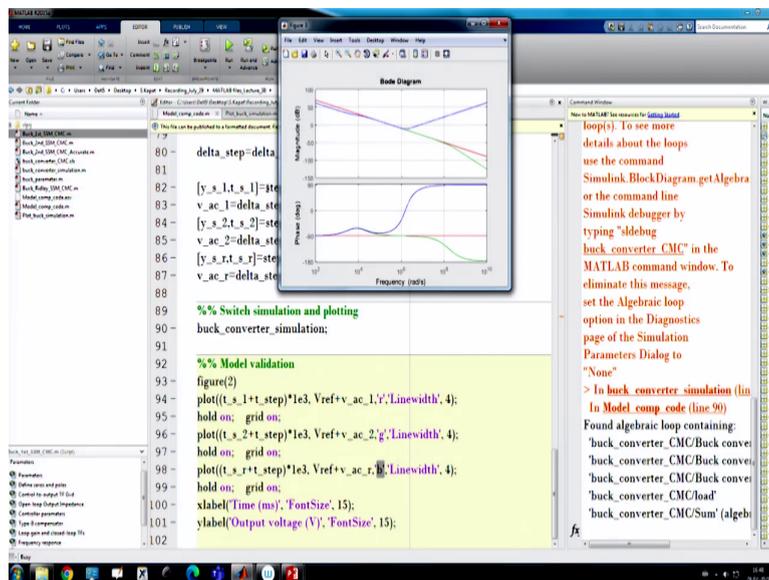
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And then we will get the transient response of the using three different models. That means, we will take the transient response load. We will say the reference transient response. It is changing from 0 to that means; we are applying a reference step of point 1. So, it is changing

from 1 volt to 1.1 volt at 3 millisecond duration. So, all this we have discussed in voltage mode control itself how to obtain match the model.

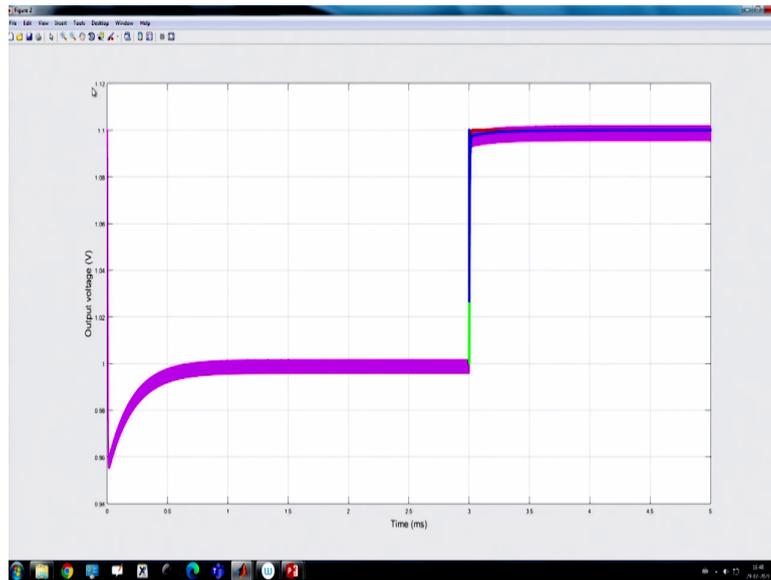
So, here we are getting the step response of the closed loop system for all three cases and for the first case method 1. We are using red colour for method 2 we are using green colour and for Ridley method we are using blue colour and actual switch simulation we are using magenta colour. So, let us run and see what happens. So, it will ask for ramp voltage. Let us say we are setting a very low ramp voltage 0.001 and we will run it will ask for bandwidth.

(Refer Slide Time: 48:48)



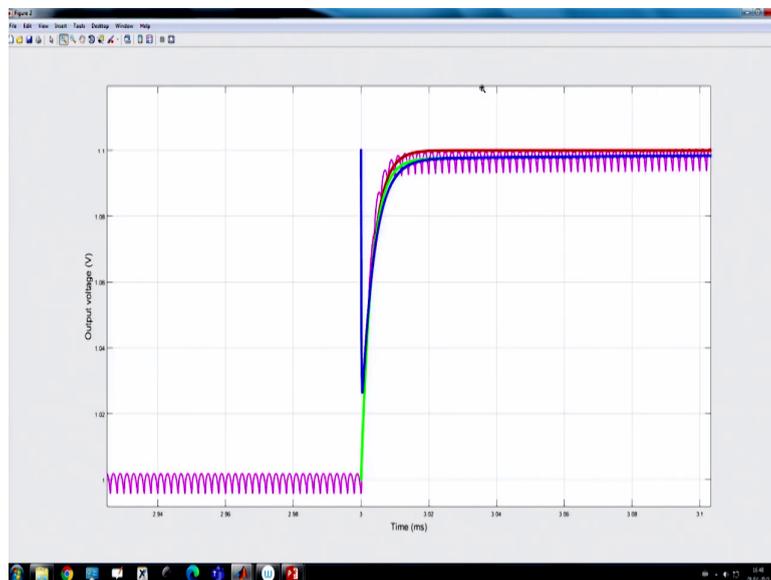
So, let us say 50 kilohertz is my bandwidth then it will design.

(Refer Slide Time: 48:51)

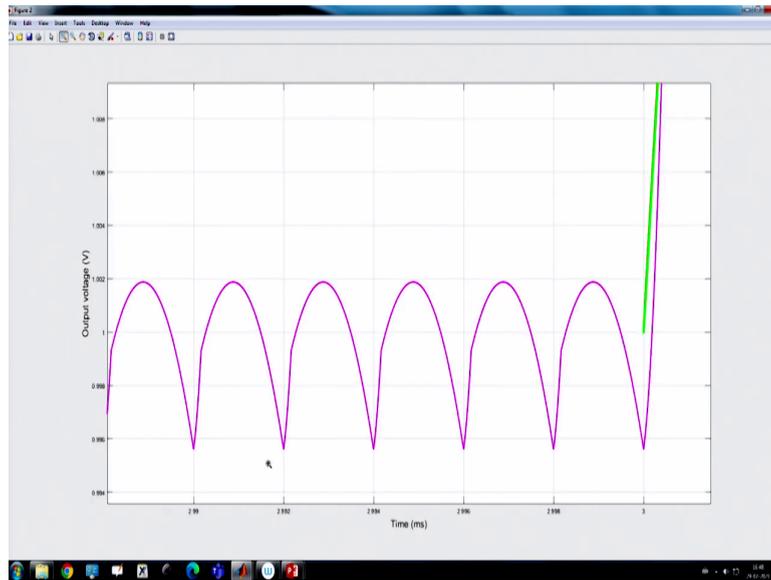


So, I will go one by step by step. This is my transient simulation matching.

(Refer Slide Time: 48:56)

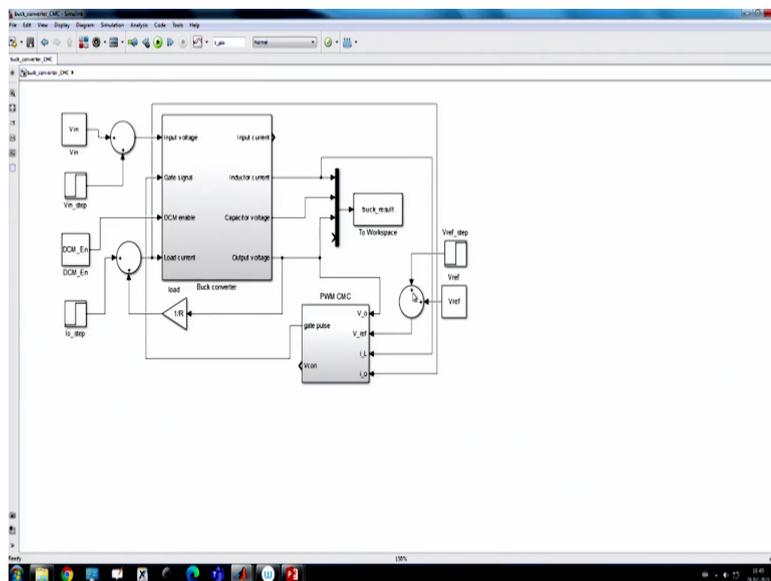


(Refer Slide Time: 49:00)



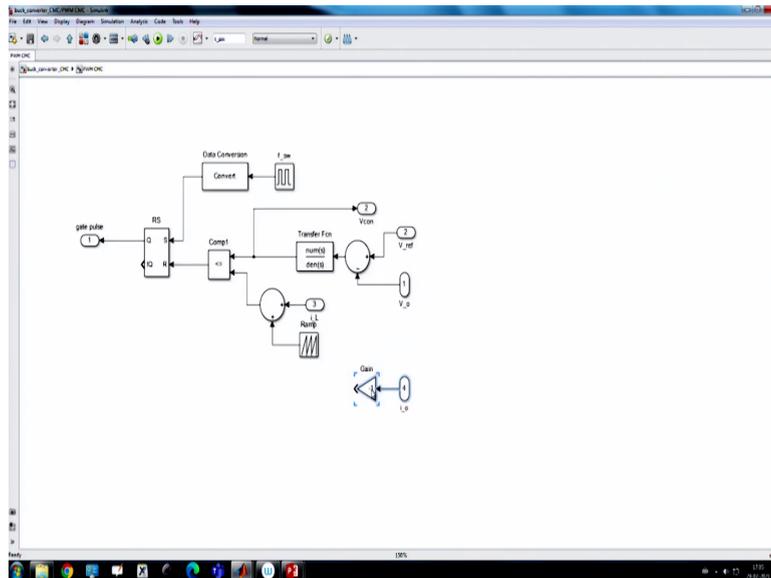
So, let us go back. So, the magenta colour is the actual switch simulation that we are getting from a closed loop and if you go back to the closed loop that means, I have a buck converter which is implemented closed loop implementation.

(Refer Slide Time: 49:12)



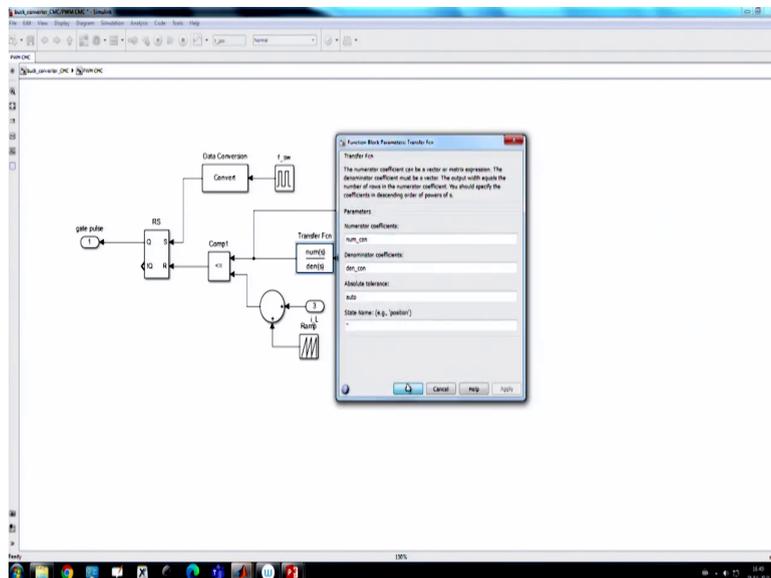
So, this is my buck converter I am calling it.

(Refer Slide Time: 49:17)



And I am using there is no. So, this is my ramp, so I need to use ramp, this is my inductor current.

(Refer Slide Time: 49:24)



This transfer function I am getting from I plugin from the MATLAB code, but here there is a provision for load feed forward, but I am not adding yet ok. Next, so this is my MATLAB model ok.

(Refer Slide Time: 49:41)

```

80 delta_step=delta_Vref;
81
82 [y_s_1,t_s_1]=step(G_cl_1,(t_sim-t_step));
83 v_ac_1=delta_step*y_s_1;
84 [y_s_2,t_s_2]=step(G_cl_2,(t_sim-t_step));
85 v_ac_2=delta_step*y_s_2;
86 [y_s_r,t_s_r]=step(G_cl_r,(t_sim-t_step));
87 v_ac_r=delta_step*y_s_r;
88
89 %% Switch simulation and plotting
90 buck_converter_simulation;
91
92 %% Model validation
93 figure(2)
94 plot(t_s_1+t_step)*1e3, Vref+v_ac_1,'LineWidth',4);
95 hold on; grid on;
96 plot(t_s_2+t_step)*1e3, Vref+v_ac_2,'LineWidth',4);
97 hold on; grid on;
98 plot(t_s_r+t_step)*1e3, Vref+v_ac_r,'LineWidth',4);
99 hold on; grid on;
100 xlabel('Time (ms)','FontSize',15);
101 ylabel('Output voltage (V)','FontSize',15);
102

```

Command Window:

```

Set ramp voltage in V 0.001
Select BW in kHz ... 50

```

(Refer Slide Time: 49:47)

```

80 delta_step=delta
81
82 [y_s_1,t_s_1]=ste
83 v_ac_1=delta_ste
84 [y_s_2,t_s_2]=ste
85 v_ac_2=delta_ste
86 [y_s_r,t_s_r]=ste
87 v_ac_r=delta_ste
88
89 %% Switch simulation and plotting
90 buck_converter_simulation;
91
92 %% Model validation
93 figure(2)
94 plot(t_s_1+t_step)*1e3, Vref+v_ac_1,'LineWidth',4);
95 hold on; grid on;
96 plot(t_s_2+t_step)*1e3, Vref+v_ac_2,'LineWidth',4);
97 hold on; grid on;
98 plot(t_s_r+t_step)*1e3, Vref+v_ac_r,'LineWidth',4);
99 hold on; grid on;
100 xlabel('Time (ms)','FontSize',15);
101 ylabel('Output voltage (V)','FontSize',15);
102

```

Bode Diagram:

Command Window:

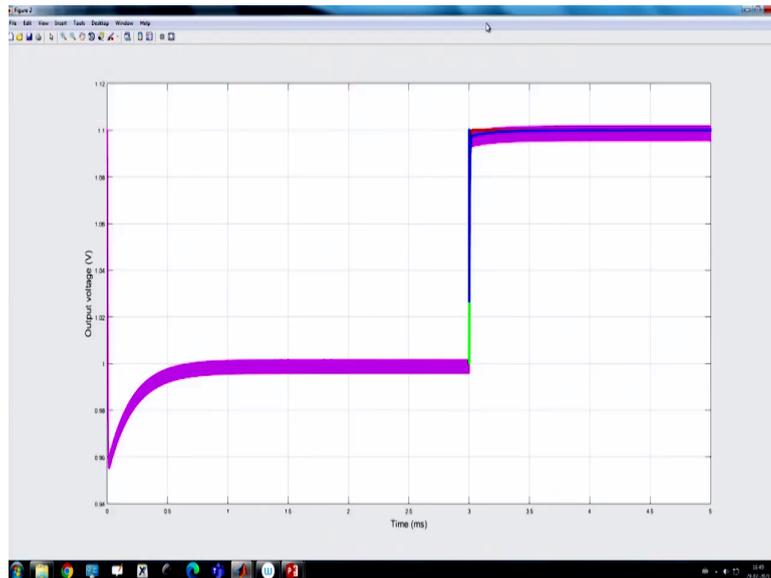
```

loop(s). To see more
details about the loops
use the command
Simulink.BlockDiagram.getAlgebraic
or the command line
Simulink.debugger by
typing 'sdebug
buck_converter_CMC' in the
MATLAB command window. To
eliminate this message,
set the Algebraic loop
option in the Diagnostics
page of the Simulation
Parameters Dialog to
'None'
> In buck_converter_simulation (lin
In Model_comp_code (line 90)
Found algebraic loop containing:
'buck_converter_CMC/Buck conver
'buck_converter_CMC/Buck conver
'buck_converter_CMC/load'
'buck_converter_CMC/Sum' (algebr

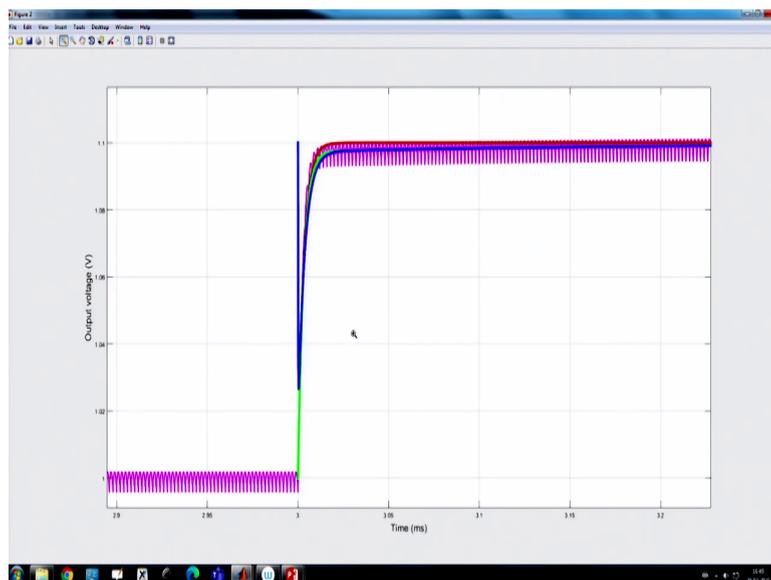
```

Now, I am just once more, I am running. So, it will ask for ramp voltage and let us see 50 kilohertz bandwidth.

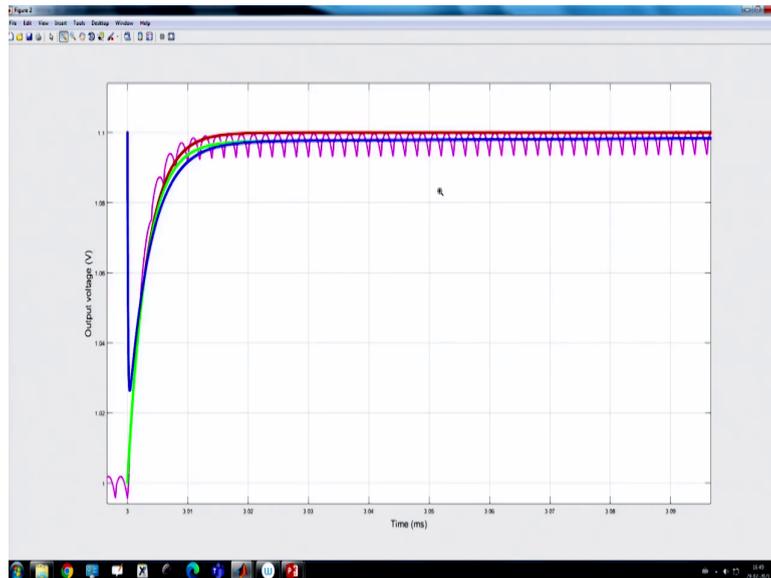
(Refer Slide Time: 49:51)



(Refer Slide Time: 49:55)

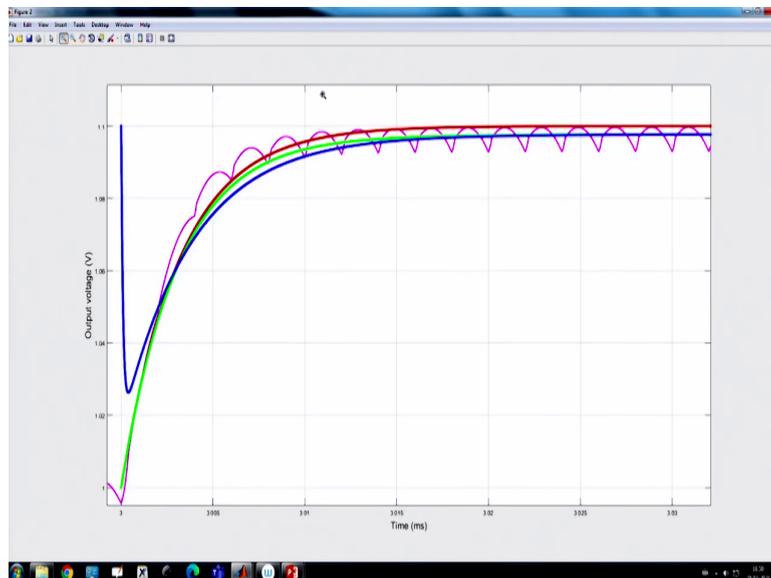


(Refer Slide Time: 49:58)



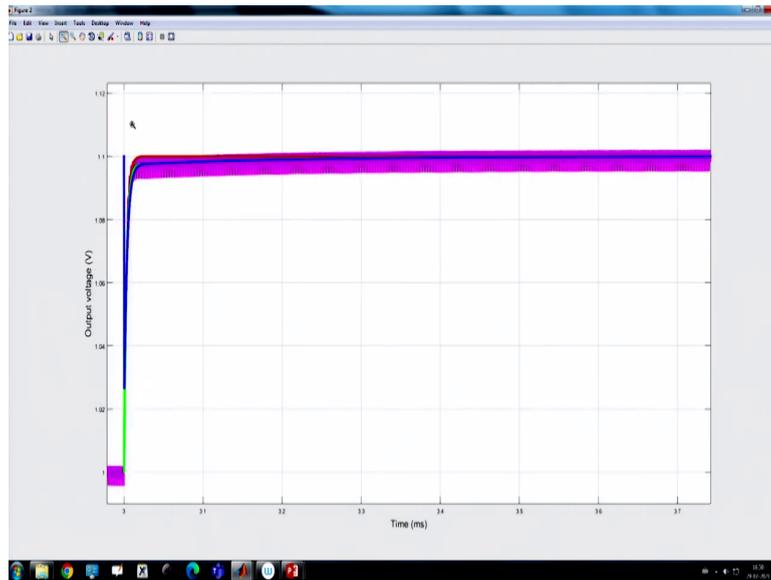
So, you will see that the red one is coming from my first-order model and which is there is a slight mismatch in terms of ripple voltage.

(Refer Slide Time: 50:12)

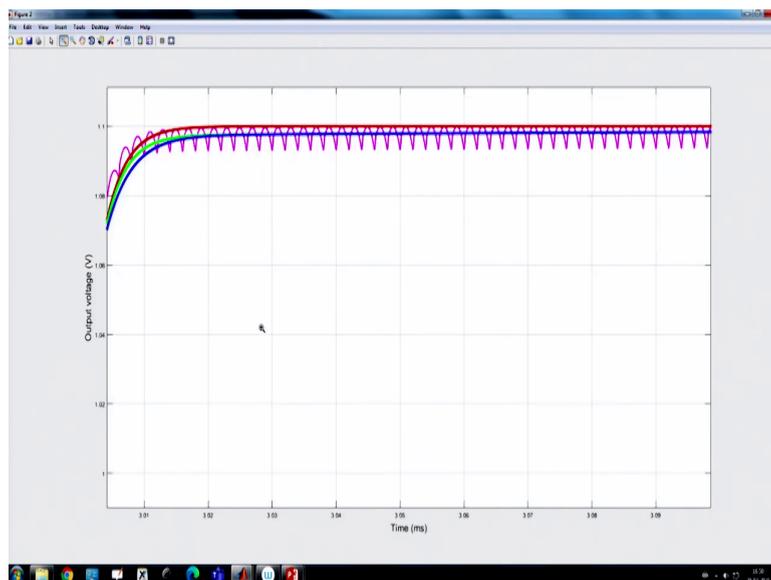


Otherwise it is matching green one is coming from the accurate model ok and the blue one is coming from Ridley model.

(Refer Slide Time: 50:16)

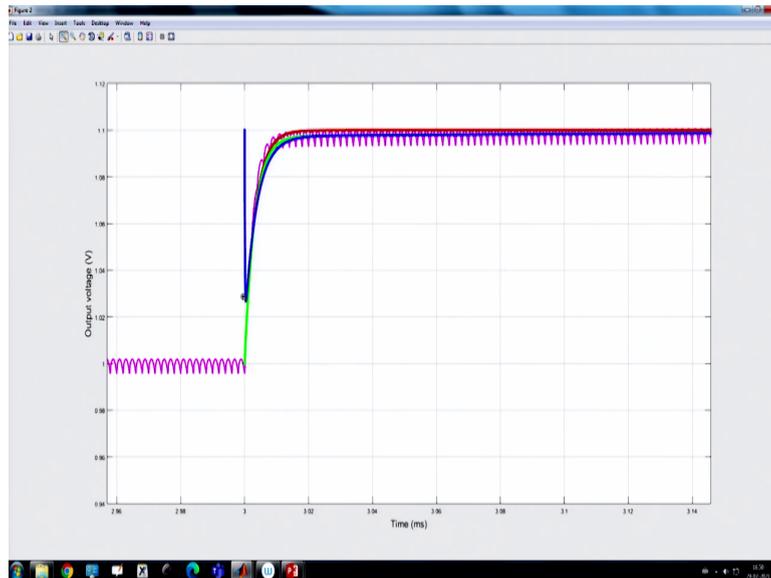


(Refer Slide Time: 50:19)

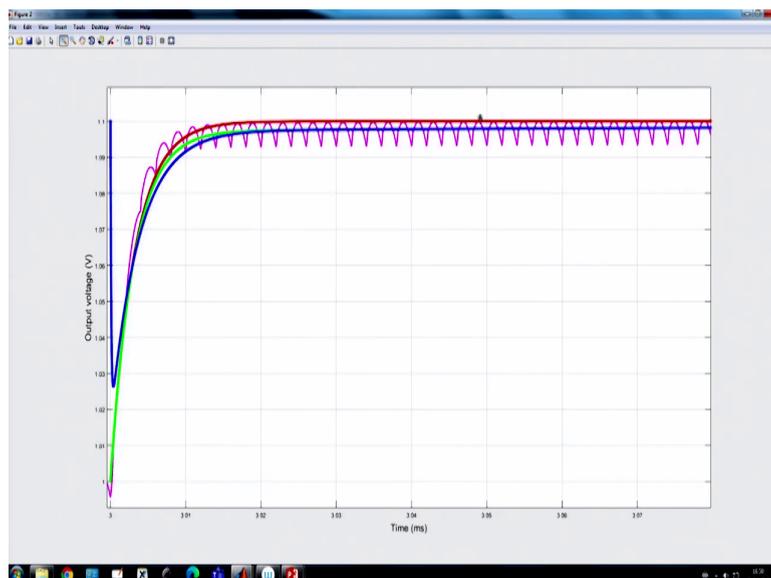


So, if you see you know we are interested in this part.

(Refer Slide Time: 50:20)

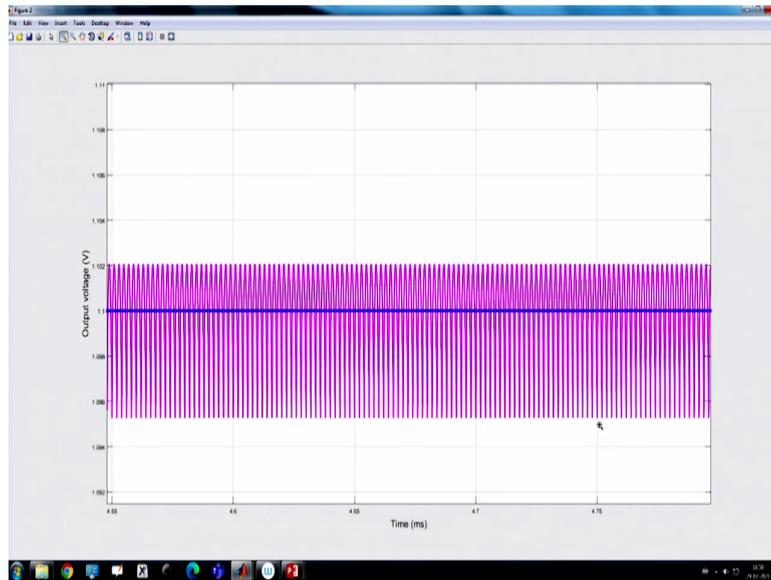


(Refer Slide Time: 50:23)



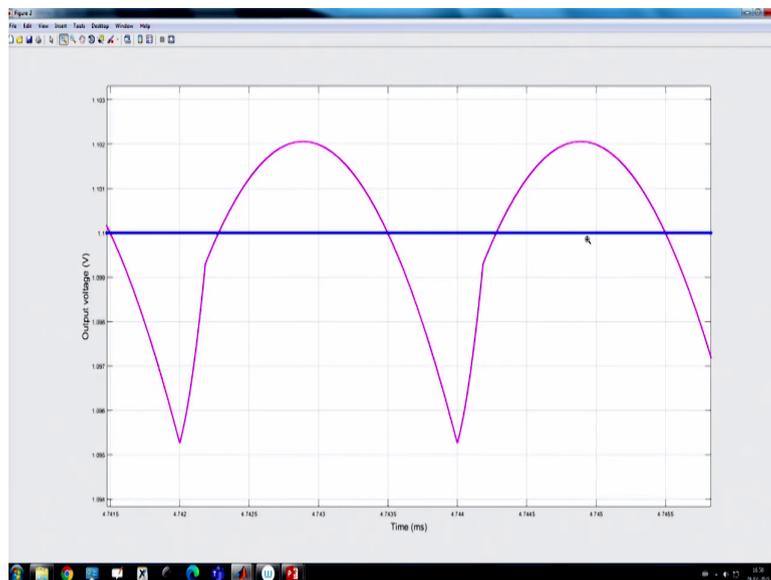
That means we are interested in this particular part. So, all these three models are matching more or less fine only.

(Refer Slide Time: 50:32)



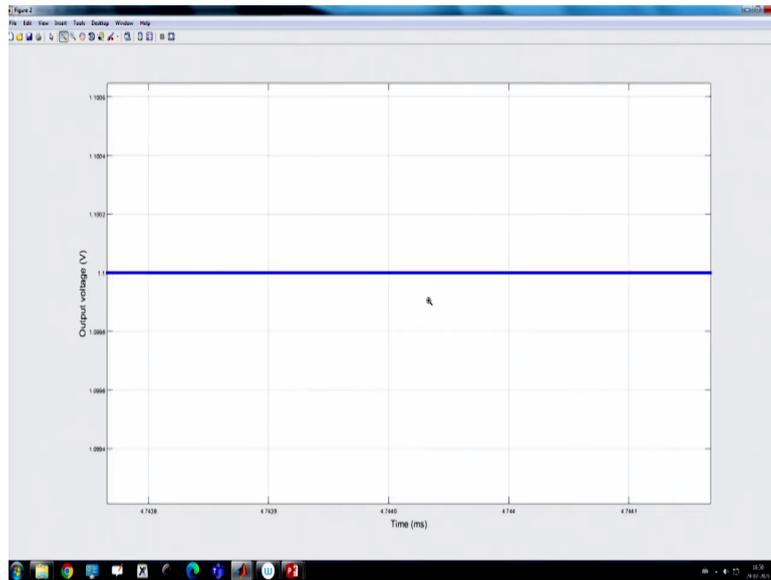
And if you wait for a long time, they will eventually merge together.

(Refer Slide Time: 50:34)

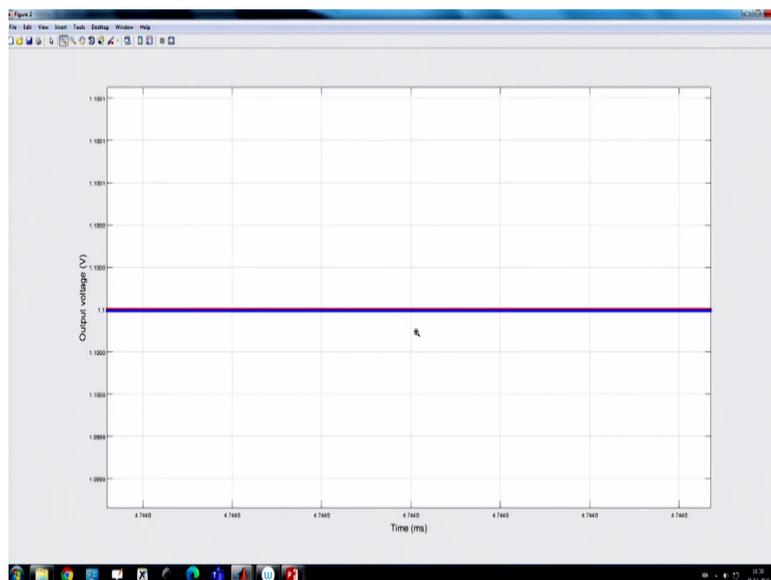


All three model will actually merge together.

(Refer Slide Time: 50:35)

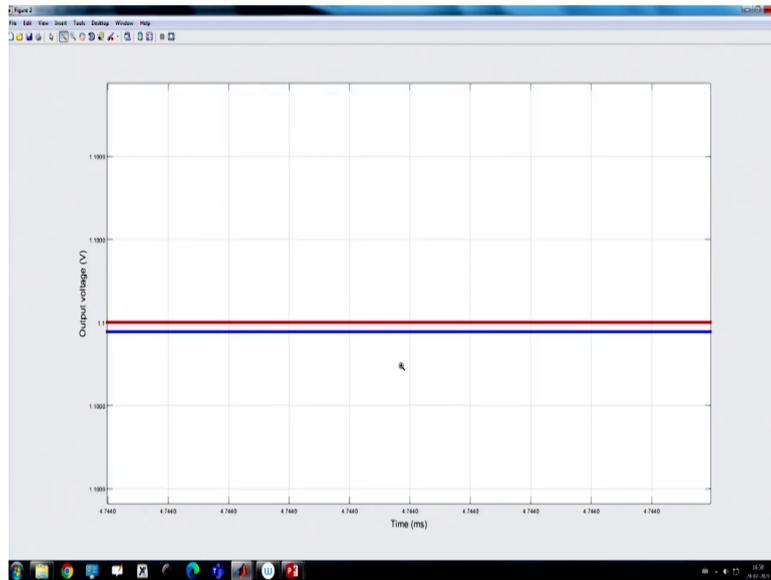


(Refer Slide Time: 50:36)

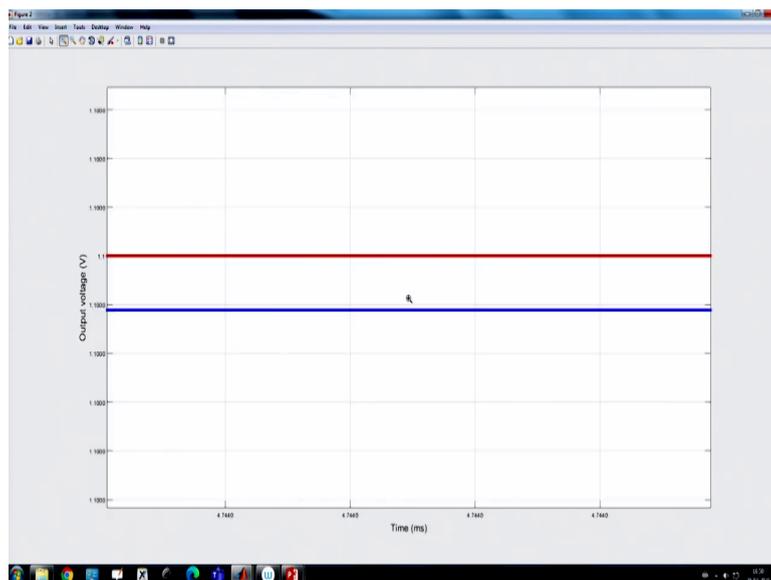


So, they will merge together.

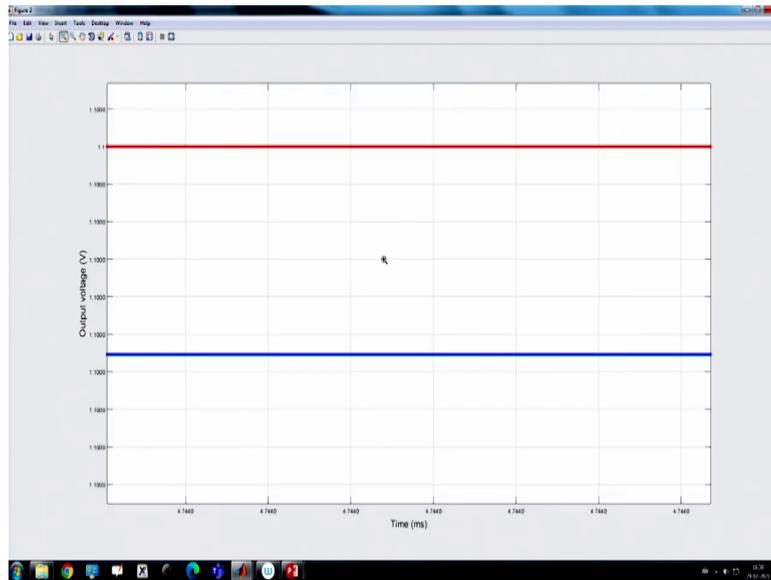
(Refer Slide Time: 50:37)



(Refer Slide Time: 50:38)

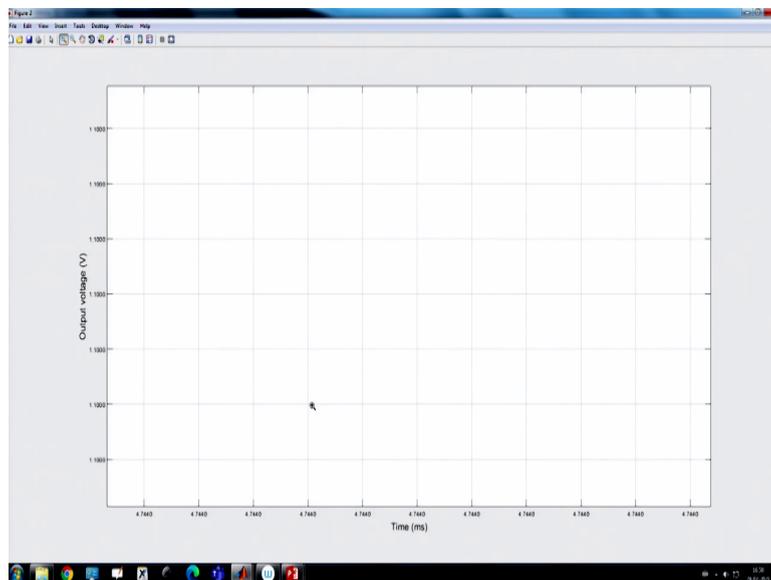


(Refer Slide Time: 50:39)



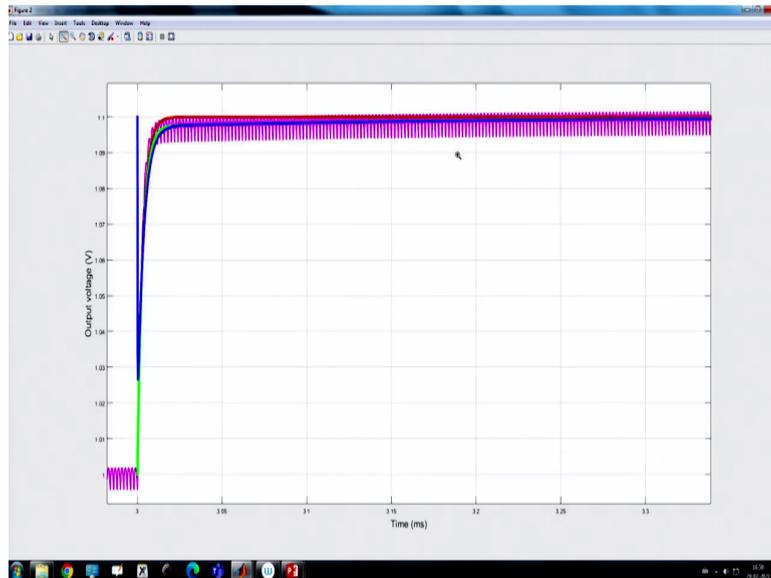
All three models are there ok.

(Refer Slide Time: 50:41)



So, you can see there is a green color stress is there. So, they will merge together.

(Refer Slide Time: 50:50)



But there is a slight difference in the transient performance, but that is fine and here we are running at 12 volt ok. So, 12 volt input and 1 volt out, now I want to see what happens? In the same condition if we add a ramp compensation. First let us see what happens, if we want to increase the bandwidth to one feed rather than 1 tenth.

(Refer Slide Time: 51:09)

```
close all; clear; clc;

%% Parameters
buck_parameter: Vin=12; Vref=1;
D=Vref/Vin; M1={Vin-Vref}/L;
R=1; f_sw=1/T;
r_eq=r_L+r_1; alpha=(R+r_eq)/R;
k2=(0.5-D)*(T/L);

%% Modulator Gain
V_m=input('Set ramp voltage in V ');
Fm=1/V_m; Mc=V_m/T;

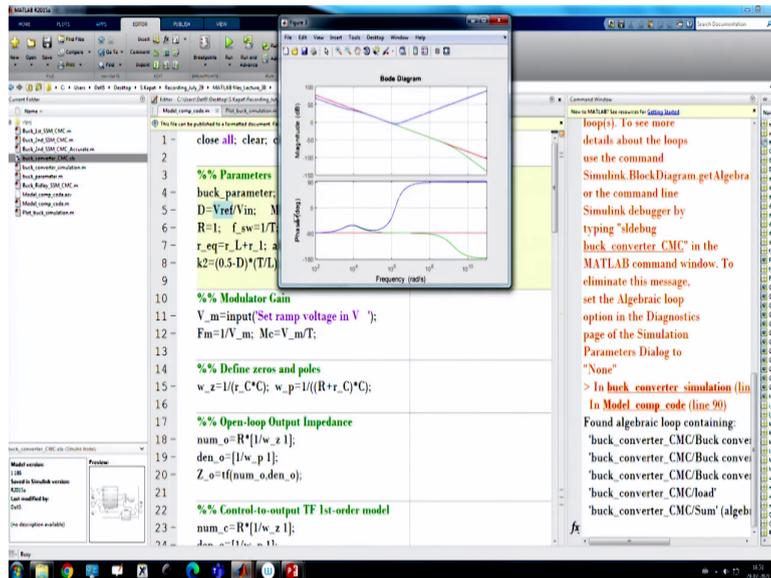
%% Define zeros and poles
w_z=1/(r_C*C); w_p=1/((R+r_C)*C);

%% Open-loop Output Impedance
num_o=R*(1/w_z);
den_o=[1 w_p];
Z_o=tf(num_o,den_o);

%% Control-to-output TF 1st-order model
num_c=R*(1/w_z);
den_c=[1 f_sw];
```

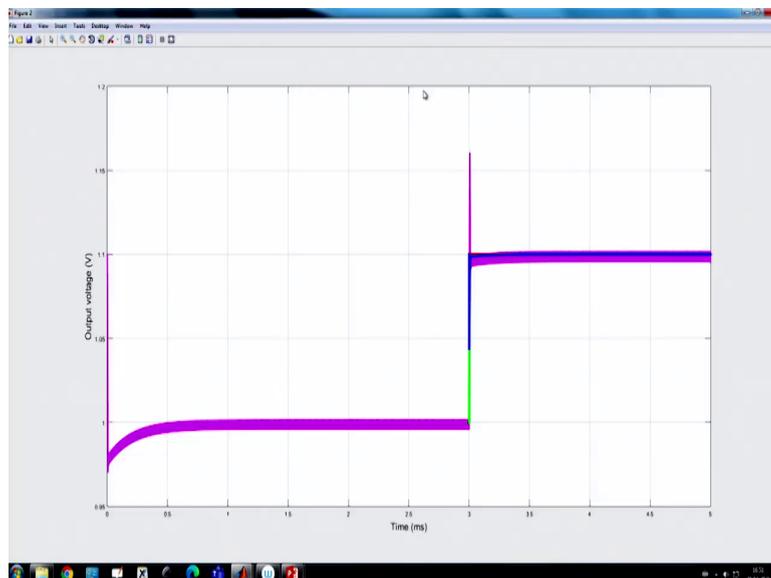
So, again, we call we are using a very negligible amount of ramp because 12 volt 1 volt does not require any ramp and now we are going for 100 kilohertz.

(Refer Slide Time: 51:18)



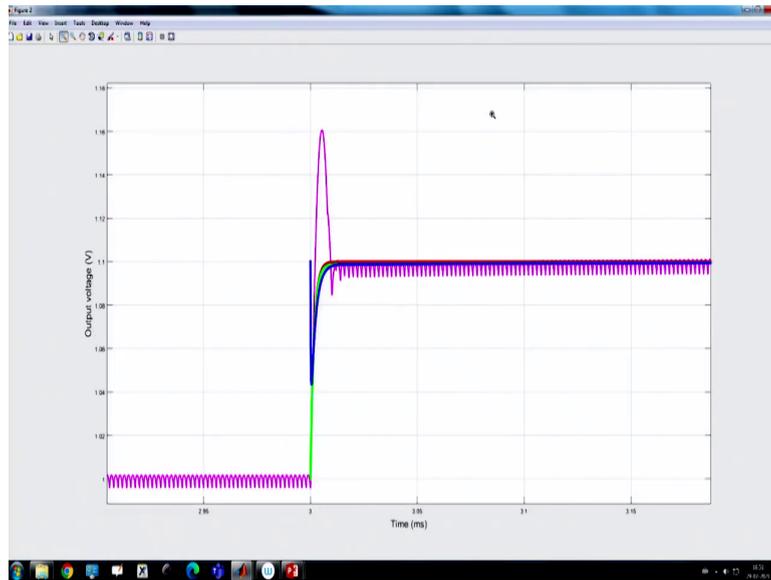
So, we are trying to push the bandwidth.

(Refer Slide Time: 51:21)

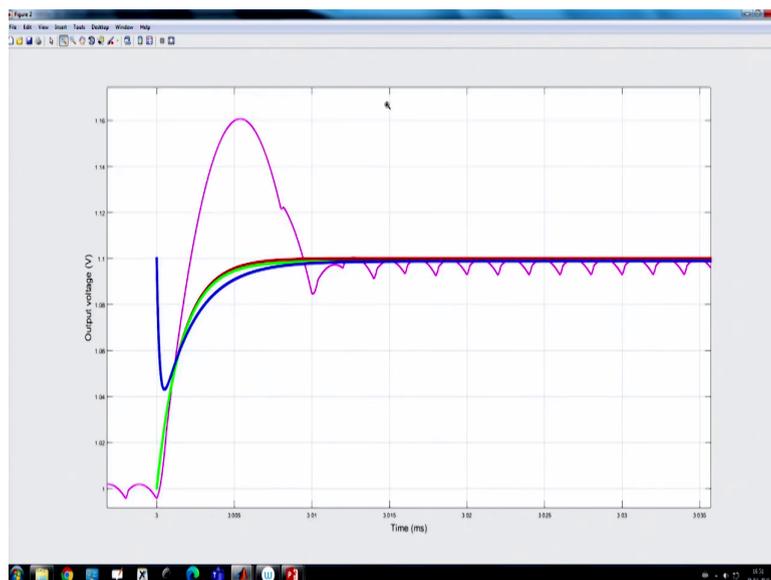


And we want to see how fast.

(Refer Slide Time: 51:25)



(Refer Slide Time: 51:28)



So, you see, none of the model actually capture this behaviour actual switch simulation because in actual switch simulator there is a huge overshoot because the model is not valid. I mean, it is inaccurate when it tries to push the bandwidth you know at close to one fifth of the switching frequency; that means, we should not try to do that.

(Refer Slide Time: 51:47)

```

1 close all; clear; clc;
2
3
4 %% Parameters
5 buck_parameter; Vin=12; Vref=1;
6 D=Vref/Vin; M1=(Vin-Vref)/L;
7 R=1; f_sw=1/T;
8 r_eq=r_L+r_1; alpha=(R+r_eq)/R;
9 k2=(0.5-D)*(T/L);
10
11 %% Modulator Gain
12 V_m=input('Set ramp voltage in V ');
13 Fm=1/V_m; Mc=V_m/T;
14
15 %% Define zeros and poles
16 w_z=1/(r_C*C); w_p=1/((R+r_C)*C);
17
18 %% Open-loop Output Impedance
19 num_o=R*(1/w_z);
20 den_o=[1 w_p];
21 Z_o=tf(num_o,den_o);
22
23 %% Control-to-output TF 1st-order model
24 num_c=R*(1/w_z);
25 J_den_c=[1 f_sw, 0, 0];
  
```

Another way if we try to increase let us say 0.001 let us say our 500 kilohertz is my switching frequency. So, I want to obtain 500 divided by let us say sixth 8 times instead of one tenth I will 1 8.

(Refer Slide Time: 52:03)

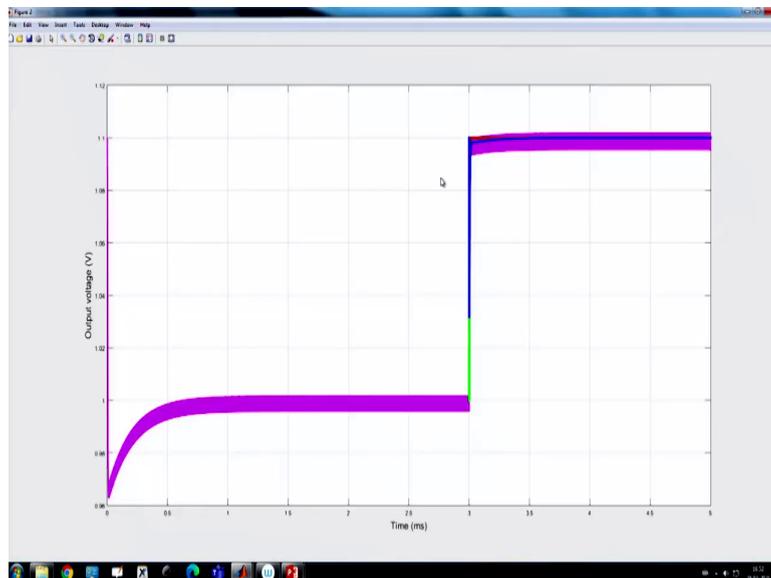
Command Window:

```

> In buck_converter_simulation (lin
In Model_comp_code (line 90)
Found algebraic loop containing:
'buck_converter_CMC/Buck conver
'buck_converter_CMC/Buck conver
'buck_converter_CMC/Buck conver
'buck_converter_CMC/load
'buck_converter_CMC/Sum' (algebr
  
```

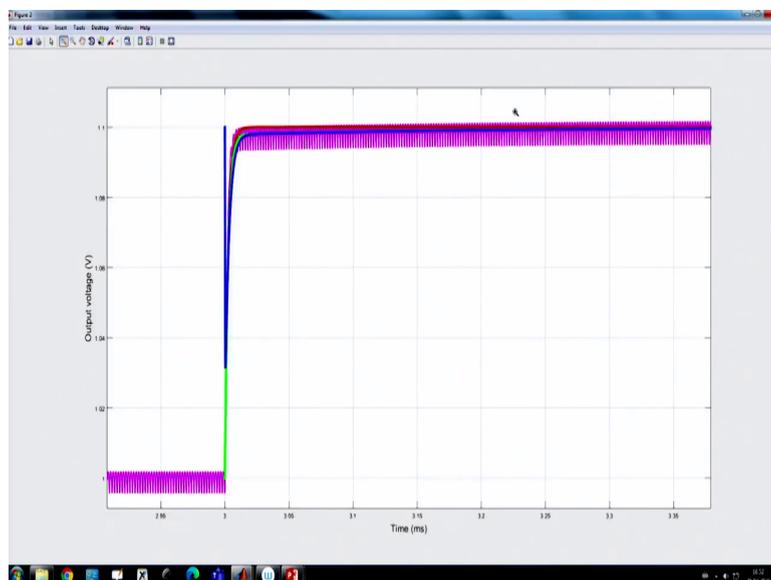
And let us see how far they match.

(Refer Slide Time: 52:07)

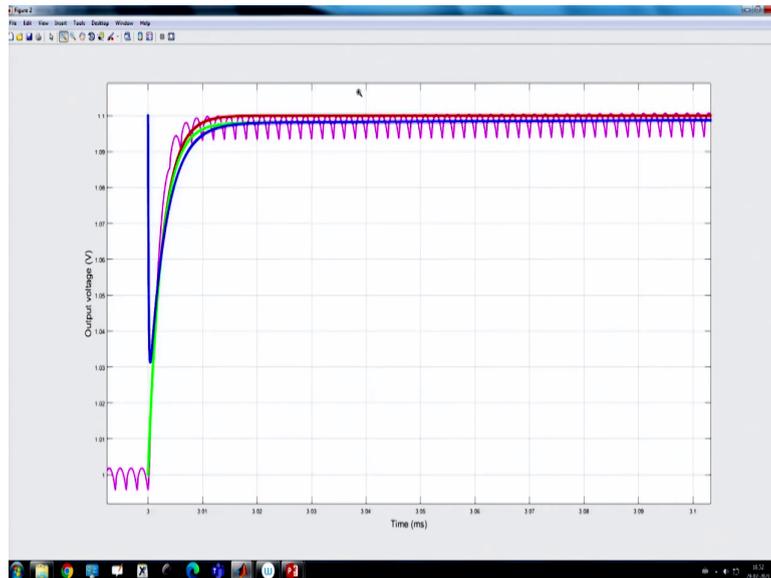


So, 1 8 they matches somewhat better.

(Refer Slide Time: 52:12)

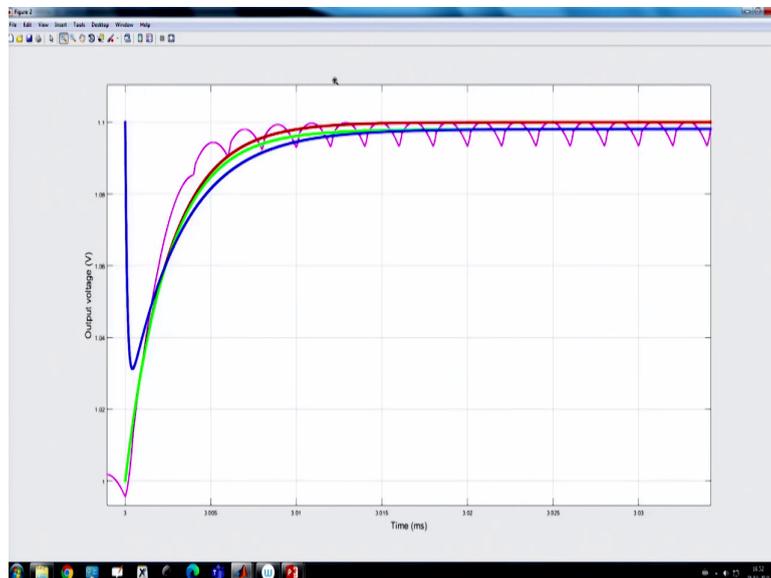


(Refer Slide Time: 52:15)



That means, we can push a bandwidth a little higher, like a 1 with 1 eighth in voltage mode control we got 1 tenth. So, in current mode control, we can go like 1 eighth or so right.

(Refer Slide Time: 52:26)



So, bandwidth and you see the model they are matching reasonably well. Now, we want to see what happens if we want to add a large ramp.

(Refer Slide Time: 52:34)

```

1 close all; clear; clc;
2
3 %% Parameters
4 buck_parameter; Vin=12; Vref=1;
5 D=Vref/Vin; M1=(Vin-Vref)/L;
6 R=1; f_sw=1/T;
7 r_eq=r_L+r_1; alpha=(R+r_eq)/R;
8 k2=(0.5*D)*(T/L);
9
10 %% Modulator Gain
11 V_m=input('Set ramp voltage in V ');
12 Fm=1/V_m; Mc=V_m/T;
13
14 %% Define zeros and poles
15 w_z=1/(r_C*C); w_p=1/((R+r_C)*C);
16
17 %% Open-loop Output Impedance
18 num_o=R*(1/w_z+1);
19 den_o=[1/w_z p 1];
20 Z_o=tf(num_o,den_o);
21
22 %% Control-to-output TF 1st-order model
23 num_c=R*(1/w_z+1);
24 J_m_s=-11 f_m_s, s, 1);
  
```

So, we want to consider the ramp is we have use a over compensated ramp for 10 volt I am giving ramp when it is not required, but I want to check whether the model is valid or not and 50 kilo hertz is my bandwidth.

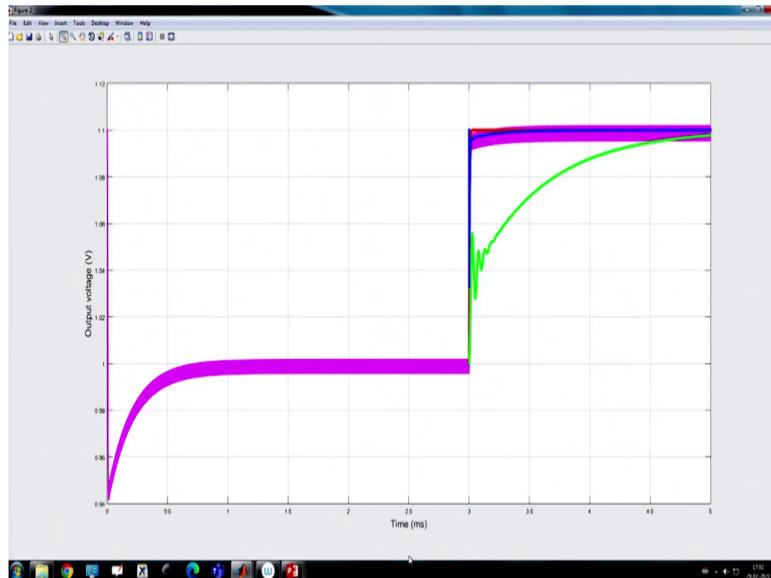
(Refer Slide Time: 52:48)

```

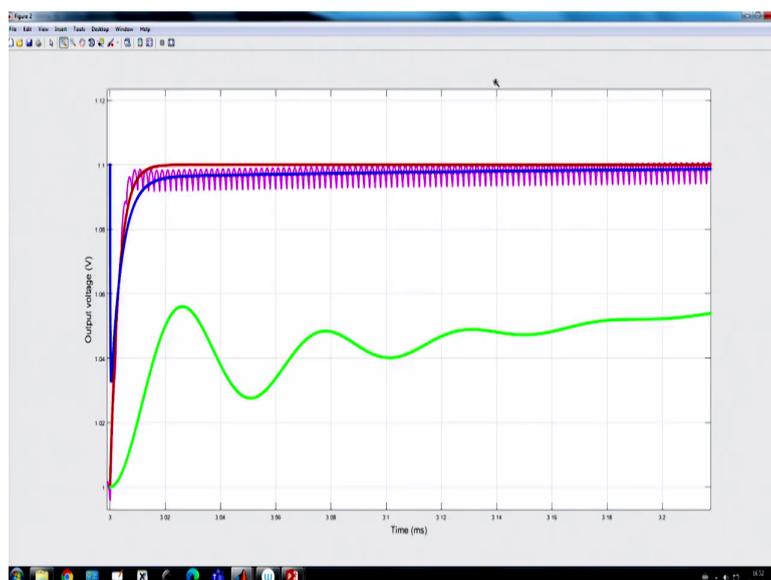
> In buck_converter_simulation (lin
In Model_comp_code (line 90)
Found algebraic loop containing:
'buck_converter_CMC/Buck convert
'buck_converter_CMC/Buck convert
'buck_converter_CMC/Buck convert
'buck_converter_CMC/load
'buck_converter_CMC/Sum' (algebraic
  
```

I am setting now I am trying to match.

(Refer Slide Time: 52:51)

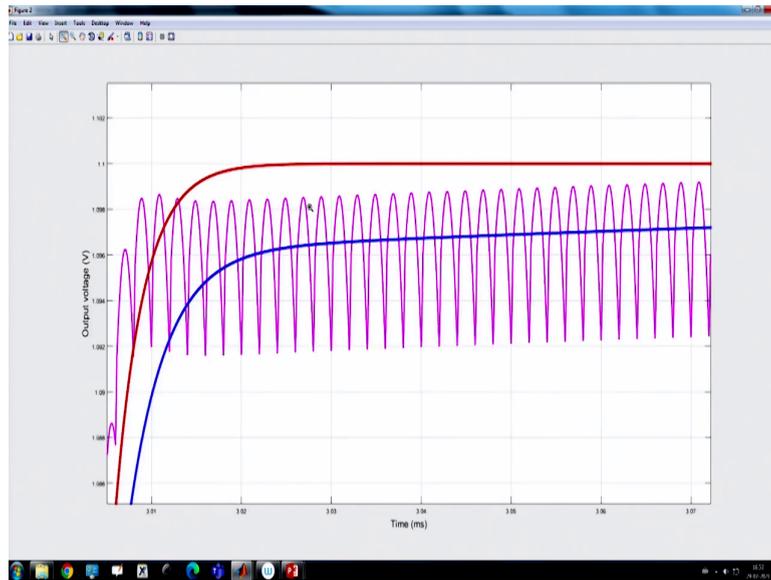


(Refer Slide Time: 52:56)



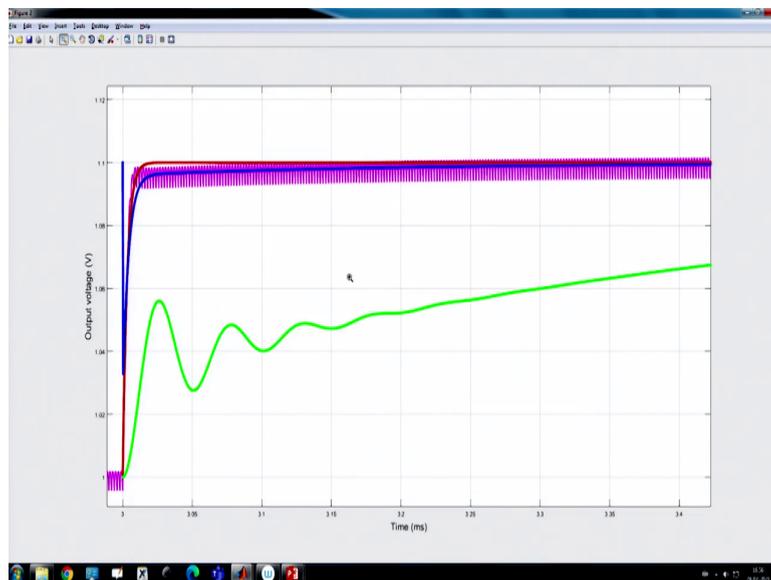
You see that the Ridley model is matching fine ok and the actual the first-order model matching fine.

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With slight mismatch but the Ridley model is more accurate.

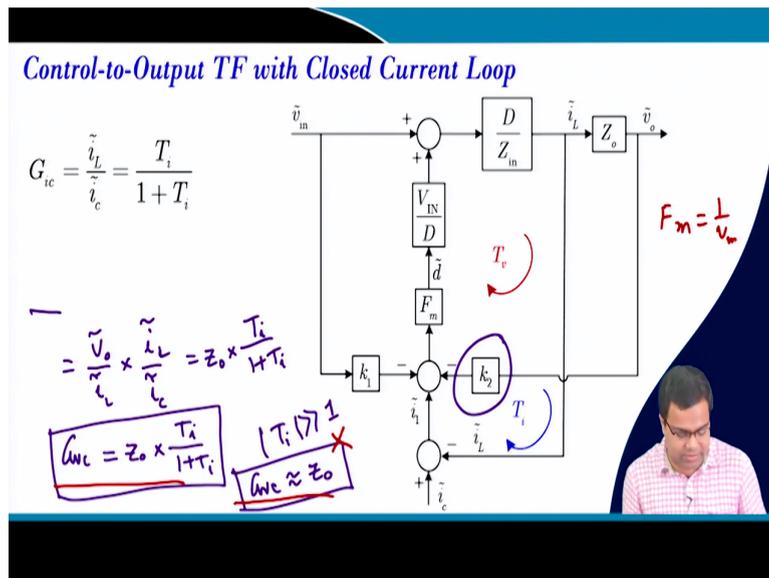
(Refer Slide Time: 53:11)



But first-order is also good enough, but accurate model is not matching because if you go back and if you check the accurate model, you see the current loop gain if you consider the current loop gain in this expression the current loop here because it is T_v by T_v . So, even if we ignore this effect in the current loop F_m will come in the forward path that means, the current loop can be approximated as if this k_2 is set to 0.

Then the current loop can be written as the current loop transfer function is simply $F_m V_m$ in Z in right. So, if you and what is F_m it is 1 by V_m . So, if you take a large ramp voltage, then 1 by V_m is small. So, that the modulator gain is small and if the modulator gain is small, then the current loop gain decreases and if the current loop gain decreases, then our discussion here. So, this approximation does not work.

(Refer Slide Time: 54:18)



That means, if I take this approximation, this does not work because current loop gain is coming close to 1 and you cannot ignore. So, there is a significant deviation between this and this model, because here this model the modulator gain is simply 1 by V_m in this method 2 accurate method.

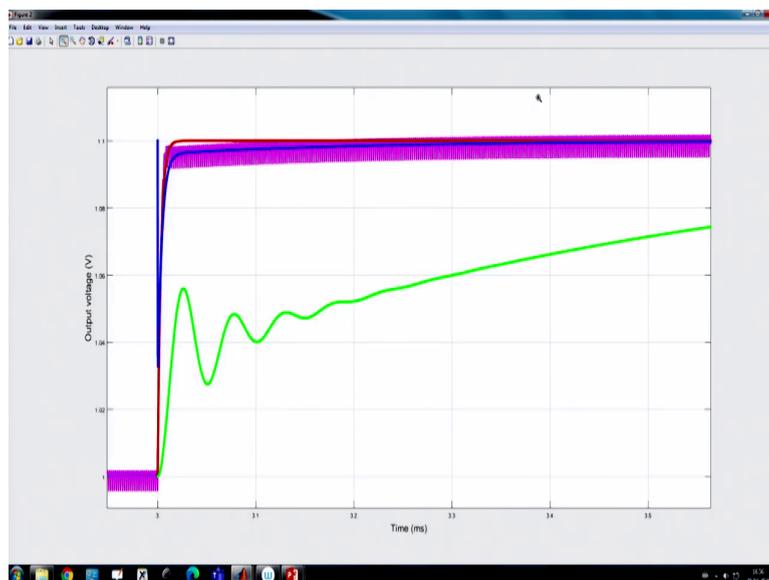
That means, this may that is why there are so many models in current mode control because that fundamentally differs in terms of modulator gain in case of Ridley model the modulator gain is basically it is the. So, in case of Ridley model, so if I go back Ridley model modulator gain was like a F_m was taken as M_c plus 1 M_c is the capital M_c into T that means, it is both the addition of the slope of the ramp if we add it.

And this is because what was the analogy behind this the analogy behind this that if you take a peak current reference and this is my peak current reference, this is my control current. And if you consider inductor current, right? So, we can take the inductor current slope m_1 is like in voltage mode control. So, what we do?

We just take this slope of the ramp 1 by M c into T is my modulator gain in voltage mode control. So, here the analogy is that if there is no ramp composition in this method, F m is simply 1 by m 1 T that means, as if it is like a voltage mode control, the ramp is replaced by inductor current. So, the slope of the inductor current will act like a slope of the ramp.

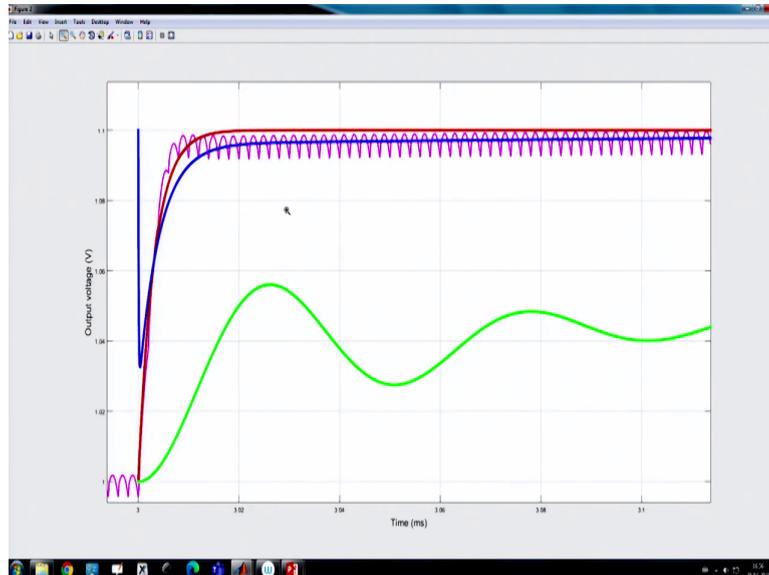
So, which is consistent logically consistent, but if we add ramp then as I told, if we add ramp then the overall current slope will be inductor current rising slope plus ramp slope and that is exactly what is happening. So, that means, this model even, but if you consider the accurate mode in this model, even the modulator gain is decreasing. Because we are adding ramp plus supply the ramp of the slope of the rising slope, but still matching is quite nice because it includes the sampling effect ok.

(Refer Slide Time: 56:42)



So, that means, we can conclude that.

(Refer Slide Time: 56:47)



Even with ramp compensation, our first-order model is reasonably fine, but the Ridley model is more accurate because of the sampling effect.

(Refer Slide Time: 56:56)

Recommendations on CMC Design in SMPC

- Step 1: Design using first-order model – sufficient without ramp compensation
- Step 2: With ramp compensation, design using step~1 should be followed
- Step 3: For both steps, Ridley model should be used to verify frequency response of the loop transfer functions and closed-loop transient response
- Step 4: Finally, MATLAB or other professional software tools should be used to verify AC transient simulation of the design using step 1

And so that means, we consider all and what are the recommendation. So, we can design using first-order model. If there is no ramp compression, this is perfectly fine, if we consider ramp compensation we still can design it, but we should validate for both cases with Ridley

model for frequency response matching and the transient response that we have checked the response coming from the transfer function like AC analysis plus DC offset. And all should be checked with actual switch simulation and all we have done right. So, the final recommendation we can simply design the current mode control using first-order model.

(Refer Slide Time: 57:34)

Small-Signal Control-to-Output TF of a CMC Buck Converter

$$K_{\text{loop}}(s) = G_{\text{vc}} \times G_c$$

$$G_{\text{vc}}(s) = \frac{R \left(1 + \frac{s}{\omega_{\text{esr}}} \right)}{\left(1 + \frac{s}{\omega_p} \right)}$$

$$\omega_{\text{esr}} = \frac{1}{r_c C}$$

$$\omega_p = \frac{1}{(R + r_c) C}$$

Now, with this the control to output transfer function, we want to design current mode control. So, the current mode control now, we are designing with a first-order model. So, this is a control to output transfer function. Then we are designing a compensator, and this is my loop transfer function. So, what is the loop transformation? It is a product of the control to output transformation into the controller and this is my control to output transfer function for the first-order model and we know all this expression already we have discussed.

(Refer Slide Time: 58:03)

Loop-Gain of a CMC Buck Converter

$$K_{\text{loop}}(s) = G_{vc} \times G_c$$

$$\therefore K_{\text{loop}}(s) = \frac{R \left(1 + \frac{s}{\omega_{\text{esr}}} \right)}{\left(1 + \frac{s}{\omega_p} \right)} \times G_c$$

- What should be the structure of the compensator?
- Depend on open-loop poles/zeros and loop shaping objectives

Now, the loop gain analysis. So, we want to design this compensator and we want to shape the loop. So, what should be the structure of the compensator? It depends on a number of poles and 0. So, we have one zero one pole

(Refer Slide Time: 58:17)

Open Loop Pole/Zeros: Buck Converter

$$K_{\text{loop}}(s) = \frac{R \left(1 + \frac{s}{\omega_{\text{esr}}} \right)}{\left(1 + \frac{s}{\omega_p} \right)} \times G_c$$

- **ESR zero** – Located at high frequency for lower ESR
- **Single real pole** – Closer to imaginary axis at higher R, making it slower transient response
- **DC gain** – High at light load, leading load dependent DC gain, resulting poor load regulation

So, the first thing we have a ESR zero which is located at high frequency. We can cancel by using a controller pole we have a single real pole and where which actually make the response slower if the load resistance is high and at DC we have to also a DC gain in that loop transfer function the DC gain is a load resistance dependent.

(Refer Slide Time: 58:45)

Primary Loop Shaping Objectives

$$K_{\text{loop}}(s) = \frac{R \left(1 + \frac{s}{\omega_{\text{esr}}} \right)}{\left(1 + \frac{s}{\omega_p} \right)} \times G_c$$

- To cancel ESR zero – to eliminate high frequency ripple (1P)
- To cancel single pole – to arbitrary place CL pole (1Z)
- To consider an integrator – to eliminate SS error (1P)



So, it has poor load regulation. So, the primary loop shaping objective we want to cancel ESR zero by considering one pole in the controller. We want to cancel single pole by considering a 0 in the controller. So, the controller requires one zero one pole and then we need to introduce one integrator because we want to reach 0 steady state error or very reduced steady state error and the primary loop shaping objective.

(Refer Slide Time: 59:10)

Primary Loop Shaping Objectives

$$K_{\text{loop}}(s) = \frac{R \left(1 + \frac{s}{\omega_{\text{esr}}} \right)}{\left(1 + \frac{s}{\omega_p} \right)} \times G_c$$

*one zero
two poles including
the pole at origin*

- To consider compensator gain – to meet desired DC gain and crossover frequency
- A type-II compensator needed; PI enough for ideal buck



And then we need to set the compensator gain DC gain to meet desired frequency crossover frequency. So, a type 3 that means, we need how many poles. So, we need one zero and two

poles; two pole that means, with including the pole at origin ok. That means, type 2 compensator is needed type PI may be sufficient if there is no ESR 0 because that we cannot do not need to cancel.

So, for an ideal buck converter or if the ESR is very, very low which is going far away if the ESR goes beyond our much beyond our control bandwidth or even it is go close to the switching frequency then simple PI controller should be enough. That is why in current mode control people use either type 2 or PI. So, they will be more or less synonymous if the pole of the type 2 compensator one of the pole which is used to cancel the ESR zero it is placed much high frequency.

(Refer Slide Time: 60:26)

Perfect Compensation – Buck Converter

$$K_{loop}(s) = \frac{R \left(1 + \frac{s}{\omega_{est}}\right)}{\left(1 + \frac{s}{\omega_p}\right)} \times G_c$$

$$G_c = \frac{k_c \left(1 + \frac{s}{\omega_{cz}}\right)}{s \left(1 + \frac{s}{\omega_{cp}}\right)}$$

$\approx \frac{k_c \left(1 + \frac{s}{\omega_{cz}}\right)}{\omega_{cz} \frac{s}{s}, \omega_{cp} = \omega_{est}}$
 PI

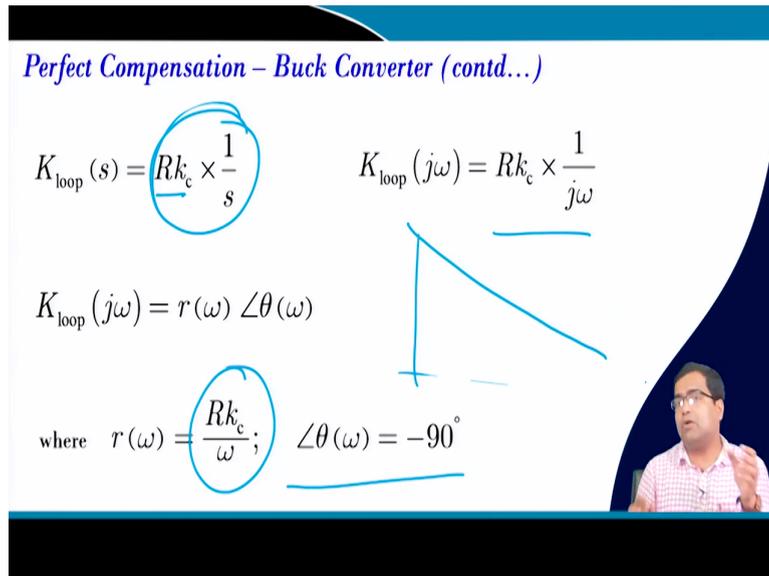
So, that effect is negligible. We need a type two compensator if this pole is kept far away. If this effect is negligible, then this can be approximated as a simply $k_c \frac{1 + s/\omega_{cz}}{s}$. This is the structure of a PI controller. This is the case when the pole of the controller is placed far away and which is used to conceal the ESR 0. If the ESR 0 is left far away right. So, in our condition, the controller 0 is used to cancel the pole of this plant and the controller pole is used to cancel the ESR 0.

(Refer Slide Time: 61:04)

Perfect Compensation – Buck Converter (contd...)

$$K_{\text{loop}}(s) = Rk_c \times \frac{1}{s} \quad K_{\text{loop}}(j\omega) = Rk_c \times \frac{1}{j\omega}$$
$$K_{\text{loop}}(j\omega) = r(\omega) \angle \theta(\omega)$$

where $r(\omega) = \frac{Rk_c}{\omega}$; $\angle \theta(\omega) = -90^\circ$



And if we write the loop transfer function, it is Rk_c because if you go back, if you cancel this and cancel this using the controller. That means, if we cancel this with the controller, this pole and if we cancel this using controller 0 then you will have this term, this term and this term.

So, this is nothing but our loop transfer function and if we write the frequency response by replacing s equal to $j\omega$ then you can separate the real part real part is very simple and imaginary part is only at the denominator. So, you will get a phase of minus 90 degree.

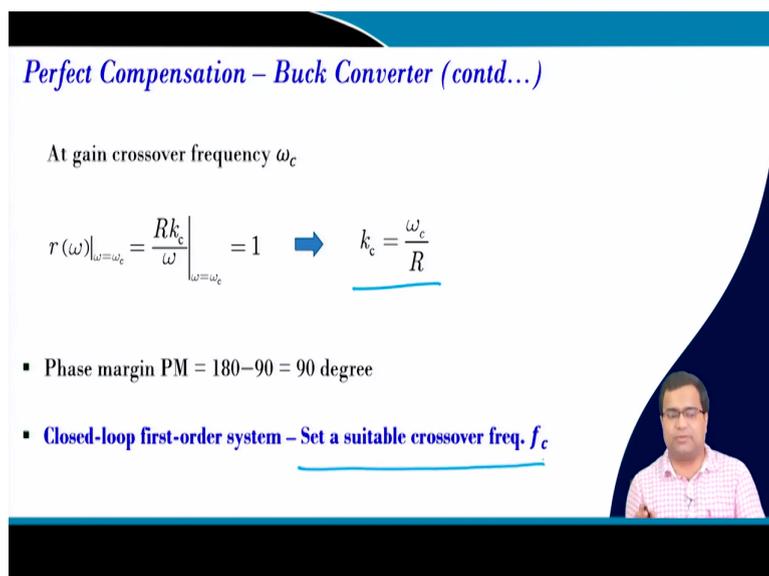
(Refer Slide Time: 61:47)

Perfect Compensation – Buck Converter (contd...)

At gain crossover frequency ω_c

$$r(\omega) \Big|_{\omega=\omega_c} = \frac{Rk_c}{\omega} \Big|_{\omega=\omega_c} = 1 \quad \Rightarrow \quad k_c = \frac{\omega_c}{R}$$

- Phase margin $PM = 180 - 90 = 90$ degree
- Closed-loop first-order system – Set a suitable crossover freq. f_c



So, we are getting a first-order system loop transfer system is a first-order. Whichever minus 20 degree per decade, the gain it is just simply a straight line. The gain will start falling by minus 20 degree per decade ok. Now, we need to select what will be my crossover frequency.

So, at crossover frequency, our gain should be 1 and this gives us the controller gain is simply ω_c by R and then phase margin is always 90 degrees. So, we need to close loop set. So, we have to select the suitable crossover frequency. Now, let us consider one MATLAB case study. Here we are talking about now we under identify the first-order model is good enough.

(Refer Slide Time: 62:21)



(Refer Slide Time: 62:28)

```

MATLAB R2013a
File Edit View System Tools Home
Modeling Simulink
Current Folder: C:\Users\...
Name: Buck_2A_100V_CMC.m
44 - [Gm.Pm,Wcg,Wcp] = margin(G_loop);
45
46 %% Transient parameters and transient response
47 - t_sim=5e-3; t_step=3e-3;
48 - delta_Io=20; delta_Vin=0; delta_Vref=0;
49
50 - [y_s,t_s]=step(Z_oc,(t_sim-t_step));
51 - v_ac=-delta_Io*y_s;
52
53 - buck_converter_simulation;
54
55 - figure(2)
56 - plot(t_s+t_step)*1e3, Vref+v_ac,'LineWidth',4);
57 - hold on; grid on;
58 - xlabel('Time (ms)', 'FontSize', 15);
59 - ylabel('Output voltage (V)', 'FontSize', 15);
60
61
62 % display('f_cgf in kHz')
63 % f_cgf=Wcg/(2*pi*1e3)
64 % display('Phase margin in degree')
65 % Pm
66
Command Window: fx >>
  
```

So, now, we will go for the first model, now you want to make load transient.

(Refer Slide Time: 62:34)

```

1 close all; clear; clc;
2
3 %% Parameters
4 buck_parameter; Vin=12; Vref=1;
5 [t] z_eq=r_L+r_L; alpha=(R+r_eq)/R;
6 f_sw=1T; V_m=0.0001;
7
8 %% Define zeros and poles
9
10 w_z=L/(r_L*C); w_p=1/((R+r_L)*C);
11
12 %% Control-to-output TF Gvd
13 num_c=R*(1/w_z 1);
14 den_c=[1 w_p 1];
15 Gvc=tf(num_c,den_c);
16
17 %% Open-loop Output Impedance
18 num_o=R*(1/w_z 1);
19 den_o=[1 w_p 1];
20 Z_o=tf(num_o,den_o);
21
22 %% Controller parameters
23 %% Type-II compensator
24 f_c=...

```

That means, here we are initially. It is 1 that means, 1 ohm resistance 12 volt input it is in the parameter file.

(Refer Slide Time: 62:43)

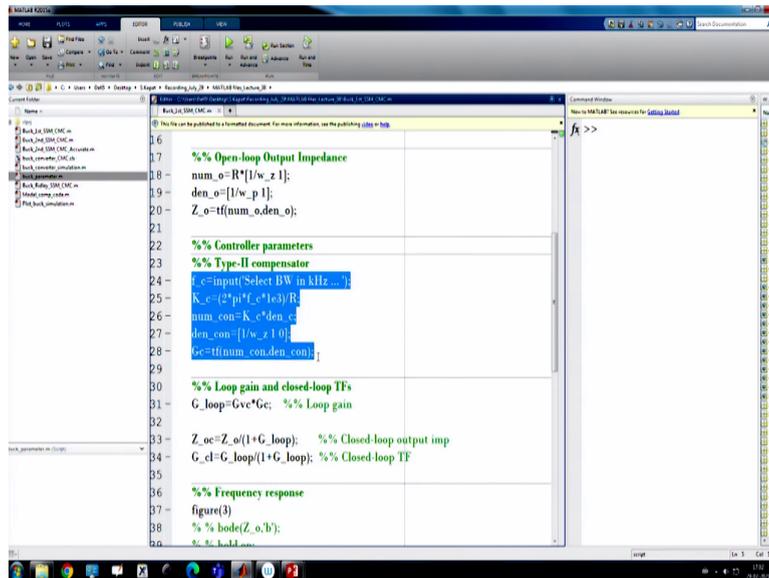
```

1 L=0.5e-6; % output inductance
2 C=200e-6; % output capacitance
3 T=2e-6; % switching time period
4 r_L=5e-3; % inductor DCR
5 r_1=5e-3; % High-side MOSFET on resistance
6 r_d=5e-3;
7 v_d=0*0.55;
8 r_2=r_1; % Low-side MOSFET on resistance
9 r_C=1e-3; % capacitor ESR
10 Vin=12; % input voltage
11 Vref=1; % reference output voltage
12
13

```

If you go to the parameter file input is 12 volt I can take it out. So, I can keep input voltage is 12 volt reference volt is 1 volt initially load resistance 1 ohm and all this derivation we have discussed.

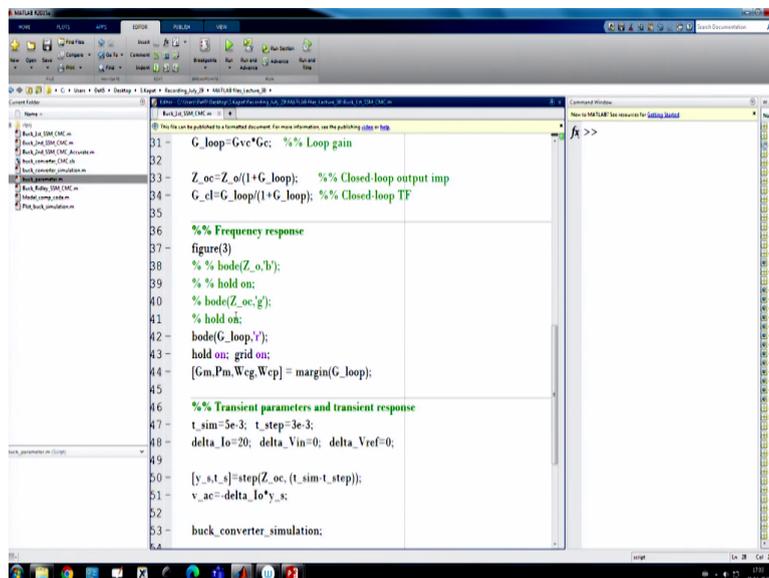
(Refer Slide Time: 62:58)



```
16  
17 %% Open-loop Output Impedance  
18 num_o=R*(1/w_z 1);  
19 den_o=[1 w_p 1];  
20 Z_o=tf(num_o,den_o);  
21  
22 %% Controller parameters  
23 %% Type-II compensator  
24 f_c=input('Select BW in kHz ...');  
25 K_c=(2*pi*f_c*10^3)/R;  
26 num_con=K_c*den_o;  
27 den_con=[1 w_z 1 0];  
28 Gc=tf(num_con,den_con);  
29  
30 %% Loop gain and closed-loop TFs  
31 G_loop=Gvc*Gc; %% Loop gain  
32  
33 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp  
34 G_cl=G_loop/(1+G_loop); %% Closed-loop TF  
35  
36 %% Frequency response  
37 figure(3)  
38 %% % bode(Z_o,'b');  
39
```

So, the compensator is designed using the method I have discussed.

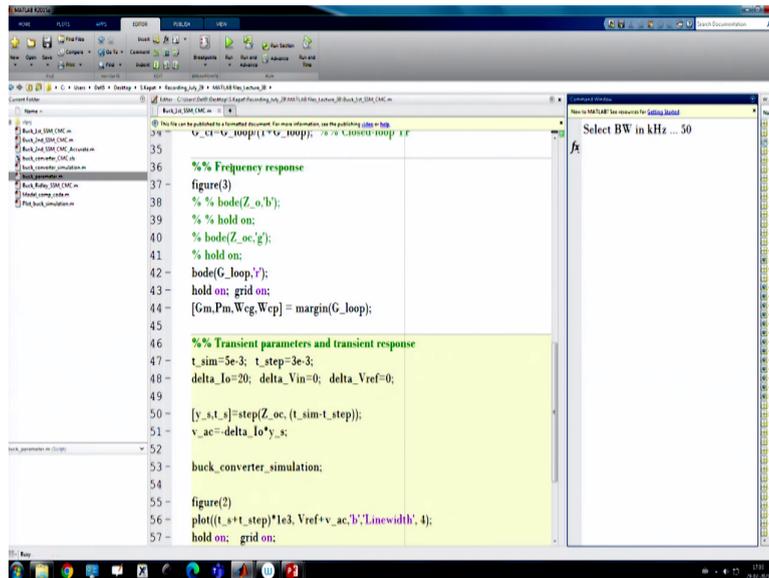
(Refer Slide Time: 63:02)



```
31 - G_loop=Gvc*Gc; %% Loop gain  
32  
33 - Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp  
34 - G_cl=G_loop/(1+G_loop); %% Closed-loop TF  
35  
36 %% Frequency response  
37 figure(3)  
38 %% % bode(Z_o,'b');  
39 %% % hold on;  
40 %% bode(Z_oc,'g');  
41 %% hold on;  
42 bode(G_loop,'r');  
43 hold on; grid on;  
44 [Gm,Pm,Wcg,Wep]=margin(G_loop);  
45  
46 %% Transient parameters and transient response  
47 t_sim=5e-3; t_step=3e-3;  
48 delta Io=20; delta_Vin=0; delta_Vref=0;  
49  
50 [y_s,t_s]=step(Z_oc,(t_sim-t_step));  
51 v_ac=delta_Io*y_s;  
52  
53 buck_converter_simulation;  
54
```

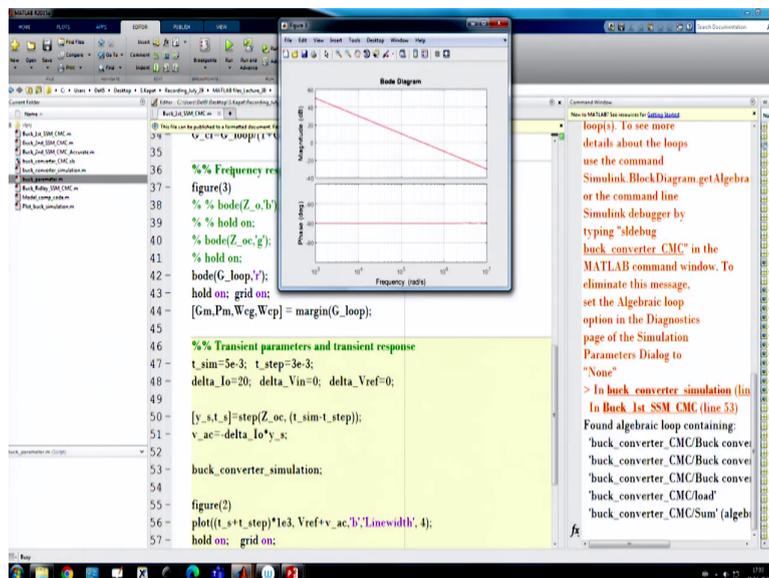
Then you can draw the loop transfer function and then it will plot we are making the load transient response. So, it is a 20 ampere load step we have applied ok.

(Refer Slide Time: 63:13)



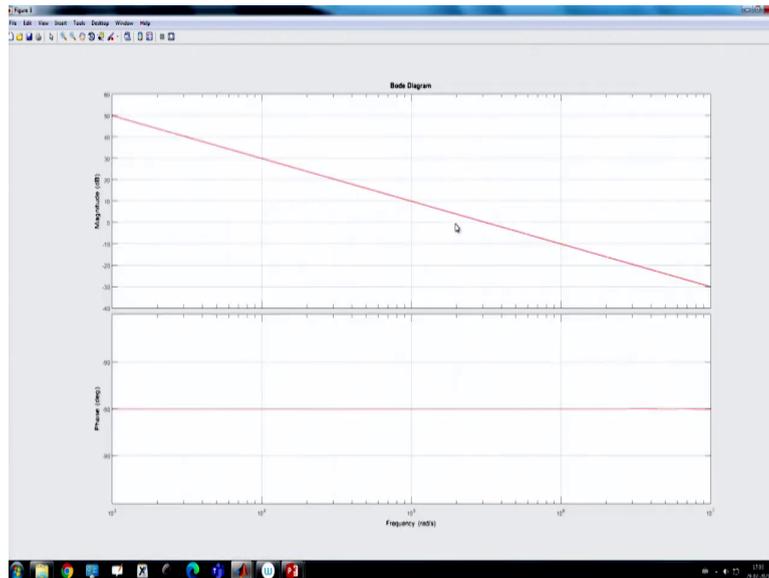
So, if you run it will ask for bandwidth. So, if we say 50 kilohertz.

(Refer Slide Time: 63:17)



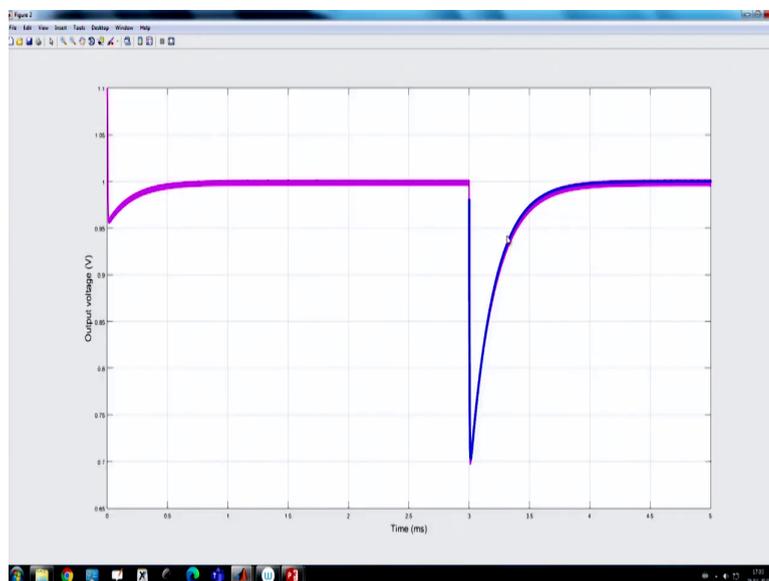
Then let us go and check.

(Refer Slide Time: 63:24)



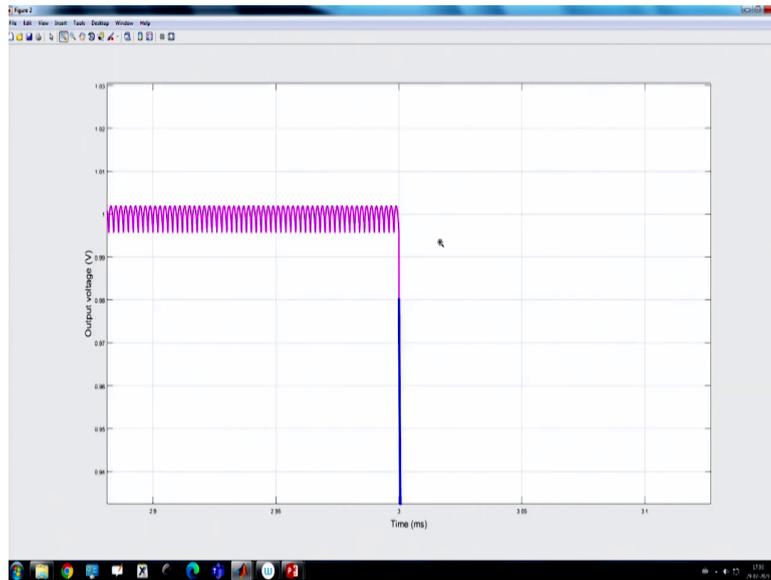
So, you can see the loop transfer function clearly shows that it is just an integrator like a one over s with some gain. It is falling and your phase is 90 minus 90 degrees.

(Refer Slide Time: 63:36)



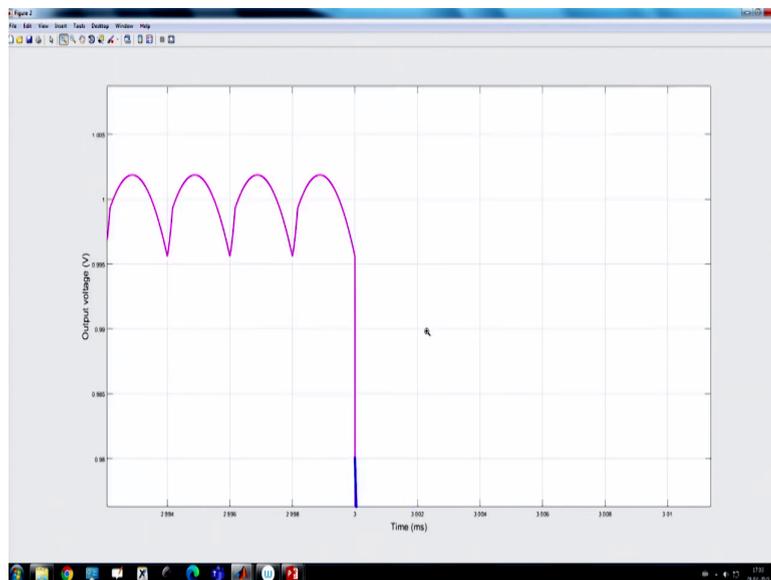
So, phase margin is 90 degree and if you go to the response of the output voltage.

(Refer Slide Time: 63:40)

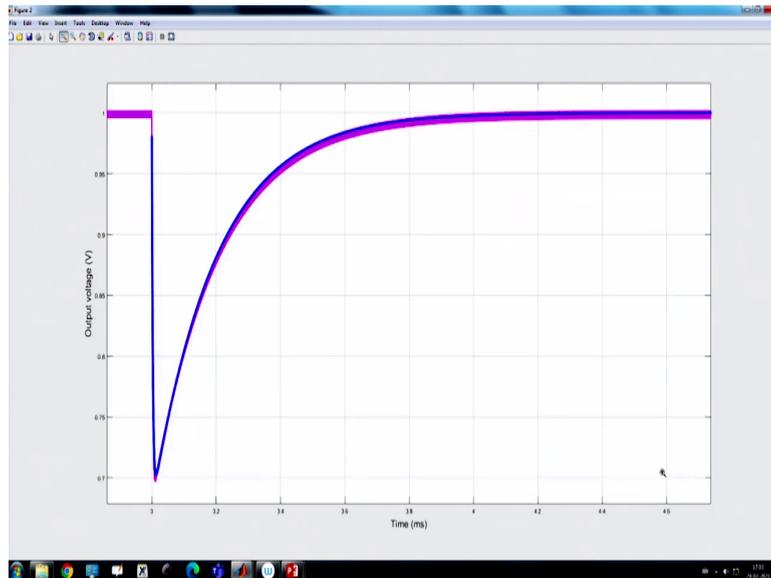


So, there is an ESR jump that is why the model is capturing that ESR jump.

(Refer Slide Time: 63:42)



(Refer Slide Time: 63:46)



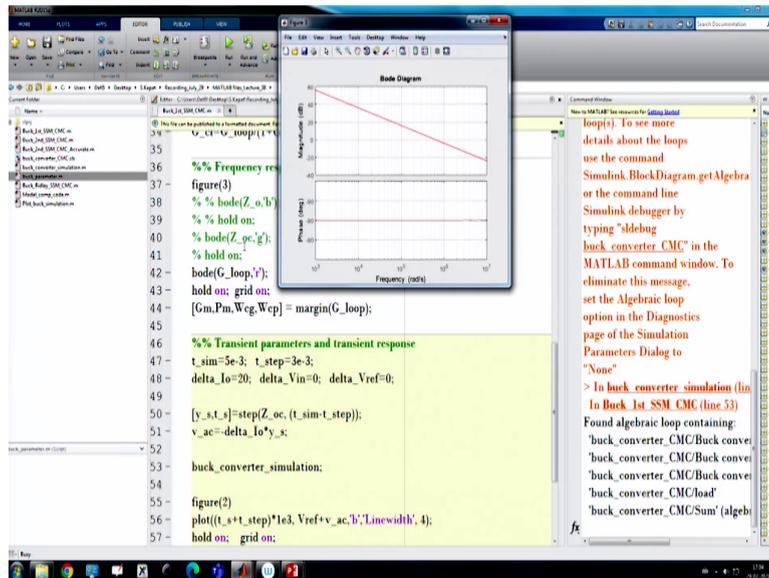
And then it is accurately capturing the behaviour of the closed loop system.

(Refer Slide Time: 63:49)



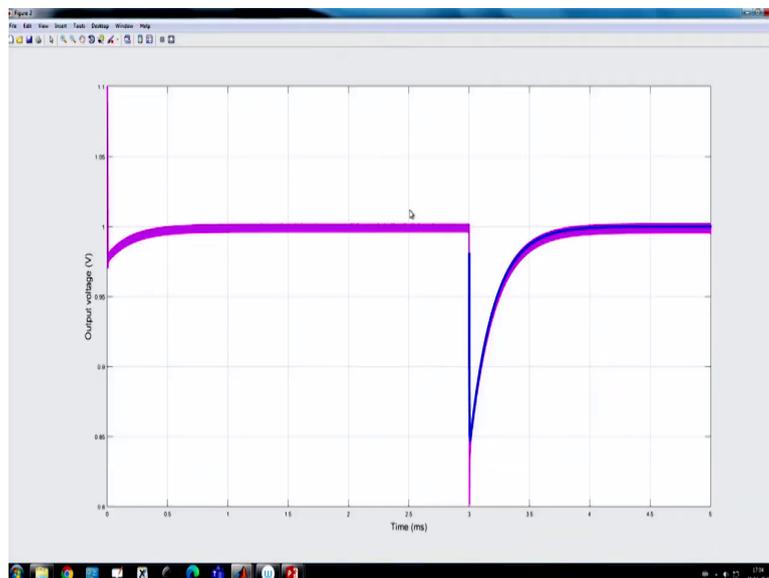
Under load transient response is more or less capturing.

(Refer Slide Time: 64:04)



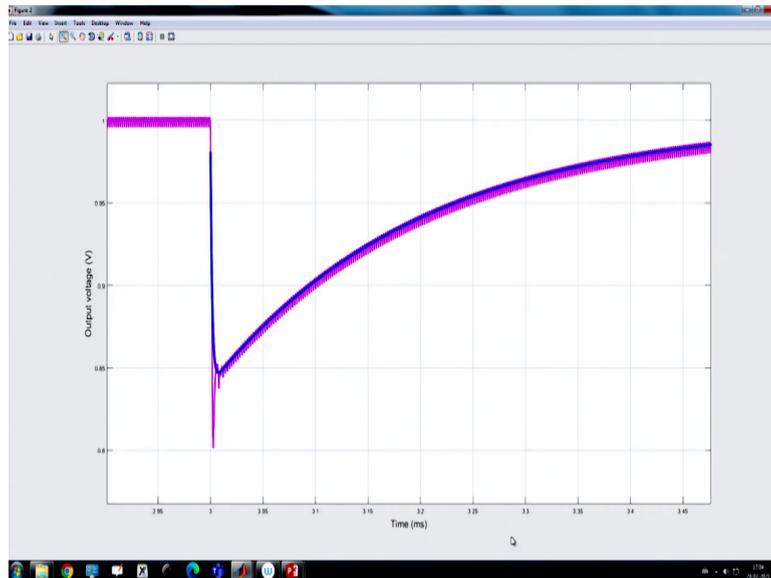
And if you want to increase, let us say one feed.

(Refer Slide Time: 64:07)

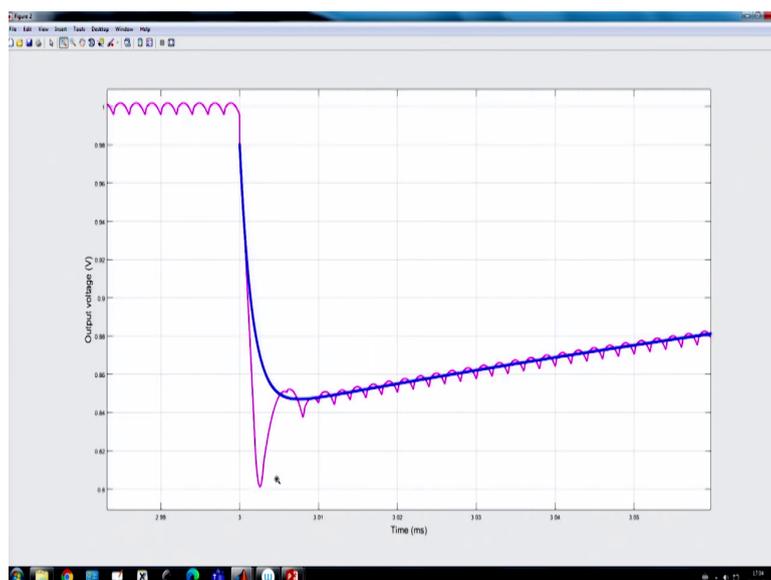


Or that means, 100 kilohertz is my crossover frequency. You see the model start diverging.

(Refer Slide Time: 64:12)

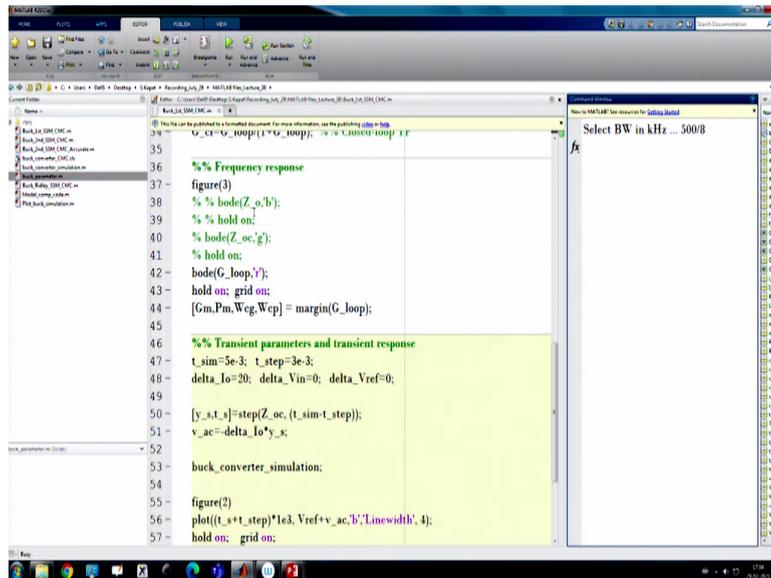


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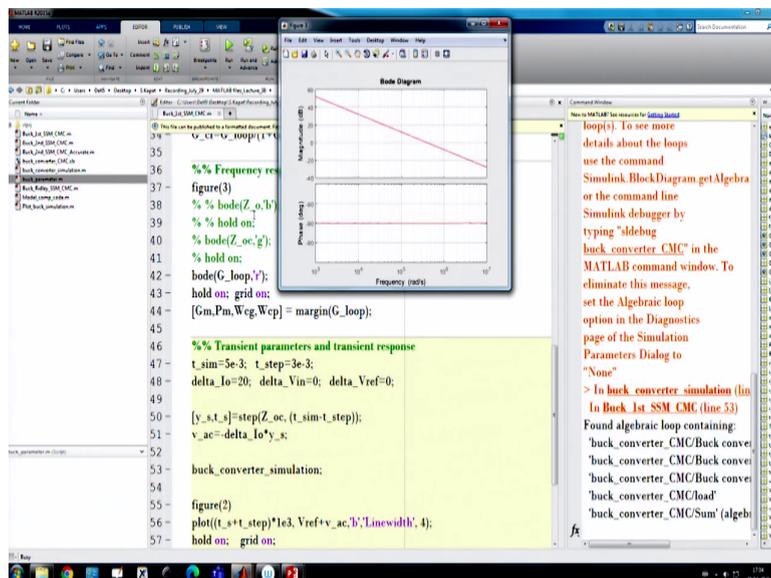
Because of your actual switch simulation, you have a huge peak, which cannot be captured by your model because the model will not be valid.

(Refer Slide Time: 64:23)



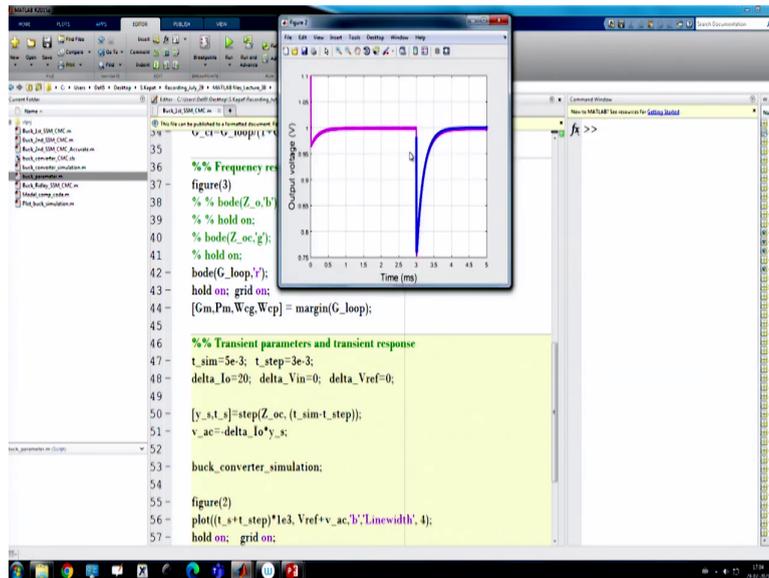
So, again I have told that you can select that. Let us say 1 eighth will be good number.

(Refer Slide Time: 64:28)



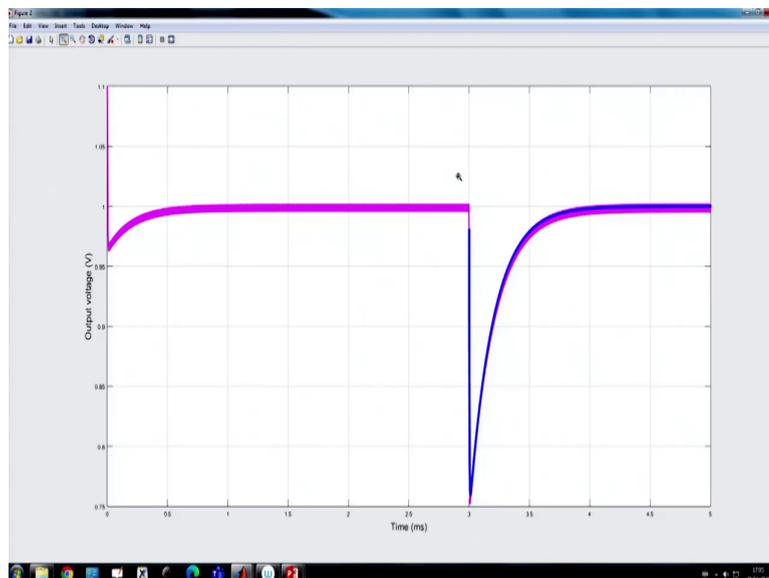
So, 500 divided by 8 may be a number.

(Refer Slide Time: 64:31)



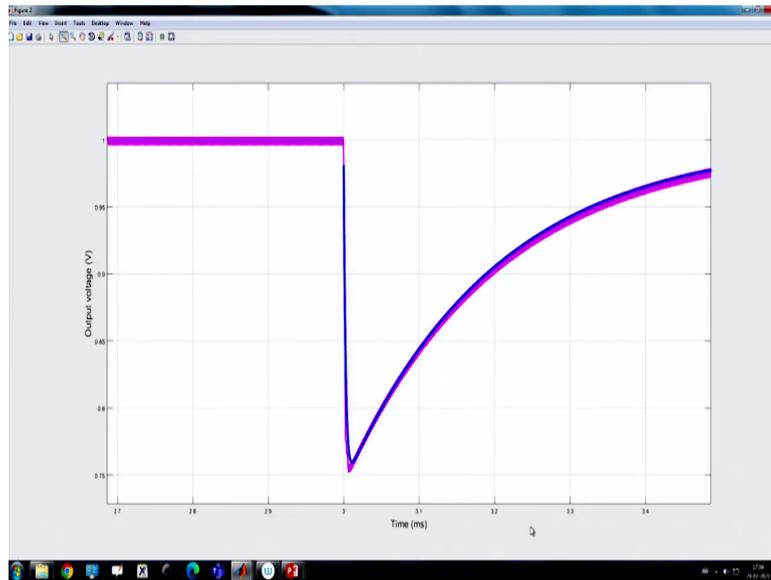
So, 18 can be a good choice and it matches reasonably well.

(Refer Slide Time: 64:36)



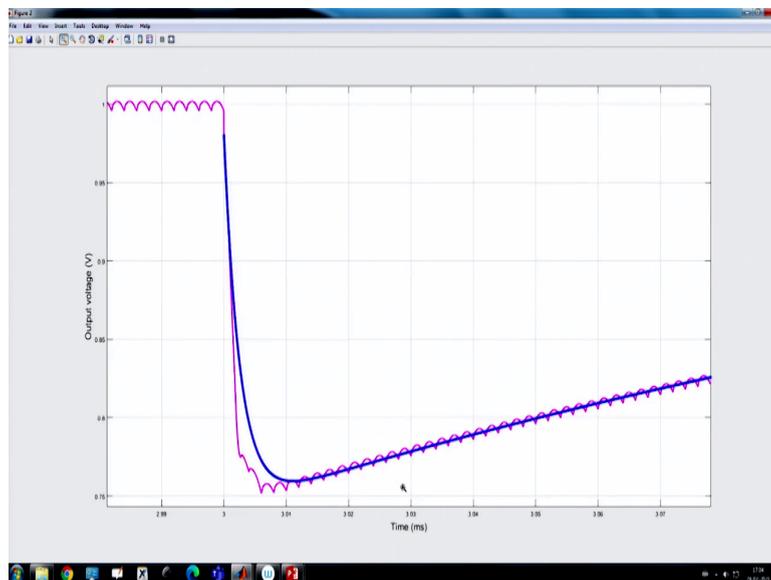
So, in current mode control, you can slightly increase the bandwidth.

(Refer Slide Time: 64:39)



And that it becomes 1.8 ok slightly, but still there is a mismatch.

(Refer Slide Time: 64:44)



So, I think one seven sorry 1.8 to 1.9 or I can say 1 tenth is the perfect thing that we are working. So, we have seen that response of the current mode control.

(Refer Slide Time: 64:58)

Output Impedance with Load Current Feed-Forward

$i_c' = G_{ic}(-\tilde{v}_o)$

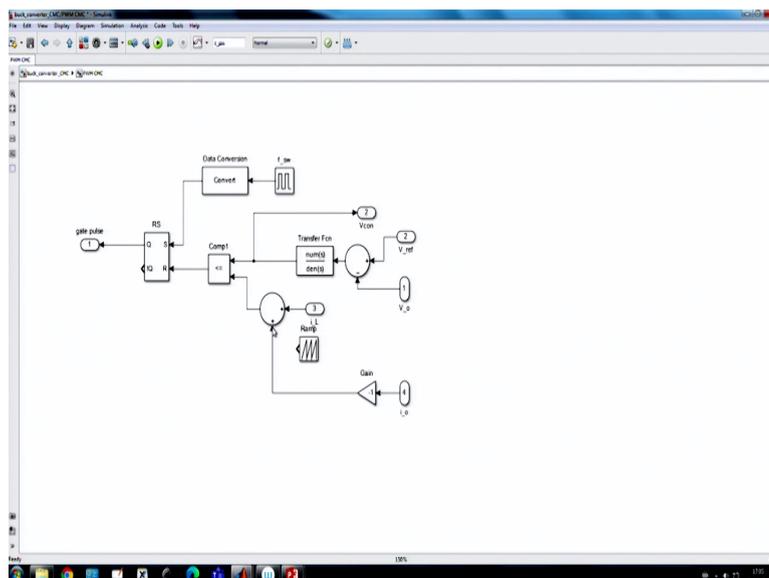
$$K_{loop} = G_c G_{ic} z_o$$

$$\tilde{v}_o = z_o (\tilde{i}_L - \tilde{i}_o) = z_o \left[G_{ic} (\tilde{i}_c' + \tilde{i}_o) - \tilde{i}_o \right]$$

$$= z_o \left[G_{ic} \left\{ G_c (-\tilde{v}_o) \right\} \right] + (G_{ic} - 1) z_o \tilde{i}_o$$

Now, that means, we have discussed open loop impedance, now the output impedance. So, current mode control one of the problem that is very slow transient response. If you want to increase the bandwidth, then the model is not valid. So, it is sluggish because it is approximately like a first-order over damp system. So, if you want to speed up the transient what you can do you can simply incorporate the load current feed forward.

(Refer Slide Time: 65:25)



That means, instead of ramp, I can simply add load current feed forward because the load can be added with the ref this reference current or it can be subtracted from the load inductor current that is why minus 1 gain is given.

(Refer Slide Time: 65:39)

```

35 % v_c1 = v_loop(1 + v_loop); % v_c100000 loop 1 f
36
37 %% Frequency response
38 figure(3)
39 %% bode(Z_o,'l');
40 %% hold on;
41 %% bode(Z_oc,'g');
42 %% hold on;
43 bode(G_loop,'r');
44 hold on; grid on;
45 [Gm,Pm,Wcg,Wcp] = margin(G_loop);
46
47 %% Transient parameters and transient response
48 t_sim=5e-3; t_step=3e-3;
49 delta_lo=20; delta_Vin=0; delta_Vref=0;
50
51 [y_s,t_s]=step(Z_oc,(t_sim-t_step));
52 v_ac=-delta_lo*y_s;
53
54 buck_converter_simulation;
55
56 figure(2)
57 plot((t_s+t_step)*1e3,Vref+v_ac,'LineWidth',4);
58 hold on; grid on;

```

So, now, if we run it with 50 kilohertz.

(Refer Slide Time: 65:42)

```

36 %% Frequency res
37 figure(3)
38 %% bode(Z_o,'l')
39 %% hold on;
40 %% bode(Z_oc,'g');
41 %% hold on;
42 bode(G_loop,'r');
43 hold on; grid on;
44 [Gm,Pm,Wcg,Wcp] = margin(G_loop);
45
46 %% Transient parameters and transient response
47 t_sim=5e-3; t_step=3e-3;
48 delta_lo=20; delta_Vin=0; delta_Vref=0;
49
50 [y_s,t_s]=step(Z_oc,(t_sim-t_step));
51 v_ac=-delta_lo*y_s;
52
53 buck_converter_simulation;
54
55 figure(2)
56 plot((t_s+t_step)*1e3,Vref+v_ac,'LineWidth',4);
57 hold on; grid on;

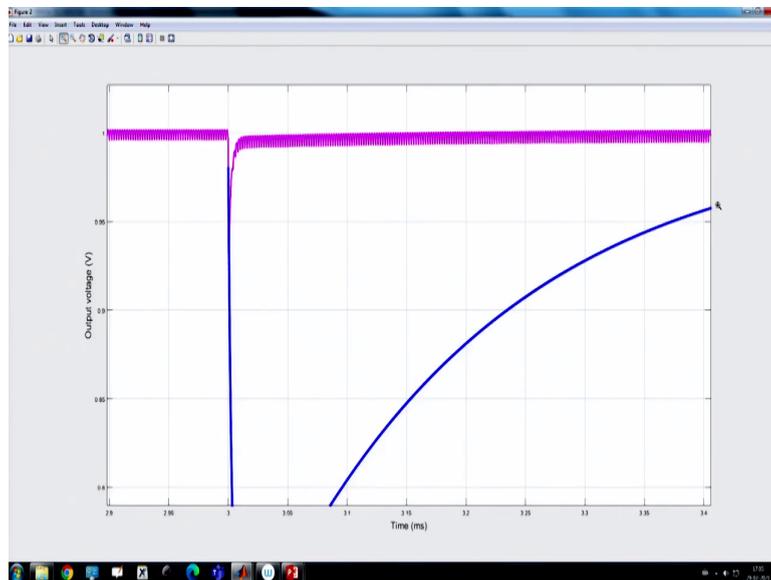
```

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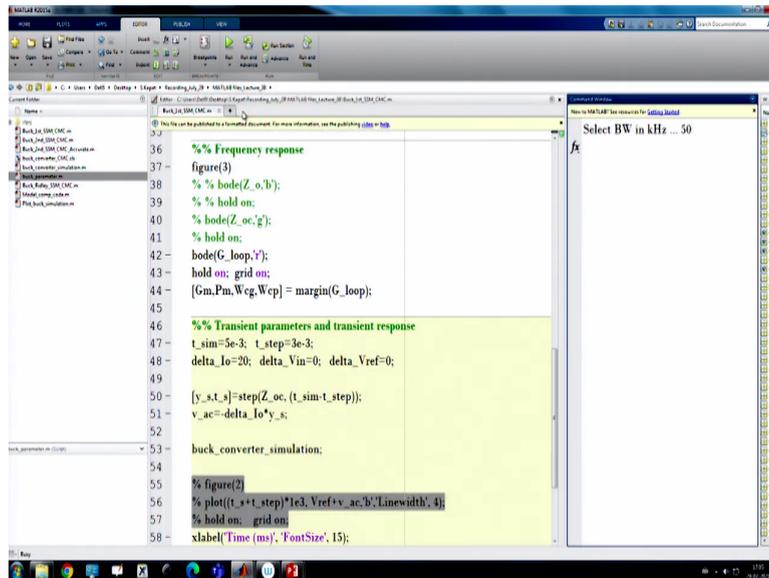
So, I have now added there, but now the match closed loop response will not match because of the model.

(Refer Slide Time: 65:49)



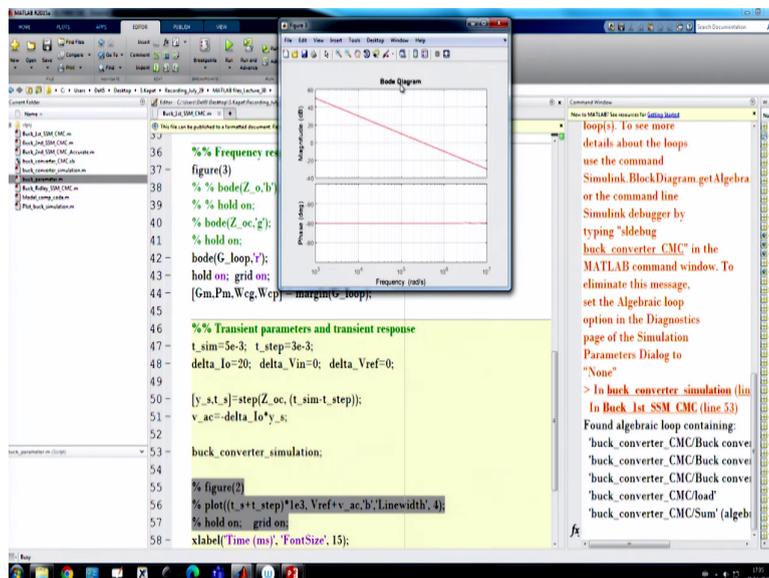
I have not considered this, you know. So, let us remove this because we can we have not discussed this part. So, this will match fine; that means, I will comment it.

(Refer Slide Time: 65:59)



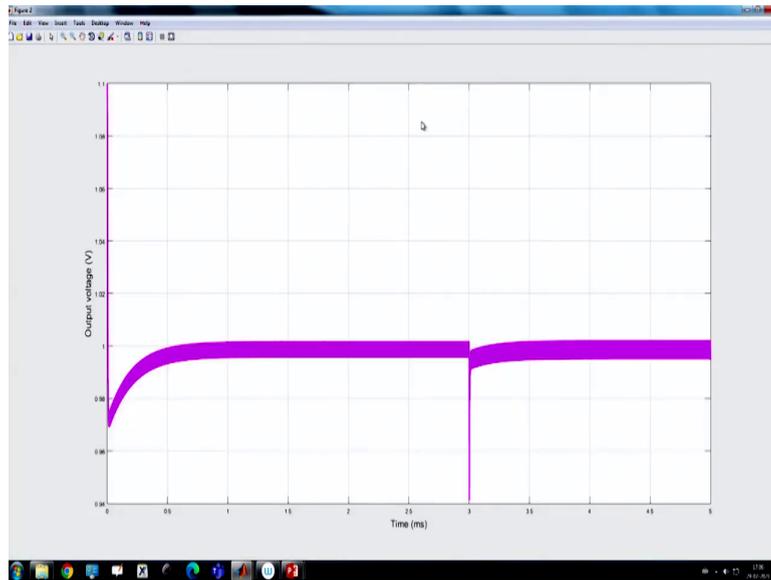
I will run it again 50 kilohertz.

(Refer Slide Time: 66:01)



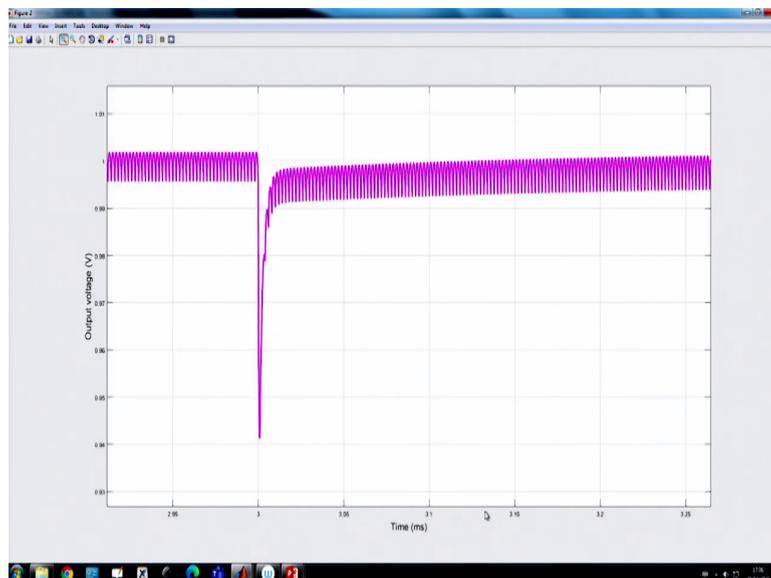
So, the response will be very fast.

(Refer Slide Time: 66:04)

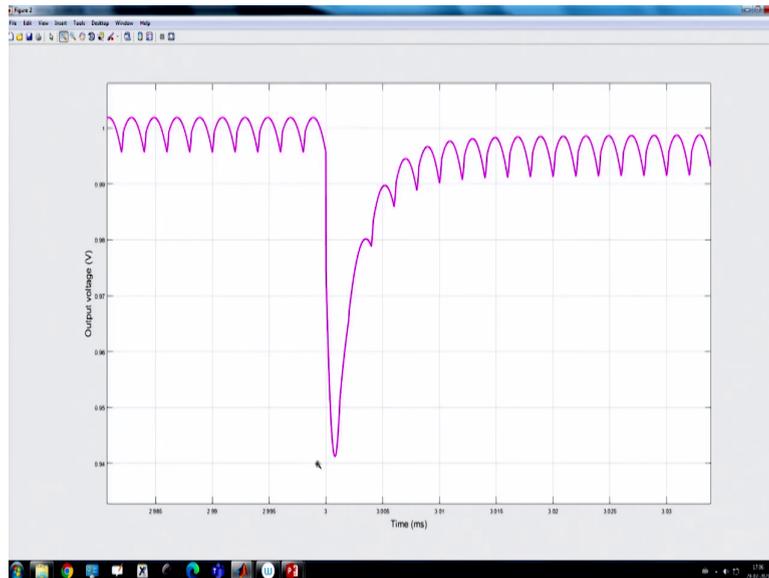


When you talk about load you know load current feed forward.

(Refer Slide Time: 66:09)



(Refer Slide Time: 66:11)



It can be significantly improved over without load current feed forward ok. So, the load current feed forward if you consider then as if we are adding this load current with the reference current which is coming out of the controller or we can subtract from the inductor current right we can subtract. So, if you do that then we can write that v_0 expression in terms of Z_0 into this i_L minus i_0 .

And what is i_L ? It is nothing but G_c into this current plus this current sorry plus this current this current. So, this is this current and then if you further substitute what is this current is G_c into because we are not talking about in load transient not talking about the reference change. So, it is 0. So, your i_1 dash is nothing but G_c into minus V_0 perturbation of this product. So, that is written here.

(Refer Slide Time: 67:14)

$$\tilde{v}_o = z_o (\tilde{i}_L - \tilde{i}_o) = z_o \left[G_{ic} \left\{ G_c (-\tilde{v}_o) \right\} \right] + (G_{ic} - 1) z_o \tilde{i}_o$$

$$= -K_{loop} \tilde{v}_o - (1 - G_{ic}) z_o \tilde{i}_o$$

$$\tilde{v}_o (1 + K_{loop}) = -(1 - G_{ic}) z_o \tilde{i}_o$$

And if you write the complete thing, you will see this is a loop. This $z_o i_c G_c$ this whole product is nothing but my loop transfer function. So, loop and if you write the complete equation.

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$$\tilde{v}_o (1 + K_{loop}) = -(1 - G_{ic}) z_o \tilde{i}_o$$

$$z_{oc} = -\frac{\tilde{v}_o}{\tilde{i}_o} = \frac{(1 - G_{ic}) z_o}{1 + K_{loop}} = 0 \quad G_{ic} = \frac{T_i}{1 + T_i}$$

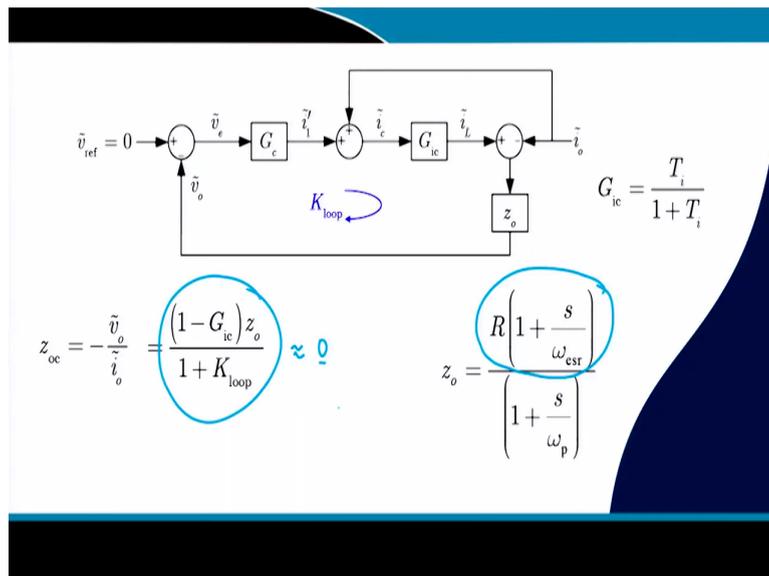
Then this is the final expression. What is the output impedance of the closed loop system minus v_o by i_o and this will be simply this. Now, we know that G_{ic} which is the transfer function between i_L and i_c right. So, in approximate first-order model, this will be

approximately equal to 1 because in our first-order approximation we have considered the inductor current perturbation to be equal to i_c perturbation and if it is so.

Then this will be simply 0 because this term will be 0. So, in the approximate sense, it shows that closed loop output impedance is 0. It behaves like an ideal voltage source. But in practice there will be slight change because that will come due to the dynamics of the current because your inductor, in order to achieve 0 output impedance, your inductor has to be negligibly small almost 0 and output capacitor has to be very large.

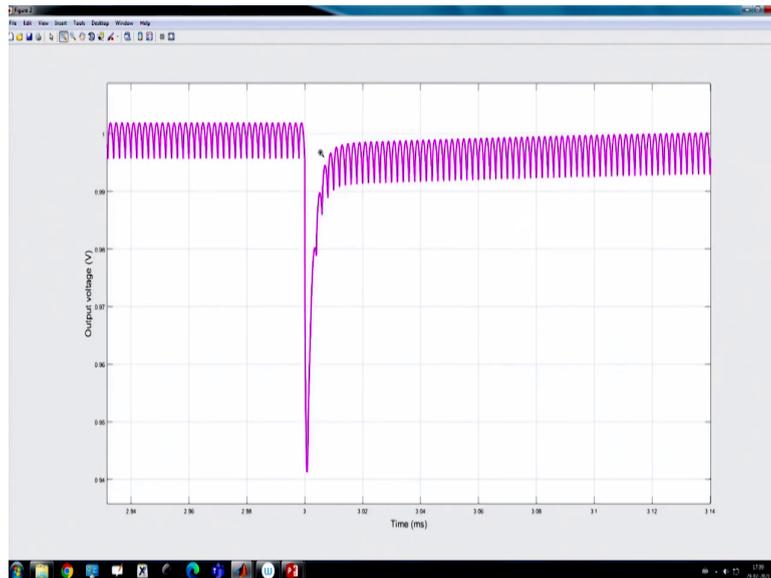
So, in order to achieve this your characteristic impedance, which is nothing but square root of L by C should be tending to 0 it should tend to 0. And this is practically not possible because neither you can reduce the inductor below a certain value your current ripple will increase nor you can increase the capacitor significantly because it will penalize your power density right.

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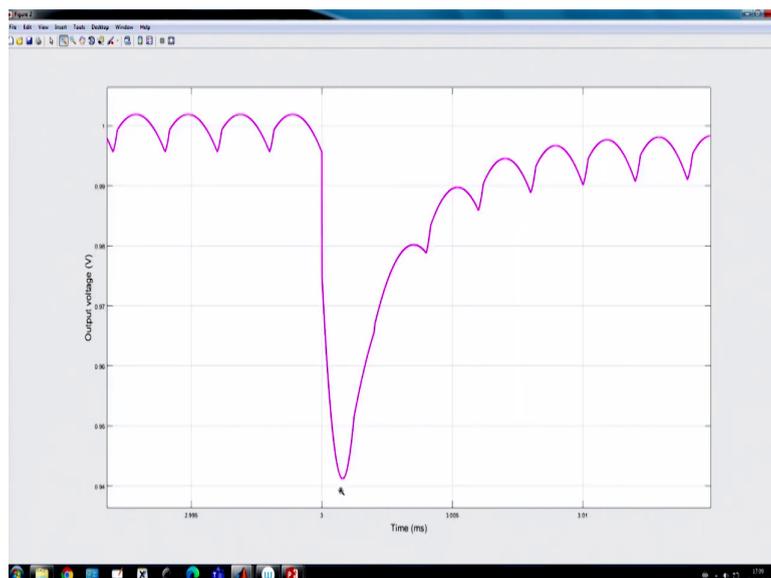


So, it is like this that means, the closed loop output impedance can be whereas, the open loop output impedance is a function of load resistance, but now the closed loop output impedance virtually become close to 0. So, there is in sync that means if there is a load step transient the output voltage as if there is no change and this is in average sense it is more or less correct; that means, if you take you know very closely if you see.

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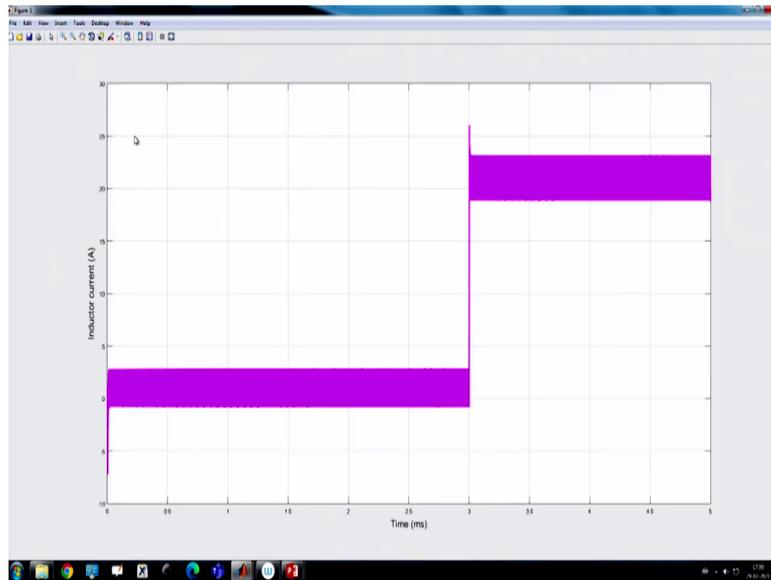


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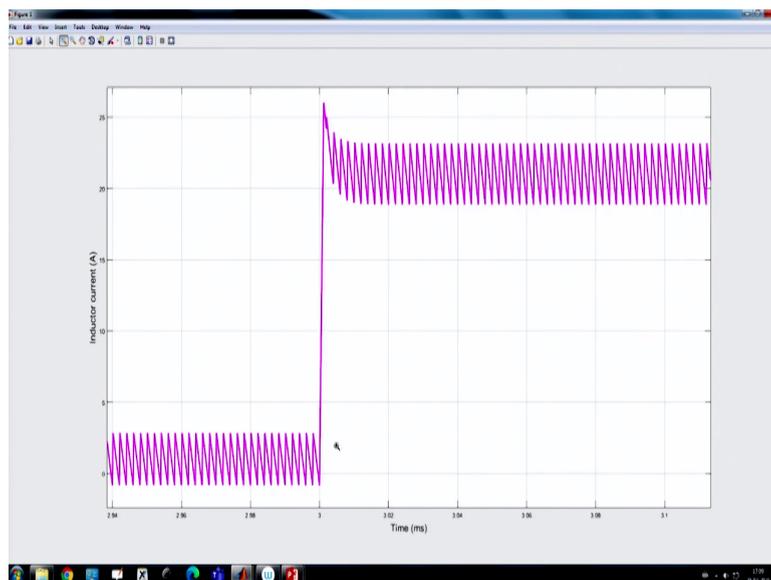


Only except for this my because this is due to the finite slew rate of the inductor and capacitor because inductor current is rising it cannot reach immediately because if you go back here.

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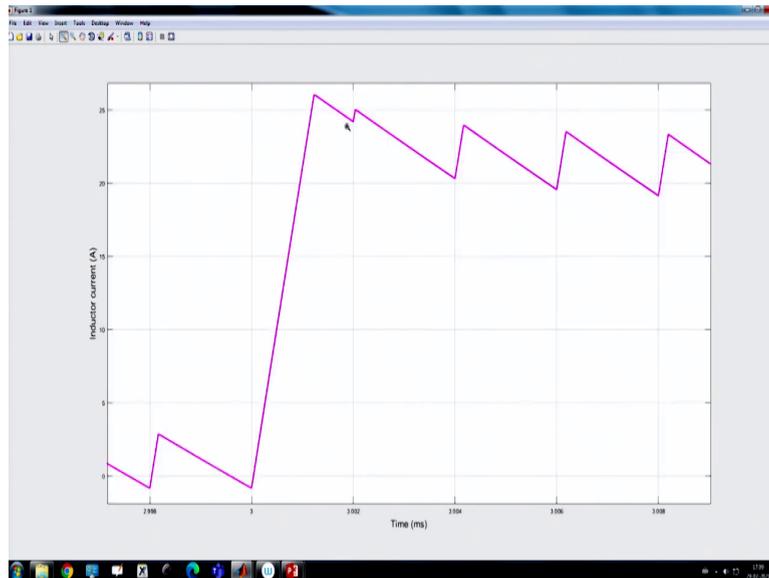


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If you just open up as I said that it takes time right inductor takes time it will slew up.

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So, slew up and if you reduce inductor values smaller and smaller than this will be reduced as even becomes smaller and smaller. So, that means, if you take if you go back to our power stage design if I can reduce my inductor value that means, if I go to my buck converter parameter.

(Refer Slide Time: 69:54)

```
1 - L=0.1e-6; % output inductance
2 - C=200e-6; % output capacitance
3 - T=2e-6; % switching time period
4 - r_L=5e-3; % inductor DCR
5 - r_L=5e-3; % High-side MOSFET on resistance
6 - r_L=5e-3;
7 - v_d=0*0.55;
8 - r_L=5e-3; % Low-side MOSFET on resistance
9 - r_C=1e-3; % capacitor ESR
10 - Vin=12; % input voltage
11 - Vref=1; % reference output voltage
12
13
```

If I take a smaller inductor let us say I am just taking 0.1 the ripple will be quite large.

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```

36 %% Frequency response
37 figure(3)
38 %% bode(Z_o,b);
39 %% hold on;
40 %% bode(Z_oc,g);
41 %% hold on;
42 bode(G_loop,'r');
43 hold on; grid on;
44 [Gm,Pm,Wcg,Wcp] = margin(G_loop);
45
46 %% Transient parameters and transient response
47 t_sim=5e-3; t_step=3e-3;
48 delta_lo=20; delta_Vin=0; delta_Vref=0;
49
50 [y_s,t_s]=step(Z_oc,(t_sim-t_step));
51 v_ac=-delta_lo*y_s;
52
53 buck_converter_simulation;
54
55 % figure(2)
56 % plot((t_s+t_step)*1e3,Vref+v_ac,'Linewidth',4);
57 % hold on; grid on;
58 xlabel('Time (ms)', 'FontSize', 15);

```

But let us say if we run it 50 kilohertz then you will see the output has almost no effect right.

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Bode Diagram

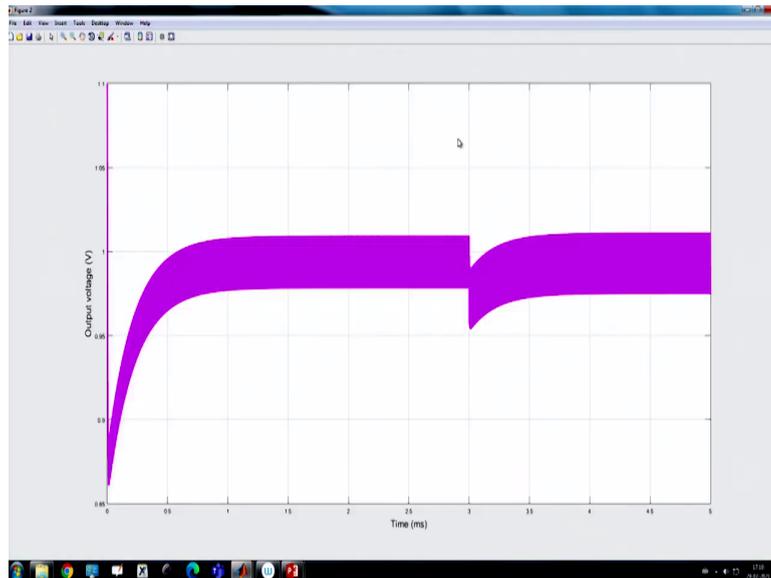
Magnitude (dB)

Phase (deg)

Frequency (rad/s)

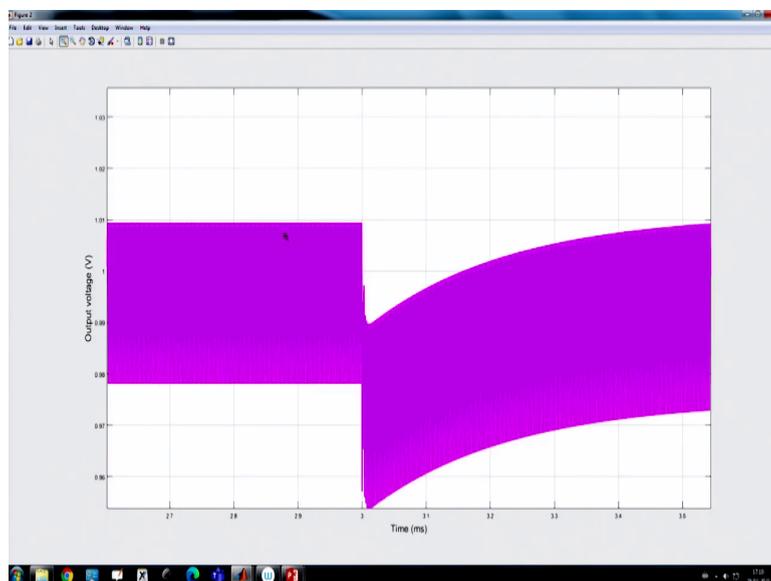
Found algebraic loop containing:
'buck_converter_CMC/Buck conver
'buck_converter_CMC/Buck conver
'buck_converter_CMC/Buck conver
'buck_converter_CMC/load'
'buck_converter_CMC/Sum' (algeb

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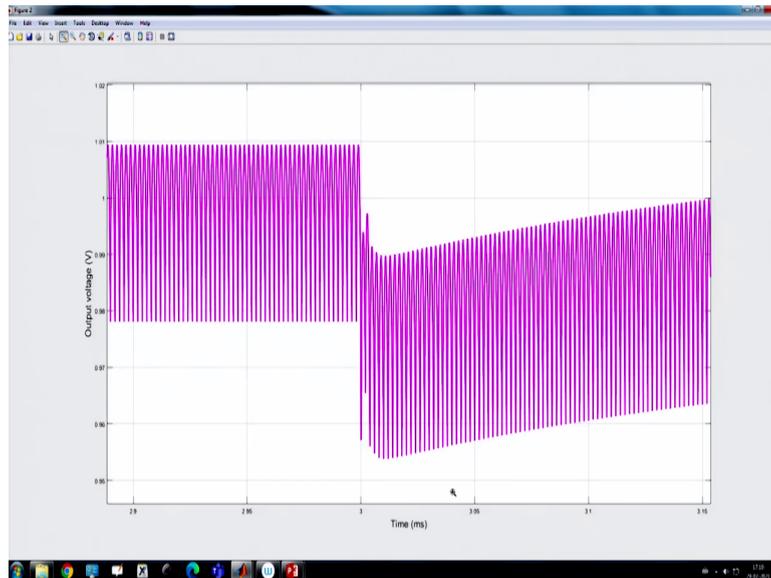


So, we will see that output will have almost no effect. That means, it virtually becomes 0 in negligible effect.

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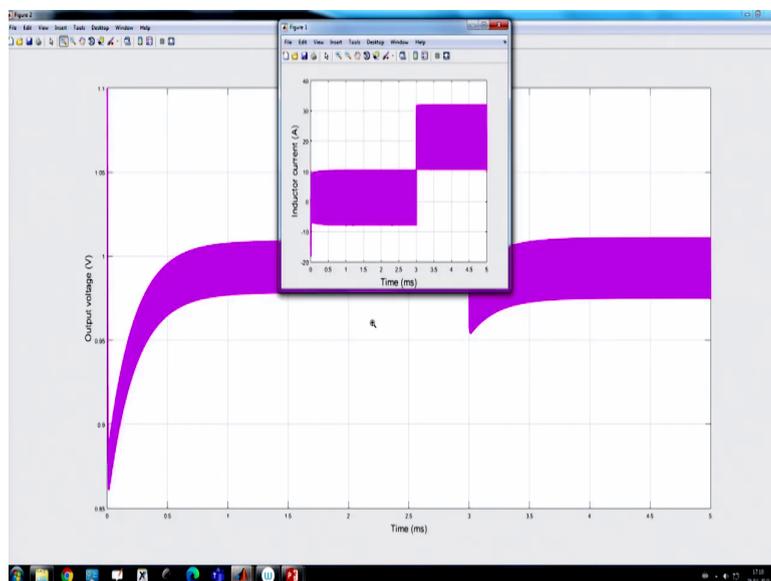


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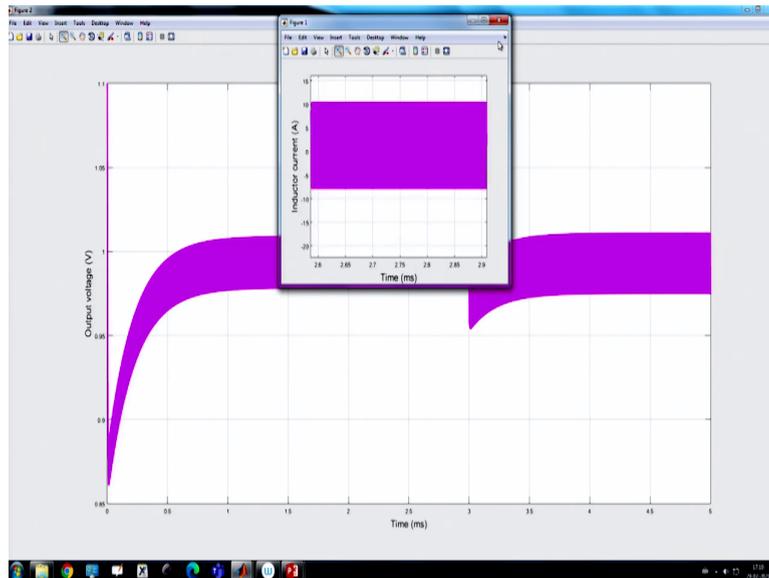
Negligible effect that means, it is in the order of you know some 5 to 10 millivolt or 20 millivolt.

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But the penalty for a such small inductor your current ripple is significantly large.

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And which is not acceptable. So, this that means, it is not exactly 0 it is close to 0 the effect is neglected.

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Summary

- Comparative study of various modeling techniques – discussed
- Model validation carried out using MATLAB transient simulation
- Loop shaping objectives and design guidelines – discussed
- Closed-loop output impedance with load current feedforward



So, in summary, we have discussed the comparative study of various modelling technique then we did model validation using different small-signal model and we value validated in MATLAB simulation and we have discussed that which model should be used for the design. And then we also discuss the loop shaping objective as well as the design of current mode

control in a buck converter, then we have discussed the closed loop output impedance with load current feed forward ok. So, with this I will finish it here.

Thank you very much.