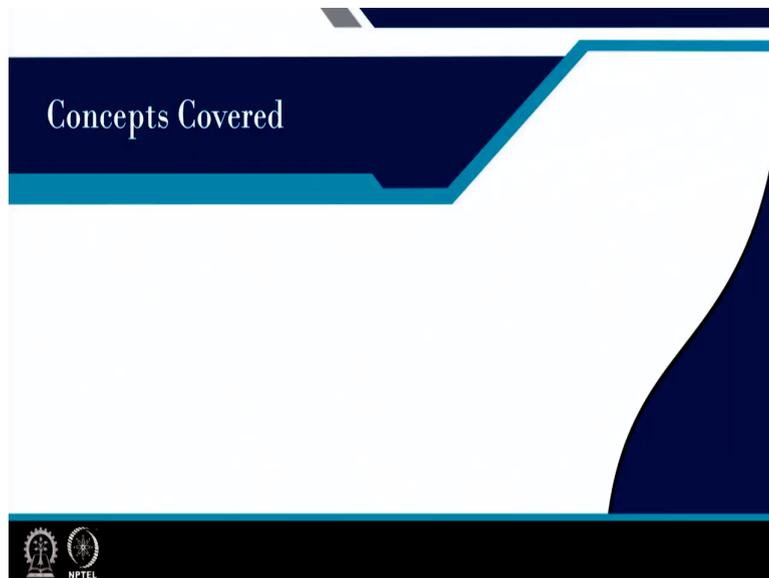


Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 07
Small-signal Design and Tuning of PWM Voltage Mode Control
Lecture - 36
Design of VMC Boost Converter and MATLAB Design Case Studies

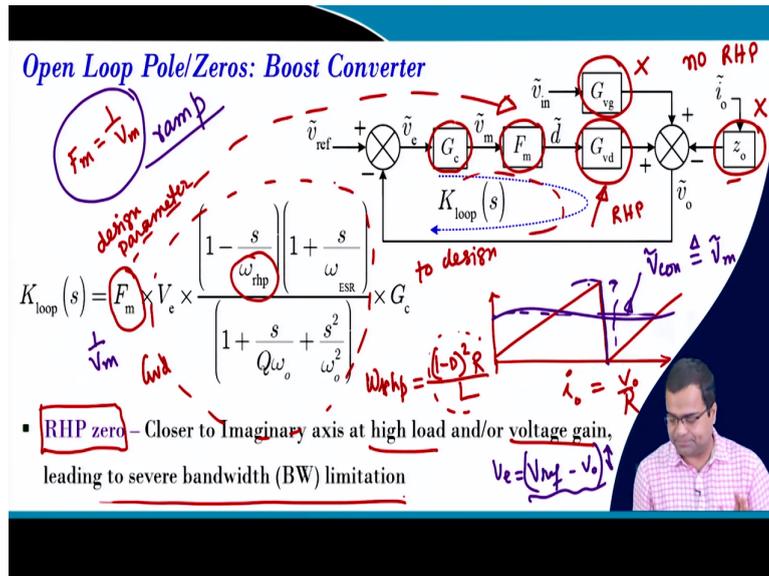
Welcome, this is Lecture Number 36. In this lecture, we are going to talk about Design of Voltage Mode Control Boost Converter and MATLAB Design Case Studies.

(Refer Slide Time: 00:36)



So, in this lecture we are going to talk about loop shaping objective in a boost converter, then PID controller design this is primarily for an ideal boost converter, then type 3 compensator design that the type 3 compensator design that is for a practical boost converter and then MATLAB case studies.

(Refer Slide Time: 00:58)



Now, first we need to you know summarize what are the open-loop poles and zero in a boost converter and this loop diagram you know this loop transfer closed loop diagram we have already seen in multiple lectures which was presented you know in the previous lectures.

So, here we have control to output transfer function. This is our duty ratio to this is G vd; that means, duty ratio to output transfer function, audio susceptibility, then output impedance and, of course, a modulator gain, then controller transfer function. So, and we know the loop transfer function and we have discussed loop shaping; that means, loop you know compensation particularly.

So, if we write the loop transfer function equation for a boost converter first it will come the modulator gain F_m which is here which is here, then this whole term this entire term is nothing but G vd ok and then this is a controller which we have to design we have to design this controller and this is given, this is also a design parameter, but you know this is also a design parameter ok.

But sometime it has a limit because you know modulator gain. What is the modulator gain? We know the modulator gain is $1/V_m$ that is the peak voltage of the sawtooth waveform and if we go to like an integrated circuit implementation, we cannot have arbitrarily large value of voltage. So, there should be a limit at the same time the voltage should not be too small, then there is a possibility of duty ratio saturation, because if the bandwidth of the loop is high then you know ok.

So, if the bandwidth of the loop is high, suppose if you this is your sawtooth waveform and we are talking about this and now we also know the control voltage like. So, this is our control voltage, and this control voltage comes from or here it is the modulator voltage. You can say this is the in this case we have denoted this as a modulator voltage.

So, this comes from the controller ok and if you increase the bandwidth of the controller when there is a transient that time there is a possibility that v_m can be quite large and it can go above this sawtooth waveform and if it goes above the sawtooth waveform, then it will lead to duty ratio saturation. And under duty ratio saturation boost converter has a severe problem because we have discussed that if the switch is on in the boost converter that inductor current continuously keeps on charging and voltage will fall right.

So; that means, if you are if you consider error voltage, suppose if you consider error voltage, which is $V_{ref} - V_0$ right. Now, if the switch is on maybe for one cycle, then this voltage will fall. Output voltage will fall; that means, this voltage will fall as a result the error this error will increase ok. So, error will increase and if the error increases, the control voltage will further go up.

So, it mean it will never come back; that means, in that case we need to put a saturation limit because we need to forcefully turn on the switch otherwise this duty ratio saturation can lead to complete collapse of the system in which the inductor current will totally you know it will go unboundedly it will saturate the inductor or it can it may damage the you know MOSFET and the other devices and the output voltage will completely collapse.

So, we need to be careful about the selection of this modulator voltage, or particularly the sawtooth waveform, is maximum value, but if you take a large value even within the specified range then you will find this F_m which is $1/V_m$. So, the DC gain of the loop will decrease and that may lead to higher steady state error.

So, that is why you have to keep in mind the tradeoff between the DC gain as well as a choice of this ramp and of course, we can raise the DC gain by increasing the integral gain that we are going to discuss, but yes, this design of this ramp should be very suitably like you need to you know seriously consider the choice of ramp particularly for a boost converter.

Now, if we talk about ok so, if we talk about. So, this is G_{vd} boost converter has a right half plane zero right, and this is nothing but this ES, this rhp zero. Now we know that this right

half plane zero because boost converter compared to buck converter it is called so-called non-minimum phase converter sometime it is also called indirect energy transfer converter.

That means, it first take the charge from the source through the inductor. It takes. It actually stores energy in the inductor in the next cycle. That energy is transferred to the capacitor. So; that means it is a two-stage process, first it takes the energy and the second stage it same you know it returned the energy to the capacitor.

So, in this process, if your whole duration is you know given for the charging the inductor, then inductor will some simply keep on charging and voltage will collapse. So and why it is non-minimum phase because in the one time when the inductor current rise then voltage falls. So, one state variably is increasing which is the inductor and the other is falling and because of that the control to output transfer function will have v_0 to d will have a right half plane zero.

But you know, in the small signal derivation, we have derived the transfer function of a boost converter. I think in lecture number 32 or 33, probably we have derived transfer function. So, here you will find a notice that the right half zero is only here in this control to output transfer function, but boost converter does not have any right half plane zero.

In the audio susceptibility, there is no right half plane zero in the output impedance there is no right half plane is zero. So, there is no RHP. So, no RHP, but only this one. We have RHP ok. So; that means, this problem of the non-minimum phase actually comes into picture for the control to output transfer function.

So, when you talk about open loop output impedance there is no RHP, but when you design closed loop output impedance, then this loop comes into picture and that loop consist of G_{vd} . So, as a result the loop transfer function that the crossover frequency of this loop transfer function is limited by the RHP.

And in this class we are going to see that whether what should be achievable crossover frequency in presence of a right half plane zero ok. So; that means, the right half plane zero is a very problem, I mean it is a real challenge for design of a boost converter control ok and we also saw what was the expression of right half zero this RHP.

So, this expression was 1 minus D whole square R by L; that means, for light R equal to small when the load current is high because we know the load current is nothing but v 0 by R; that means, when the load current is high for a given output voltage, the resistance is smaller and if the resistance is smaller this quantity become smaller.

That means it comes closer to the imaginary axis, and it comes closer to the imaginary axis we have discussed in the previous lecture that the right half plane zero introduces a undershoot behaviour. And that becomes more and more prominent when the right half plane zero comes closer to the imaginary axis.

So, it is very problematic when the RHP zero comes closer to the imaginary axis, but we cannot do anything for the boost converter when the load current increases. Similarly, when the duty ratio increases; that means, 1 minus D become smaller or D become larger than also right half plane zero comes to close to the imaginary axis.

So, for higher load current or higher voltage gain it leads to severe bandwidth problem due to the RHP zero location and which comes close to the imaginary axis and that we are going to see, now the open loop boost converter we have an RHP zero.

(Refer Slide Time: 10:11)

Open Loop Pole/Zeros: Boost Converter

$$K_{loop}(s) = F_m \times V_c \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right) \left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

$Q = f(R, Z_o, D)$

$\omega_{ESR} = \frac{1}{r_e C}$

$\omega_o = \begin{cases} \frac{1}{\sqrt{LC}} & \text{buck conv.} \\ \frac{(1-D)}{\sqrt{LC}} & \text{boost conv.} \end{cases}$

stable poles complex conj.

$F_m = \frac{1}{V_m}$

- ESR zero - Located at high frequency for lower ESR
- Double poles - Complex conjugate for higher R, poor PM
- High DC gain - higher bandwidth BW, but poor PM
- High modulator gain - higher DC gain, BW, but poor PM

Next, the ESR zero generally ESR zero depends on you know we have discussed that ESR zero it depends on what? It depends on simply ESR into r. So, this is same as the buck

converter. So; that means, if you take a smaller e_s smaller ESR, then ESR zero will be placed at very high frequency.

So, it has you know very insignificant impact in the control bandwidth as long as the ESR is very small, but boost converter has double pole same as the buck converter, but there is a compared to the buck converter here we have an interesting you know here if you take ω_0 ; that means, the natural frequency.

It was for buck converter it was $1/\sqrt{LC}$ this is for buck converter right that we have studied multiple time and it was quite easy because if L and C are fixed at the stage of design stage then this ω_0 is more or less you know known ok. So, it is not going to vary significantly unless there is a certain variation in the component value of the inductor or capacitor, but in case of a boost converter it is nothing, but $1 - D$ by square root of LC.

That means it is for the boost converter; that means you have another problem. The problem is that this double pole, this double pole moves along because it is the frequency of the double pole and if the duty ratio is large, then $1 - D$ small. So, this double pole comes closer to you know it will come closer to imaginary side, but it is a left-hand side you know if you talk about this is a stable pole this stable pole and generally complex conjugate they are generally complex conjugate.

But the frequency of this complex conjugate pole, the natural frequency it is basically $1 - D$ by square root of LC though L by C may be constant. We can assume they to be constant, but duty ratio can vary so; that means, when there is a if you try to increase the duty ratio of the buck-boost converter, then $1 - D$ become smaller. So, in the if you take this; that means, if you this complex conjugate pole and if this frequency corresponds to ω_0 .

So, as duty ratio increases, this will move along this line; that means this pole location will get shifted as duty ratio increases and if it comes closer and closer than we have real problem right because you have a double frequency pole and there is a phase crossover frequency which is coming at the low frequency.

So, we have to compensate otherwise you can have a very shortage of phase margin and also we saw this Q factor of the buck converter; it was a function of load resistance and the

characteristic impedance that we saw in boost converter you will see it is also a function of duty ratio right.

So; that means, if the load resistance decreases, this Q factor also increases; that means, you have more and more peaking effect as the load resistance decreases, but the location of this pole complex pole. It depends on the duty ratio if the duty ratio is fixed this location remains fixed; that means, this axis placement remains fixed, but the Q factor which decides that peaking effect that changes with the load resistance.

So, in light load condition, you have severe you know poor phase margin. That means, at high load condition we have a right half plane zero problem because we saw that it is coming close to imaginary axis RHP zero at light load. The poles have a shortage of you know; that means it is it has a poor phase margin; that means, it will be more and more oscillatory.

So, that makes the boost converter design very difficult. Another thing you need a high DC gain because you need to increase bandwidth, but we will see what is the achievable bandwidth and higher modulator gain higher DC gain, but poor phase margin and we saw that higher modulator gain also has a problem because higher modulator gain means what it is a lower V_m right.

So, it is a lower V_m and smaller V_m can easily saturate the duty ratio and that can be detrimental unless you put a duty ratio limit or current limit. But we are talking about voltage mode control, so here we need to put a duty ratio limit; that means, there should be a saturation limit, so that the control voltage should not exceed the maximum voltage.

(Refer Slide Time: 15:22)

Primary Loop Shaping Objectives

$$K_{\text{loop}}(s) = F_m \times V_e \times \frac{\left(1 - \frac{s}{\omega_{\text{rhp}}}\right) \left(1 + \frac{s}{\omega_{\text{ESR}}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

- To cancel ESR zero – to eliminate high freq. ripple (1P)
- To place a stable pole at RHP zero – for flat gain (1P)
- To cancel double poles – to offset sharp phase fall (2Z)



Now, the primary loop shaping objective, we need to cancel this ESR zero right and what we did for the buck converter the same way the controller requires a pole to compensate or to cancel this ESR zero. The next part to place a stable pole at RHP zero right that means, because we cannot cancel, we should not or we never attempt to cancel an unstable zero with an unstable pole.

Because it will make the whole system unstable right and unstable pole zero cancellation can make the internal stable a system unstable right, it should be internally unstable. But what we can do, we can place a stable pole which in coincidence with the RHP zero because in the gain response they will cancel each other in terms of gain, but phase will be different.

Now to cancel double pole although I would not say cancellation because exact cancellation is not possible is a load resistance dependency duty ratio dependency term but we can compensate for it ok. So, next so, initially we will consider the exact you know perfect compensation and we need to figure out what is the critical bandwidth and that formula can be used then you can go for more realistic design where you may not need to cancel this double pole rather you can design based on the worst case.

And the same way we discuss for the buck converter, we can try it out; that means, in the previous lecture we have discussed. So, initially we want to develop analytical design step where we will assume perfect compensation, then we want to relax; that means, we want to put some additional phase to compensate if you cannot cancel this double pole right.

(Refer Slide Time: 17:18)

Primary Loop Shaping Objectives

$$K_{loop}(s) = F_m \times V_e \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right) \left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

Handwritten notes:
 - **Type-III**
 - **Controller**
 - **2 zeros** → double poles
 - **3 poles** → one pole at origin, one pole at ω_{ESR} to cancel zero
 - **Ideal boost** $r_c = 0$
 - **practical PID controller** 2 zeros + 2 poles

- To consider an integrator – to eliminate SS error (IP)
- To consider compensator gain – to meet desired DC gain
- **A type-III compensator needed**



Now, the primary loop shaping objective here again as we discussed, we also need to consider an integrator because we need to achieve zero steady state error in final value; that means, at limit t tends to infinity, but in general we can achieve very close to the desired output voltage because it is within the specified limit.

And we need to consider a compensator DC gain and that is used to meet the desired bandwidth as well as we have to provide sufficient gain, so that the steady state error can be reduced. So, you need how many poles we discuss 3 poles and we discussed suppose 2 zeros, so out of 2 zeros, this is to compensate your double pole right.

So, I am talking about this is the controller and out of this one pole at origin sorry the other pole one pole at ω_{RHP} in coincidence; it is a stable pole all are stable pole and one pole to cancel ESR zero ok. So; that means, this is type 3 compensator when we have discussed in the previous lecture the type 3 compensator has 2 zeros and 3 poles including the polar origin.

Now, if we take an ideal buck, a boost converter, this will not be there; that means r_c equal to 0. So, this 0 will not be there for the ideal boost converter; that means, we do not need this additional pole ok. So, we do not need for ideal boost converter what do we need the controller we need 2 zeros; we need 2 zeros plus 2 poles, one pole at origin and one pole in coincidence with the RSP zero and 2 zeros to anticipate the double pole.

So, in this case 2 zero 2 poles, we know that it is nothing but the PID control right PID controller or it is basically a practical PID controller. So, a practical PID controller has 2 zero 2 pole. So, for an ideal boost converter or you can say if the ESR is really very small, it has in significant effect, then a PID controller is sufficient to compensate a boost converter, but if the ESR zero is not very high.

Because you know we cannot assume that the ESR should be close to zero, it has some value and it also has a large capacitor in that case, we typically require a type 3 compensator, ok. So, type 3 compensator, but you know in many cases it the PID controller and type 3 compensators are interchangeably used, but it is not true, because the type 3 compensator has 2 zeros and 3 poles whereas, the PID controller has 2 zeros and 2 poles.

So, but they can be used interchangeably if the ESR zero at very high frequency, then even if we use another pole to compensate, that also goes very high frequency. So, it essentially or very close to the type 3 compensator will is almost becoming a PID controller because even if you take the type 3 compensator the pole at very high frequency we have discussed that a second order system can be approximated to a first order system if the poles are widely separated.

So, in that says a type 3 compensator can be approximated to a PID controller if one of the pole is very high frequency compared to the remaining poles.

(Refer Slide Time: 21:45)

PID Control Tuning in VMC Boost Converter

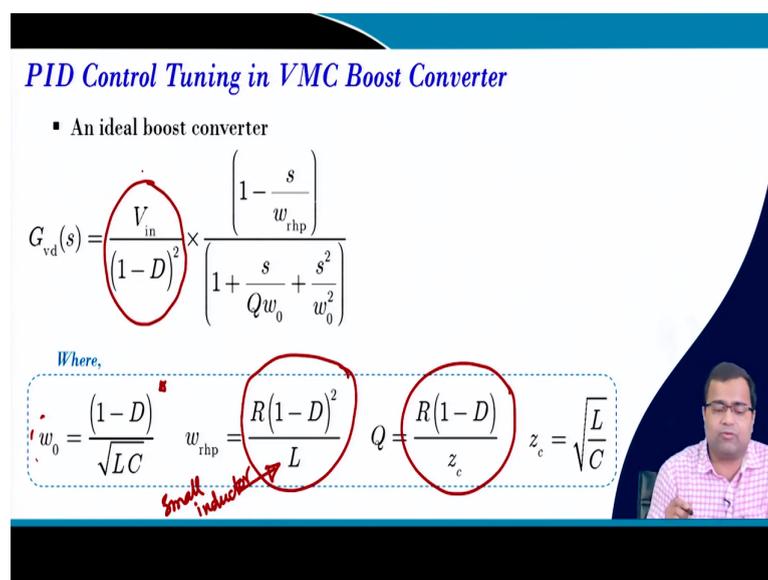
- An ideal boost converter

$$G_{vd}(s) = \frac{V_{in}}{(1-D)^2} \times \frac{\left(1 - \frac{s}{w_{rhp}}\right)}{\left(1 + \frac{s}{Qw_0} + \frac{s^2}{w_0^2}\right)}$$

Where,

$$w_0 = \frac{(1-D)}{\sqrt{LC}} \quad w_{rhp} = \frac{R(1-D)^2}{L} \quad Q = \frac{R(1-D)}{z_c} \quad z_c = \sqrt{\frac{L}{C}}$$

Small inductor →



So, for an ideal boost converter, it turns out to be that you know. Now we will find here Q and that we discuss. So, Q is nothing but $R \sqrt{L/C}$ right and that we learn that this Q is the quality factor Q factor ok. And here we also saw the RHP zero location right. So, that is a function of R, D and L, another point in the power stage design we should keep in mind for a boost converter it is better to use a small inductor.

In fact, we have discussed in lecture number 7 in the power stage design for a buck converter we saw a smaller inductor try to speed up the transient response even for the open loop system and it also reduces the overshoot undershoot the natural overshoot undershoot.

So, for a buck converter we have a special requirement because the RHP zero. If you want to place the RHP zero far away in the right-hand side to reduce its effect, it is better to use a smaller inductor, but there should be a limit of the minimum value of the inductor because smaller inductor can increase the conduction loss.

Because it can increase the RMS current or that means, particularly, the ripple current is increased. So, that means, but we need to choose smaller inductor within our specified ripple limit as well as the loss point of view right. But we also saw the natural frequency is a function of $1/(1-D) \sqrt{L/C}$ that one also we saw ok.

(Refer Slide Time: 23:24)

▪ Practical PID controller

$$G_c = K_p + \frac{K_i}{s} + \frac{K_d s}{\tau_D s + 1}$$

$$= K_i \left[\frac{1 + k_1 s + k_2 s^2}{s(\tau_D s + 1)} \right]$$

where

$$k_1 = \frac{K_p}{K_i} + \tau_D \quad k_2 = \frac{K_d + K_p \tau_D}{K_i}$$


Now for a PID practical PID controller we have discussed that it can be transformed into K i into this form and we have sufficiently discussed this PID controller in the previous lecture in two previous two lectures like lecture number 33 as well as 34 right, sorry 34 35 yeah.

(Refer Slide Time: 23:46)

$$K_{\text{loop}}(s) = F_m \times G_{\text{vd}}(s) \times G_c(s)$$

$$= \underbrace{\frac{F_m K_i}{(1-D)^2}}_{K_L} \times \underbrace{\frac{\left(1 - \frac{s}{w_{\text{tbp}}}\right)}{1 + \frac{s}{Q w_0} + \frac{s^2}{w_0^2}}}_{D_p(s)} \times \frac{\overbrace{(1 + k_1 s + k_2 s^2)}^{N_c(s)}}{s(\tau_D s + 1)}$$

Next, in the loop transfer function, we can write the whole loop transfer function; that means, you know if you what is G vd. So, if you go to the G vd we have this particular constant term V in by 1 minus D whole square and if you go to the compensator, then compensator has a K i term here right. So, K i is a constant term coming out ok so, constant term ok.

Now if you multiply so, this constant term this term was part of our you know G vd and this term is the modulator gain and this term is our controller ok. So, all these three parts are you know discussed ok. So, this whole term I am taking as K L ok. Now, we have a right half plane zero and double pole and what is our initial perfect compensation assumption that we want to cancel this pole because it is a stable pole by controller zeros 2 zeros are taken.

(Refer Slide Time: 25:05)

$$K_{loop}(s) = F_m \times G_{vd}(s) \times G_c(s)$$

$$= \underbrace{\frac{F_m V_{in} K_i}{(1-D)^2}}_{K_L} \times \frac{1 - \frac{s}{w_{rhp}}}{1 + \frac{s}{Qw_0} + \frac{s^2}{w_0^2}} \times \frac{N_c(s)}{s(\tau_D s + 1)}$$

$\tau_D = \frac{1}{w_{rhp}}$

discrete jumps at every switching event

Design Steps

- Cancel open-loop (stable) pole by controller zero $D_p(s) = N_c(s)$
- Place derivative filter pole in coincidence with RHP zero $\tau_D = \frac{1}{w_{rhp}}$

So, design step cancels stable pole zero cancellation by putting the controller zero to be equal to the plane pole. We need to place a derivative filter in coincidence with the RHP zero, but one thing generally derivative filter. We think that the time constant of the derivative filter is much smaller, but here we are taking 1 by RHP zero. So, in that sense, it is not actually a PID controller because we are making the derivative action much slower, which is constrained by the RHP zero location.

But this is like intentionally we are putting a zero, which you know not only attenuates the; that means we are actually not taking the full derivative action, but why there is also a reason? The reason is that if you take a practical boost converter output voltage ripple, ok. So, if you take a practical output. So, when the switch is on it falls, then suddenly there is a jump, then it tries to go up then jumps sorry and then it goes like this. So, this discrete jump happens at every switching event.

So, these are the discrete jumps which occur at every switching event. This is actually if you take a derivative action with a smaller time derivative time constant at filter time constant, then you may inject the differentiation of this; that means impulsive action, it can cause actuator or your error amplifier can get saturated.

Because if you try to differentiate this discrete jump, then it can really cause trouble. So, that is why in a boost converter, it is better to use a low bandwidth derivative filter, so that you can substantially reduce this effect and that is exactly what we are doing.

(Refer Slide Time: 27:21)

Loop gain becomes,

$$K_{loop}(s) = \frac{F_m V_{in} K_i}{(1-D)^2} \times \frac{\left(1 - \frac{s}{w_{rhp}}\right)}{s \left(1 + \frac{s}{w_z}\right)}$$

Frequency response $s = j\omega$

$$K_{loop}(j\omega) = \frac{F_m V_{in} K_i}{(1-D)^2} \times \frac{\left(1 - \frac{j\omega}{w_{rhp}}\right)}{\frac{j\omega}{w_{rhp}} \left(1 + \frac{j\omega}{w_{rhp}}\right)}$$

Handwritten notes and annotations:

- $\frac{1 - \frac{s}{w_{rhp}}}{s \left(1 + \frac{s}{w_{rhp}}\right)}$ with $\frac{s}{w_{rhp}}$ and $s = j\omega$ written next to it.
- $\frac{1 - \frac{j\omega}{w_{rhp}}}{\frac{j\omega}{w_{rhp}} \left(1 + \frac{j\omega}{w_{rhp}}\right)}$ with $\frac{1}{w_{rhp}} \times \left(\frac{s}{w_{rhp}}\right) \left(1 + \frac{s}{w_{rhp}}\right)$ written next to it.
- $j\omega \times (1 + j\omega)$ written below the frequency response equation.
- A red circle highlights the $\frac{j\omega}{w_{rhp}}$ term in the denominator of the frequency response equation.
- A red arrow points from the $\frac{1 - \frac{s}{w_{rhp}}}{s \left(1 + \frac{s}{w_{rhp}}\right)}$ term to the $\frac{1 - \frac{j\omega}{w_{rhp}}}{\frac{j\omega}{w_{rhp}} \left(1 + \frac{j\omega}{w_{rhp}}\right)}$ term.



Next, the loop gain becomes now so what we did, we have cancelled this term using this term right and then we place this tau D to be 1 by rhp zero right. So, then loop transfer function becomes like this 1 minus s_rhp zero. So, this omega z is nothing, but omega rhp right rhp and this is the product of the earlier term.

Now so, the loop if you write now we are talking about the frequency response frequency response. So, we plot s equal to j omega ok and if you substitute s equal to j omega here, then we will get this and this can be written. Now, initially what we did we initially it was 1 minus s by omega rhp divided by s into 1 plus s into omega rhp.

Now, you will find that this particular term particularly this term and this term you have something like s by omega RHP right and if you substitute s equal to j omega then you will get j of omega by omega rhp. So, both of them are in this form. So, in order to make consistent, I also want to write this in this form.

So, what I do, I will also write 1 minus s omega RHP then I divided by a rhp term 1 plus s omega RHP and since we are dividing so, we have to rhp so, multiplication. So, that, so this is intentionally made, and this is exactly what is done here so, we have taken this term ok other things are same.

(Refer Slide Time: 29:41)

Consider, $w_n = \frac{w}{w_{rhp}}$

$\frac{w}{w_{rhp}} = w_n$

$K_L = \frac{F_m V_{in} K_i}{(1-s)^2 w_{rhp}} \angle \phi_1 = 90^\circ$

$K_{loop}(jw_n) = K_L \times \frac{(1-jw_n)}{jw_n(1+jw_n)}$

$|K_{loop}(jw_n)| = \left| \frac{K_L}{w_n} \right| \triangleq r(w_n)$

$\angle K_{loop}(jw_n) = -90^\circ - \tan^{-1} \left(\frac{2w_n}{1-w_n^2} \right)$

Handwritten notes:

- $x_1 = (1-jw_n) = r_{x1} \angle \theta_{x1}$
- $y_1 = jw_n = r_{y1} \angle \theta_{y1}$
- $y_2 = (1+jw_n) = r_{y2} \angle \theta_{y2}$
- $r_{x1} = \sqrt{1+w_n^2}$
- $r_{y1} = w_n$
- $r_{y2} = \sqrt{1+w_n^2}$
- $\frac{x_1}{y_1 y_2} = \frac{r_{x1} \angle \theta_{x1}}{(r_{y1} \angle \theta_{y1})(r_{y2} \angle \theta_{y2})}$
- $\frac{1}{w_n}$

Now, our next step is very clear since we have something like omega by omega rhp. So, we will take this a ratio of 2 frequency we call it as a normalized frequency omega n. So, it is normalized with respect to the RSP zero ok. Now you write down the loop transfer function in terms of normalized frequency. If I do, then if you go back, what is there. So, this term will be 1 minus j omega n and this whole term will be j omega n multiplied by 1 plus j omega n.

So, this will be in the denominator and this is exactly you are getting and we know the KL, what is KL, KL is nothing, but F m V in K i divided by this. So, it is F m V in K i 1 minus D whole square omega rhp ok. Now this number it is a complex number right and this complex number can be written in the polar form. So, this is something like a polar form.

So, in the polar form you will see. Let us say x 1 is equal to 1 minus j omega n, x 2 is equal to or y 1 sorry. Let us say y 1 is equal to j omega n and y 2 is equal to 1 plus j omega n. Now if I take the complex number, which is x 1 by y 1 y 2 right. I can write this complex number to be r x 1 theta x 1 in the polar form r y 1 theta y 1, then r y 2 theta y 2 in this form right because this is a complex number.

So, I can write individual complex number in it is polar form because I can write like this is r 1 like a theta r x 1 theta x 1, this also I can write r y 1 theta y 1 and this I can write r y 2 theta y 2 right. Now, from this number, what is r x 1 so; that means, it is coming from here r x 1 is simply 1 plus omega n square root and r y 1 is simply omega n and r y 2 is simply it is also 1 plus omega n square.

So, now if you take this particular number what you will get, you will get; that means, simply $1/\omega_n$ is the magnitude term and that is K_L/ω_n because there is a constant term as well. So, the amplitude is clear. What is a phase angle, what is the angle of this θ_x , θ_y $1/\omega_n$ is nothing but what because it is associated with this complex number.

So, it is nothing but 90 degree because it is just an imaginary quantity ok, but since it is in the denominator, you know that this particular term this can be written as hold on. So, if I write it now erase this part ok. So, what we can write here.

(Refer Slide Time: 34:08)

Consider, $w_n = \frac{w}{w_{rhp}}$

$$K_{loop}(jw_n) = K_L \times \frac{(1 - jw_n)}{jw_n(1 + jw_n)}$$

** Polar form*

$$|K_{loop}(jw_n)| = \left(\frac{K_L}{w_n} \right) \triangleq r(w_n)$$

$$\angle K_{loop}(jw_n) = -90^\circ - \tan^{-1} \left(\frac{2w_n}{1 - w_n^2} \right)$$

Handwritten notes in red and blue ink:

- $\frac{r_{x1}}{r_{y1} r_{y2}} \times \angle \theta_{x1} - \angle \theta_{y1} - \angle \theta_{y2}$
- $\angle \theta_{y1} = 90^\circ$
- $x_1 = (-jw_n) = r_{x1} \angle \theta_{x1}$
- $(Y_1 = jw_n) = r_{y1} \angle \theta_{y1}$
- $Y_2 = (1 + jw_n) = r_{y2} \angle \theta_{y2}$
- $\frac{x_1}{Y_1 Y_2} = \frac{r_{x1} \angle \theta_{x1}}{(r_{y1} \angle \theta_{y1}) (r_{y2} \angle \theta_{y2})}$
- $r_{x1} = \sqrt{1 + w_n^2}$
- $r_{y1} = w_n$
- $r_{y2} = \sqrt{1 + w_n^2}$
- $\frac{1}{w_n}$

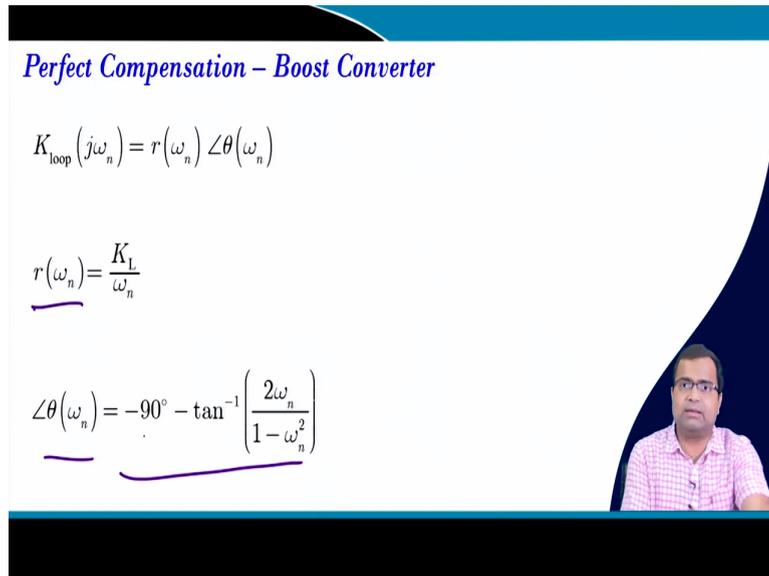
So, if I take this I can write $r_{x1} / (r_{y1} r_{y2})$ multiplied by $\theta_{x1} - \theta_{y1} - \theta_{y2}$. Because in polar form if you know two complex numbers are multiplied then their angle can be added and if they are divided angle can be subtracted by that way so we are getting this angle ok.

(Refer Slide Time: 34:45)

Perfect Compensation – Boost Converter

$$K_{loop}(j\omega_n) = r(\omega_n) \angle\theta(\omega_n)$$

$$r(\omega_n) = \frac{K_L}{\omega_n}$$

$$\angle\theta(\omega_n) = -90^\circ - \tan^{-1}\left(\frac{2\omega_n}{1-\omega_n^2}\right)$$


Next for perfect compensation our this is my magnitude, and this is my angle ok.

(Refer Slide Time: 34:58)

Select Crossover Frequency based on Phase Margin: $\omega_c = k \times \omega_{rhp}$

At gain crossover frequency ω_c

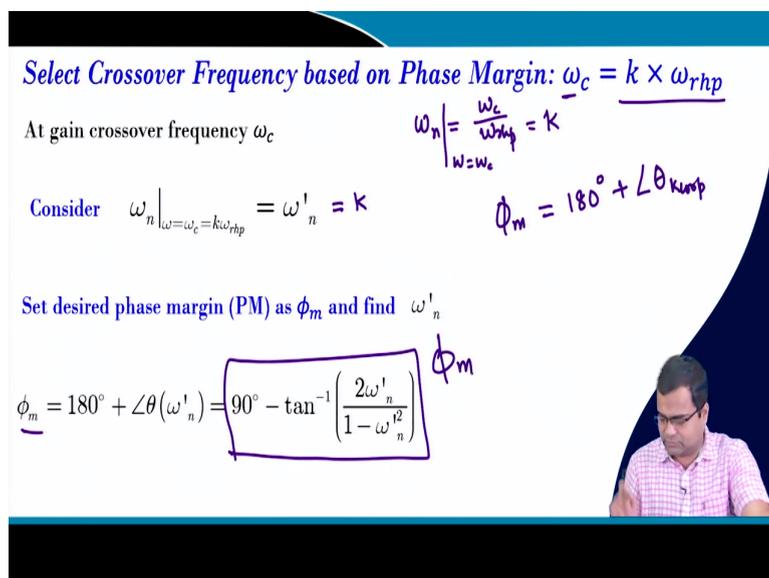
Consider $\omega_n|_{\omega=\omega_c=k\omega_{rhp}} = \omega'_n = k$

Set desired phase margin (PM) as ϕ_m and find ω'_n

$$\phi_m = 180^\circ + \angle\theta(\omega'_n) = 90^\circ - \tan^{-1}\left(\frac{2\omega'_n}{1-\omega'^2_n}\right)$$

Handwritten notes on the slide:

$$\omega_n|_{\omega=\omega_c} = \frac{\omega_c}{\omega_{rhp}} = k$$

$$\phi_m = 180^\circ + \angle\theta_{kmp}$$


Now we need to select the crossover frequency. Suppose if we set the crossover frequency k times rhp zero, then at crossover frequency what is our normalized frequency, our normalized frequency at crossover frequency is nothing, but what omega c by omega rhp right.

And this is something nothing, but we took it is something k because we took omega c k times rhp right. So, here we can simply write K. We can simply write K ok now. So, select

crossover frequency. So, you need to there are two ways of designing. one you set the crossover frequency, then phase margin. Everything else will be computed.

So, we have to first find out the phase margin. What is the phase margin? The phase margin is nothing but 180 degree plus the angle of the loop transfer function; that means, you are you are talking about the loop. So, it will be your k loop ok and what was the angle that we got. So, this is an angle. So, if you add it then 180 minus 90 it will be 90. So, our phase margin is 90 minus this. So, this is our phase margin ok.

(Refer Slide Time: 36:33)

Select Crossover Frequency based on Phase Margin: $\omega_c = k \times \omega_{rhp}$

At gain crossover frequency ω_c $\omega_n = k$

Find ω'_n from $\phi_m = 180^\circ + \angle\theta(\omega'_n) = 90^\circ - \tan^{-1}\left(\frac{2\omega'_n}{1-\omega_n'^2}\right)$ $0 < \phi_m < 90^\circ$

Find the crossover frequency using the following equation

$r(\omega'_n) = \frac{K_L}{\omega'_n} = 1 \Rightarrow \omega'_n = k = \frac{\omega'_c}{\omega_{rhp}} = K_L$ *Can we set $\omega_c = \omega_{rhp}$? $k=1?$ $\phi_m=0^\circ$*

$K_L = \frac{F_m V_{in} K_i}{(1-D)^2 \omega_{rhp}} \Rightarrow \omega_c = \frac{K_i F_m V_{in}}{(1-D)^2} \Rightarrow K_i = \frac{\omega_c (1-D)^2}{F_m V_{in}}$

So, at crossover frequency again how to find because our phase margin is F m. So, there are two ways of solving. We can find crossover frequency for a given phase margin or we can provide crossover frequency. Then phase margin will be computed because only 1 degree of freedom is there. In a buck converter in type 3 compensator what we did by introducing an additional pole we had two choices; that means, we could customize both phase margin and crossover frequency ok.

Because even if you set phase margin we can even customize the crossword frequency can be varied, provided that your model is valid. But in boost converter you have only one choice either you provide phase margin and that to the phase margin should be greater than 0 degree and less than 90 degree you cannot, because of this because this angle cannot contribute it should contribute less than 90 degree otherwise your phase margin will be negative ok.

So, you can achieve phase margin up to I mean very low value, which is greater than 0 must be greater than 0 and it can be smaller than 90 degrees. That means, here we are trying to find crossover frequency using the following equation. So, at crossover frequency, it is 1. So, we can find out that k that we took equal to K L and what is omega.

So, what is k, omega c by omega rhp and that we have selected here right and what is K L? We know the K L; that means, we have given earlier. So, from this equation, if you substitute, then you can find out that; that means, if you substitute K L, this K L expression here, then omega rhp, omega rhp in the denominator will get cancel.

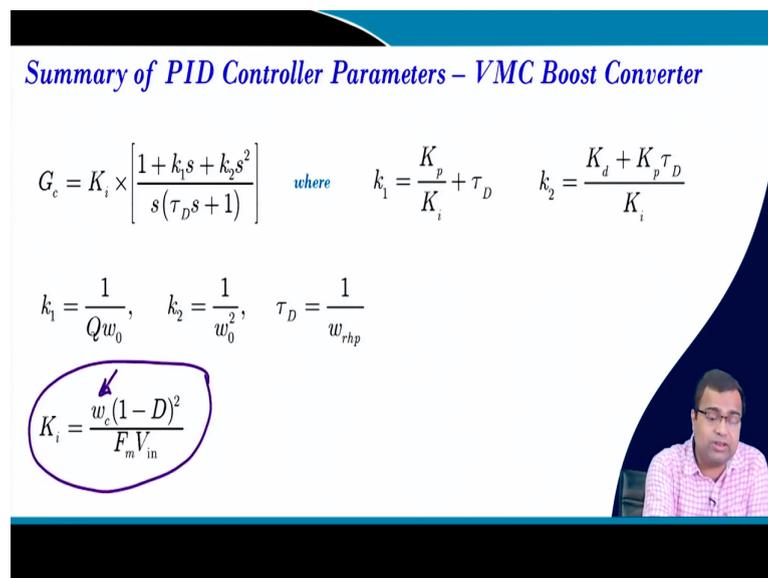
And your crossover frequency can be obtained from here. You can find out the integral gain. The integral gain can be obtained for a given either phase margin or it can be obtained for a given cutoff frequency and we will see in MATLAB case study we can design in either way; that means, either you set the cut of frequency or you set the phase margin, ok.

(Refer Slide Time: 39:05)

Summary of PID Controller Parameters – VMC Boost Converter

$$G_c = K_i \times \left[\frac{1 + k_1 s + k_2 s^2}{s(\tau_D s + 1)} \right] \quad \text{where} \quad k_1 = \frac{K_p}{K_i} + \tau_D \quad k_2 = \frac{K_d + K_p \tau_D}{K_i}$$

$$k_1 = \frac{1}{Qw_0}, \quad k_2 = \frac{1}{w_0^2}, \quad \tau_D = \frac{1}{w_{rhp}}$$

$$K_i = \frac{w_c(1-D)^2}{F_m V_{in}}$$


So, in a summary of design, we have a PID controller. We choose the controller parameter to compensate this you know K p K i K d. So, this we do not know. We need to know that who gives you crossover frequency, either crossover frequency directly given to you or you have to find out from the phase margin.

(Refer Slide Time: 39:29)

Perfect Compensation – Boost Converter

$$K_{loop}(s) = F_m \times V_e \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right) \left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

$$G_c = \frac{k_c \left(1 + \frac{s}{\omega_{cz1}}\right) \left(1 + \frac{s}{\omega_{cz2}}\right)}{s \left(1 + \frac{s}{\omega_{cp1}}\right) \left(1 + \frac{s}{\omega_{cp2}}\right)}$$

$\omega_{cz1}\omega_{cz2} = \omega_o^2$, $\omega_{cz1} + \omega_{cz2} = \frac{\omega_o}{Q}$,
 $\omega_{cp2} = \omega_{ESR}$, $\omega_{cp1} = \omega_{rhp} = \omega_p$

So, perfect compensation of a boost converter now the same thing in the PID controller. Whatever we do, PID controller it was for the ideal boost converter. For a practical boost converter, we have an extra zero ESR zero and we have an extra pole.

(Refer Slide Time: 39:52)

Perfect Compensation – Boost Converter (contd...)

$$K_{loop}(s) = F_m \times V_e \times k_c \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)}{s \left(1 + \frac{s}{\omega_{rhp}}\right)}$$

$$K_{loop}(s) = \frac{F_m V_e k_c}{k_L} \times \frac{\omega_p}{s} \times \frac{\left(1 - \frac{s}{\omega_{rhp}}\right)}{\left(1 + \frac{s}{\omega_{rhp}}\right)}$$

$V_e = \frac{V_m}{(1-D)^2}$

$$K_{loop}(j\omega_n) = k_L \times \frac{(1 - j\omega_n)}{j\omega_n (1 + j\omega_n)}, \quad \omega_n = \frac{\omega}{\omega_{rhp}}$$

So, after this pole zero cancellation, the method is exactly the same as earlier. It is the same as earlier. So, no change so that means, we are not going to discuss this because only thing this loop transfer function is exactly equal to the earlier one only this practical parasitics will come because here V_e will be V in by $1 - D$ ok it is $1 - D$ square.

So, that you can find out from the model that you are derived earlier, ok. So, this is a V e right. So, if you go back in the previous yes we can see 1 minus D term is there and there can be alpha term because of the ESR I parasitic of the inductor DCR as well as RDS one ok.

(Refer Slide Time: 40:46)

Perfect Compensation – Boost Converter (contd...)

$$K_{\text{loop}}(j\omega_n) = r(\omega_n) \angle\theta(\omega_n)$$

$$r(\omega_n) = \frac{k_L}{\omega_n}$$

$$\angle\theta(\omega_n) = -90^\circ - \tan^{-1}\left(\frac{2\omega_n}{1-\omega_n^2}\right)$$


But the rest of the compensation is exactly the same.

(Refer Slide Time: 40:47)

Perfect Compensation – Boost Converter (contd...)

At gain crossover frequency ω_c

$$r(\omega_n) \Big|_{\omega=\omega_c} = \frac{k_L}{\omega_n} \Big|_{\omega=\omega_c} = 1 \Rightarrow k_L = \frac{\omega_c}{\omega_{rhp}}$$

$$\Rightarrow k_L = \frac{\omega_c}{\omega_{rhp}} = \frac{F_m V_c k_c}{\omega_{rhp}}$$

$$\Rightarrow k_c = 2\pi \times \frac{1}{F_m V_c} \times f_c$$


We can provide the crossover frequency and to find out the phase margin ok.

(Refer Slide Time: 40:51)

Case Study 1: $\omega_c = \omega_{rhp}$

At gain crossover frequency ω_c

$$r(\omega_n) \Big|_{\omega_n=1} = \frac{k_L}{\omega_n} \Big|_{\omega_n=1} = 1 \Rightarrow k_c = 2\pi \times \frac{1}{F_m V_c} \times f_c$$

$$\angle \theta(\omega_n) = -90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1 - \omega_n^2} \right) = -90^\circ - 90^\circ = -180^\circ$$

▪ Phase margin PM = $180 - 180 = 0$ degree **Not a stable design!!**

Handwritten notes:
 $\omega_c > \omega_{shp} \Rightarrow \phi_m < 0^\circ$
 unstable
 For stable design $\omega_c < \omega_{shp}$
 $\omega_c = \min \left\{ \frac{2\omega_{shp}}{70}, \frac{\omega_{shp}}{k} \right\}$ x72



Now first we want to see after getting all these now can we select the crossover frequency up to rhp zero, because my last question what we have raised if we provide crossover frequency then phase margin will be automatically calculated ok. So, if we go back, whether it is ideal or practical ultimately end result is same.

So, this is the end result, yes if you go back here, can we set; can we set omega c is equal to omega rhp; that means, K equal to 1 can you set? In this case, what is omega n dash? It is nothing, but that K, if you set K equal to 1 this, what will be this term, it will be you know if you find out 1 minus it will be infinity 0.

So, your phase margin is 0; that means, you can obtain from there your phase margin will be 0 degree and it is not a stable design ok. And that is exactly what we can find out; that means, if your rhp zero is set to this and you can find out that your phase margin is 0 degree and it is not a stable design.

And if you go cut off frequency if you choose omega c greater than omega rhp zero then you can get phase margin to be less than 0 degree; that means, it is unstable design; that means, one thing is clear for stable design for stable design phase margin. Sorry for stable design. Your omega c must be smaller than rhp zero and that is the bottom line of this boost converter. That means your crossover frequency can must be smaller than, but how small that we will decide, ok.

So; that means, we started with the beginning that the crossover frequency for a buck converter we can go up we can go up to one-tenth of the switching frequency beyond that we saw some model validity is a problem. So, one-tenth is a standard term rule because the model is valid up to that point quite accurately.

But in a boost converter, you cannot even go up to one-tenth of the switching frequency as long as your rhp zero is lower than that. Our crossover frequency in a boost converter should be I would say it should be a minimum of you know I will show you either you know $2\pi f_s$ by 10 or I would say $\omega_{rhp\ zero}$ by some factor K where K should be greater than 1 here and we will take this.

(Refer Slide Time: 44:08)

Case Study 2: Design with 60 degree PM

At gain crossover frequency ω_c

$$PM = \angle \theta(\omega_n) \Big|_{\omega=\omega_c} - (-180^\circ) = 90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1-\omega_n^2} \right) \Big|_{\omega=\omega_c}$$

$$PM_{\text{desired}} = 90^\circ - \tan^{-1} \left(\frac{2\omega_n}{1-\omega_n^2} \right) \Big|_{\omega=\omega_c} = 60^\circ$$

$$\left(\frac{2\omega_n}{1-\omega_n^2} \right) \Big|_{\omega=\omega_c} = \tan 30^\circ \approx 0.577$$


So, if you design a 60 degree phase margin then you can find out that.

(Refer Slide Time: 44:15)

Case Study 2: Design with 60 degree PM

At gain crossover frequency ω_c

$$\left(\frac{2\omega_n}{1-\omega_n^2} \right) \Big|_{\omega=\omega_c} = \tan 30^\circ \approx 0.577 \triangleq p$$

$$\omega_n \Big|_{\omega=\omega_c} = -\frac{1}{p} \pm \sqrt{1 + \left(\frac{1}{p}\right)^2} = -1.733 \pm \sqrt{4}$$

$$\omega_n \Big|_{\omega=\omega_c} = 0.267 \quad \rightarrow \quad \omega_c = 0.267 \times \omega_{rhp} \approx \frac{\omega_{rhp}}{4}$$

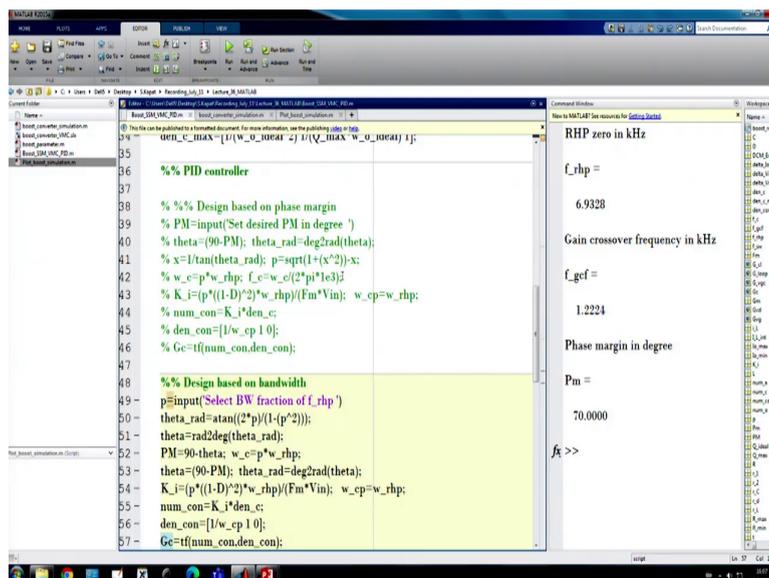
Gain crossover frequency limited to 1/4th of the RHP zero frequency

Handwritten notes:
 $\omega_c = \frac{\omega_{rhp}}{5}$
 $f_c = \frac{f_{rhp}}{5}$



The bandwidth will come to p times. It can be shown that rhp zero by so. For 60 degree phase margin, we will get roughly one - fourth of the bandwidth will be one-fourth of the rhp zero; that means, if your rhp zero is very close to the imaginary axis, then overall closed-loop bandwidth will be very, very low and you will get really very slow response from the boost converter, ok.

(Refer Slide Time: 44:41)



```

35 % den_w_max = 1/(w_0_max*(z-1)/(Q_max*w_0_mean)+1);
36
37 %% PID controller
38 %% Design based on phase margin
39 PM=input('Set desired PM in degree ');
40 theta=(90-PM); theta_rad=deg2rad(theta);
41 % x=1/tan(theta_rad); p=sqrt(1+(x^2));x;
42 % w_cp=p*w_rhp; Lc=w_cp*(2*pi*1e3);
43 % K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
44 % num_con=K_i*den_c;
45 % den_con=[1/w_cp 1 0];
46 % Ge=tf(num_con,den_con);
47
48 %% Design based on bandwidth
49 p=input('Select BW fraction of f_rhp ');
50 theta_rad=atan((2*p)/(1-(p^2)));
51 theta=rad2deg(theta_rad);
52 PM=90-theta; w_cp=p*w_rhp;
53 theta=(90-PM); theta_rad=deg2rad(theta);
54 % K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
55 % num_con=K_i*den_c;
56 % den_con=[1/w_cp 1 0];
57 % Ge=tf(num_con,den_con);

```

Command Window Output:

```

RHP zero in kHz
f_rhp =
    6.9328

Gain crossover frequency in kHz
f_gcf =
    1.2224

Phase margin in degree
Pm =
    70.0000

fx >>

```

Now, let us go to the MATLAB because we want to show design case study.

(Refer Slide Time: 44:47)

```

26 - num_a=1/(1-(1-D)^3)*1;
27 - den_c=[1/(w_o_ideal^2) 1/(Q_ideal*w_o_ideal) 1];
28 - Gvg=tf(num_a,den_c);
29
30 %% Modulator and Controller parameters
31 - V_m=10; Fm=1/V_m;
32 - Q_max=((1-D)*R_max)/z_c;
33 - w_rhp_min=(R_min*(1-D)^2)/L;
34 - den_c_max=[1/(w_o_ideal^2) 1/(Q_max*w_o_ideal) 1];
35
36 %% PID controller
37
38 %% Design based on phase margin
39 % PM=input('Set desired PM in degree ');
40 % theta=(90-PM); theta_rad=deg2rad(theta);
41 % x=1/tan(theta_rad); p=sqrt(1+(x^2))-x;
42 % w_c=p*w_rhp; f_c=w_c/(2*pi*1e3);
43 % K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
44 % num_con=K_i*den_c;
45 % den_con=[1/w_cp 1 0];
46 % Gc=tf(num_con,den_con);
47
48 %% Design based on bandwidth

```

Command Window:

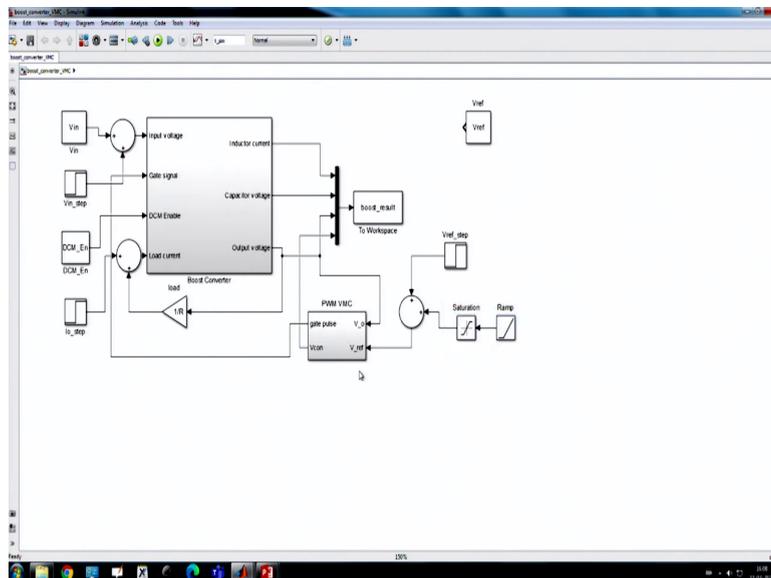
```

RHP zero in kHz
f_rhp =
6.9328
Gain crossover frequency in kHz
f_gcf =
1.2224
Phase margin in degree
Pm =
70.0000
fx >>

```

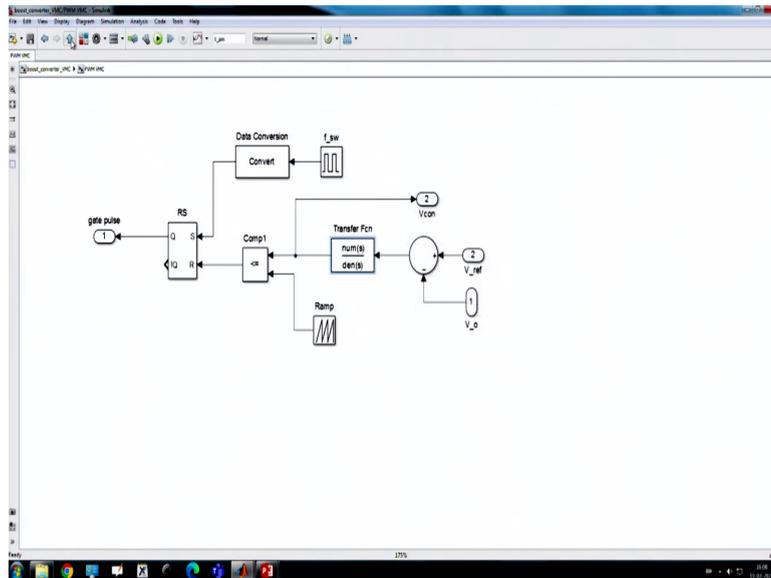
So, here we want to design this boost converter, because you know all the models are known to you. So, here we have a boost converter voltage mode control.

(Refer Slide Time: 44:56)



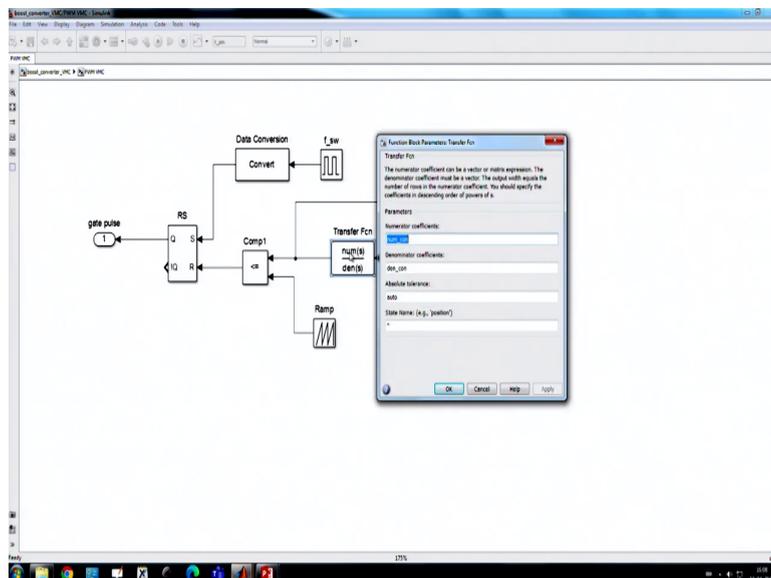
And I have shown you earlier that this is our boost converter circuit, and it is a startup logic.

(Refer Slide Time: 45:04)



Then this if you go to the compensating the sawtooth waveform.

(Refer Slide Time: 45:09)



This is our controller transfer function which we are setting from the MATLAB file ok. And here we can apply transient to supply we can apply transient to low step we can apply transient to the reference step whatever we want ok and it has a fixed load resistance that we are applying, now let us go for the design. So, first thing we want to design based on bandwidth; that means, here if I run this code first let us run it. So, if you carefully observe first.

(Refer Slide Time: 45:42)

```

41 % x=1/tan(theta_rad); beta=sqrt(1+(x^2));x;
42 % w_cp=p*w_rhp; Lc=w_c/(2*pi*1e3);
43 % K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
44 % num_con=K_i*den_c;
45 % den_con=[1/w_cp 1 0];
46 % Gc=tf(num_con,den_con);
47
48 %% Design based on bandwidth
49 p=input('Select BW fraction of f_rhp ');
50 theta_rad=atan((2*p)/(1-(p^2)));
51 theta=rad2deg(theta_rad);
52 PM=90-theta; w_cp=p*w_rhp;
53 theta=(90-PM); theta_rad=deg2rad(theta);
54 K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
55 num_con=K_i*den_c;
56 den_con=[1/w_cp 1 0];
57 Gc=tf(num_con,den_con);
58
59 %% Loop gain and closed-loop TFs
60 G_loop=Gvd*Fm*Gc; %% Loop gain
61
62 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
63 G_cl=G_loop/(1+G_loop); %% Closed-loop TF

```

If I run it will ask for select bandwidth which is the cutoff crossover frequency fraction of RHP zero; that means, we have learned that the crossover frequency must be smaller than $f_{z\text{ rhp}}$ but how small.

(Refer Slide Time: 46:03)

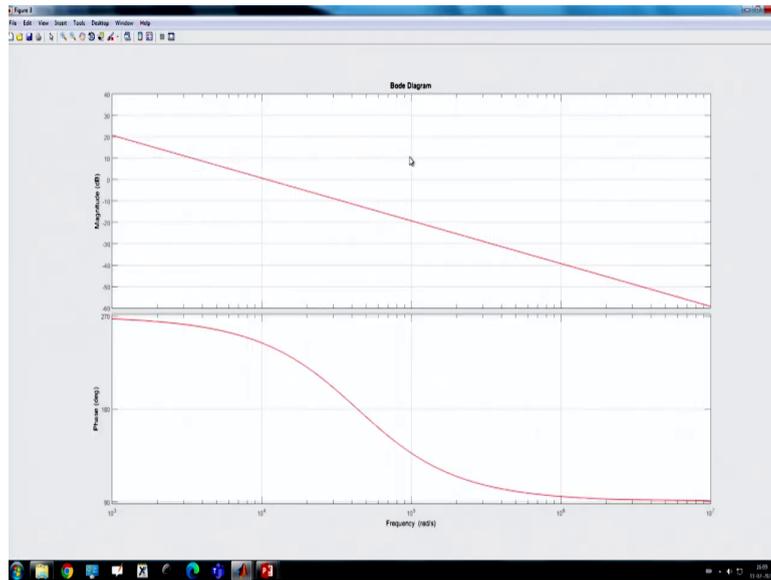
```

41 % x=1/tan(theta_rad); beta=sqrt(1+(x^2));x;
42 % w_cp=p*w_rhp; Lc=w_c/(2*pi*1e3);
43 % K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
44 % num_con=K_i*den_c;
45 % den_con=[1/w_cp 1 0];
46 % Gc=tf(num_con,den_con);
47
48 %% Design based on bandwidth
49 p=input('Select BW fraction of f_rhp ');
50 theta_rad=atan((2*p)/(1-(p^2)));
51 theta=rad2deg(theta_rad);
52 PM=90-theta; w_cp=p*w_rhp;
53 theta=(90-PM); theta_rad=deg2rad(theta);
54 K_i=(p*(1-D)^2)*w_rhp/(Fm*Vin); w_cp=w_rhp;
55 num_con=K_i*den_c;
56 den_con=[1/w_cp 1 0];
57 Gc=tf(num_con,den_con);
58
59 %% Loop gain and closed-loop TFs
60 G_loop=Gvd*Fm*Gc; %% Loop gain
61
62 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
63 G_cl=G_loop/(1+G_loop); %% Closed-loop TF

```

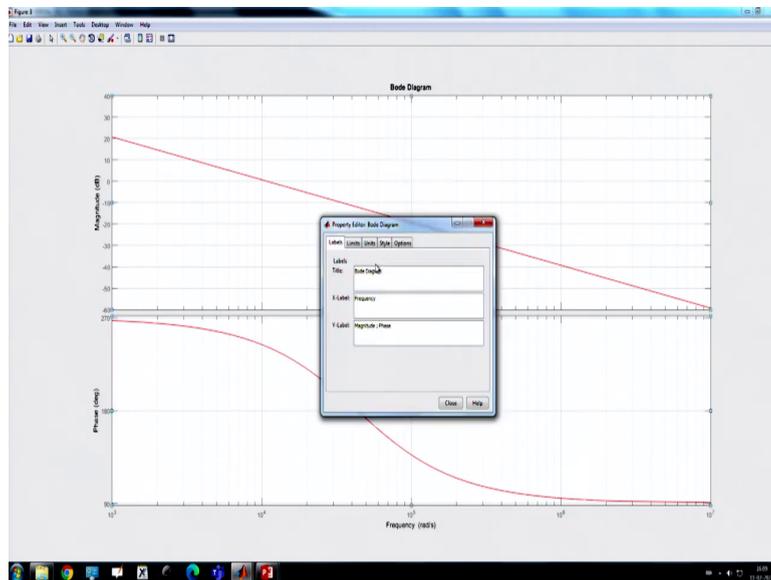
So, if I let us say we start with one-fifth or one-fourth.

(Refer Slide Time: 46:03)



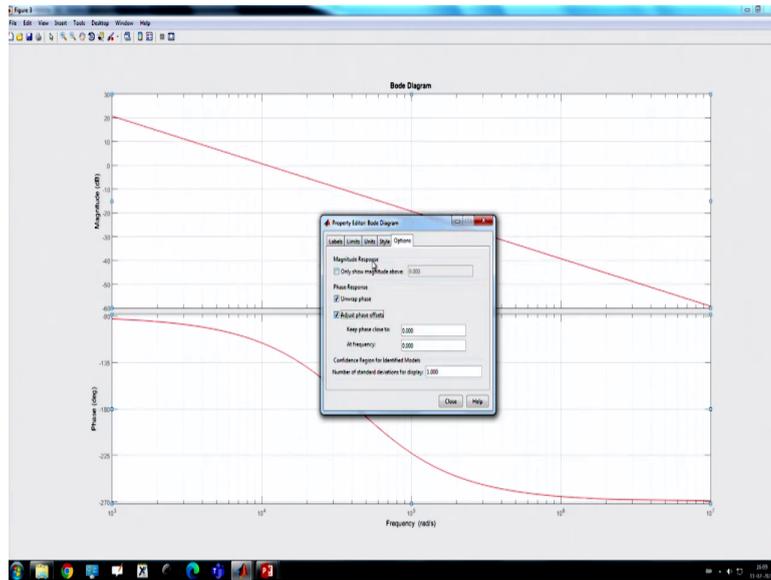
So, if one-fourth, then roughly one - fourth should give around 60 degree phase margin that we have learned.

(Refer Slide Time: 46:13)

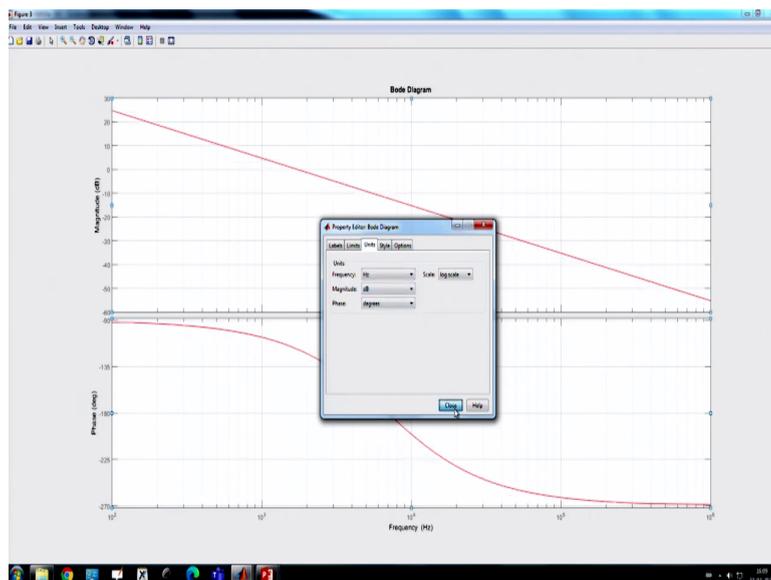


So, let us do that let us see because this is our loop gain plot.

(Refer Slide Time: 46:17)



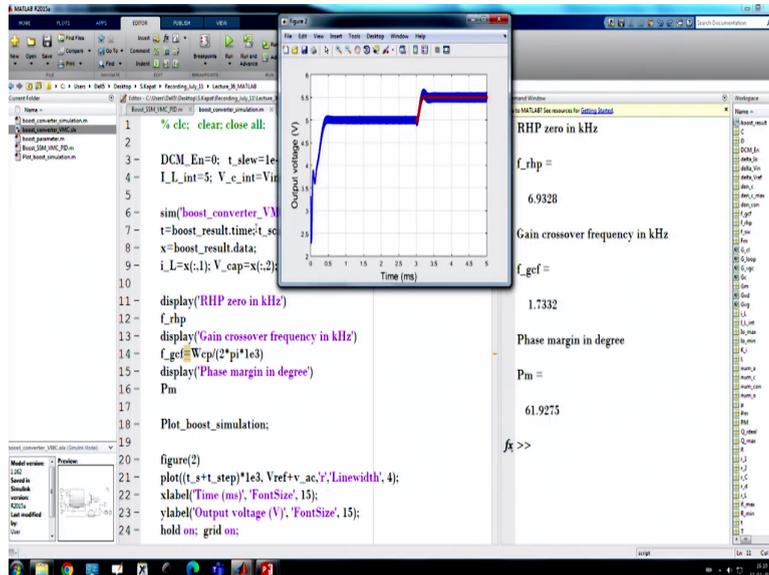
(Refer Slide Time: 46:19)



And here we have to change to hertz ok. So, we have to see what are the stable. So, you see.

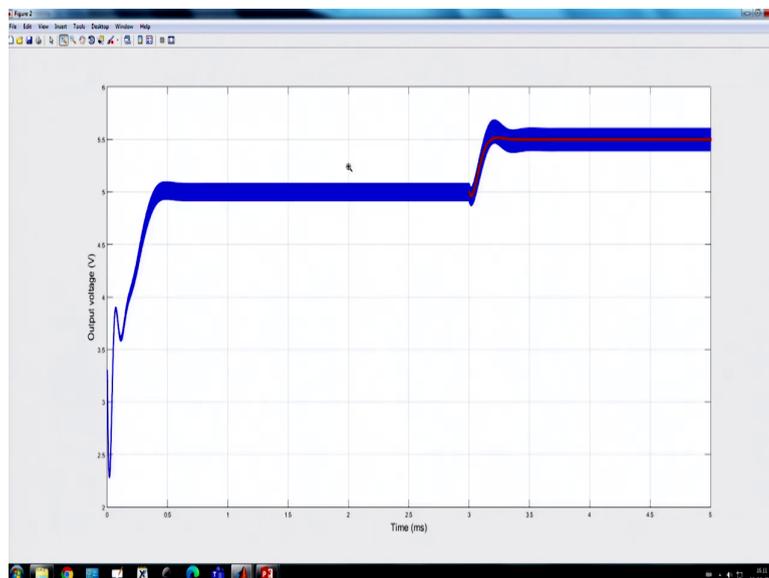
Now we want to see whether the response matches with the model or not.

(Refer Slide Time: 47:30)

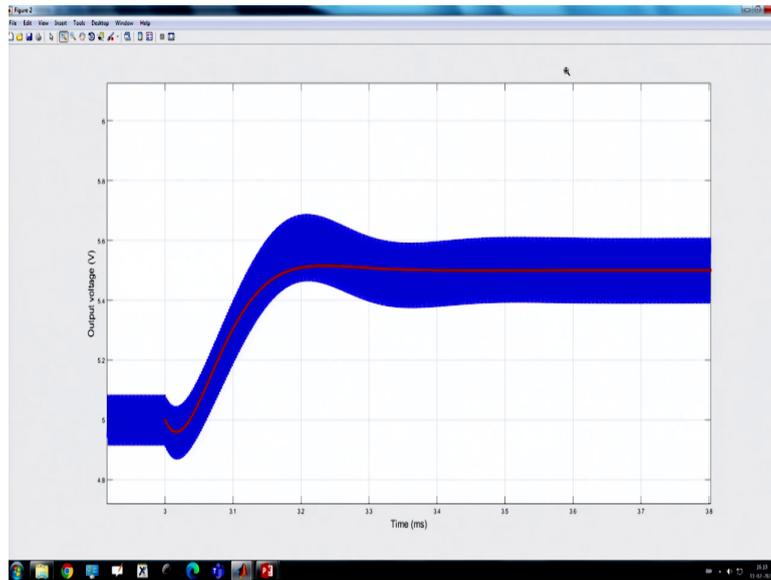


Because we have learned that whatever small-signal model we will derive, we should check that model ok.

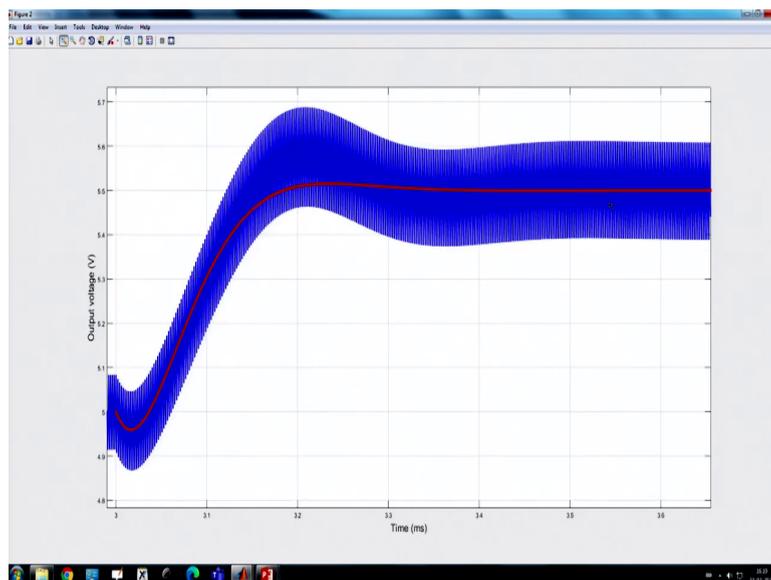
(Refer Slide Time: 47:33)



(Refer Slide Time: 47:36)

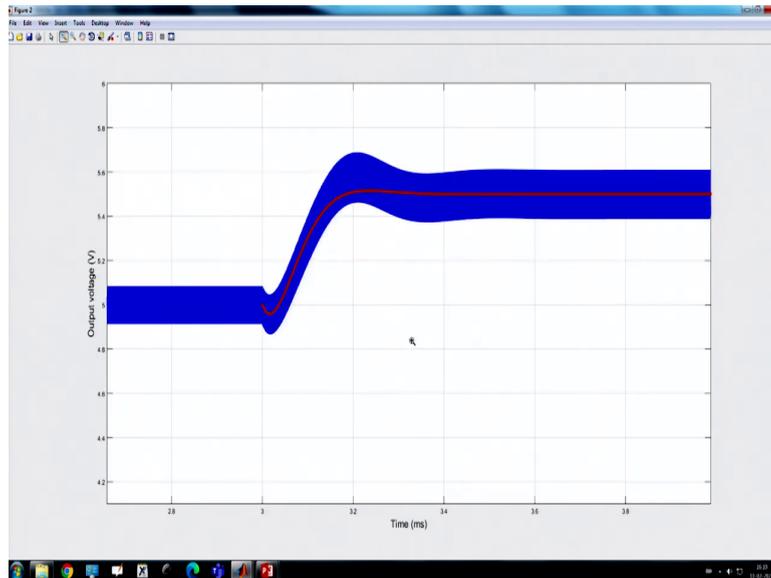


(Refer Slide Time: 47:40)

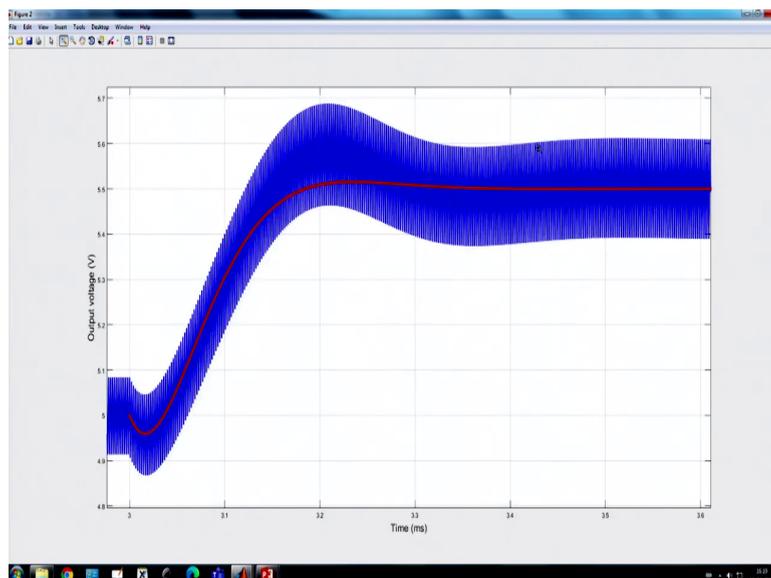


So, this is we have applied a reference transient response ok.

(Refer Slide Time: 47:43)

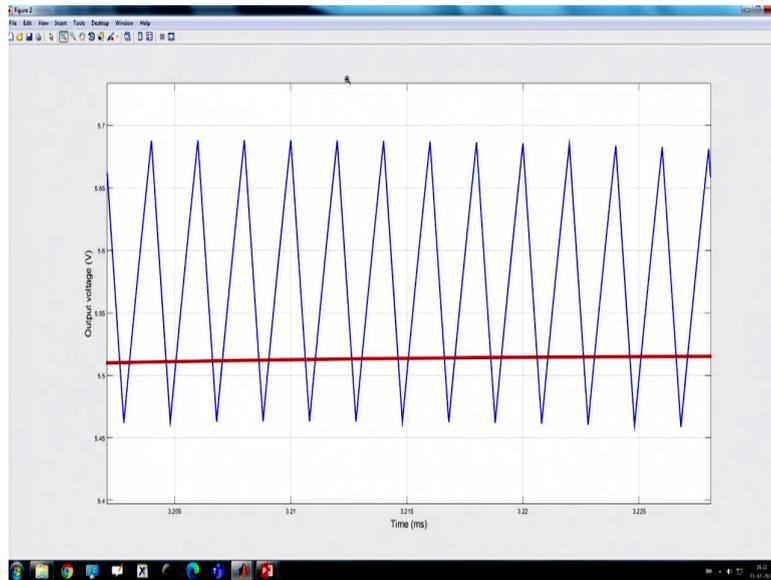


(Refer Slide Time: 47:47)

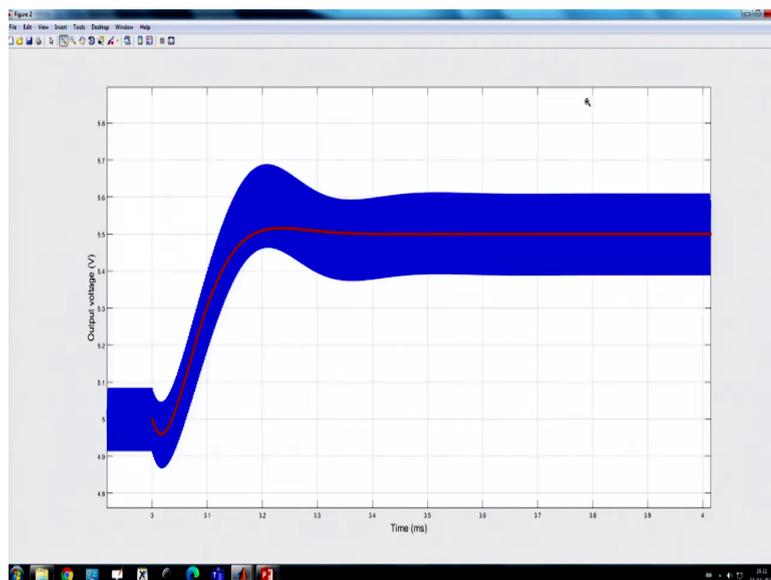


And this is actually that means, the red one indicate the response which is obtained from the small-signal model and the blue one is the response which is actual switch simulation that means if you go to actual switch simulation these are the current.

(Refer Slide Time: 48:01)

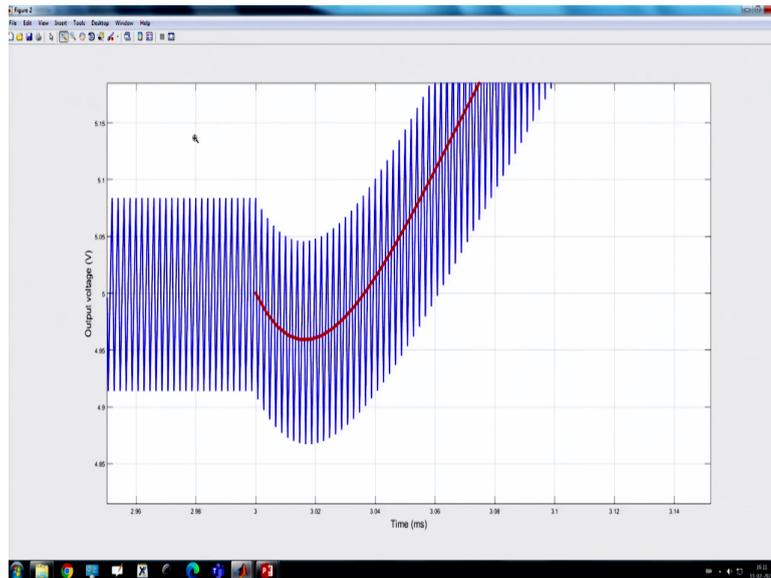


(Refer Slide Time: 48:08)



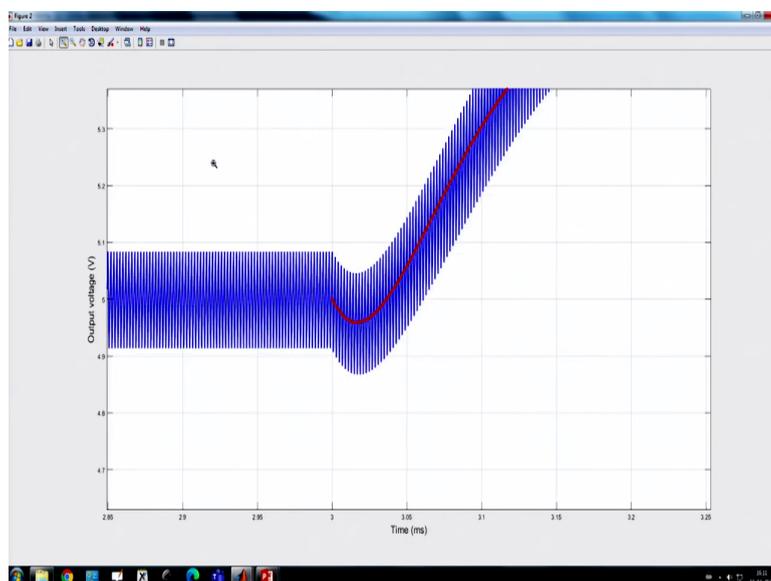
Sorry, voltage waveform and you see there is a undershoot behaviour right.

(Refer Slide Time: 48:10)



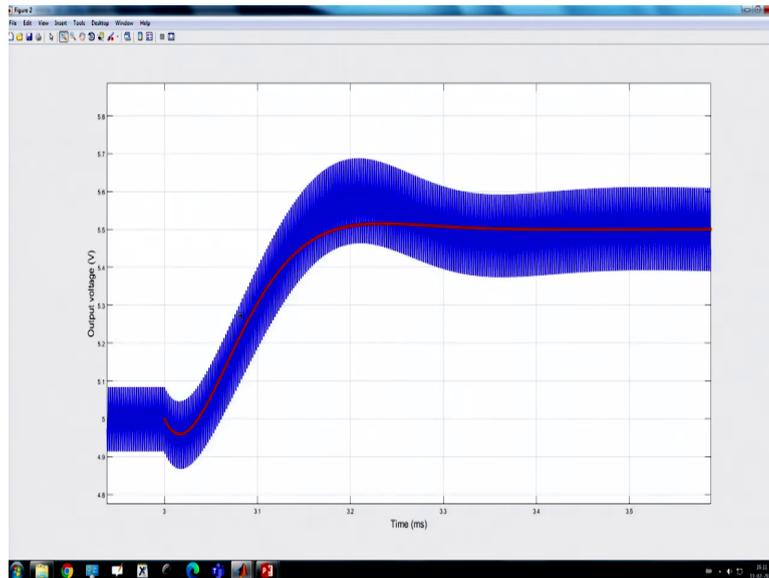
Non-minimum phase behaviour; that means, the voltage was supposed to increase.

(Refer Slide Time: 48:13)



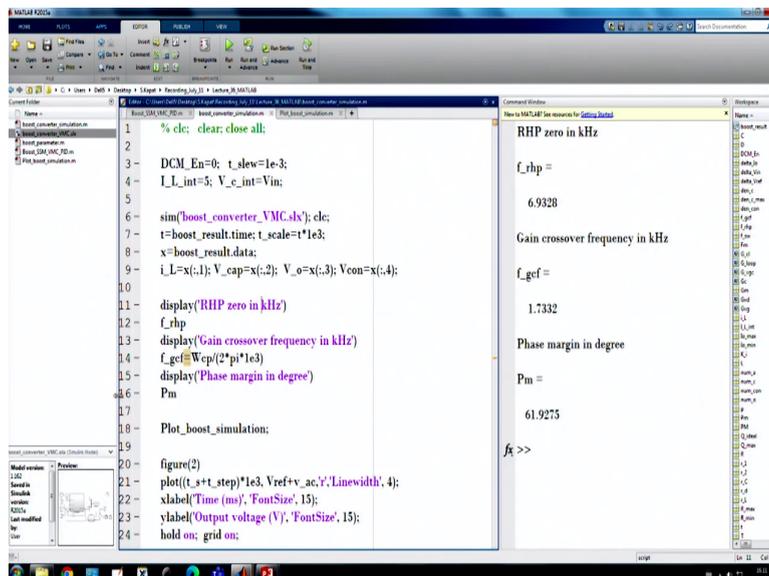
But it is not increasing at the beginning it is first going something you know.

(Refer Slide Time: 48:18)



So, it is initially going down; that means this part it is going down if we take the cursor. So, this part is going down. So, you see this is also and then it is increasing ok, but eventually it is increasing. So, initially there is an undershoot. So, this is the effect due to the right half plane zero non-minimum phase behaviour.

(Refer Slide Time: 48:42)



Now if we want to you know we want to in fact we chose a very small.

(Refer Slide Time: 48:49)

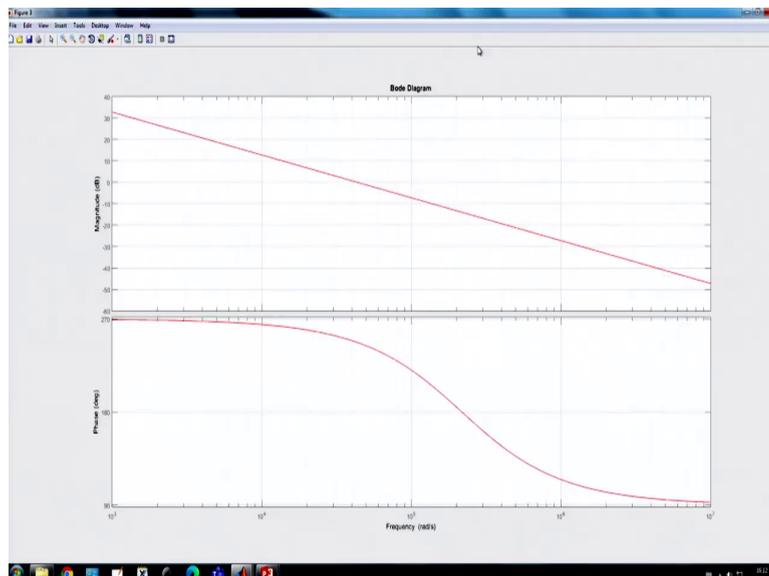
```

1  clc; close all; clear;
2
3  %% Parameters
4  boost_parameter; Vin=3.3; Vref=5; R=1;
5  D=(Vref-Vin)/Vref;
6  Io_min=0.5; R_max=Vref/Io_min;
7  Io_max=10; R_min=Vref/Io_max;
8
9  f_sw=1T; w_sw=2*pi*f_sw;
10 z_c=sqrt(L/C); w_o_ideal=(1-D)/sqrt(L*C);
11 Q_ideal=((1-D)*R)/z_c;
12 w_rhp=(R*(1-D)^2)/L;
13 f_rhp=w_rhp/(2*pi*1e3);
14
15 %% Control-to-output TF Gvd
16 num_c=(Vin/((1-D)^2))*1/w_rhp;
17 den_c=[1/(w_o_ideal^2) 1/(Q_ideal*w_o_ideal) 1];
18 Gvd=tf(num_c,den_c);
19
20 %% Open-loop Output Impedance
21 num_o=(1/((1-D)^2))*L;
22 den_o=[1/(w_o_ideal^2) 1/(Q_ideal*w_o_ideal) 1];
23 Z_o=tf(num_o,den_o);

```

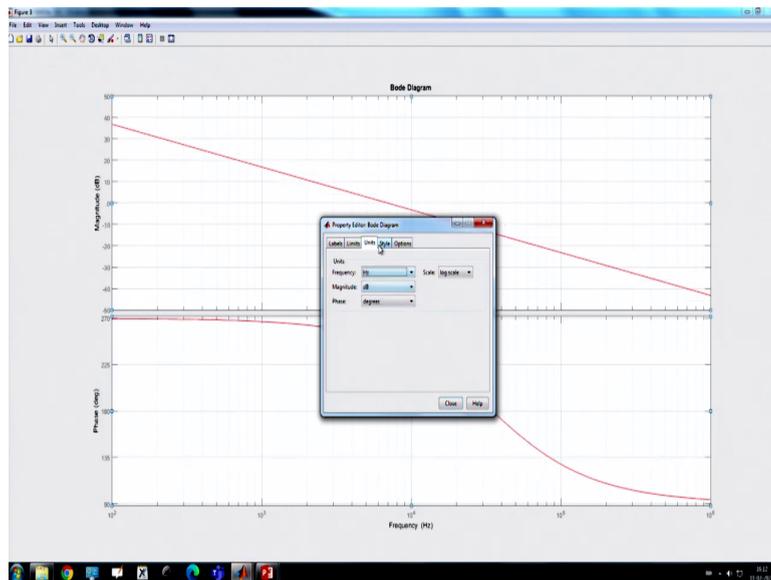
So, now, you we resist load resistance because with we have discussed that right half plane zero depends on the load resistance for a smaller load resistance, it is very close to the imaginary axis. Now, we have increased the load resistance by five times and we want to see what happens.

(Refer Slide Time: 49:08)

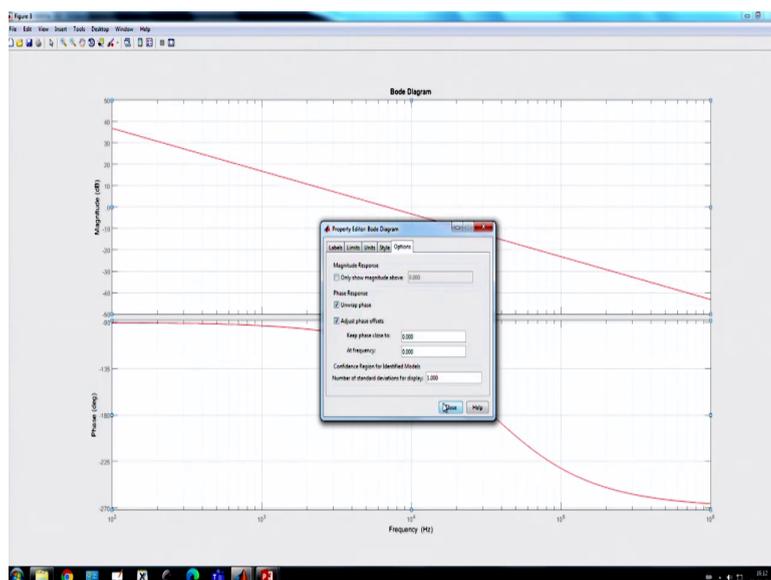


So, now, we are also taking one - fifth of the switching frequency.

(Refer Slide Time: 49:14)

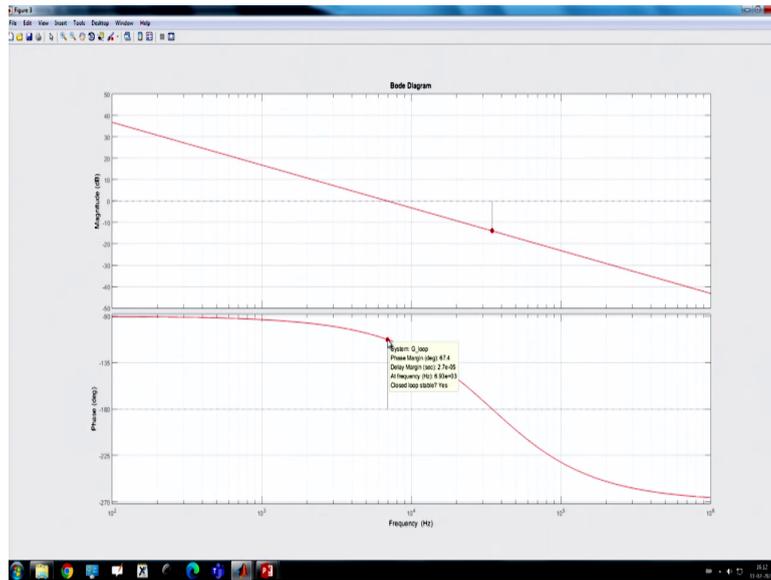


(Refer Slide Time: 49:16)



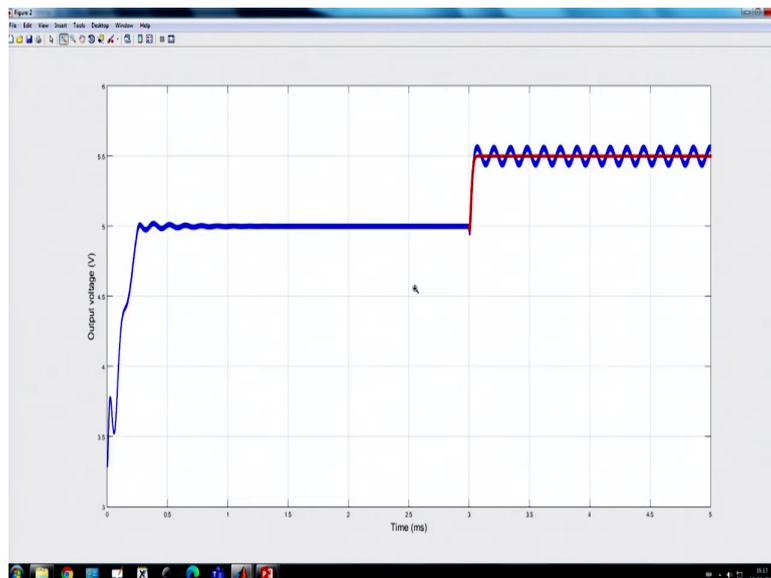
And if you take then if you go to the Bode plot and if you change this ok.

(Refer Slide Time: 49:18)

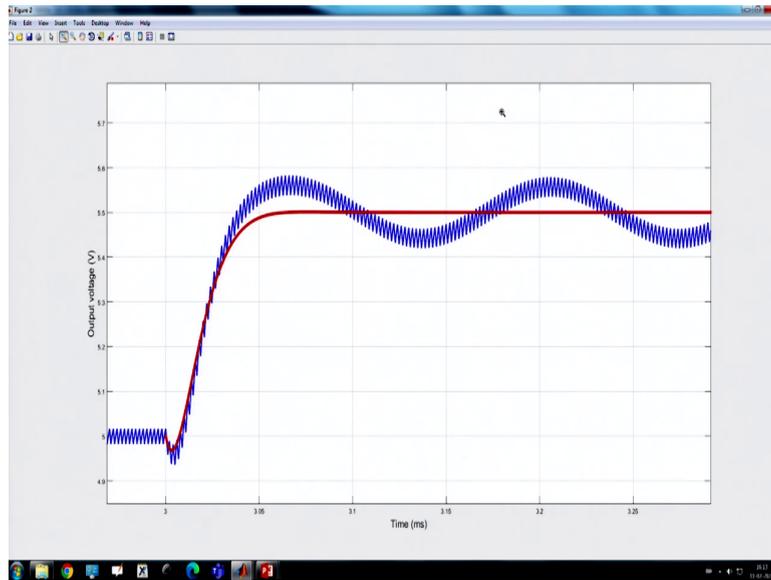


Now you will find now the bandwidth is 6.93 kilohertz. So, earlier it was 1.73 now it has increased by almost four times because RHP zero location has moved and phase margin also has increased ok because we have chosen one-fifth of the switching frequency RHP zero frequency the bandwidth ok then we will go for the results.

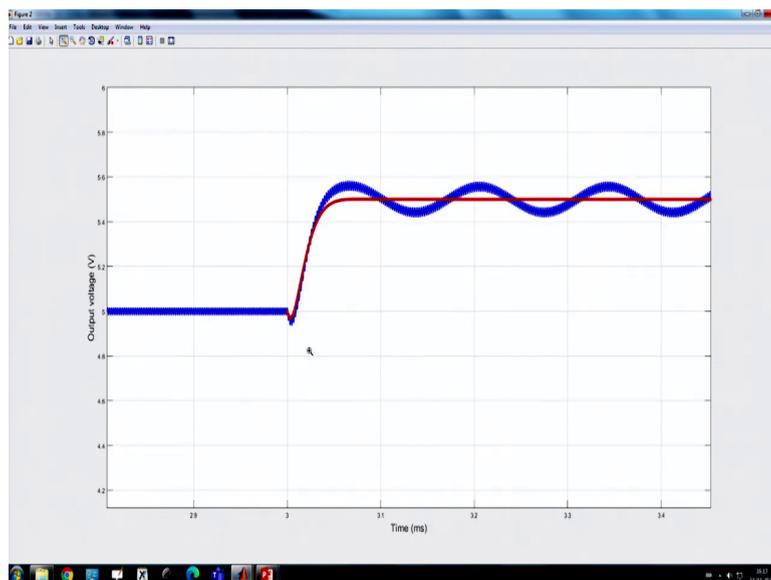
(Refer Slide Time: 49:49)



(Refer Slide Time: 49:55)

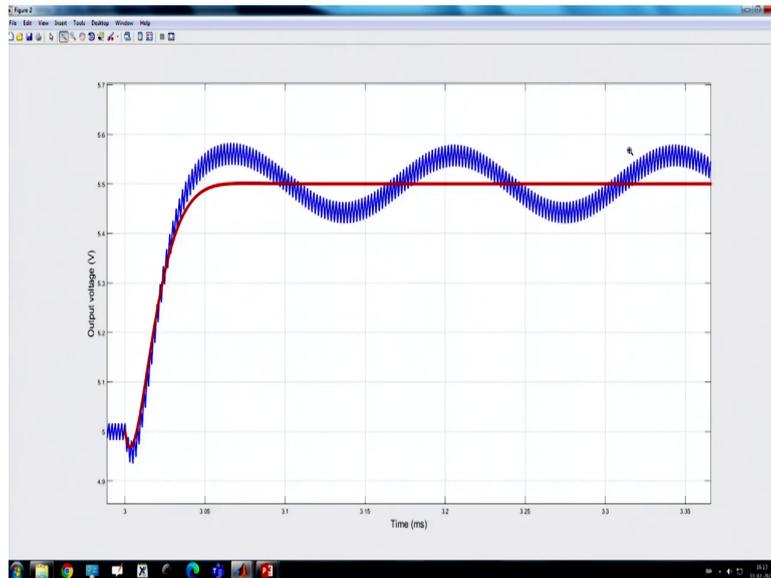


(Refer Slide Time: 49:57)



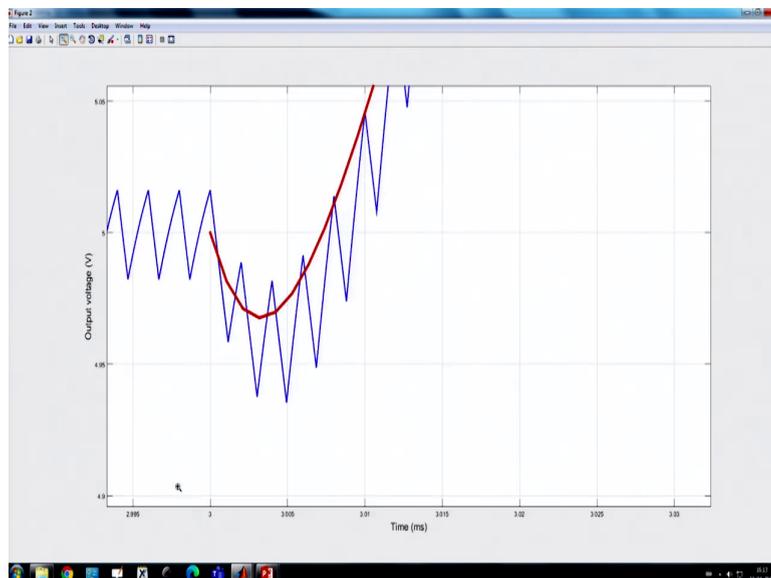
But we will see first thing.

(Refer Slide Time: 49:59)



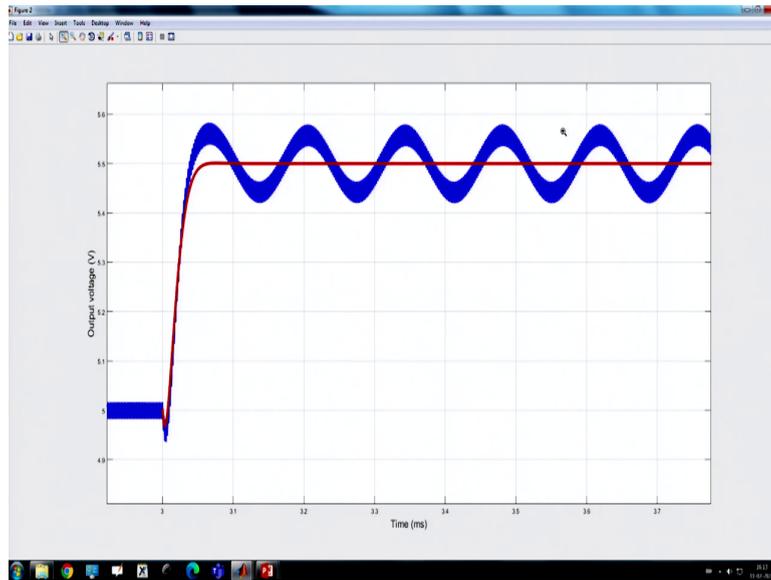
So, though there is a matching you know they are matching quite nicely up to this part.

(Refer Slide Time: 50:04)



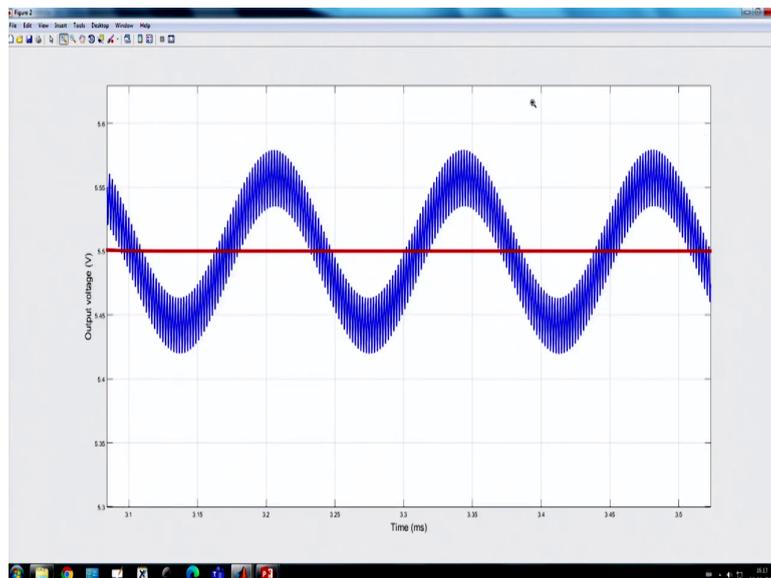
And there is also an undershoot behaviour right they are matching.

(Refer Slide Time: 50:10)

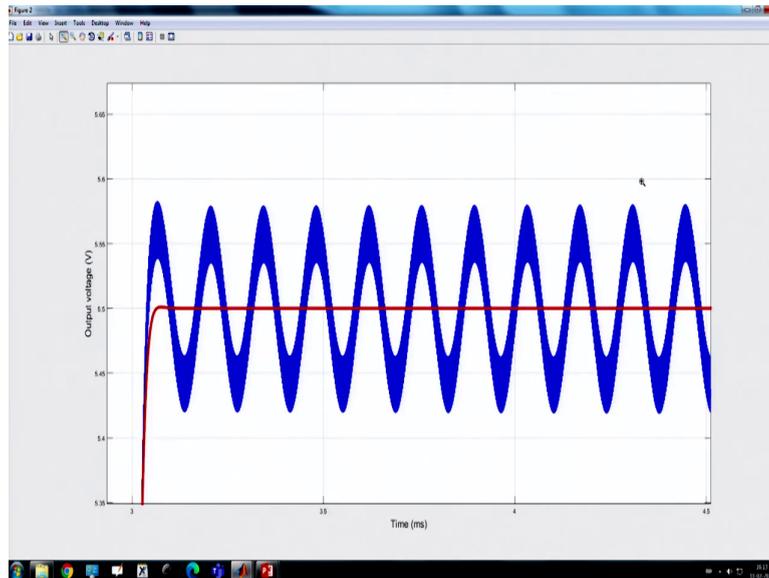


But you will find the behaviour becomes oscillatory at highest output voltage condition because this oscillatory behaviour is due to at light load sorry your resistance is low in this case though you can push the right half plane zero further right side, but this also increases the Q factor and subsequently the phase margin also decreases.

(Refer Slide Time: 50:41)



(Refer Slide Time: 50:45)



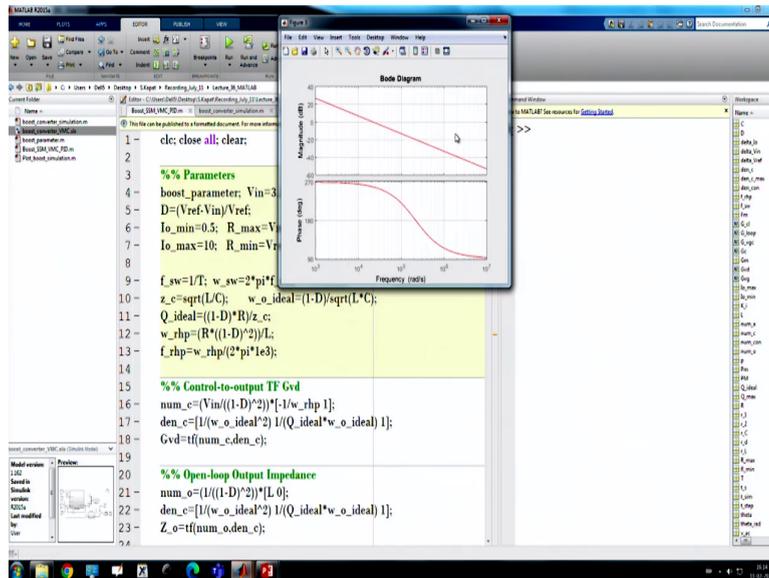
I mean though we have increased, but actual switching converter it will be hard to anticipate this effect using voltage mode control right. So; that means, then how to address this problem one of the way you should even decrease the bandwidth.

(Refer Slide Time: 50:57)

```
1- clc; close all; clear;
2
3 %% Parameters
4- boost_parameter; Vin=3.3; Vref=5; R=1;
5- D=(Vref-Vin)/Vref;
6- Io_min=0.5; R_max=Vref/Io_min;
7- Io_max=10; R_min=Vref/Io_max;
8
9- f_sw=1/T; w_sw=2*pi*f_sw;
10- z_c=sqrt(L/C); w_o_ideal=(1-D)/sqrt(L*C);
11- Q_ideal=((1-D)*R)/z_c;
12- w_rhp=(R*(1-D)^2)/L;
13- f_rhp=w_rhp/(2*pi*1e3);
14
15 %% Control-to-output TF Gvd
16- num_c=(Vin*((1-D)^2))*[-1/w_rhp 1];
17- den_c=[1/(w_o_ideal^2) 1/(Q_ideal*w_o_ideal) 1];
18- Gvd=tf(num_c,den_c);
19
20 %% Open-loop Output Impedance
21- num_o=(1/((1-D)^2))*[L 0];
22- den_o=[1/(w_o_ideal^2) 1/(Q_ideal*w_o_ideal) 1];
23- Z_o=tf(num_o,den_o);
```

That means, even if you try to decrease the bandwidth let us say we take one-tenth of the switching frequency.

(Refer Slide Time: 50:59)



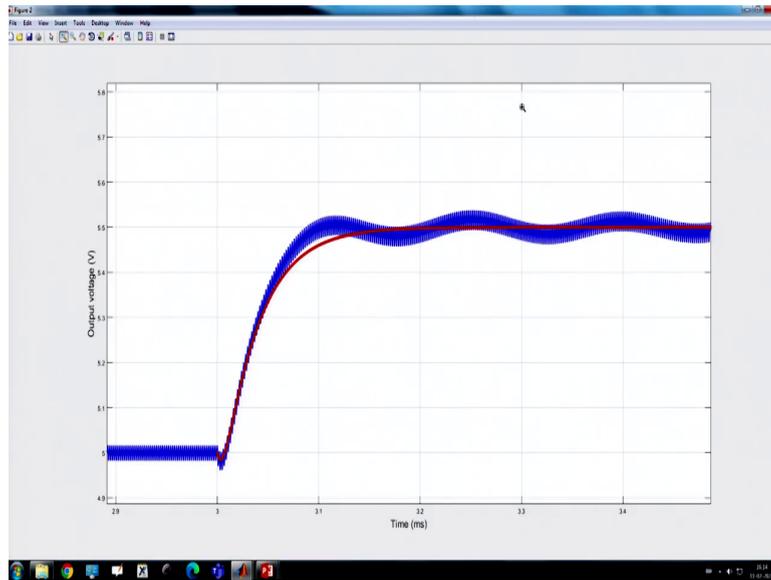
And if you simulate; that means, we are further reducing.

(Refer Slide Time: 51:05)

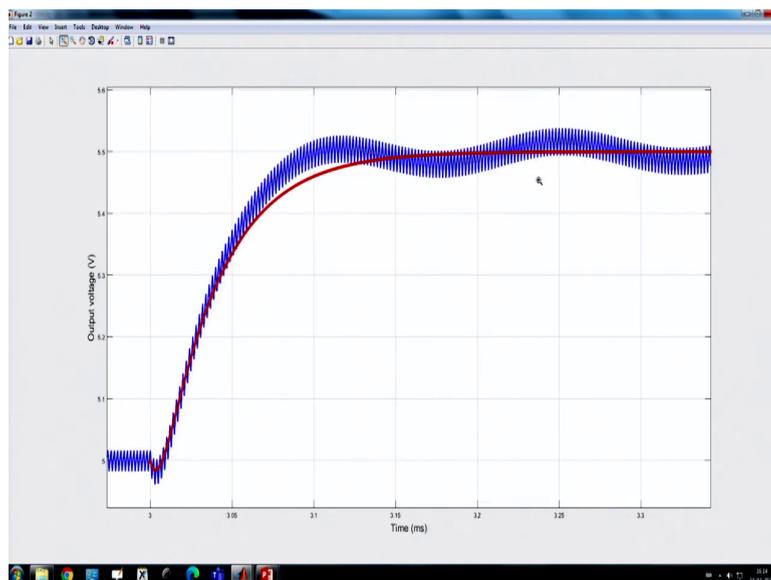


So, you can see that can be damped out somewhat.

(Refer Slide Time: 51:11)

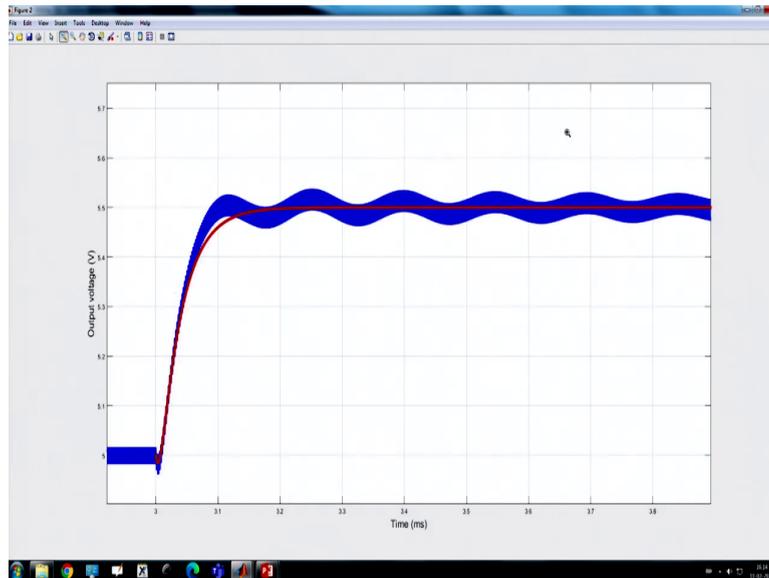


(Refer Slide Time: 51:13)



If you slow down the response we are slowing down.

(Refer Slide Time: 51:18)



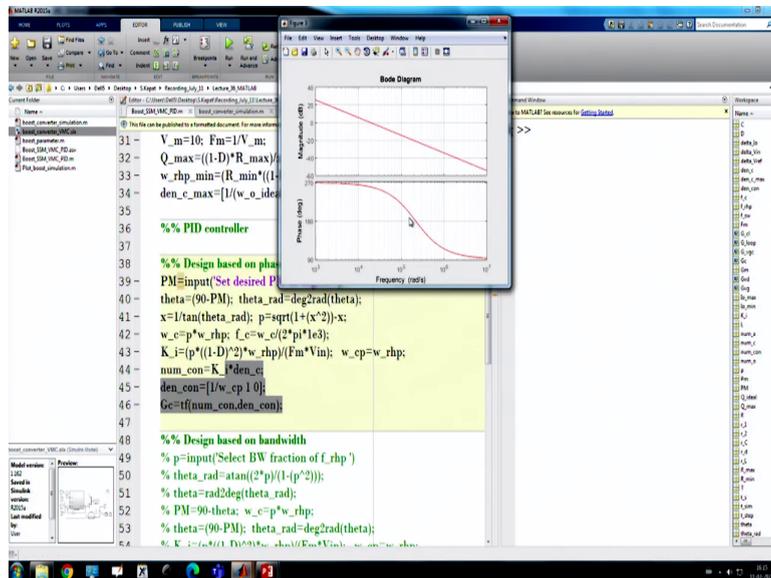
But in this case we took the bandwidth to be one-tenth of the RHP zero, which is pretty low, but still because earlier if you want to push up to one-fifth, then the problem was the actual switching converter; that means, switch actual switch model it gives oscillatory behaviour ok. Because the model validity comes into picture and that is difficult to compensate so and we will see current mode control, which can be useful. Another one I want to show suppose if you want to go for design based on phase margin.

(Refer Slide Time: 51:55)

```
31 - V_m=10; Fm=1/V_m;
32 - Q_max=((1-D)*R_max)/z_c;
33 - w_rhp_min=(R_min*(1-D^2))/L;
34 - den_c_max=[1/(w_o_ideal^2) 1/(Q_max*w_o_ideal) 1];
35
36 %% PID controller
37
38 %% Design based on phase margin
39 - PM=input('Set desired PM in degree ');
40 - theta=(90-PM); theta_rad=deg2rad(theta);
41 - x=1/tan(theta_rad); p=sqrt(1+(x^2));
42 - w_cp=p*w_rhp; z_c=w_cp*(2*p^1.63);
43 - K_i=(p*(1-D)^2*w_rhp)/(Fm*Vin); w_cp=w_rhp;
44 - num_con=K_i*den_c;
45 - den_con=[1/w_cp 1 0];
46 - Ge=tf(num_con,den_con);
47
48 %% Design based on bandwidth
49 - % p=input('Select BW fraction of f_rhp ');
50 - % theta_rad=atan(2*p)/(1-(p^2));
51 - % theta=rad2deg(theta_rad);
52 - % PM=90-theta; w_cp=p*w_rhp;
53 - % theta=(90-PM); theta_rad=deg2rad(theta);
54 - % K_i=(p*(1-D)^2*w_rhp)/(Fm*Vin); w_cp=w_rhp;
```

Let us say now we enable this and earlier it was asking for fraction of switching frequency RHP zero.

(Refer Slide Time: 52:07)



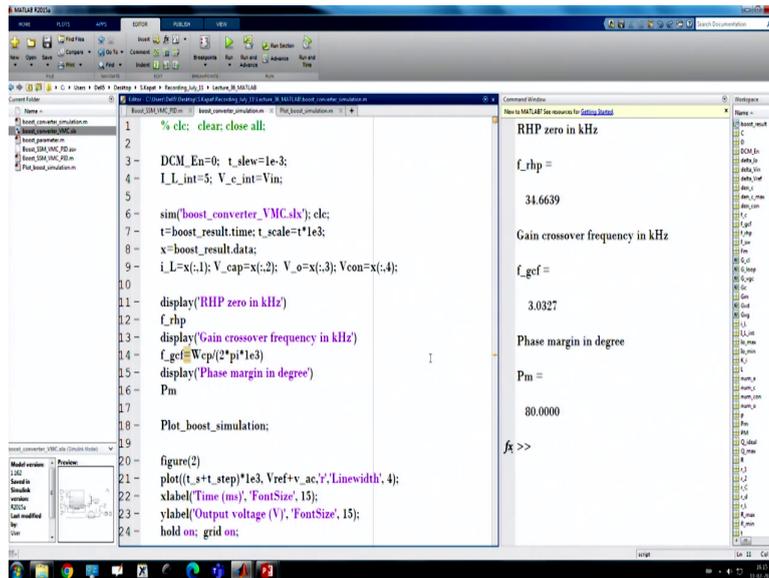
Now, we are asking for phase margin. Suppose we want to achieve let us say 80 degree phase margin which is too large ok; that means, we have given too much phases.

(Refer Slide Time: 52:14)



And you can achieve this behaviour.

(Refer Slide Time: 52:19)



```
1 % clear; clear; close all;
2
3 DCM_En=0; t_slew=1e-3;
4 I_L_int=5; V_c_int=Vin;
5
6 sim('boost_converter_VMC.slx'); clc;
7 t=boost_result.time; t_scale=*1e3;
8 x=boost_result.data;
9 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3); Vcon=x(:,4);
10
11 display('RHP zero in kHz')
12 f_rhp
13 display('Gain crossover frequency in kHz')
14 f_gc=@Wcp(2*pi*1e3)
15 display('Phase margin in degree')
16 Pm
17
18 Plot_boost_simulation;
19
20 figure(2)
21 plot(t,:t_step)*1e3, Vref+v_ac,'Linewidth', 4);
22 xlabel('Time (ms)', 'FontSize', 15);
23 ylabel('Output voltage (V)', 'FontSize', 15);
24 hold on; grid on;
```

Command Window Output:

```
RHP zero in kHz
34.6639
Gain crossover frequency in kHz
3.0327
Phase margin in degree
Pm =
80.0000
fx >>
```

And if you see the RHP zero, it is roughly 34 kilohertz and sorry the RHP zero and crossover frequency is 3 kilo hertz; that means, it less than it is around 1 by 11 times one eleventh of the RHP zero ok. So, this is a gain crossover frequency rhp zero in kilo hertz and the phase margin. So, phase margin which is obtained from Bode plot is also 80 degree and we have set 80 degree. So, it is consistent, ok.

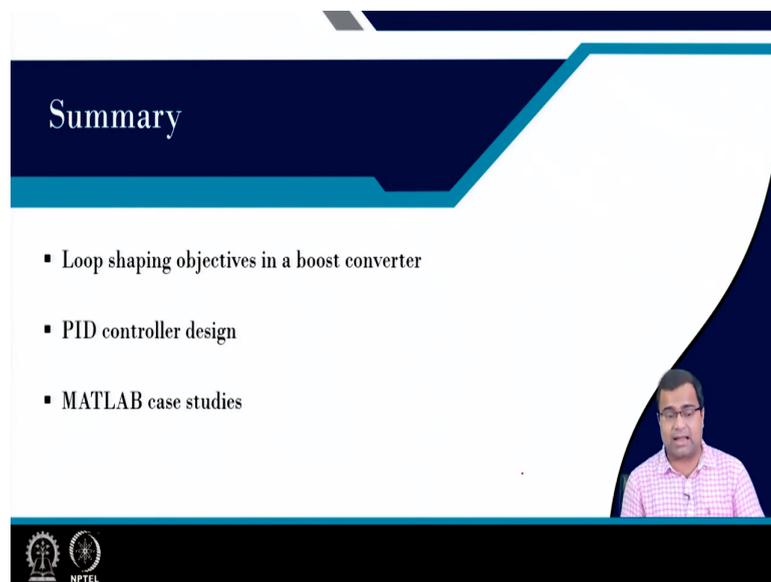
So, we have learned the boost converter design and general thumb rule for the boost converter is generally for voltage mode control the crossover frequency is chosen that rhp zero by 5. So, this is the thumb rule in fact, if you write in you know in hertz it is again f rhp zero by 5.

So, this consideration is coming by virtue of this compensation that so; that means, this gives us a constant because we cannot increase phase margin. It will penalize the crossover frequency. So, it cannot increase the crossover frequency phase margin will be penalize ok.

So, this will be a constant in a boost converter design. We will see in a subsequent lecture you know when you go to current mode control. We can achieve higher crossover frequency even though the RHP zero location will be same under current mode control, but still you can push the cutoff frequency because what we found in the switching actual switch simulation oscillator behaviour if you want to push the bandwidth.

Because actually because of poorly damped system, but in current mode control since you are directly controlling inductor current. So, it will give an over damp response. You can even push the bandwidth high

(Refer Slide Time: 54:04)



Summary

- Loop shaping objectives in a boost converter
- PID controller design
- MATLAB case studies

NPTEL

So, with this we have discussed loop shaping objective, PID controller design, then we also discussed type 3 compensator design, we showed MATLAB case studies. So, with this I want to finish here.

Thank you very much.