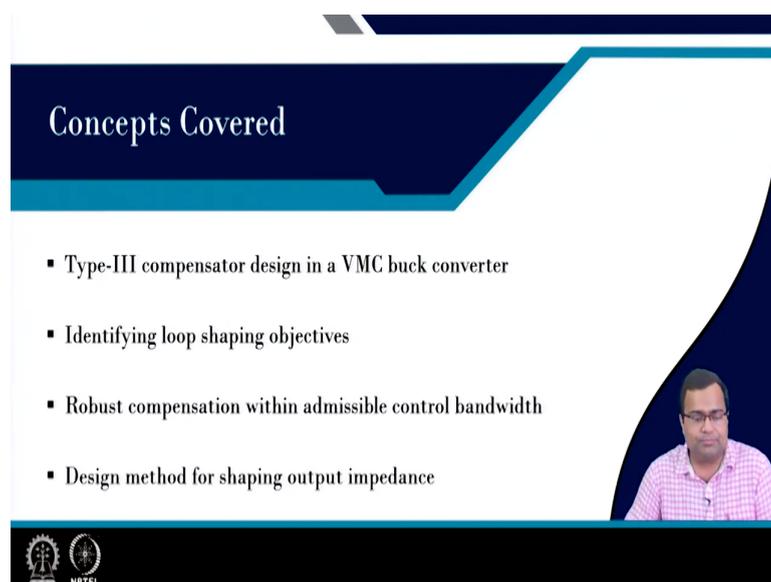


Control and Tuning Methods in Switched Mode Power Converters
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Module - 07
Small-signal Design and Tuning of PWM Voltage Mode Control
Lecture - 35
Shaping Output Impedance of a Buck Converter
Under VMC

Welcome this is lecture number 35, in this lecture we are going to talk about design of voltage mode control Buck Converter and the design is primarily intended to shape the loop for closed loop performance as well as the output impedance ok.

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Concepts Covered

- Type-III compensator design in a VMC buck converter
- Identifying loop shaping objectives
- Robust compensation within admissible control bandwidth
- Design method for shaping output impedance

NPTEL

So, in this we are talking about type 3 compensator design in a voltage mode control, then identifying loop shaping objective, then robust compensation within admissible control bandwidth, then design method for shaping output impedance.

(Refer Slide Time: 00:58)

Loop-Gain Analysis of a Buck Converter

$$G_{vd} = \frac{V_{in}}{\alpha} \times \frac{\left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

where

$$\alpha = \frac{(R + r_e)}{R}, \quad \omega_{ESR} = \frac{1}{r_c C}, \quad \omega_o = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{(R + r_e)}{(R + r_c)}}, \quad \approx \frac{1}{\sqrt{LC}}$$

$$Q = \alpha \left[\frac{(r'_c + r'_e)}{z_c} + \frac{R}{z_c} \right]^{-1}, \quad z_c = \sqrt{\frac{L}{C}}$$

So, Loop Gain Analysis of a Buck Converter this loop structure we have already seen in the previous lecture. So, I am not going to spend time on it and the control to output transfer function; that means, this particular transfer function that we know it is here ok. Where alpha we have discussed R plus r equivalent that is a total resistance DC r plus Rd S on ESR zero, then this is the LC filter pole and this can be approximated to be 1 by square root of LC because if r e and r c are close to each other or particularly if the load resistance is higher than r e and r c which is the case.

Now one thing we will keep in mind this is a Q factor which consist of parasitic; that means, ESR r equivalent characteristic impedance, where the characteristic impedance is nothing but square root of L by C, but this is the one which can affect the Q c.

That means, and this you can see if you compare this 2 term this term will be negligible when R is very large or light load condition ok and under high load condition when the R is small, then this will be dominating like R Z c by R will be a dominating factor ok. So, we have to keep in mind for practical compensation point of view.

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Primary Loop Shaping Objectives

$$K_{loop}(s) = F_m \times \frac{V_m}{\alpha} \times \frac{\left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

- To cancel ESR zero – to eliminate high frequency ripple (1P)
- To cancel double poles – to offset sharp phase fall (2Z)
- To consider an integrator – to eliminate SS error (1P)

Next our primary loop shaping objective, so here our primary loop shaping we have to cancel we are planning to cancel ESR zero. So, we need to consider that means there we need to have a 1 pole of the controller to cancel the ESR zero. We need to cancel double pole to offset the sharp fall right due to the double pole. In fact, this is the one which we will consider when you talk about robust compensation.

Because a double pole may not be perfectly known because of variation in the load resistance, but we still need 2 zeros to compensate that controller zeros ok. We need to consider one integrator in the controller which will try to eliminate the steady state error or it will provide sufficient DC gain.

(Refer Slide Time: 03:11)

Primary Loop Shaping Objectives

$$K_{loop}(s) = F_m \times \frac{V_{in}}{\alpha} \times \frac{\left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

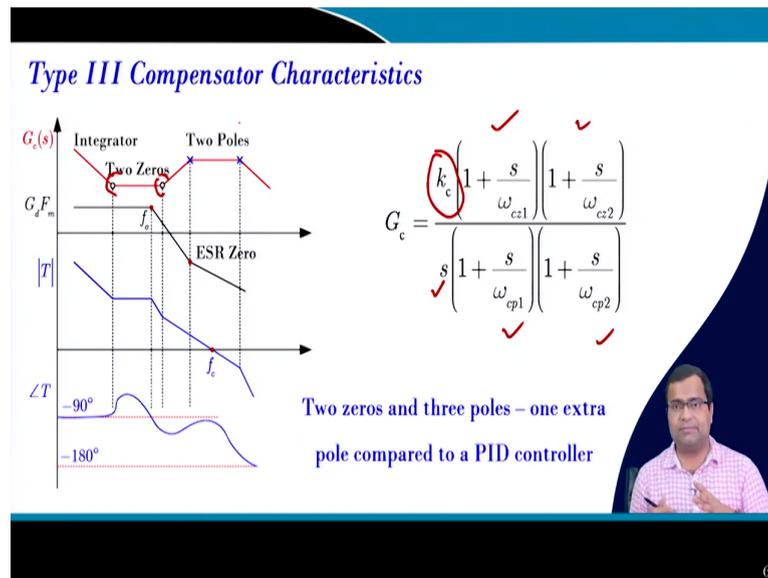
- To consider a simple pole – to achieve desired phase margin (1P)
- To set compensator DC gain – to achieve desired gain crossover freq.
- Controller summary: 3 poles (including one at origin) and 2 zeros
- A type-III compensator needed: PID enough for ideal buck

So, we need one more pole in the controller ok. We also need to consider another simple pole, because in the previous lecture, we saw another extra pole will give a degree of freedom ok.

Which was not available in the PID controller, so we could not shape the transient response under wide operating range and now we have it because in type 3 compensator. So, one more pole and then we need to set the gain of the controller to achieve desired gain crossover frequency.

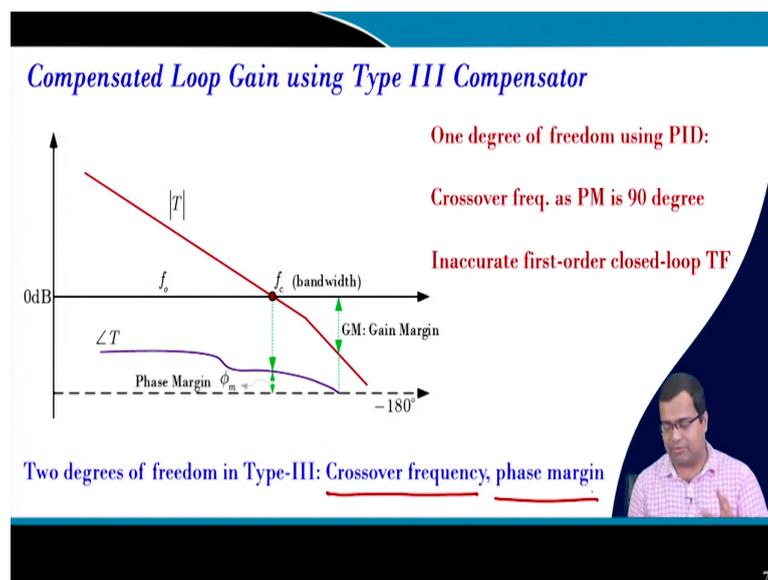
So, we have we can, you know; that means we need 3 poles and 2 and this is nothing but type 3 compensator and PID controller may be enough for an ideal buck, but for a practical buck we need a type 3 compensator. Another point, here we can independently control we will see both crossover frequency as well as the phase margin ok.

(Refer Slide Time: 04:08)



So, type 3 compensator characteristics we have discussed it has 2 zeros and 1 pole at origin and other 2 poles ok and this is the controller DC gain sorry not DC gain that is a controller gain. And first it has an integrator, then there are 2 zeros right and 2 poles and we will want to see how can we how are we going to place those poles and zeros. So, it has an one extra pole compared to PID controller, and that is why it gives another degree of freedom.

(Refer Slide Time: 04:40)



And we can measure the phase response phase margin and we will see that 2 degree of freedom will be given one crossover frequency and phase margin. Which is not there because

PID controller has 1 degree of freedom, because it always analytically shows 90 degree phase margin, but practically we show that model is not accurate enough even for a low you know control bandwidth.

(Refer Slide Time: 05:05)

Perfect Compensation – Buck Converter

$$K_{loop}(s) = F_m \times \frac{V_m}{\alpha} \times \frac{\left(1 + \frac{s}{w_{ESR}}\right)}{\left(1 + \frac{s}{Qw_o} + \frac{s^2}{w_o^2}\right)} \times G_c$$

$$G_c = k_c \times \frac{\left(1 + k_1s + k_2s^2\right)}{s \left(1 + \frac{s}{w_{cp1}}\right) \left(1 + \frac{s}{w_{cp2}}\right)}$$

$$\left(1 + k_1s + k_2s^2\right) = \left(1 + \frac{s}{Qw_o} + \frac{s^2}{w_o^2}\right)$$

$$w_{cp1} = w_{ESR}, \quad w_{cp2} \triangleq w_p$$

(Handwritten notes in red: Nc(s) and Dp(s) are circled in the transfer function. The controller numerator is also circled in red.)

Now, the perfect compensation if you take the loop transfer function, then the controller is a type 3 compensator where it has a numerator double 0 2 0 and which is like your zero of the controller is designed to compensate for the denominator of this plant.

So, we are doing zero cancellation and one of the pole is used to cancel the ESR zero. So, this is what exactly I told here. We are trying to cancel the second one. The first pole we are using the ESR zero and since we have an additional pole of the controller.

So, which is taken as a w_p omega p and we have to find out where to place the additional pole that we will discuss. But that means, the controller has 2 zero is used to cancel the double pole, but we will talk about the robust compensation and control has 1 zero; 1 pole is used to cancel ESR zero another pole is the flexible pole ok.

(Refer Slide Time: 06:13)

Perfect Compensation – Buck Converter

$$K_{loop}(s) = \frac{F_m V_{in} k_c}{\alpha \omega_p} \times \frac{1}{s \left(1 + \frac{s}{\omega_p}\right)}$$

$$K_{loop}(j\omega) = \frac{F_m V_{in} k_c}{\alpha \omega_p} \times \frac{1}{K_L \left(\frac{j\omega}{\omega_p}\right) \left(1 + \frac{j\omega}{\omega_p}\right)}$$

$$K_{loop}(j\omega_n) = K_L \times \frac{1}{j\omega_n (1 + j\omega_n)}$$

where $\omega_n = \frac{\omega}{\omega_p}$

Handwritten notes: $S = j\omega$, $\omega = \omega_n$, $K_L = \frac{F_m V_{in} k_c}{\alpha \omega_p}$

Next now, if we substitute S equal to j omega because we want to do frequency response carry out, then these terms along with the controller k c; that means, the loop transfer function will look like this right. So, this is a loop transfer function after cancellation, this is a simple pole of the controller which we want to place and here we can consider another p and we can multiply with, because we want to write in the form of S by w p controller pole ok that exactly we did here right. So, we took this.

Now, you substitute S equal to j omega. Our next task we want to replace this omega by omega p is equal to omega n it is something like a used for normalization, because it is a ratio of 2 frequency.

Where omega is the instantaneous frequency related to the frequency response and omega p is the pole of the simple pole of the controller which we want to place. So, the normalize loop gain will take the form KL into this where KL; what is KL that we have discussed? KL is nothing but wait. So, we want to write what is KL k l is nothing but Fm V in k c by alpha omega p. So, you should keep this. This is the normalized frequency.

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Perfect Compensation – Buck Converter

$$K_{loop}(j\omega_n) = K_L \times \frac{1}{j\omega_n} \frac{1}{1 + j\omega_n}$$

$$K_L = \frac{F_m V_{in} k_c}{\alpha \omega_p}$$

$$K_{loop}(j\omega_n) = r(\omega_n) \angle \theta(\omega_n)$$

$$r(\omega_n) = \frac{K_L}{\omega_n} \times \frac{1}{\sqrt{1 + \omega_n^2}}$$

$$\angle \theta(\omega_n) = -90^\circ - \tan^{-1}(\omega_n)$$

$$\omega_n = \frac{\omega_c}{\omega_p}$$

The diagram also shows a polar plot of the loop gain with handwritten annotations. The magnitude is represented by the product of $\frac{1}{\omega_n}$ and $\frac{1}{\sqrt{1 + \omega_n^2}}$. The phase is the sum of -90° and $-\tan^{-1}(\omega_n)$.

Now, this is what we have already written. Now consider loop gain in polar form. So, this is the Cartesian form and we need to convert into polar form, because we have 2 complex number this one and this one 2 complex numbers. So, you can treat like is a X and Y right, so there are 2 complex number. See, in the polar form, we want to write in terms of amplitude and angle, right magnitude and angle.

So, what is the magnitude if you take this polar form? K_L is already in terms of magnitude ok. So, there is no it is it is a it is a real number right and it is of course a positive quantity, then for this term X that means this term your magnitude is simply 1 by omega n and for this term your magnitude is simply one by 1 plus omega n the square root of 1 by omega, so this is an overall magnitude of this. Then what is the phase you see? The phase due to this is nothing but minus 90 degrees, because it is a 1 by S term 1 by g omega right.

This is a complex number minus 90 degrees and the phase due to this term; that means, you know if you use a different colour, the phase due to this the theta contribution it is sorry theta is nothing but this quantity minus of this and minus 90 degrees is coming due to 1 by j omega n it is coming due to this ok. And you know the in polar form the phase. If they are product, then the phase can be added, if the 2 complex numbers if you write a product of 2 complex number the phase will be the sum of their individual complex number phase of the ok.

(Refer Slide Time: 10:06)

Perfect Compensation – Buck Converter

$$K_{loop}(j\omega_n) = K_L \times \frac{1}{j\omega_n(1 + j\omega_n)}$$

$$K_L = \frac{F_m V_{in} k_c}{\alpha \omega_p}$$

$r(\omega_n) = \omega_n = \frac{\omega_c}{\omega_p}$

At gain crossover frequency (f_c)

$$r(\omega_n) \Big|_{\omega=\omega_c} = 1 \Rightarrow K_L = \left(\frac{\omega_c}{\omega_p}\right) \times \sqrt{1 + \left(\frac{\omega_c}{\omega_p}\right)^2}$$

So, at gain crossover frequency our magnitude is 1 because in polar form we have a magnitude which is a function of omega and it is nothing but what is omega n, omega n is nothing but omega by omega p where it is an instantaneous frequency and it is a pole frequency that is our controller pole. Now at gain crossover frequency, that is nothing but r of omega c by omega p right, that is equal to 1. And what is this? That means, this if you take if you go back, what is r, r equal to this; that means, at gain crossover frequency, this quantity is 1 ok.

That means this whole quantity will be 1 which is computed at omega n is equal to omega c by omega p, that is a gain crossover frequency. Then this quantity will become unity. This is exactly what we did and from there we can write this omega n is nothing but omega of c by omega p and there was another term here. So, this ratio so square root of this, so this is this is a very important relationship ok.

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Perfect Compensation – Buck Converter

$$K_{loop}(j\omega_n) = K_L \times \frac{1}{j\omega_n(1+j\omega_n)}$$

$$K_L = \frac{F_m V_{in} k_c}{\alpha \omega_p}$$

(1) decide PM
 (2) decide $f_c(\omega_c)$

phase contribution

Phase margin (PM): $PM = 180^\circ + \left. \angle \theta(\omega_n) \right|_{\omega=\omega_c} = 90^\circ - \tan^{-1} \left(\frac{\omega_c}{\omega_p} \right)$

* Set f_c and calculate f_p for a given PM

* Set PM and calculate f_p for given f_c

$\frac{\omega_c}{\omega_p} = \tan(90^\circ - PM)$
 $\omega_p = f(\omega_c)$

Next that means, we got one relationship next phase margin. So, at gain crossover frequency, our phase margin can be computed 180 degree plus the phase contributed by this, phase contributed by this which is theta of this is a phase contribution, we are talking about the phase contribution ok. And this can be written as 90 degree minus tan inverse of omega c by omega p. From here from this relationship we can write omega c by omega p is nothing but phase 90 degree minus phase margin hold on, so you can write simply write it as a 90 degree minus phase margin this whole thing tan of this ok.

So, here if we decide omega c then we can find out omega p ok. That means we can find out omega p as a function of omega c from this relationship ok. That means we can now, if we set the crossover frequency, then we can calculate f p for a given phase margin if the phase margin is given; that means, 2 things you have to keep in mind. First we have to decide that means, decide your phase margin PM ok sorry phase margin phase margin.

Then number one to decide the gain crossover frequency or you can say omega c in radian per second right, if you decide phase margin, then you will get a relationship between f p and f omega c and if you set omega c, then you can find out omega p from this relationship. So that means you can get this 2 value for a given phase margin as well as for a given cutoff frequency.

(Refer Slide Time: 13:40)

Perfect Compensation – Buck Converter

$$G_c = k_c \times \frac{\Delta(s)}{s \left(1 + \frac{s}{w_p}\right) \left(1 + \frac{s}{w_{esr}}\right)}$$

Calculate controller pole

$$w_p = \frac{w_c}{\tan(90^\circ - PM)}$$

Calculate controller DC gain

$$k_c = \frac{\alpha w_c}{F_m V_{in}} \times \sqrt{1 + \left(\frac{w_c}{w_p}\right)^2}$$

That means calculate the controller pole. It can be computed from this relationship; that means, if you provide the phase margin, then you can relate omega p in terms of omega c and if you provide omega c then you can compute the controller gain, because it is coming from that KL relationship right. So, it can it is coming from you know this relationship this relationship ok.

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Case Study: $\omega_p = \omega_c$, but what should be ω_c ?

At gain crossover frequency ω_c

$$r(\omega_n) \Big|_{\omega_n=1} = \frac{k_c}{\omega_n \sqrt{1 + \omega_n^2}} \Big|_{\omega_n=1} = 1 \quad \Rightarrow \quad k_c = \frac{\alpha \omega_c}{F_m V_{in}} \times \sqrt{2}$$

Handwritten notes: $w_n = 1$, $w = \omega_c$

$$\angle \theta(\omega_n = 1) = -90^\circ - \tan^{-1}(\omega_n) = -90^\circ - 45^\circ = -135^\circ$$

Handwritten notes: $\sqrt{2}$, 45°

- Phase margin PM = 180 - 135 = 45 degree

So now, what happens if we set the controller pole at the crossover frequency, then it can be found from this r omega n it is equal to 1 since we are setting omega p equal to omega c. So,

omega n is equal to 1 where omega c omega equal to omega c crossover frequency in this case. So, this is also 1 this is also this is 2.

So, we will get k c will be alpha omega c F m into square root of 2 because of this term will be square root of 2 1 plus 1 square root ok. And if you find out, the phase margin first of all angle will be omega n is 1. So, it is 45 degree 45 degree, so here it is minus this is 45 degree this contribution and if you calculate phase margin, it will be 45 degree ok.

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More Practical Compensation Case

$$Q = \left[\frac{\gamma_e + \gamma_c}{z_c} + \frac{z_c}{R} \right]^{-1}$$

$$K_{loop}(s) = F_m \times \frac{V_m}{\alpha} \times \frac{\left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

$$G_c = \frac{k_c \left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}{s \left(1 + \frac{s}{\omega_p}\right) \left(1 + \frac{s}{\omega_{ESR}}\right)}$$

At Rmin
 $Q_p = \frac{\gamma_e + \gamma_c}{z_c} = \frac{z_c}{\gamma_e + \gamma_c}$

Use earlier results and calculate k_c under perfect compensation

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```

1
2 figure(1)
3 plot(t_scale, I, 'b', 'LineWidth', 2); hold on; grid on;
4 xlabel('Time (ms)', 'FontSize', 15);
5 ylabel('Inductor current (A)', 'FontSize', 15);
6
7 figure(2)
8 plot(t_scale, V_o, 'b', 'LineWidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11

```

(Refer Slide Time: 15:14)

```

1 %clc; close all; clear;
2 clc;
3 %% Parameters
4 buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 R=1; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6 Io_min=0.5; R_max=Vref/Io_min;
7
8 f_sw=1/T; w_sw=2*pi*f_sw;
9 z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 Q=alpha/((r_C+r_eq/z_c)+(z_c/R));
12
13 %% Define zeros
14 w_z=1/(r_C*C); w_z1=1/(R+r_C*C); w_z2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin*alpha)*1/(w_z1);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq*alpha)*1/(w_z2*w_z)((1/w_z)+(1/w_z2)) 1];
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 Z_o=tf(num_o,den_o);

```

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```

22 num_o=(r_eq*alpha)*1/(w_z2*w_z)((1/w_z)+(1/w_z2)) 1];
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 Z_o=tf(num_o,den_o);
25
26 %% Audio suseptibility
27 num_c=(D/alpha)*1/(w_z1);
28 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
29 Gvg=tf(num_c,den_c);
30
31 %% Modulator and Controller parameters
32 V_m=10; Fm=1/V_m;
33 %Q_max=alpha/((r_C+r_eq/z_c)+(z_c/R_max));
34 Q_max=alpha/((r_C+r_eq/z_c));
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_c/k_x;
45 K_x=((alpha*Fm*(1/(R+r_C*C)))/(cos(theta)*w_cp));

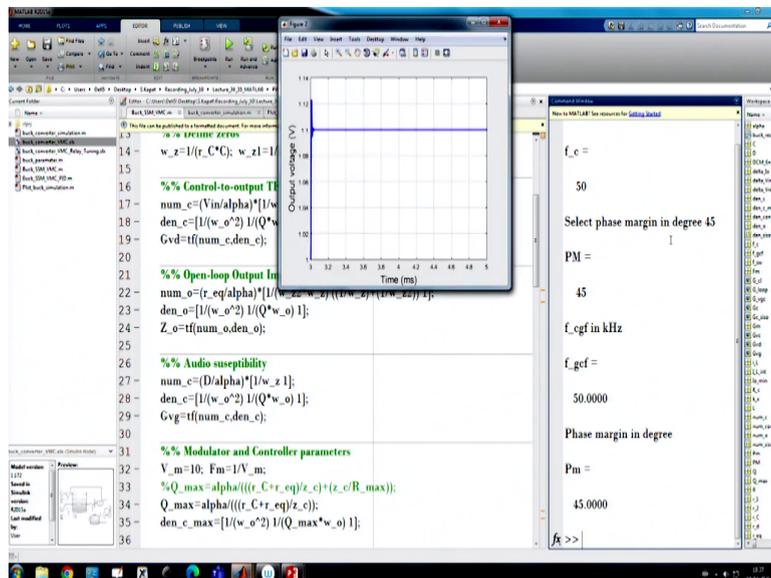
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So more that means, now we want to go to mat lab simulation you have to check. So now, we are asking for so this is the expression that we have derived all the transfer functions are given numerator denominator controller gain. Now, we are designing the compensator; that means, it will ask for cut off frequency or crossover frequency and we will ask for phase margin; that means, a 2 possibility right.

know there are 2 zeros 2 poles are there one and also one 3 poles one at origin and one to cancel ESR. Another is the flexible controller pole which we got from the analytically right. So, let us design and check the mat lab simulation.

So, suppose we want to select the bandwidth; that means bandwidth same as so that means, let us say first. So, there are 2 things here. If we set the bandwidth let us say 50 kilohertz it is in kilohertz 50 kilohertz our switching frequency is 500 kilohertz. So that means your crossover frequency is showing 50 kilohertz and phase margin just 1 minute yeah phase margin.

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If we set let us say 45 degree because we know if the crossover frequency is 50 kilohertz and if you set the phase margin 45 degree, then you should get the controller pole at the same location right that you have learned ok. So that means, gain crossover frequency is 50 kilohertz phase margin is 45 degree now, we want to see. What is my controller pole? because let me find out what is my omega c.

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```

1 %clear; close all; clear;
2 clc;
3 %% Parameters
4 buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 R=0.1; r_eq=r_L+r_r; alpha=(R+r_eq)/R;
6 Io_min=0.5; R_max=Vref/Io_min;
7
8 f_sw=1/T; w_sw=2*pi*f_sw;
9 z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_c)));
11 Q=alpha/((r_c+R+r_eq)/z_c+(z_c/R));
12
13 %% Define zeros
14 w_x=1/(r_c*C); w_x1=1/((R+r_c)*C); w_x2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin*alpha)*[1/w_x 1];
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq*alpha)*[1/(w_x2*w_x) ((1/w_x)+(1/w_x2)) 1];
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24

```

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```

28 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
29 Gv=tf(num_c,den_c);
30
31 %% Modulator and Controller parameters
32 V_m=10; Fm=1/V_m;
33 %Q_max=alpha/((r_c+R+r_eq)/z_c+(z_c/R_max));
34 Q_max=alpha/((r_c+R+r_eq)/z_c);
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_c/k_x;
45 K_c=((alpha*w_c)/(Fm*Vin))*(sqrt(1+(k_x^2)));
46 num_con=K_c*den_c;
47 den_con=[1/(w_x*w_cp) (1/w_x)+(1/w_cp) 1 0];
48 Ge=tf(num_con,den_con);
49
50 %% Lead compensator
51 % K_c=10000; w_cp=2.38e3; f_c=15;

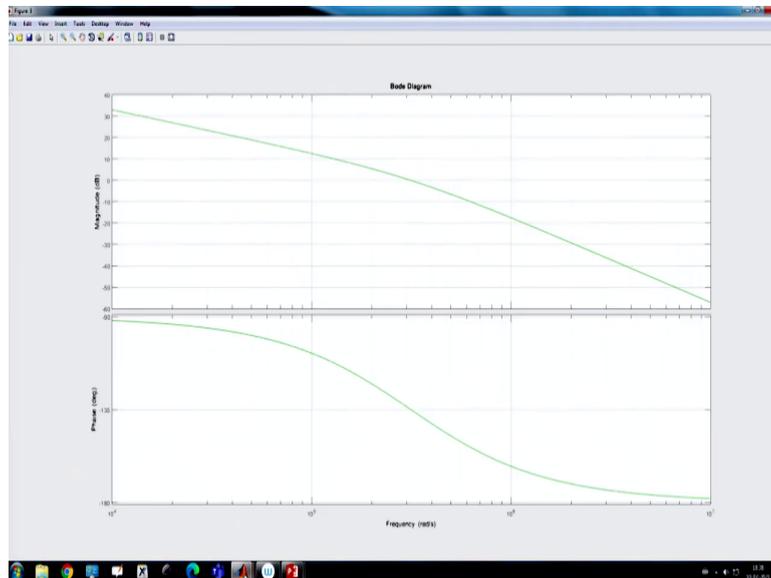
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So, this is in radian per second crossover frequency what is my controller pole, so they are exactly equal it is coming from the mathematical expression here ω_{cp} and what is K_x here, the ratio of the $2K_x$ is equal to 1.

Because we have learned that from this expression that this K_x which is a normalized frequency is equal to 1. When if it is set to same, they are same, then it will give it to 45 degree phase margin. That means we are checking the other way we are setting 45 degree phase margin and we can see that ω_n is coming to be 1 so which is consistent right. So

now, we obtain 45 degree phase margin and this is the response of the converter coming from you know.

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(Refer Slide Time: 18:45)

```

61 % den_con=[1/w_c 1];
62 % Gc=tf(num_con,den_con);
63
64 %% Loop gain and closed-loop TFs
65 G_loop=Gvd*Fm*Gc; %% Loop gain
66
67 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
68 G_cl=G_loop/(1+G_loop); %% Closed-loop TF
69 G_vgc=Gvg/(1+G_loop); %% Closed-loop audio sus.
70
71 %% Frequency response
72 figure(3)
73 % bode(Z_o,'b');
74 % hold on;
75 % bode(Z_oc,'-b');
76 % hold on;
77 bode(G_loop,'b');
78 hold on; grid on;
79 [Gm,Pm,Wcg,Wcp]=margin(G_loop);
80 grid on;
81
82 %% Transient parameters and transient response
83 t_sim=5e-3; t_step=3e-3;
84 J_s, I_s, I_o, J_d, I_d, V_s, V_o, V_e, f=0;

```

Command Window:

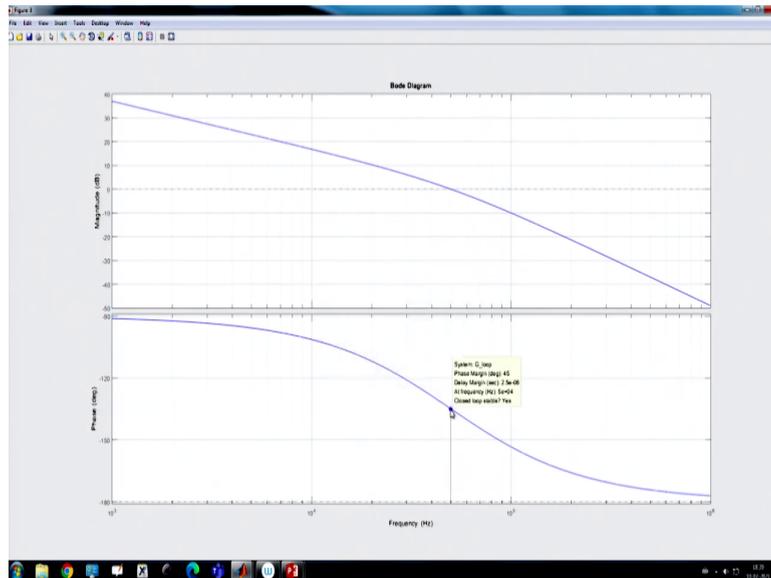
```

f_c =
    50
Select phase margin in degree 45
PM =
    45
f_cgf in kHz
f_cgf =
    50.0000
Phase margin in degree
Pm =
    45.0000
fx >>

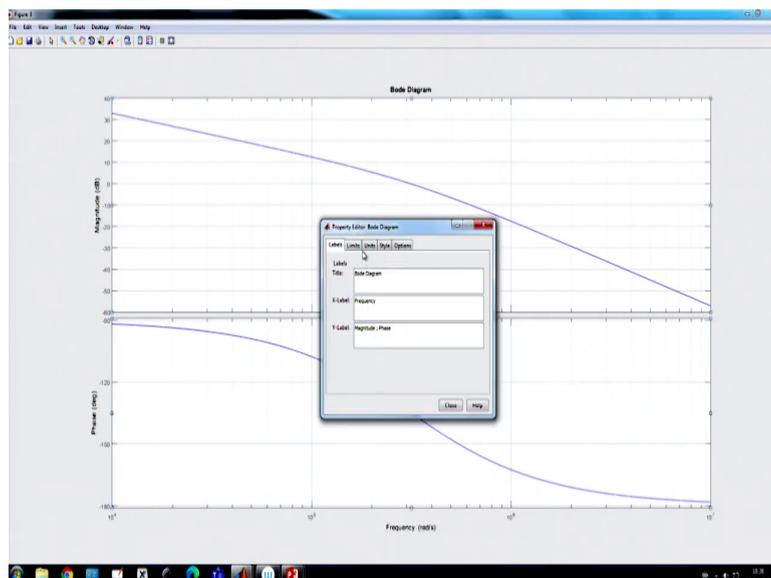
```

And if we check the Bode plot ok. So, ok so let us use a different ok. So, let us once more because green colour may not look good so Bode plot ok. So, again, we are running 50 kilohertz 50 kilohertz and 45 degree phase margin, ok.

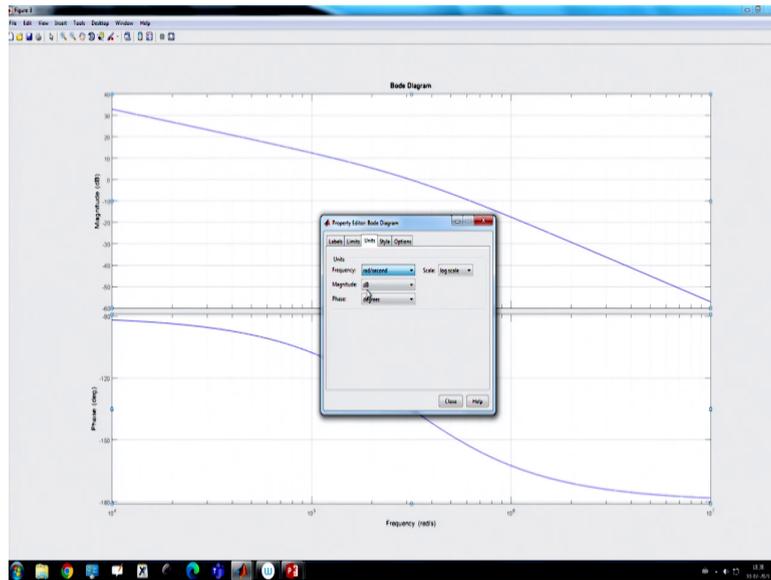
(Refer Slide Time: 18:51)



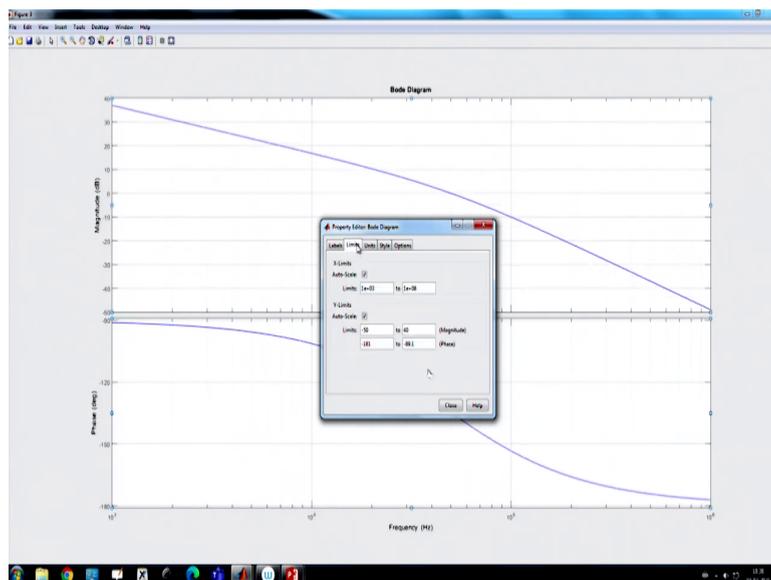
(Refer Slide Time: 18:57)



(Refer Slide Time: 19:01)

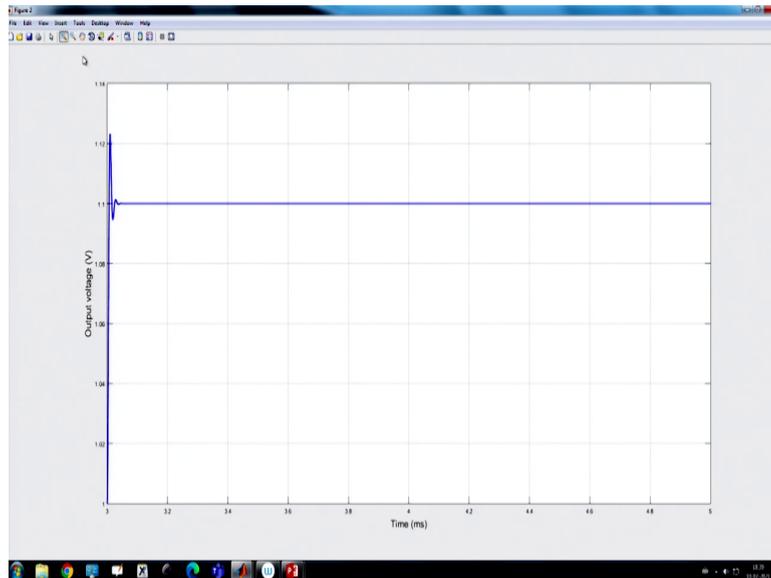


(Refer Slide Time: 19:05)

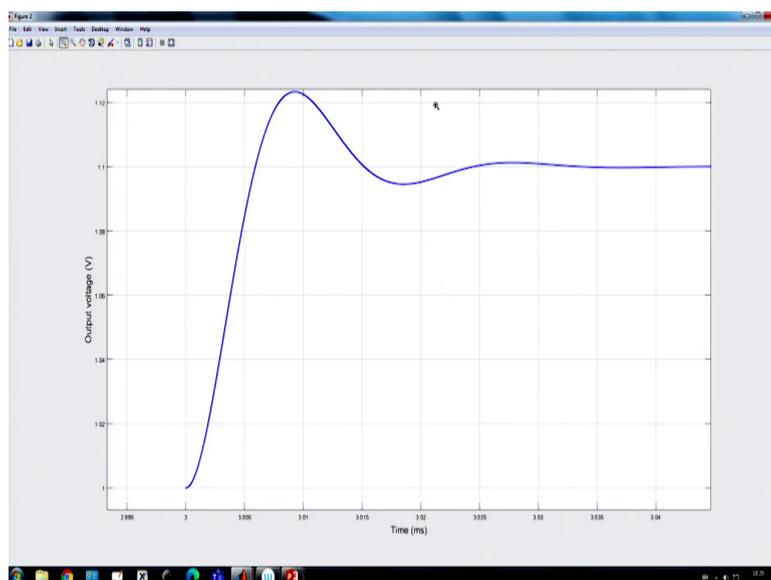


Now, Bode plot if you look at the Bode plot and if you want to plot all the. So, it is in radian per second, so we have to convert into Hz first. So, it is in Hz now and we can you know we want to find out what are the stability margin. So, you see, we are getting 50 kilohertz crossover frequency 45 degree phase margin. So, it is perfectly consistent with a given specification.

(Refer Slide Time: 19:21)

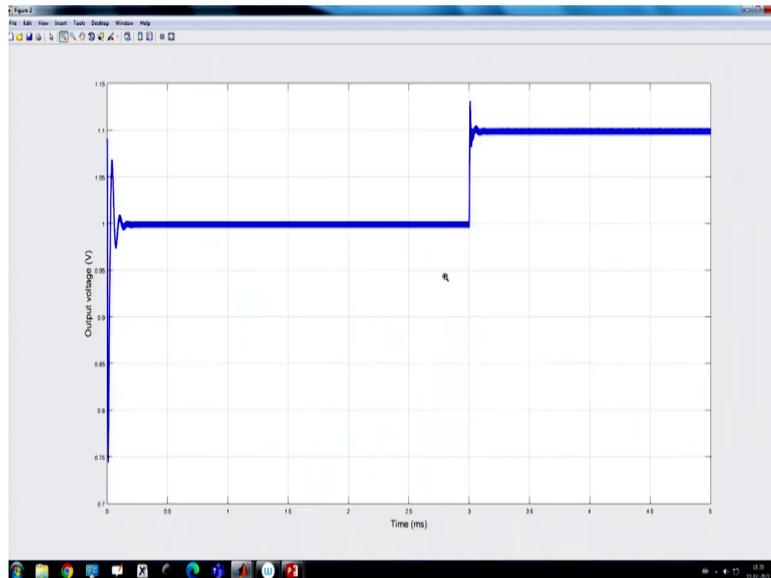


(Refer Slide Time: 19:25)

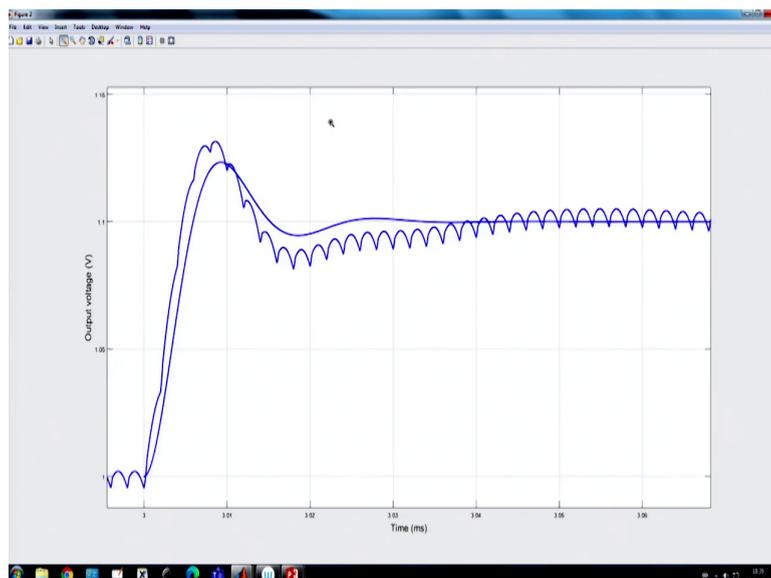


And this is a response that means, this is the reference transient performance. I mean, it has a high overshoot like around 20 percent overshoot. But we need to check whether the response matches with the converter response or not that we want to check ok.

(Refer Slide Time: 19:41)



(Refer Slide Time: 19:47)

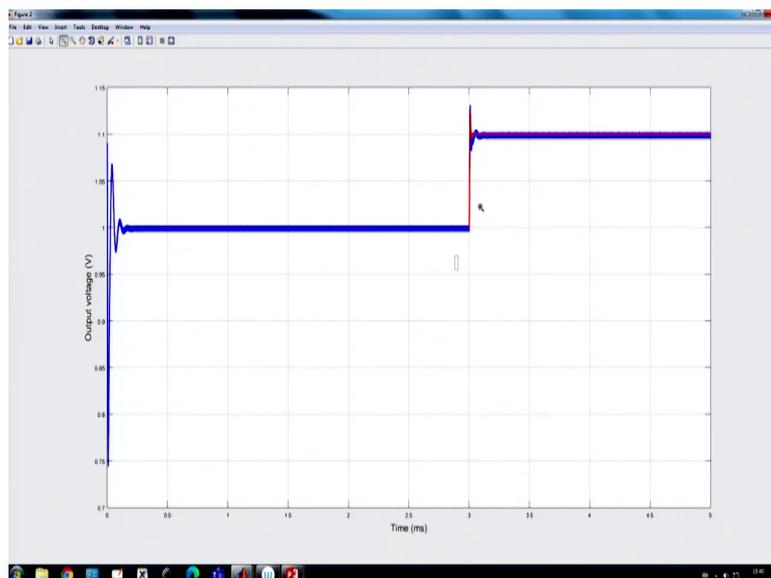


So, it looks like they are not matching quite accurately, because even though because even we have set we have set 110 bandwidth but it is not perfectly matching ok.

(Refer Slide Time: 20:06)

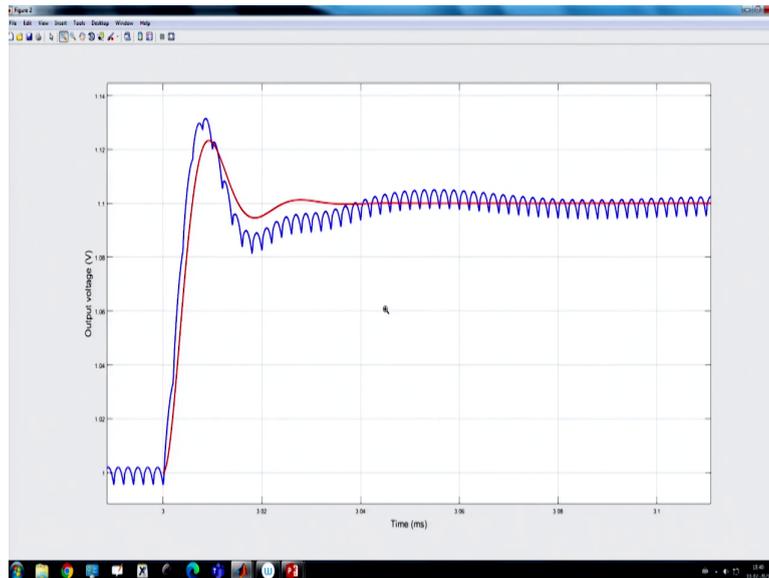
```
77- bode(G_loop,'b');
78- hold on; grid on;
79- [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80- grid on;
81
82- %% Transient parameters and transient response
83- t_sim=5e-3; t_step=3e-3;
84- delta_ilo=0; delta_Vin=0; delta_Vref=0.1;
85
86- [y_s,t_s]=step(G_cl,(t_sim-t_step));
87- v_ac=delta_Vref*y_s;
88
89- figure(2)
90- plot((t_s+t_step)*1e3,Vref+v_ac,'l','Linewidth',2);
91- xlabel('Time (ms)', 'FontSize', 15);
92- ylabel('Output voltage (V)', 'FontSize', 15);
93- hold on; grid on;
94
95- display('f_cgf in kHz')
96- f_cgf=Wcg/(2*pi)*1e3;
97- display('Phase margin in degree')
98- Pm
```

(Refer Slide Time: 20:18)



So, I think we should use a different colour here ok. So, that means they are not matching perfectly. So, if we hold on read on and again reevaluate ok.

(Refer Slide Time: 20:22)

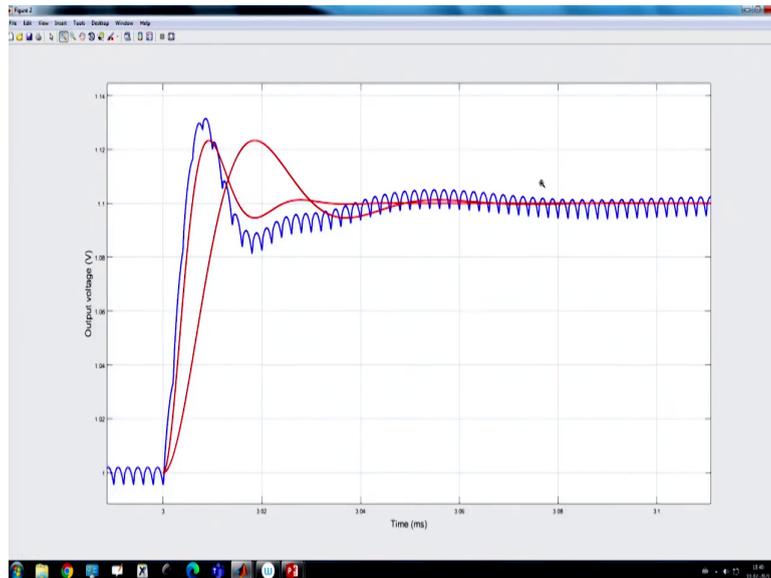


So, if we set that, the response obtained from the small signal model is not exactly matching with the response obtain from actual switch simulation. So, they are deviating; that means, the model is not perfectly valid ok. But if we reduce the cut off frequency, suppose if we redesign the 1, if we reduce to let us say we make earlier to 110.

(Refer Slide Time: 20:47)

```
77 - bode(G_loop,'b');
78 - hold on; grid on;
79 - [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80 - grid on;
81
82 - %% Transient parameters and transient response
83 - t_sim=5e-3; t_step=3e-3;
84 - delta_Io=0; delta_Vin=0; delta_Vref=0.1;
85
86 - [y_s,t_s]=step(G_cl,(t_sim-t_step));
87 - v_ac=delta_Vref*y_s;
88
89 - figure(2)
90 - plot(t_s+t_step*1e3,Vref+v_ac,'l',Linewidth,2);
91 - xlabel('Time (ms)',FontSize,15);
92 - ylabel('Output voltage (V)',FontSize,15);
93 - hold on; grid on;
94
95 - display('f_cg in kHz')
96 - f_cg=Wcp/(2*pi)*1e3;
97 - display('Phase margin in degree')
98 - Pm
99
```

(Refer Slide Time: 20:51)



Now, if we make 120 th; let us say we are making 25 kilohertz and 45 degree phase margin, this is a response which you obtain from you know. So, now we want to obtain from this ok.

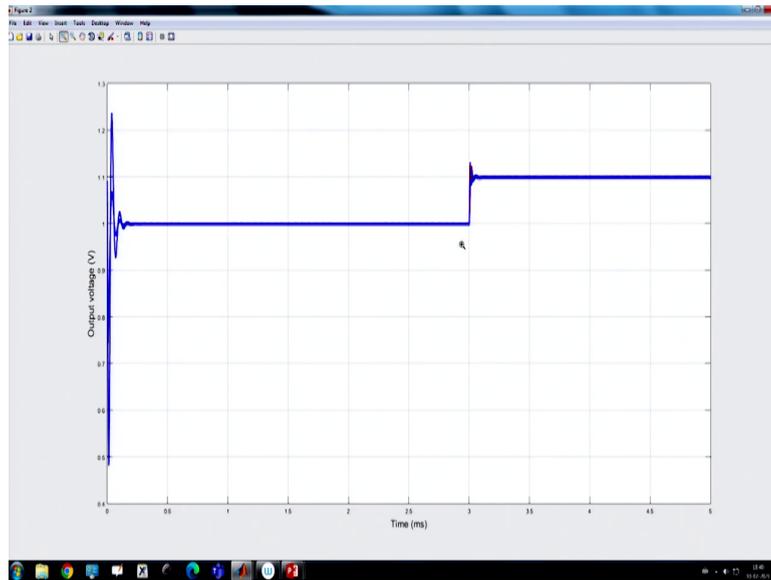
(Refer Slide Time: 21:00)

```
1 % c; clear; close all;
2
3 DCM_En=0;
4 I_L_in=1; V_c_in=1.1;
5
6 sim('buck_converter_VMC.slx');
7 t=buck_result.time_t_scale*1e3;
8 x=buck_result.data;
9 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3); Vcon=x(:,4);
10
11 Plot_buck_simulation;
```

Command Window

```
Warning: buck_converter_VMC
contains 1 algebraic
loop(s). To see more
details about the loops use
the command
Simulink.BlockDiagram.getAlgebraic
or the command line
Simulink.debugger by typing
'sldebug
buck_converter_VMC' in the
MATLAB command window. To
eliminate this message, set
the Algebraic loop option
in the Diagnostics page of
the Simulation Parameters
Dialog to 'None'
> In buck_converter_simulation (line
Found algebraic loop containing:
'buck_converter_VMC/Buck convert
'buck_converter_VMC/Buck convert
'buck_converter_VMC/Buck convert
'buck_converter_VMC/load'
'buck_converter_VMC/Sum' (algebr
```

(Refer Slide Time: 21:03)



(Refer Slide Time: 21:08)



So, it looks like now, the second case they are matching somewhat reasonably.

(Refer Slide Time: 21:19)

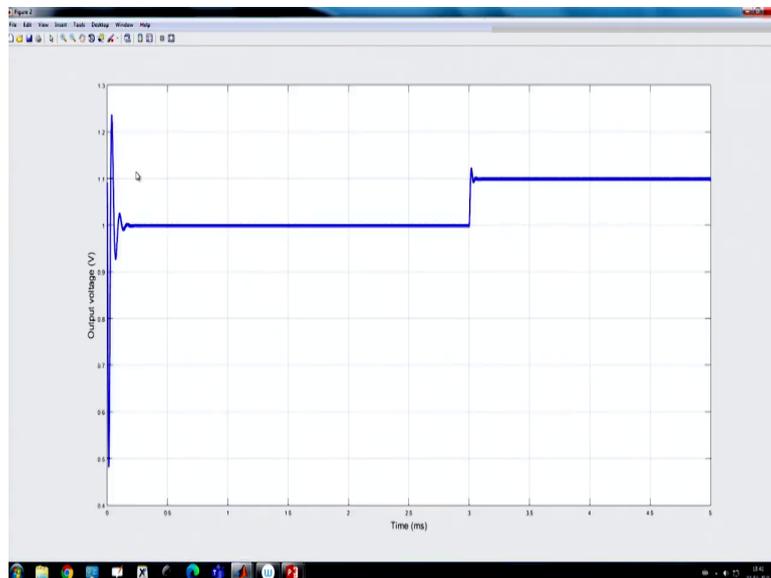
```

1  clc; close all; clear;
2  clc;
3  %% Parameters
4  buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5  R=0.1; r_eq=L+r_L; alpha=(R+r_eq)/R;
6  Io_min=0.5; R_max=Vref/Io_min;
7
8  f_sw=1/T; w_sw=2*pi*f_sw;
9  z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 Q=alpha/((r_C+r_eq/z_c)+(z_c/R));
12
13 %% Define zeros
14 w_x=1/(r_C*C); w_x1=1/((R+r_C)*C); w_x2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin*alpha)*[1/w_x 1];
18 den_c=[(w_o^2) 1]/(Q*w_o 1);
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq*alpha)*[1/(w_x2*w_x) ((1/w_x)+(1/w_x2)) 1];
23 den_o=[(w_o^2) 1]/(Q*w_o 1);
24 G_o=tf(num_o,den_o);

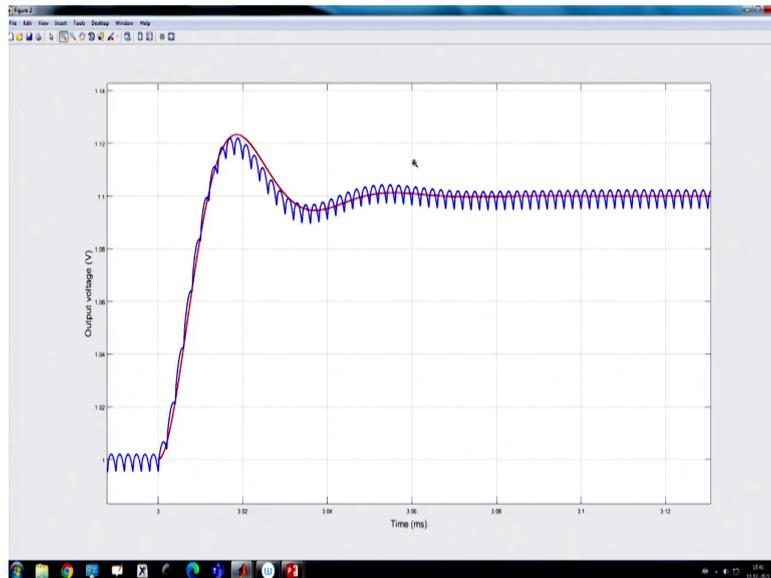
```

So, let us plot once more because you know we have ok. So, we have you know we have setting 25 kilohertz, 45 degree phase margin and this is the response.

(Refer Slide Time: 21:32)

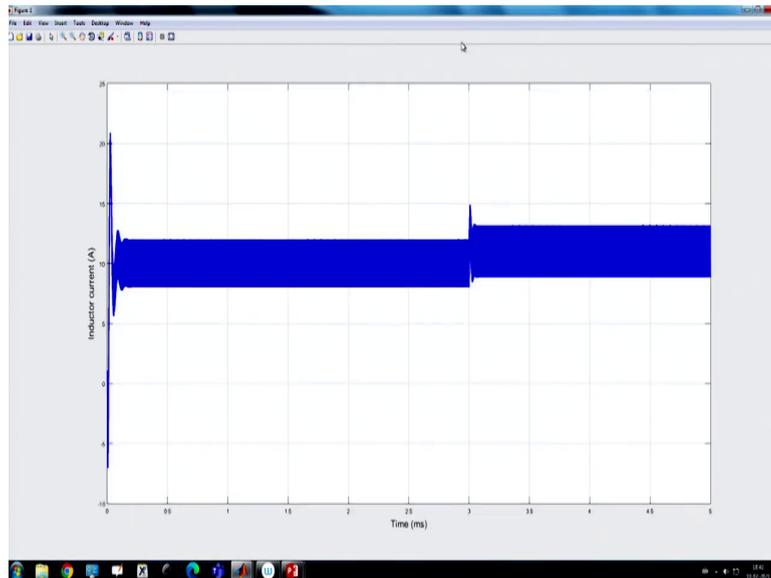


(Refer Slide Time: 21:39)

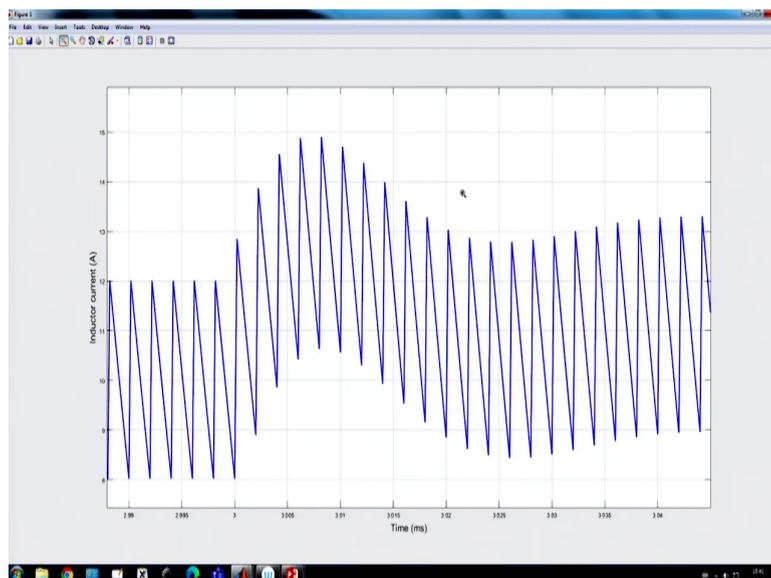


And if we do actual switch simulation, so they are matching, you know, somewhat reasonably accurate and not exactly matching, but yeah with reason. So, in this case at least the validity is coming to be 120 th of the switching frequency.

(Refer Slide Time: 21:50)



(Refer Slide Time: 21:54)



Because maybe the step size is large, that is why because it will be clear from the current waveform the reason is nothing but the duty ratio perturbation is somewhat larger. So, they are not exactly matching but closely matching ok.

(Refer Slide Time: 22:04)

The image shows a MATLAB script for a buck converter simulation. The script includes the following code:

```

1 = clc; close all; clear;
2 = clc;
3 = %% Parameters
4 = buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 = R=0.1; r_eq=L+r_l; alpha=(R+r_eq)/R;
6 = Io_min=0.5; R_max=Vref/Io_min;
7 =
8 = f_sw=1/T; w_sw=2*pi*f_sw;
9 = z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 = w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_c)));
11 = Q=alpha/((r_c*C+r_eq/z_c)+(z_c/R));
12 =
13 = %% Define zeros
14 = w_x=1/(r_c*C); w_x1=1/((R+r_c)*C); w_x2=r_eq/L;
15 =
16 = %% Control-to-output TF Gvd
17 = num_c=(Vin/alpha)*1/(w_x1);
18 = den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 = Gvd=tf(num_c,den_c);
20 =
21 = %% Open-loop Output Impedance
22 = num_o=(r_eq/alpha)*1/(w_x2*w_x)((1/w_x)+(1/w_x2) 1);
23 = den_o=[1/(w_o^2) 1/(Q*w_o) 1];

```

On the right side, there is a 'Command Window' with the following settings:

- Select BW in kHz 50
- f_c = 50
- Select phase margin in degree 60
- PM = 60
- f_x

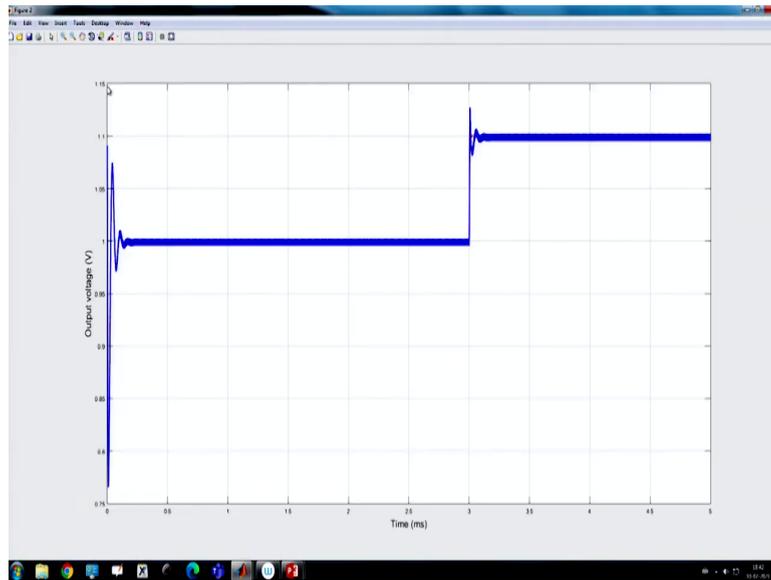
(Refer Slide Time: 22:10)

The image shows the same MATLAB script as above, but with a plot window overlaid. The plot shows the output voltage over time. The y-axis is labeled 'Output voltage (V)' and ranges from 0 to 1.2. The x-axis is labeled 'Time (ms)' and ranges from 0 to 5. The plot shows a steady-state output voltage of approximately 1.0 V.

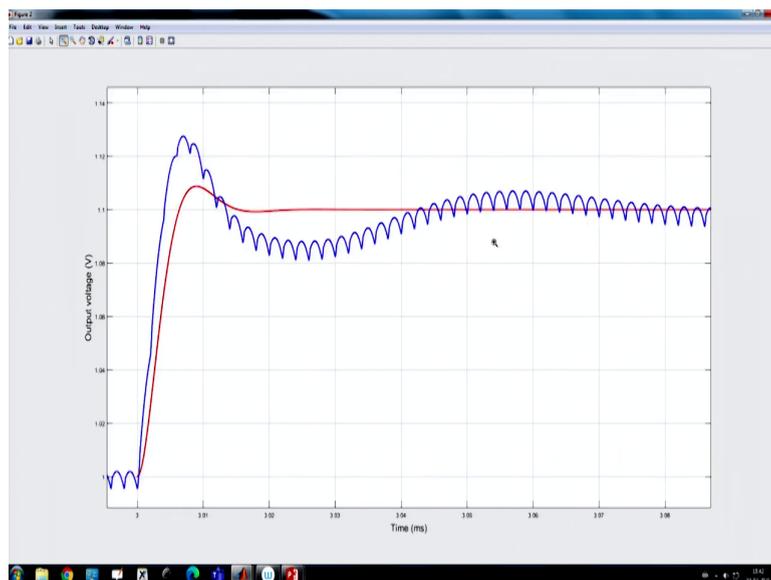
The 'Command Window' on the right has the following settings:

- f_c = 50
- Select phase margin in degree 60
- PM = 60
- f_cg_f in kHz
- f_gf = 50.0000
- Phase margin in degree
- Pm = 60.0000
- f_x >>

(Refer Slide Time: 22:18)

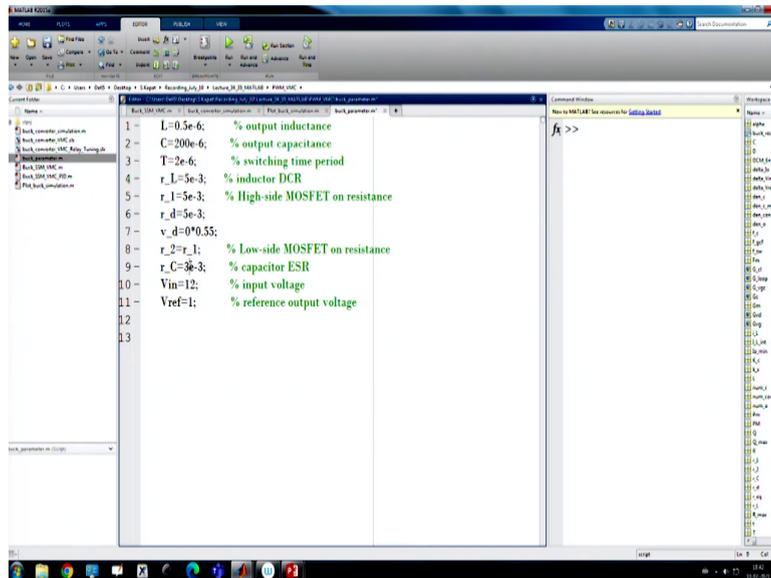


(Refer Slide Time: 22:21)



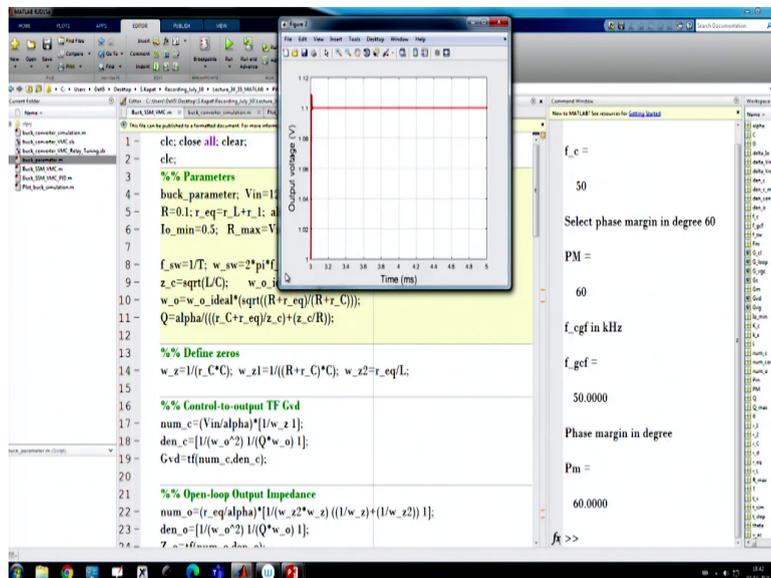
Now, we want to design for a higher phase margin. Suppose we do 50 kilohertz and provide 60 degree phase margin ok. So, now, if we run the simulation sixty degree phase margin, so we want to check that how far the response is valid. See response is not valid because for maybe the step size is large. So, actual response of the converter is somewhat deviating from the response obtained from the small-signal model ok. So, we probably have to reduce the bandwidth, I am sorry you know; we have to probably ah.

(Refer Slide Time: 22:46)

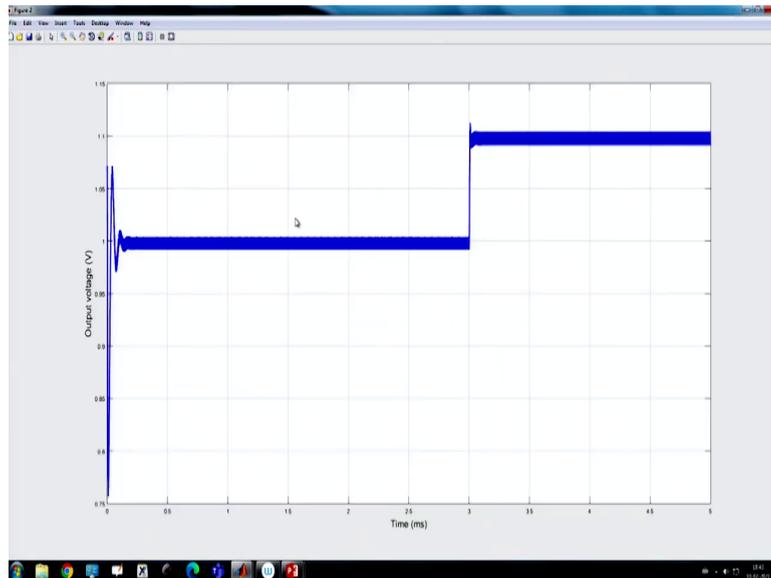


Say if you go to the parameter file, our ESR is too small. So, if we take a little larger ESR then hopefully, it will because we need to properly damp.

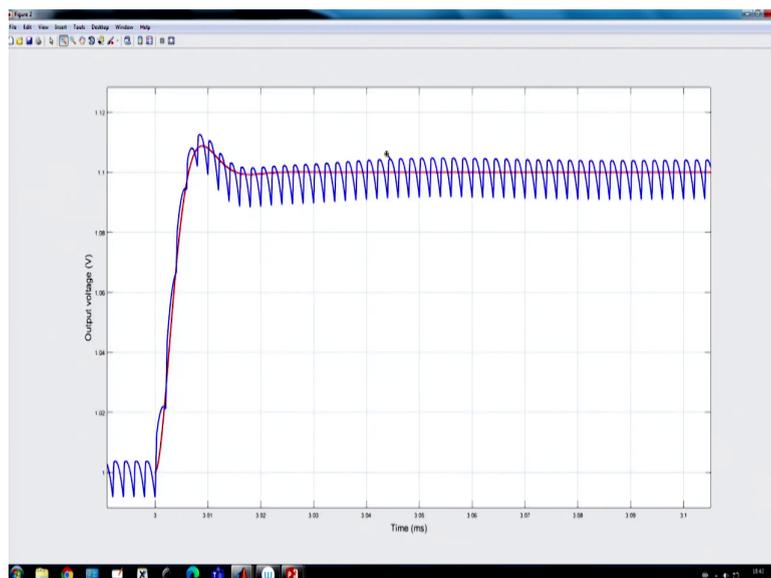
(Refer Slide Time: 22:57)



(Refer Slide Time: 23:05)



(Refer Slide Time: 23:11)



So, now 50 kilohertz with 60 degree phase margin and if we run it, then I feel that they will match because it was like close to ideal one the ESR was too small. So, now it is 110 switching frequency is matching; that means, the matching depends on also ESR for a little larger ESR. You know we can go a little bit like you know, up to 110 switching frequency you can get reasonably good matching.

But if you are reducing the ESR, then the validity of the model will even get degraded at 110, because even we have to operator slot switching frequency. But what I want to mean that it is a well damped response the closed loop performance ok.

And if you see the Bode plot, so we can we can check the Bode plot response. Now, so this is the response that you obtain for we can do more practical compensation case. So, here we have exactly canceled the poles, but suppose we do not know the exact value of resistance.

So, what will happen if you take the Q_p ; that means, if you take the Q_p because Q_p is what; earlier what was the Q our original Q . That means, you know if you check the what is there is a Q expression, if you recall our original Q was I think it was r_e plus r_c by z_c sorry plus z_c by R inverse yes.

And I told you this term will become significant when the load resistance is very high, otherwise this will primarily dominate right. Now, if we design Q_p at R_{min} ; that means, at the highest load current condition, then what is it I mean, what is going to happen ok? So, let us go back and change our compensator.

(Refer Slide Time: 25:14)

```

19- Gvd=tf(num_c,den_c);
20-
21- %% Open-loop Output Impedance
22- num_o=(r_eq/alpha)*1/(w_z2*w_z1)*((1/w_z)+(1/w_z2)) 1];
23- den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24- Z_o=tf(num_o,den_o);
25-
26- %% Audio susceptibility
27- num_c=(D/alpha)*1/w_z 1];
28- den_c=[1/(w_o^2) 1/(Q*w_o) 1];
29- Gvg=tf(num_c,den_c);
30-
31- %% Modulator and Controller parameters
32- V_m=10; Fm=1/V_m;
33- Q_max=alpha/((r_c+r_eq)/z_c)+(z_c/R_min);
34- %Q_max=alpha/((r_c+r_eq)/z_c);
35- den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36-
37-
38- %% Type - III compensator
39- f_c=input('Select BW in kHz ');
40- %f_c=50; PM=60;
41- w_c=2*pi*f_c*1e3;
42- DM=1;

```

(Refer Slide Time: 25:35)

```

1 = clc; close all; clear;
2 = clc;
3 = %% Parameters
4 = buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 = R=0.1; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6 = Io_min=0.5; R_min=Vref/Io_max;
7 =
8 = f_sw=1/T; w_sw=2*pi*f_sw;
9 = z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 = w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 = Q=alpha/((r_C+r_eq)/z_c+(z_c/R));
12 =
13 = %% Define zeros
14 = w_z=1/(r_C*C); w_z1=1/((R+r_C)*C); w_z2=r_eq/L;
15 =
16 = %% Control-to-output TF Gvd
17 = num_c=(Vin*alpha)*1/(w_z1);
18 = den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 = Gvd=tf(num_c,den_c);
20 =
21 = %% Open-loop Output Impedance
22 = num_o=(r_eq*alpha)*1/(w_z2*w_z*(1/(w_z)+1/(w_z2)) 1);
23 = den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 = Z_o=tf(num_o,den_o);

```

So, now we are trying to design a more practical that means, we want to get Q max and Q max is calculated. In one case where we are taking r min minimum value of R and let us say r min is equal to I 0 max I mean maximum load current ok. So now, we have used that q max and instead of using earlier one, so we have the response already plotted.

(Refer Slide Time: 25:58)

```

31 = %% Modulator and Controller parameters
32 = V_m=10; Fm=1/V_m;
33 = Q_max=alpha/((r_C+r_eq)/z_c+(z_c/R_min));
34 = %Q_max=alpha/((r_C+r_eq)/z_c);
35 = den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36 =
37 =
38 = %% Type - III compensator
39 = f_c=input('Select BW in kHz ');
40 = %f_c=50; PM=60;
41 = w_c=2*pi*f_c*1e3;
42 = PM=input('Select phase margin in degree ');
43 = theta=deg2rad(90-PM);
44 = k_x=tan(theta); w_cp=w_c/k_x;
45 = K_c=((alpha*w_c)/(Fm*Vin))*sqrt(1+(k_x^2));
46 = num_con=K_c*den_c_max;
47 = den_con=[1/(w_z2*w_cp) 1/(w_z)+1/(w_cp) 1 0];
48 = Gc=tf(num_con,den_con);
49 =
50 = %% Lead compensator
51 = % K_c=10000; w_cz=(2*pi*f_sw)/5;
52 = % num_con=K_c*(1/(w_cz 1));
53 = % den_con=[1/w_z 1];
54 = % Con=tf(num_con,den_con);

```

(Refer Slide Time: 26:03)

```

77- bode(G_loop,'b');
78- hold on; grid on;
79- [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80- grid on;
81
82- %% Transient parameters and transient response
83- t_sim=5e-3; t_step=3e-3;
84- delta_Io=0; delta_Vin=0; delta_Vref=0.1;
85
86- [y_s,t_s]=step(G_cl,(t_sim-t_step));
87- v_ac=delta_Vref*y_s;
88
89- figure(2)
90- plot(t_s+t_step)*1e3, Vref+v_ac,'LineWidth', 2);
91- xlabel('Time (ms)', 'FontSize', 15);
92- ylabel('Output voltage (V)', 'FontSize', 15);
93- hold on; grid on;
94
95- display('f_cgf in kHz')
96- f_cgf=Wcg/(2*pi)*1e3
97- display('Phase margin in degree')
98- Pm
99

```

So, now we want to compensate with maximum one and now, we want to you know change the colour. So, it is a now green colour and let us hold it, because everything will be erased.

(Refer Slide Time: 26:11)

```

1- %clc; close all; clear;
2- clc;
3- %% Parameters
4- buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5- R=0.1; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6- Io_max=20; R_min=Vref/Io_max;
7-
8- f_sw=1/T; w_sw=2*pi*f_sw;
9- z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10- w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_c)));
11- Q=alpha/((r_c+r_eq)/z_c+(z_c/R));
12-
13- %% Define zeros
14- w_x=1/(r_c*C); w_x1=1/((R+r_c)*C); w_x2=r_eq/L;
15-
16- %% Control-to-output TF Gvd
17- num_c=(Vin/alpha)*1/(w_x1);
18- den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19- Gvd=tf(num_c,den_c);
20-
21- %% Open-loop Output Impedance
22- num_o=(r_eq/alpha)*1/(w_x2*w_x)((1/w_x)+(1/w_x2)) 1);
23- den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24-

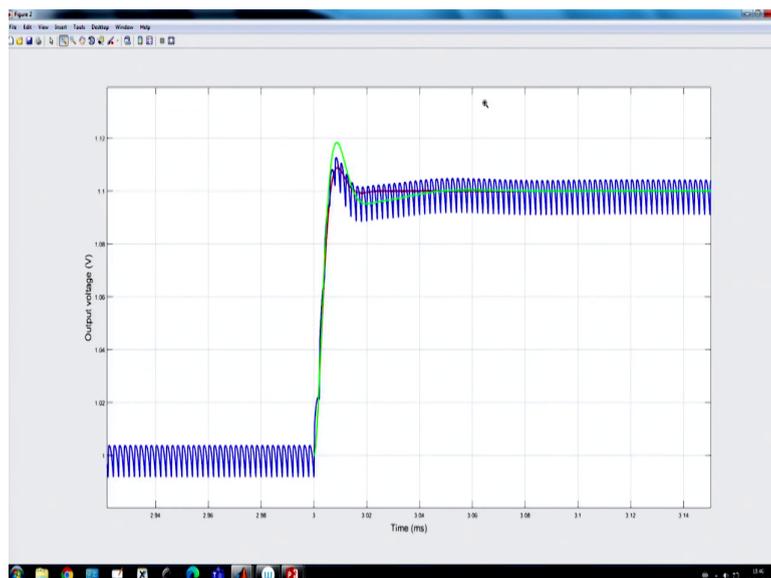
```

So, it is not defined. See, if you write the parameter file ok. So, we have to define what is I0 max. So, we will define I0 max is nothing but 20 ampere ok. Now it will ask for so 50 kilohertz and 60 degree phase margin.

(Refer Slide Time: 26:44)

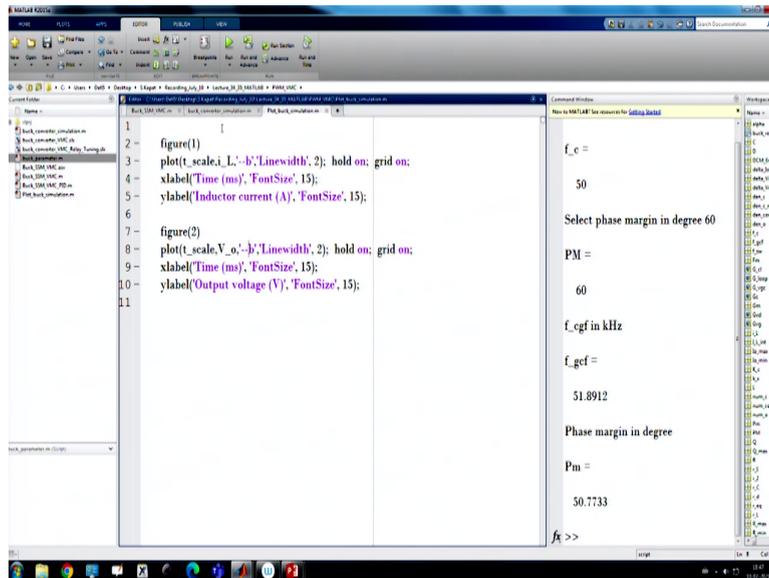


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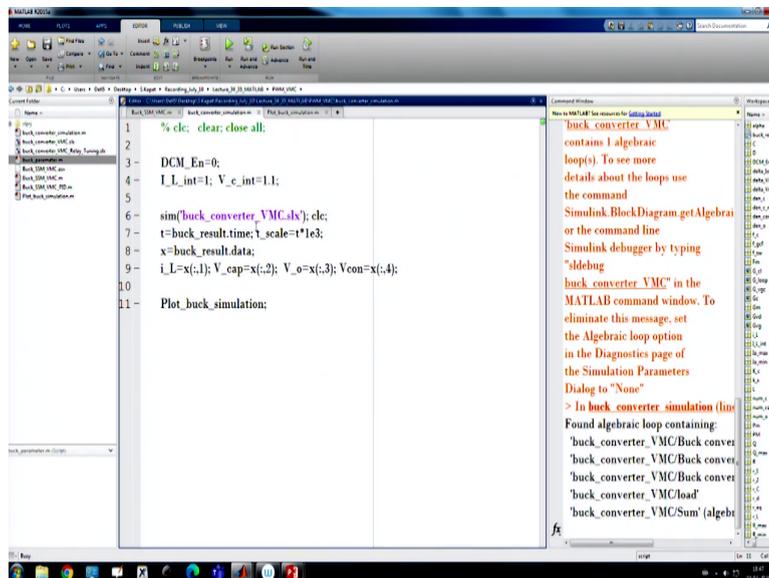
So, this is a response with compensation where we have not included with load resistance term, it is based on the lowest value load resistance.

(Refer Slide Time: 26:57)

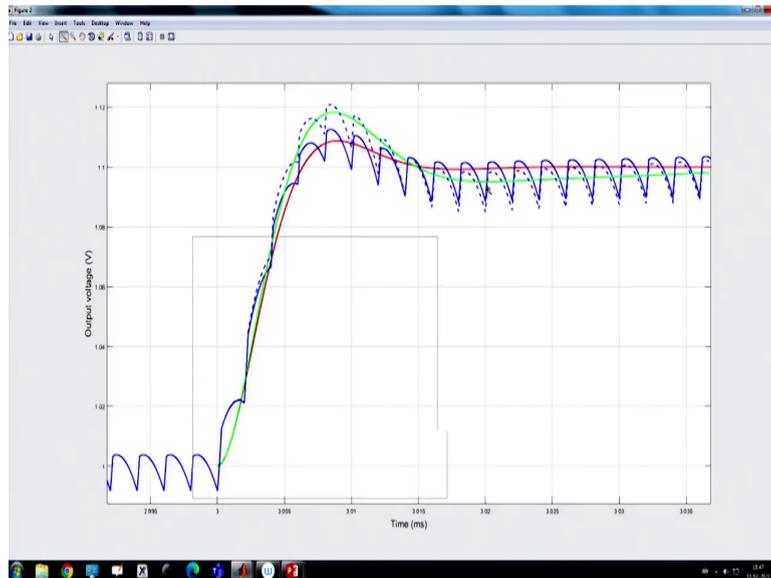


And we will see what happens with the actual converter response and there we want to use a different colour, let us say we will use you know maybe the dotted one ok.

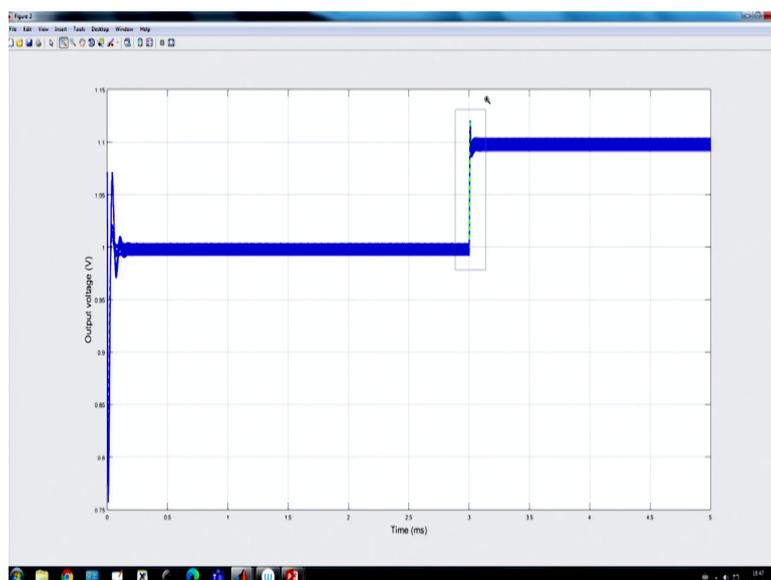
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(Refer Slide Time: 27:10)

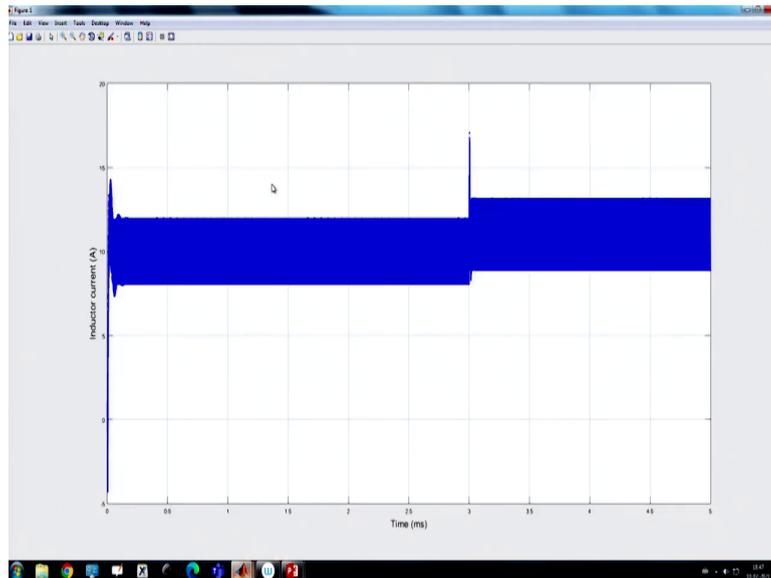


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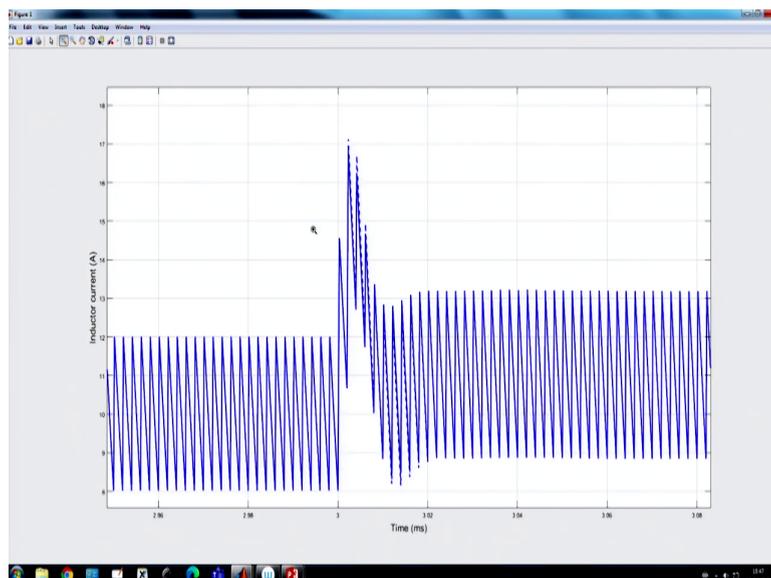


So, we will use a dotted one and see what is the response of the converter. Yes. So, it looks like the response of the converter does not change drastically; you know this is with the worst-case scenario that we are compensating. That means we are trying to compensate with exact cancellation and with robust compensation.

(Refer Slide Time: 27:31)



(Refer Slide Time: 27:36)



And there, if you see their current response inductor current response, they are also very close. That means we do not need to know exactly the value of the load resistance they are quite matching.

(Refer Slide Time: 27:43)

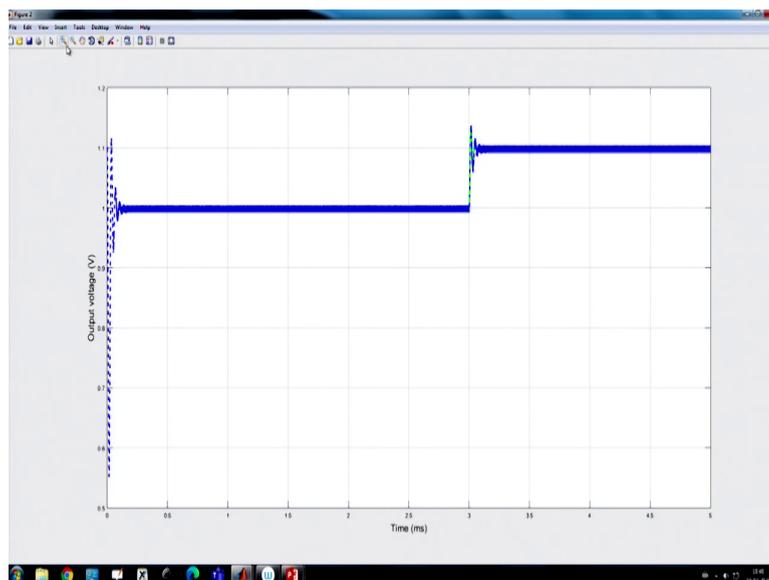
```

1  clc; close all; clear;
2  clc;
3  %% Parameters
4  buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5  R=1; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6  Io_max=20; R_min=Vref/Io_max;
7
8  f_sw=1T; w_sw=2*pi*f_sw;
9  z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 Q=alpha/((r_C+r_eq)/z_c)+(z_c/R);
12
13 %% Define zeros
14 w_x=1/(r_C*C); w_x2=1/((R+r_C)*C); w_x2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin*alpha)*1/(w_x*1);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq*alpha)*1/(w_x2*w_x2*((1/w_x)+(1/w_x2)) 1);
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 G_o=tf(num_o,den_o);

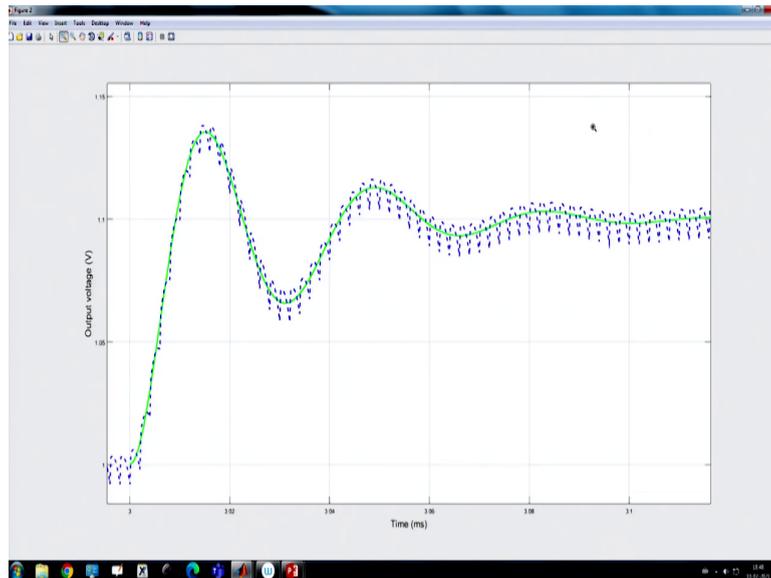
```

But now, if we want to extend this when the load is light; that means, you know under one ohm condition you want to redesign it ok. So, one case we want to use this robust compensation we want to now use 25 kilohertz cut off frequency 60 degree phase margin, and this is something using robust compensation and where this is the one with the actual switch simulation ok.

(Refer Slide Time: 28:14)



(Refer Slide Time: 28:18)



So, we want to check that how far they are matching. So they are matching nicely, but there is you know much overshoot undershoot response.

(Refer Slide Time: 28:33)

```
70
71 %% Frequency response
72 figure(3)
73 % bode(Z_o,'b');
74 % hold on;
75 % bode(Z_oc,'-b');
76 % hold on;
77 bode(G_loop,'b');
78 hold on; grid on;
79 [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80 grid on;
81
82 %% Transient parameters and transient response
83 t_sim=5e-3; t_step=3e-3;
84 delta Io=0; delta Vin=0; delta Vref=0.1;
85
86 [y_s,t_s]=step(G_cl,(t_sim-t_step));
87 v_ac=delta_Vref*y_s;
88
89 figure(2)
90 plot(t_s+t_step)*1e3, Vref+v_ac,'Linewidth', 2);
91 xlabel('Time (ms)', 'FontSize', 15);
92 ylabel('Output voltage (V)', 'FontSize', 15);
93 hold on; hold on;
```

(Refer Slide Time: 28:44)

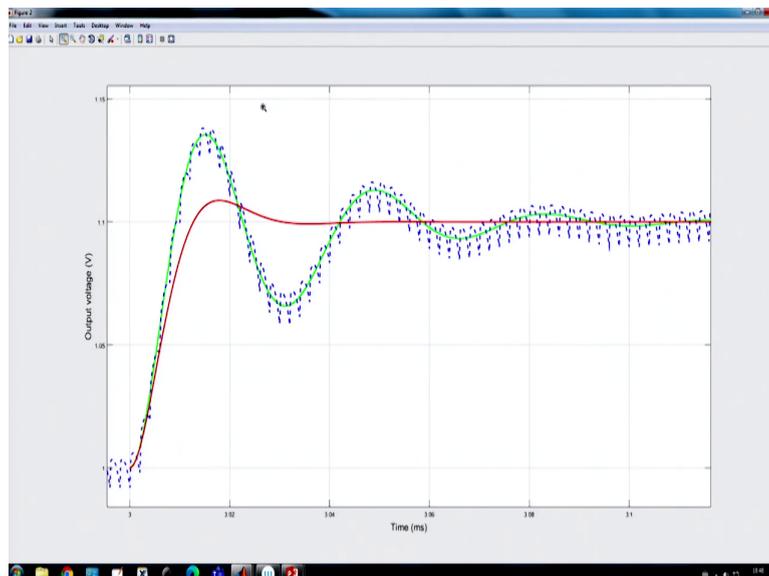
```

34 %Q_max=alpha(((r_c+r_eq)/z_c));
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o 1)];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_o*k_x;
45 K_c=((alpha*w_o)/(Fm*Vin))*sqrt(1+(k_x^2));
46 num_con=K_c*den_c;
47 den_con=[1/(w_x*w_cp) 1/(w_x) 1 0];
48 Ge=tf(num_con,den_con);
49
50 %% Lead compensator
51 % K_c=10000; w_cz=(2*pi*f_csw)/5;
52 % num_con=K_c*t[1/(w_cz 1)];
53 % den_con=[1/w_x 1];
54 % Ge=tf(num_con,den_con);
55
56 %% Lead compensator - analytical
57 % K_c=1.8/(alpha*(1/(w_o^2)+1/(Q_max*w_o)));

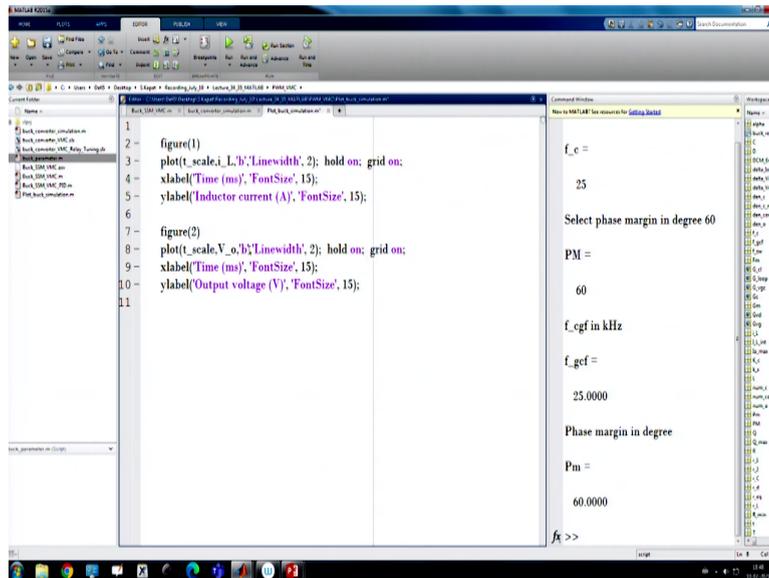
```

Now, if we want to do exact cancellation; now, we want to keep this 1; I am using red colour, so we have changing the colour to red and we are using here the compensation here we are using the denominator c the exact cancellation ok and we are running it. Again, we are using 25 kilohertz and 60 degree bandwidth ok.

(Refer Slide Time: 28:53)

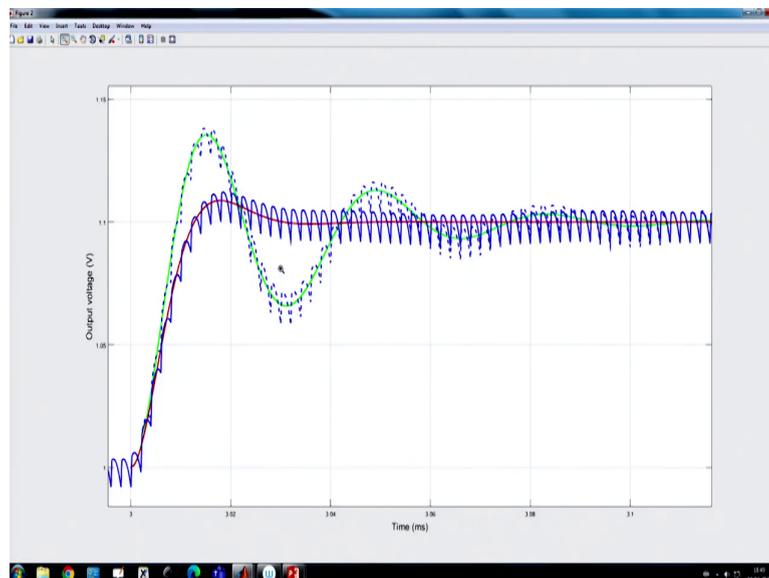


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And now we will consider the second one which is the colour now.

(Refer Slide Time: 29:07)



So, we here we are compensating using yes. So, here it looks like if we use the maximum value of the load resistance at a minimum value of load resistance, it may not be a good solution at light load. Because at light load the term this term is dominated at light load this is very low, this is dominated by this term, that means we want to use Q_p rather r_e plus r_c by z_c inverse or basically characteristic impedance r_e plus r_c this is a more robust compensation that is used for the controller pole.

So, you are not going to cancel this. We are not going to cancel this and if you do that, then let us see what happens now we want to redesign again. So, maybe we can consider another design case study where or it will be too much waveform with too much.

(Refer Slide Time: 30:09)

```

19- Gvd=tf(num_c,den_c);
20-
21- %% Open-loop Output Impedance
22- num_o=(r_eq/alpha)*1/(w_z2*w_x)((1/w_x)+(1/w_z2) 1);
23- den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24- Z_o=tf(num_o,den_o);
25-
26- %% Audio susceptibility
27- num_c=(D/alpha)*1/w_z 1;
28- den_c=[1/(w_o^2) 1/(Q*w_o) 1];
29- Gvg=tf(num_c,den_c);
30-
31- %% Modulator and Controller parameters
32- V_m=10; Fm=1/V_m;
33- %Q_max=alpha/((r_c+r_eq)/z_c)+(z_c/R_min);
34- Q_max=alpha/((r_c+r_eq)/z_c);
35- den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36-
37-
38- %% Type - III compensator
39- f_c=input('Select BW in kHz ');
40- %f_c=50; PM=60;
41- w_c=2*pi*f_c*1e3;
42- DM=input('Select alpha maximum in degrees ');

```

So, here we will set simply. So, now Q max we are see setting simply that parasitic element no load resistance dependency term ok.

(Refer Slide Time: 30:22)

```

1- clc; close all; clear;
2- clc;
3- %% Parameters
4- buck_parameter; Vin=12;
5- R=1; r_eq=r_L+r_1; alp=1;
6- Io_max=20; R_min=V_o;
7-
8- f_sw=1/T; w_sw=2*pi*f_sw;
9- z_c=sqrt(L/C); w_o=1/sqrt(L*C);
10- w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11- Q=alpha/((r_c+r_eq)/z_c)+(z_c/R);
12-
13- %% Define zeros
14- w_x=1/(r_c*C); w_z1=1/((R+r_C)*C); w_z2=r_eq/L;
15-
16- %% Control-to-output TF Gvd
17- num_c=(V_in/alpha)*1/w_z 1;
18- den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19- Gvd=tf(num_c,den_c);
20-
21- %% Open-loop Output Impedance
22- num_o=(r_eq/alpha)*1/(w_z2*w_x)((1/w_x)+(1/w_z2) 1);
23- den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24- Z_o=tf(num_o,den_o);

```

The plot shows the output voltage response over time. The y-axis is labeled 'Output voltage (V)' and ranges from 0 to 1.2. The x-axis is labeled 'Time (ms)' and ranges from 0 to 5. The plot shows a step response where the output voltage rises from 0V to approximately 1.1V and then settles to a steady-state value of about 1.0V.

(Refer Slide Time: 30:44)

```

28 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
29 Gv_g=tf(num_c,den_c);
30
31 %% Modulator and Controller parameters
32 V_m=10; Fm=1/V_m;
33 %Q_max=alpha(((r_C+r_eq)/z_c)+(z_c/R_min));
34 Q_max=alpha(((r_C+r_eq)/z_c));
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_c/k_x;
45 K_c=((alpha*w_c)/(Fm*V_m))*(sqrt(1+(k_x^2)));
46 num_con=K_c*den_c_max;
47 den_con=[1/(w_x^2*w_cp) (1/w_x)*(1/w_cp) 1 0];
48 Gc=tf(num_con,den_con);
49
50 %% % Lead compensator
51 % K_c=10000; w_x=2*pi*1e3;

```

So, now this is the one with our 25 kilohertz, 60 degree phase margin and here our actual converter response this is with exact camp compensation exact pole zero compensation. The second one, we want to do with this robust compensation, where our compensator zero it will be max ok where you see the max is set to where is that yeah it is set to Q max. So, we have just change this ok.

(Refer Slide Time: 31:06)

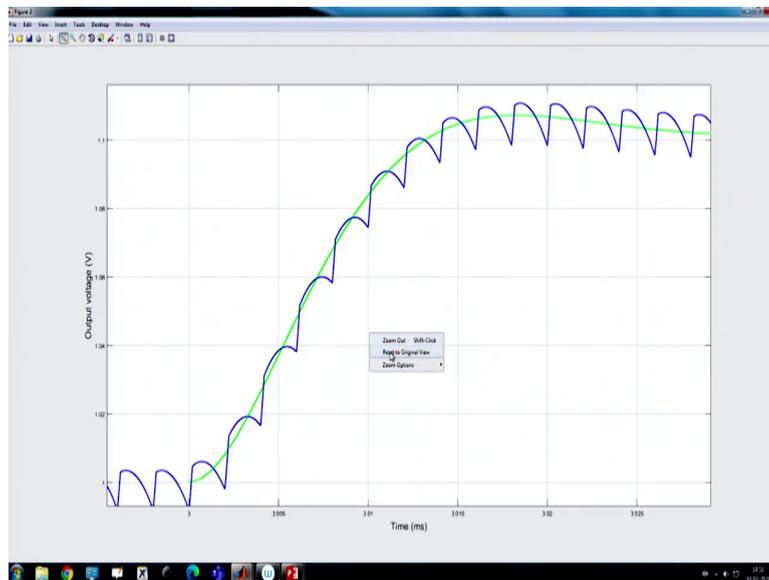
```

77 bode(G_loop,'b');
78 hold on; grid on;
79 [Gm,Pm,Wcg,Wcp]=margin(G_loop);
80 grid on;
81
82 %% % Transient parameters and transient response
83 t_sim=5e-3; t_step=3e-3;
84 delta_Io=0; delta_Vin=0; delta_Vref=0.1;
85
86 [y_s,t_s]=step(G_cl,(t_sim-t_step));
87 v_ac=delta_Vref*y_s;
88
89 figure(2)
90 plot(t_s+t_step)*1e3, Vref+v_ac,'g','LineWidth', 2);
91 xlabel('Time (ms)', 'FontSize', 15);
92 ylabel('Output voltage (V)', 'FontSize', 15);
93 hold on; grid on;
94
95 display('f_c in kHz')
96 f_gc=Wcg/(2*pi*1e3)
97 display('Phase margin in degree')
98 Pm
99

```

Now, we want to use green colour and if we can match the response to the linear model, then that should be good enough. But we can still check you know 25 kilohertz, 60 degree phase margin.

(Refer Slide Time: 31:20)



So, this is with robust compensation. Sorry it is actually eliminated all this waveform. So, this is the robust compensation method robust compensation method and what we wanted to design we should have written we should have written this ok.

(Refer Slide Time: 31:47)

```

33 %Q_max=alpha/((r_c+r_eq/z_c)+(z_c/R_min));
34 Q_max=alpha/((r_c+r_eq/z_c));
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_c*k_x;
45 K_c=((alpha*w_o)/(Fm*Vin))*(sqrt(1+(k_x^2)));
46 num_con=K_c*den_c;
47 den_con=[1/(w_o*w_cp) (1/w_o)+(1/w_cp) 1 0];
48 Gc=tf(num_con,den_con);
49
50 %% Lead compensator
51 % K_c=10000; w_cz=(2*pi*f_sw)/5;
52 % num_con=K_c*[1/w_cz 1];
53 % den_con=[1/w_cz 1];
54 % Gc=tf(num_con,den_con);
55
56 %% Lead compensator analysis

```

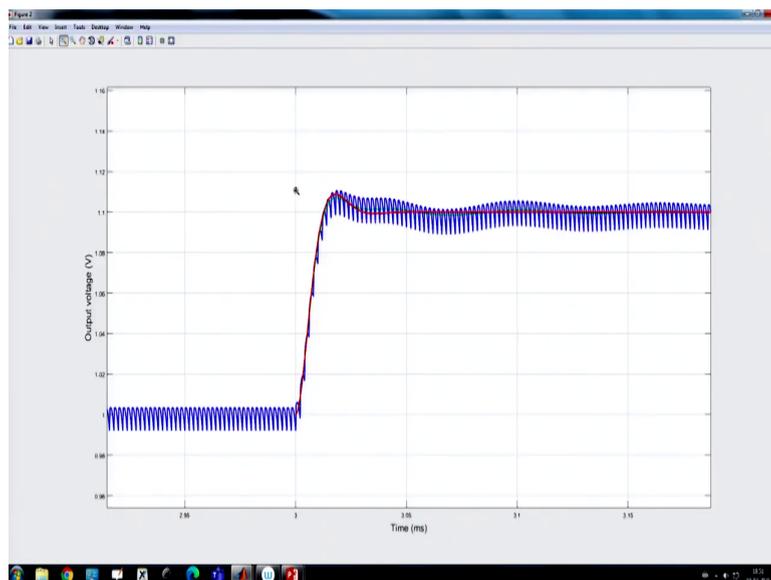
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```

1  %clear; close all; clear;
2  clc;
3  %% Parameters
4  buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5  R=1; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6  Io_max=20; R_min=Vref/Io_max;
7
8  f_sw=1/T; w_sw=2*pi*f_sw;
9  z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_c)));
11 Q=alpha/((r_c*C+r_eq/z_c)+(z_c/R));
12
13 %% Define zeros
14 w_z=1/(r_c*C); w_z1=1/(R+r_c*C); w_z2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin/alpha)*1/(w_z 1);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq/alpha)*1/(w_z2*w_z)((1/w_z)+(1/w_z2)) 1;
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 z_o=tf(num_o,den_o);

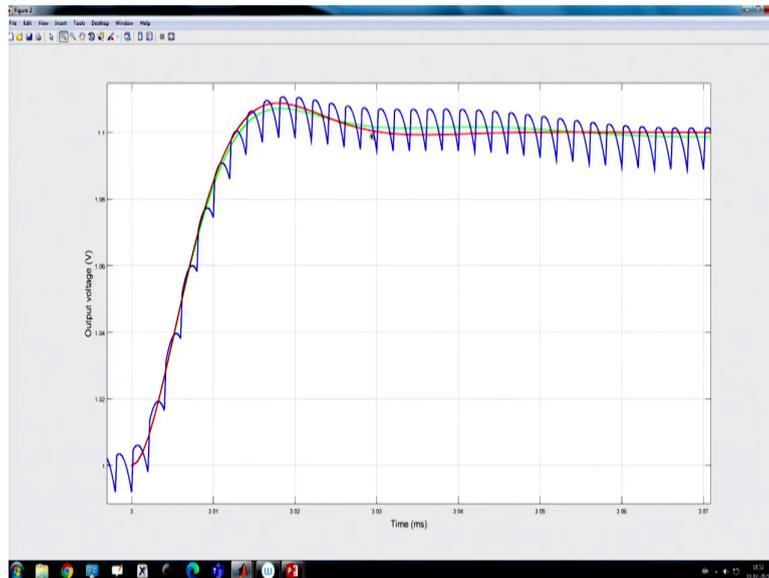
```

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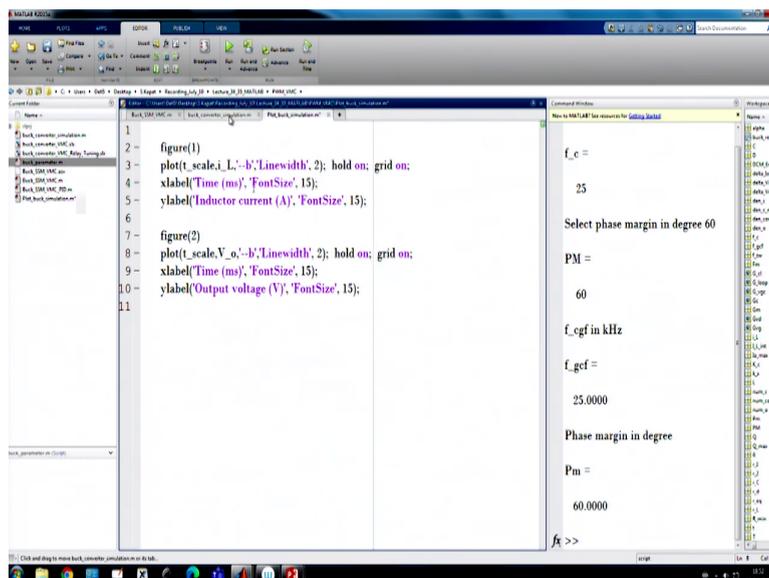


So, let us retain the whole value that means so 25 kilohertz, 60 degree phase margin yes.

(Refer Slide Time: 32:03)

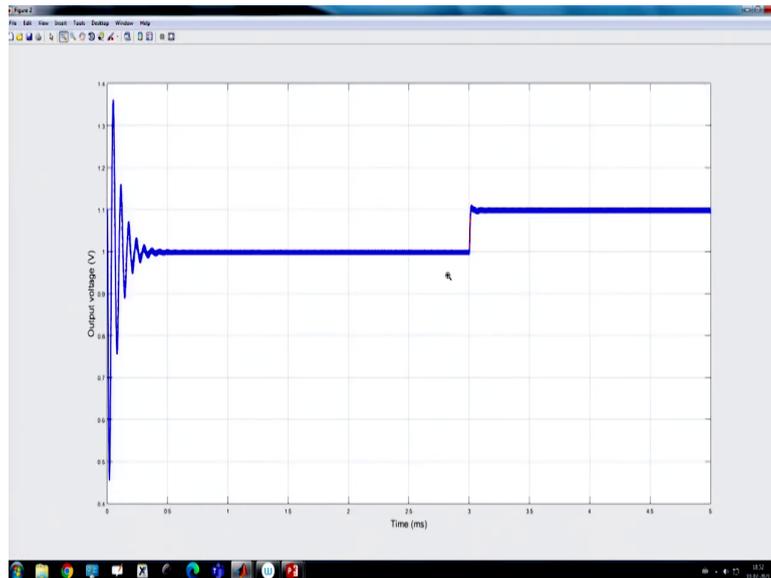


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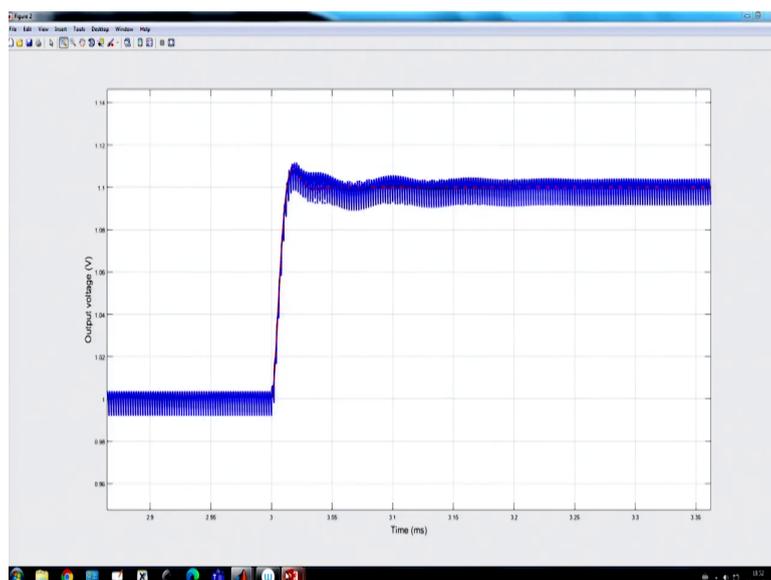


So, they are closely matching. That means exact cancellation and robust compensation. They are closely matching; you know the yellow that means your red one is exact cancellation and the green one is a robust compensation. And if we plot this you know this exact cancellation, now using a different colour this is exactly what is going to happen yeah they are closely matching.

(Refer Slide Time: 32:30)



(Refer Slide Time: 32:31)



That means it is recommended to use this compensation technique ok.

(Refer Slide Time: 32:39)

More Practical Compensation Case

$$K_{loop}(s) = F_m \times \frac{V_{in}}{\alpha} \times \frac{\left(1 + \frac{s}{\omega_{ESR}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)} \times G_c$$

$$G_c = \frac{k_c \left(1 + \frac{s}{Q_p\omega_o} + \frac{s^2}{\omega_o^2}\right)}{s \left(1 + \frac{s}{\omega_p}\right) \left(1 + \frac{s}{\omega_{ESR}}\right)}$$

$Q_p = ??$
 $Q_p = \frac{Z_c}{R_e + i_c}$

Next, more practical case study we have discussed. So, Q_p it turns out to be it would be insensitive; that means, we can use simply Z_c by R_e plus r_c . But when the load resistance is low, then that means high load condition it might be problematic, because then that term will get dominated.

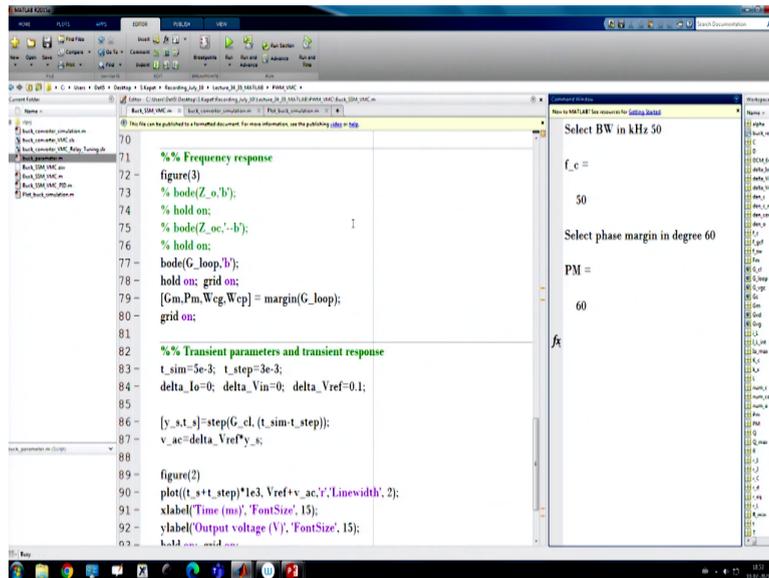
(Refer Slide Time: 33:03)

```

1  clc; close all; clear;
2  clc;
3  %% Parameters
4  buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5  R=0.05; r_eq=r_L+r_1; alpha=(R+r_eq)/R;
6  Io_max=20; R_min=Vref/Io_max;
7
8  f_sw=1/T; w_sw=2*pi*f_sw;
9  z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_c)));
11 Q=alpha/((r_c+r_eq)/z_c)+(z_c/R);
12
13 %% Define zeros
14 w_z=1/(t_c*C); w_z1=1/((R+r_c)*C); w_z2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin*alpha)*1/(w_z1);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq*alpha)*1/(w_z2*w_z)((1/w_z)+(1/w_z2)) 1];
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 Z_o=tf(num_o,den_o);

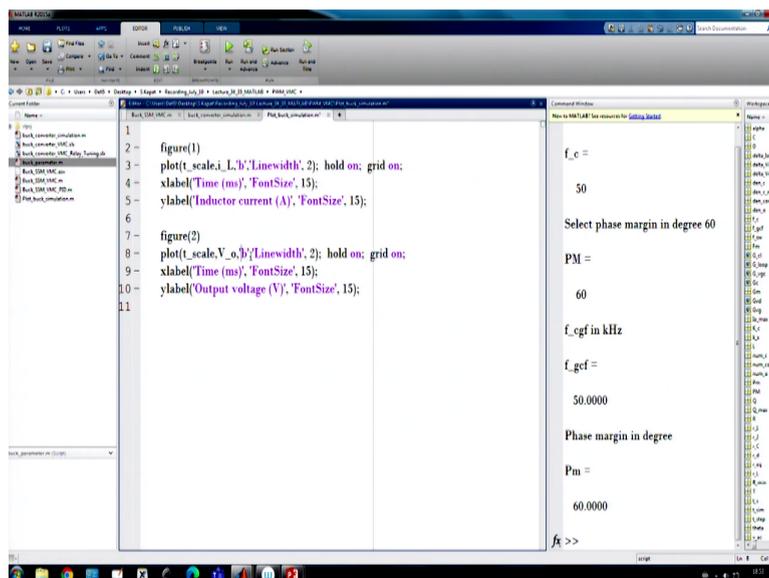
```

(Refer Slide Time: 33:21)

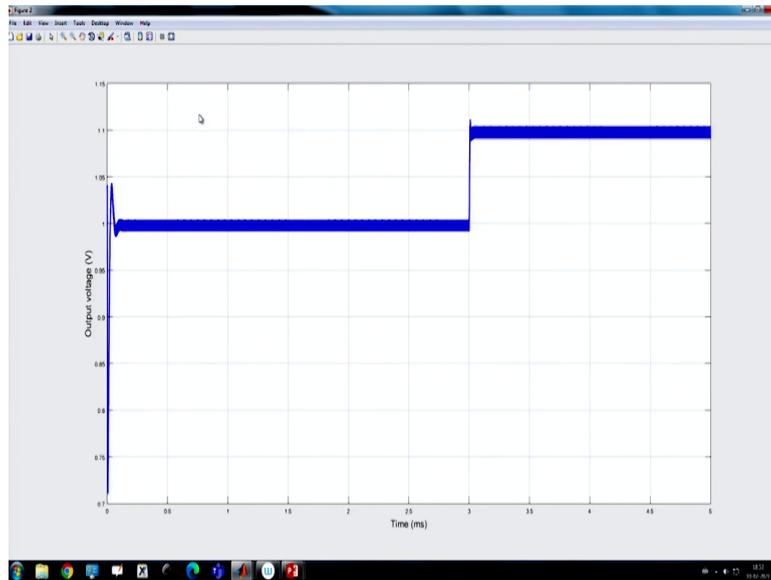


So, let us see what happen for the condition when load resistance is very low ok. So, it is like a 10 ampere we are designing or maybe we can design for, you know, the 20 ampere. So, the first case we are shown with exact cancellation and this is a response. You know we are talking about 50 kilohertz, 60 degree phase margin and if we match with this is exact cancellation.

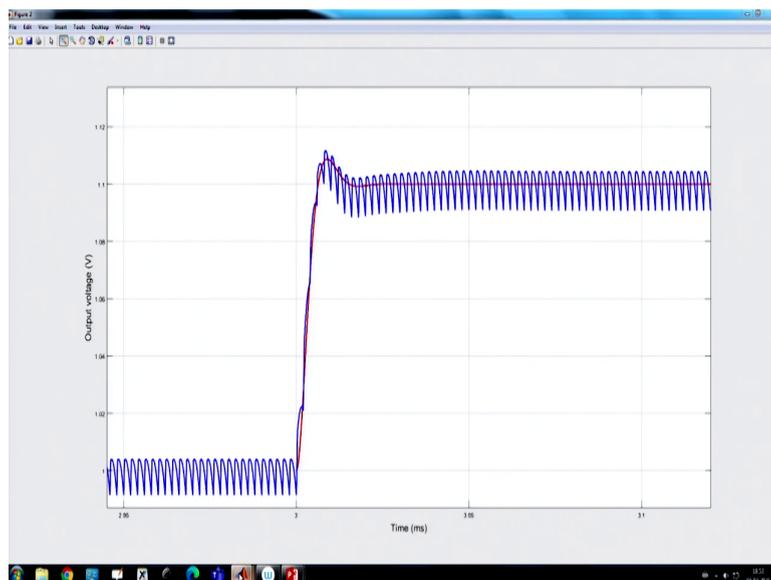
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(Refer Slide Time: 33:42)



(Refer Slide Time: 33:46)



So, we want to match the response of the system. This is under highest load current condition and the model is matching quite nicely. Now, we want to compensate using the conservative approach, which one we have selected the $Q_{q \max}$ to be this value denominator max.

(Refer Slide Time: 34:01)

```

31 %% Modulator and Controller parameters
32 V_m=10; Fm=1/V_m;
33 %Q_max=alpha(((r_C+r_eq)/z_c)+(z_c/R_min));
34 %Q_max=alpha(((r_C+r_eq)/z_c));
35 den_c_max=[1/(w_o^2) 1/(Q_max*w_o) 1];
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz ');
40 %f_c=50; PM=60;
41 w_c=2*pi*f_c*1e3;
42 PM=input('Select phase margin in degree ');
43 theta=deg2rad(90-PM);
44 k_x=tan(theta); w_cp=w_c*k_x;
45 K_c=((alpha*w_c)/(Fm*Vin))*sqrt(1+(k_x^2));
46 num_con=K_c*den_g_max;
47 den_con=[1/(w_c*w_cp) 1/(w_c) 1 1 0];
48 Gc=tf(num_con,den_con);
49
50 %% Lead compensator
51 % K_c=10000; w_cz=(2*pi*f_sw)/5;
52 % num_con=K_c*[1/w_cz 1];
53 % den_con=[1/w_cz 1];
54 % Gc=tf(num_con,den_con);

```

(Refer Slide Time: 34:08)

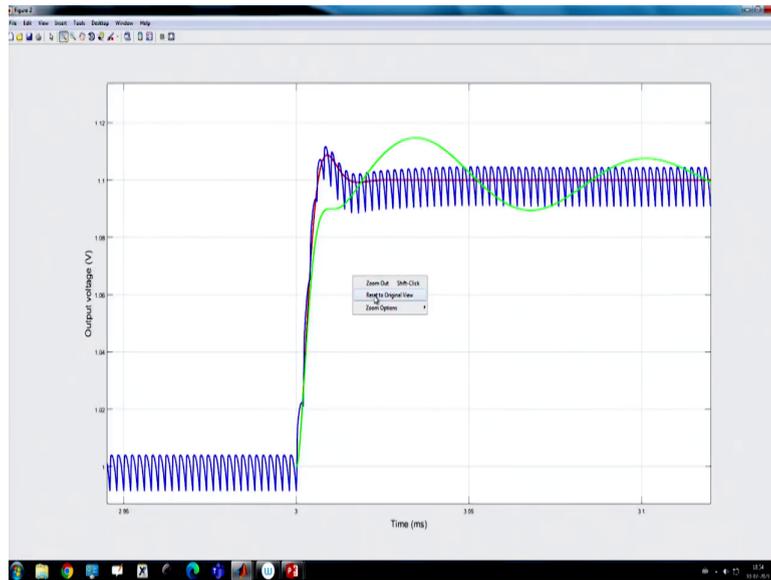
```

76 % hold on;
77 bode(G_loop,'b');
78 hold on; grid on;
79 [Gm,Pm,Wcg,Wcp]=margin(G_loop);
80 grid on;
81
82 %% Transient parameters and transient response
83 t_sim=5e-3; t_step=3e-3;
84 delta_Io=0; delta_Vin=0; delta_Vref=0.1;
85
86 [y_s,t_s]=step(G_cl, (t_sim-t_step));
87 v_ac=delta_Vref*y_s;
88
89 figure(2)
90 plot(t_s+t_step)*1e3, Vref+v_ac,'g', 'Linewidth', 2);
91 xlabel('Time (ms)', 'FontSize', 15);
92 ylabel('Output voltage (V)', 'FontSize', 15);
93 hold on; grid on;
94
95 display('f_cg in kHz')
96 f_cg=Wcp/(2*pi*1e3)
97 display('Phase margin in degree')
98 Pm
99

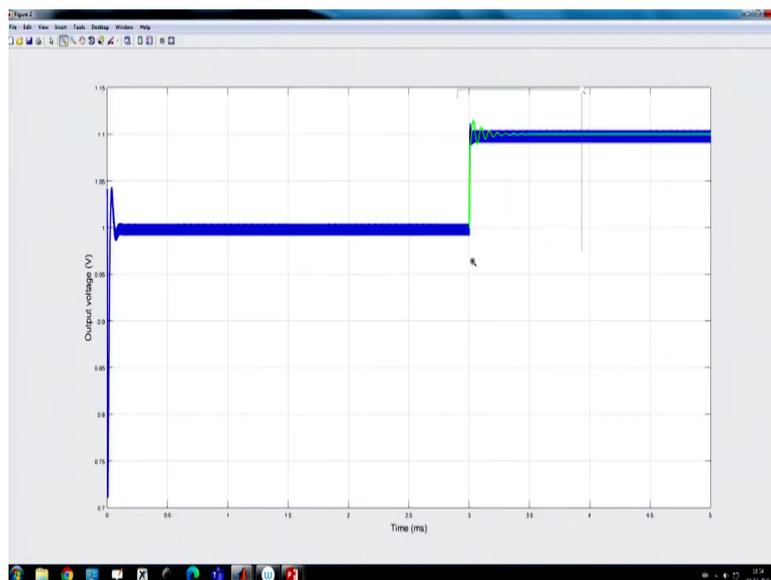
```

So, we will just use this in the compensator rather than and now, we will use a green colour and see how does the response looks like. So, 50 kilohertz, 60 degree phase margin.

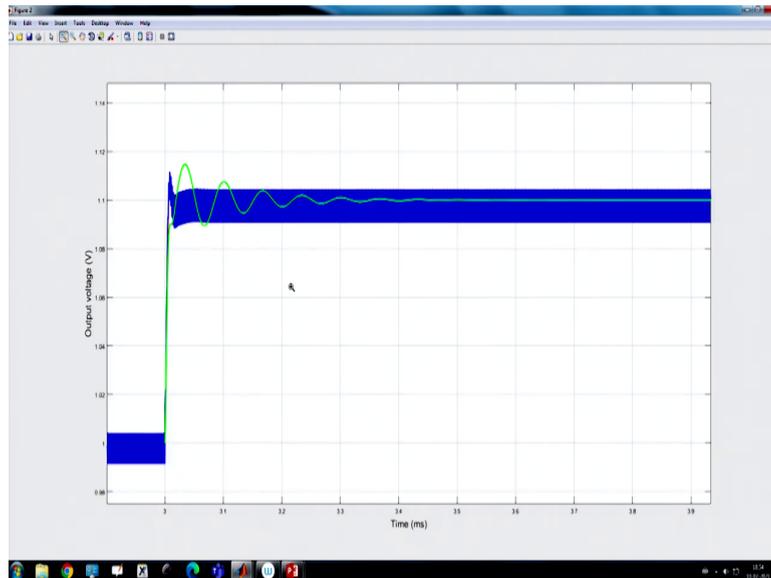
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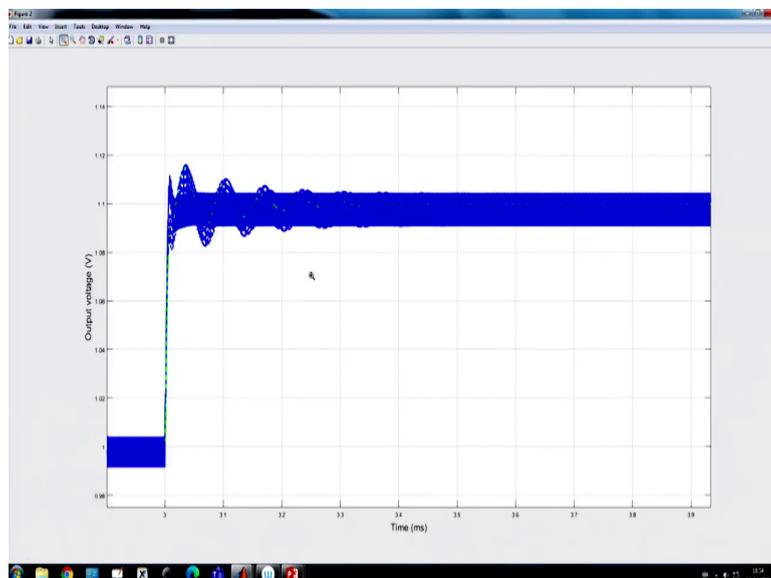


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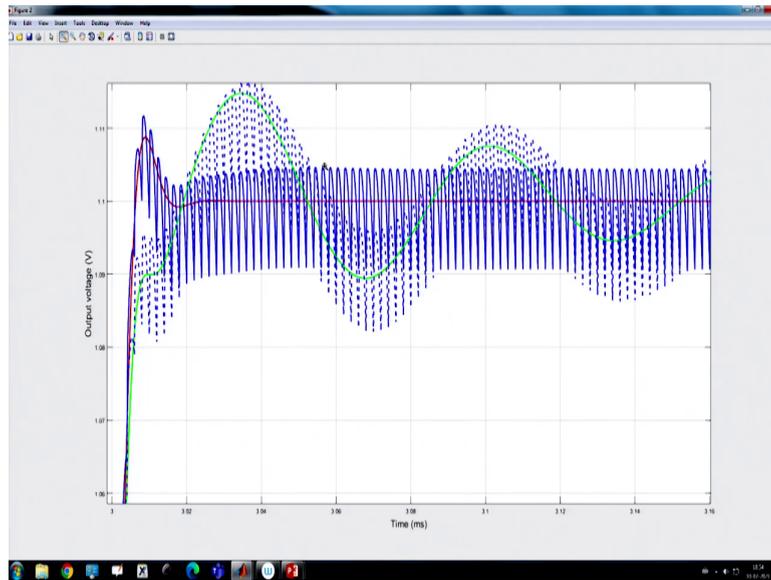


So, response is not you know it is again kind of over compensated ok. So, here probably we need to tradeoff between you know, so now if we go back to our converter response if I draw the dotted line.

(Refer Slide Time: 34:41)

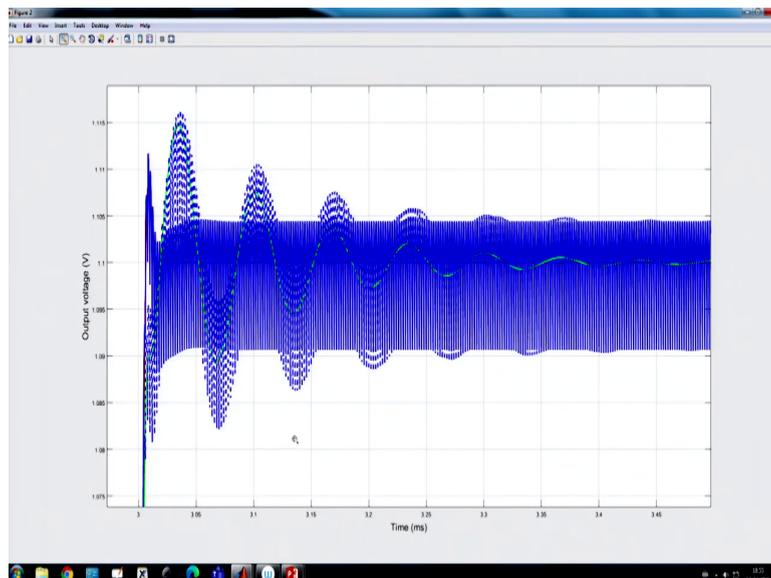


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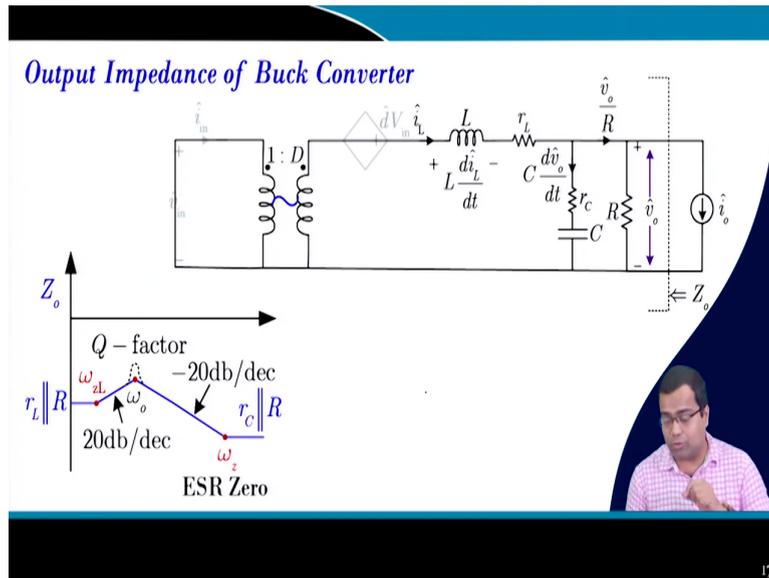
So, in terms of design point of view, so this is not a bad design, but it is more robust design. Where it may have a little longer settling time, but that is fine but at least it is insensitive to parameter variation, I slightly increase in the overshoot undershoot, otherwise in the average sense they are matching fine.

(Refer Slide Time: 35:04)

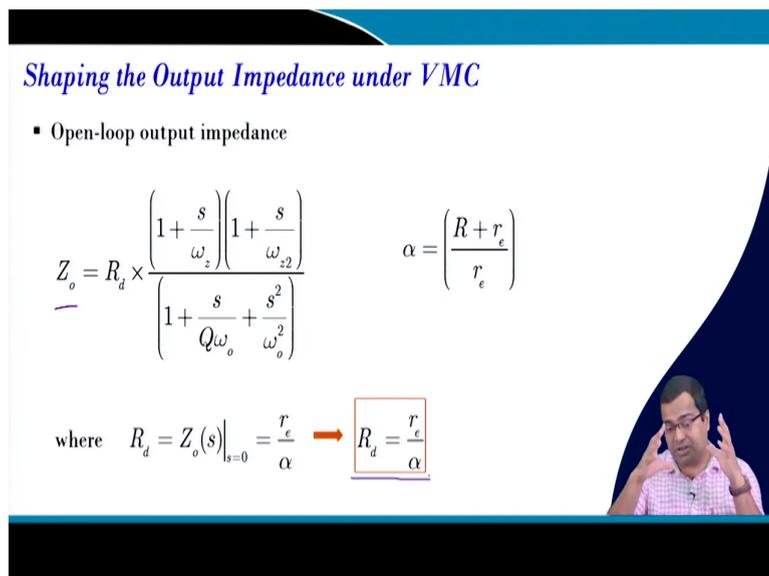


But yes you can you can expect a little longer settling time because of this compensation, but it is not significantly high ok. This may be more robust.

(Refer Slide Time: 35:16)



(Refer Slide Time: 35:20)



So, now we have to talk about output impedance. Now, if we talk about output impedance under voltage mode control, this is open loop output impedance and the closed loop output impedance can be taken as you know this is the DC characteristic and we have discussed this output impedance in lecture number 13, when we tried to model the actual source. So now, this output impedance of the converter right.

(Refer Slide Time: 35:45)

Shaping the Output Impedance under VMC

- Open-loop output impedance

$$Z_o = R_d \times \frac{\left(1 + \frac{s}{\omega_z}\right) \left(1 + \frac{s}{\omega_{z2}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

$$R_d = \frac{r_e}{\alpha} \quad \omega_z = \frac{1}{r_c C} \quad \omega_{z2} = \frac{r_e}{L}$$

$$\omega_o = \sqrt{\frac{R+r_e}{R+r_c}} \times \frac{1}{\sqrt{LC}}$$

$$Q = \alpha \left[\frac{r_c + r_e}{z_c} + \frac{z_c}{R} \right]^{-1} \quad \text{where } z_c = \sqrt{\frac{L}{C}}$$


(Refer Slide Time: 35:56)

Shaping the Output Impedance under VMC

- Open-loop output impedance

$$Z_o = R_d \times \frac{\left(1 + \frac{s}{\omega_z}\right) \left(1 + \frac{s}{\omega_{z2}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

$$R_d = \frac{r_e}{\alpha} \quad \omega_z = \frac{1}{r_c C} \quad \omega_{z2} = \frac{r_e}{L}$$

$$\omega_o = \sqrt{\frac{R+r_e}{R+r_c}} \times \frac{1}{\sqrt{LC}}$$

$$Q = \alpha \left[\frac{r_c + r_e}{z_c} + \frac{z_c}{R} \right]^{-1}$$

$Z_{oc} = \frac{z_o}{1 + K_{loop}}$



Now, open loop output impedance we have discussed different poles and zero already we have discussed. So now, we want to see what is the what is the closed loop output impedance; that means, the close loop output impedance is nothing but the close loop output impedance nothing but the open loop output impedance divided by 1 plus loop transfer function. So, so far we have used you know different type of technique.

(Refer Slide Time: 36:21)

```

1
2 figure(1)
3 plot(t_scale,L.'b','LineWidth', 2); hold on; grid on;
4 xlabel('Time (ms)', 'FontSize', 15);
5 ylabel('Inductor current (A)', 'FontSize', 15);
6
7 figure(2)
8 plot(t_scale,V.'b','LineWidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11

```

So, suppose if we consider sorry, if we consider let us say exact cancellation, for example let us consider exact cancellation, then we want to see what happens for closed loop output impedance. Because so far we have discussed step transient of the reference voltage, now we are talking about the output impedance.

(Refer Slide Time: 36:37)

```

58 % tau_L=L/r; C_tau=C*r; C'C;
59 % w_cz=((r_eq/C)-1)/(tau_L-tau_C);
60 % num_con=K_c*(1/(w_cz+1));
61 % den_con=1/(w_cz+1);
62 % Gc=tf(num_con,den_con);
63
64 %% Loop gain and closed-loop TFs
65 G_loop=Gvd*Fm*Gc; %% Loop gain
66
67 Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
68 G_cl=G_loop/(1+G_loop); %% Closed-loop TF
69 G_ygc=Gvg/(1+G_loop); %% Closed-loop audio susc.
70
71 %% Frequency response
72 figure(3)
73 bode(Z_oc,'b');
74 hold on;
75 bode(Z_oc,'-b');
76 hold on;
77 bode(G_loop,'r');
78 hold on; grid on;
79 [Gm,Fm,Wcg,Wcp]=margin(G_loop);
80 grid on;

```

So, here we have the expression of the closed loop output impedance. So, we need to plot also the Bode plot of the closed loop output impedance of different you know the loop transfer function ok.

(Refer Slide Time: 36:54)

```

70
71 %% Frequency response
72 figure(3)
73 bode(Z_o,'b');
74 hold on;
75 bode(Z_oc,'-b');
76 hold on;
77 bode(G_loop,'r');
78 hold on; grid on;
79 [Gm,Pm,Weg,Wep] = margin(G_loop);
80 grid on;
81
82 %% Transient parameters and transient response
83 t_sim=5e-3; t_step=3e-3;
84 delta_Io=20; delta_Vin=0; delta_Vref=0.1;
85
86 [y_s,t_s]=step(G_cl,(t_sim-t_step));
87 v_ac=delta_Vref*y_s;
88
89 figure(2)
90 plot(t_s+t_step)*1e3,Vref+v_ac,'Linewidth',2);
91 xlabel('Time (ms)', 'FontSize', 15);
92 ylabel('Output voltage (V)', 'FontSize', 15);
93 hold on; axis([0 10 0 1.5]);

```

(Refer Slide Time: 37:00)

```

1 = clc; close all; clear;
2 = clc;
3 %% Parameters
4 buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 R=1; r_eq=r_L+r_l; alpha=(R+r_eq)/R;
6 Io_max=20; R_min=Vref/Io_max;
7
8 f_sw=1/T; w_sw=2*pi*f_sw;
9 z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 Q=alpha/((r_C+r_eq)/z_c+(z_c/r));
12
13 %% Define zeros
14 w_x=1/(r_C*C); w_x1=1/((R+r_C)*C); w_x2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin/alpha)*(1/w_x1);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq/alpha)*1/(w_x2*w_x)((1/w_x)+(1/w_x2) 1);
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 Z_o=tf(num_o,den_o);

```

And then we want to apply. Let us say 20 ampere load step transient and let us set the load resistance to 1 ohm.

(Refer Slide Time: 37:06)

```

73 - bode(Z_o,'b');
74 - hold on;
75 - bode(Z_oc,'-b');
76 - hold on;
77 - bode(G_loop,'');
78 - hold on; grid on;
79 - [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80 - grid on;
81
82 %% Transient parameters and transient response
83 - t_sim=5e-3; t_step=3e-3;
84 - delta_lo=20; delta_Vin=0; delta_Vref=0;
85
86 - [y_s,t_s]=step(Z_oc,(t_sim-t_step));
87 - v_ac=-delta_lo*y_s;
88
89 - figure(2)
90 - plot((t_s+t_step)*1e3,Vref+v_ac,'Linewidth',2);
91 - xlabel('Time (ms)', 'FontSize',15);
92 - ylabel('Output voltage (V)', 'FontSize',15);
93 - hold on; grid on;
94
95 - display('f_cgf in kHz')
96 - f_cgf=(Wcg/(2*pi)*1e3);
  
```

(Refer Slide Time: 37:32)

```

59 - % w_cz=((r_eq/(C_o*1))/(tau_L*tau_C));
60 - % num_con=K_c*(1/(w_cz));
61 - % den_con=1/(w_cz);
62 - % Gc=1/(num_con*den_con);
63
64 %% Loop gain and closed-loop TFs
65 - G_loop=Gvd*Fm*Gc; %% Loop gain
66
67 - Z_oc=Z_o/(1+G_loop); %% Closed-loop output imp
68 - G_cl=G_loop/(1+G_loop); %% Closed-loop TF
69 - G_vge=Gvg/(1+G_loop); %% Closed-loop audio susc.
70
71 %% Frequency response
72 - figure(3)
73 - bode(Z_o,'b');
74 - hold on;
75 - bode(Z_oc,'-b');
76 - hold on;
77 - bode(G_loop,'');
78 - hold on; grid on;
79 - [Gm,Pm,Wcg,Wcp] = margin(G_loop);
80 - grid on;
81
  
```

Select BW in kHz 50

f_c = 50

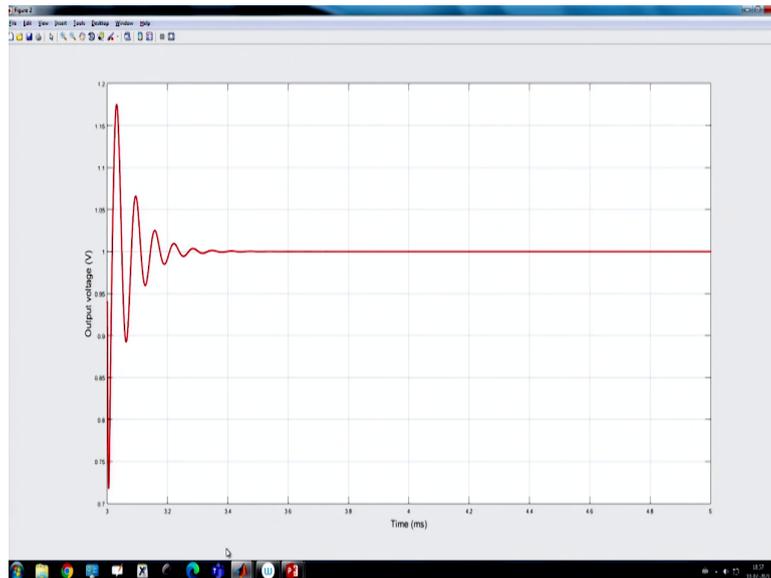
Select phase margin in degree 60

PM = 60

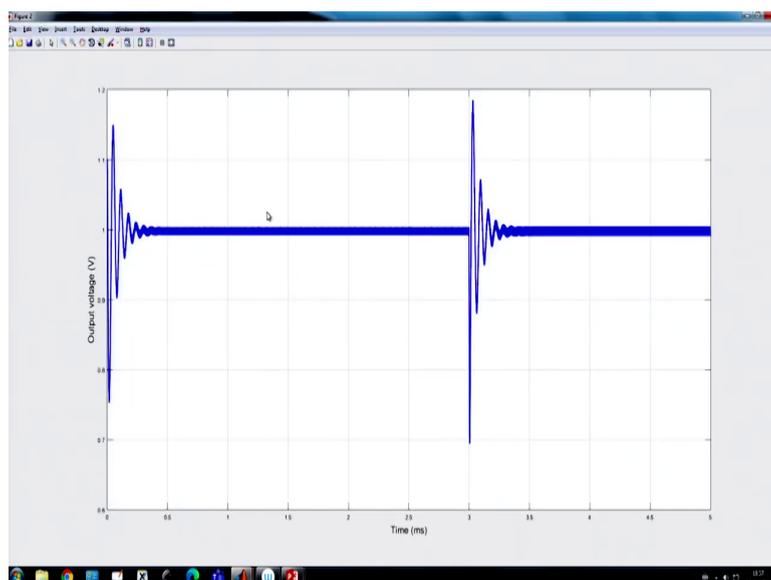
f_x

So, we are making load transient ok, where now, there is no reference transient and we have discussed that for that we need to, this will be minus of delta load step size. That means, this size ok and here it should be output impedance that means Z_{oc} ok. So, let us now try to bandwidth let us set 50, 60 degree phase margin.

(Refer Slide Time: 37:38)

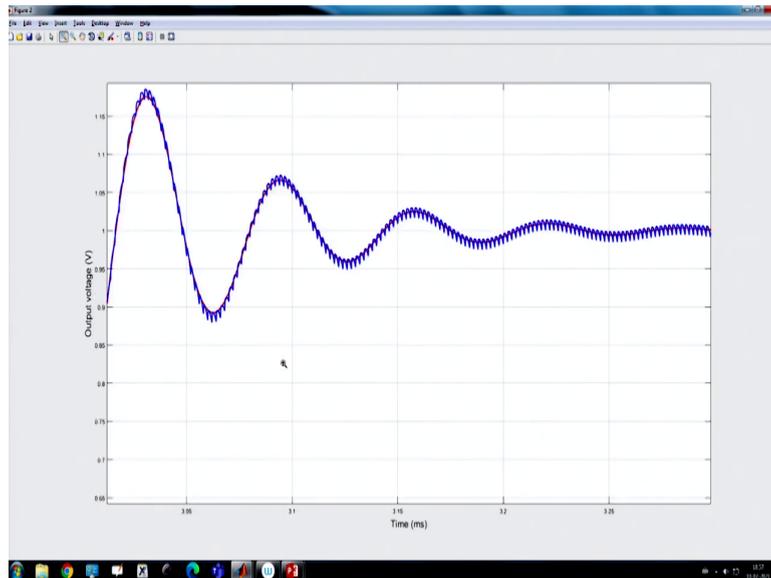


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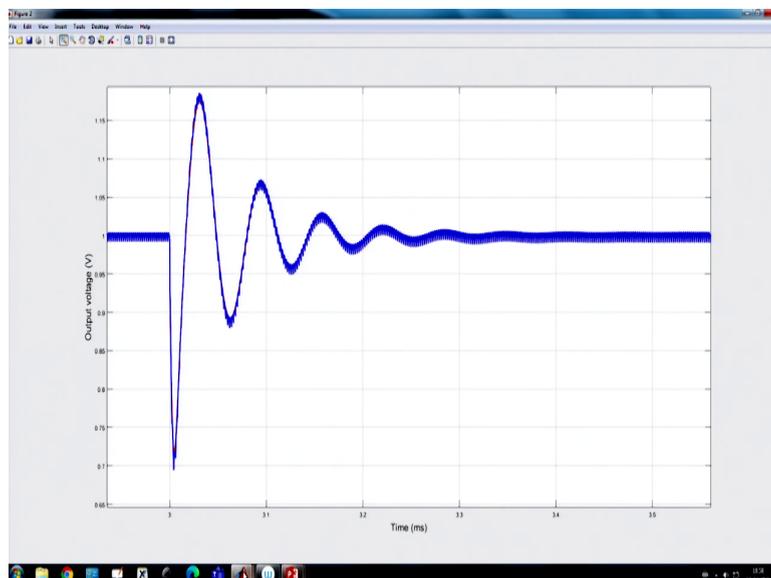
And we will see is a very much oscillatory behavior ok, is oscillatory behavior and we want to see what is the response of the system of the actual switch simulation. So, we have applied a load step transient.

(Refer Slide Time: 37:56)

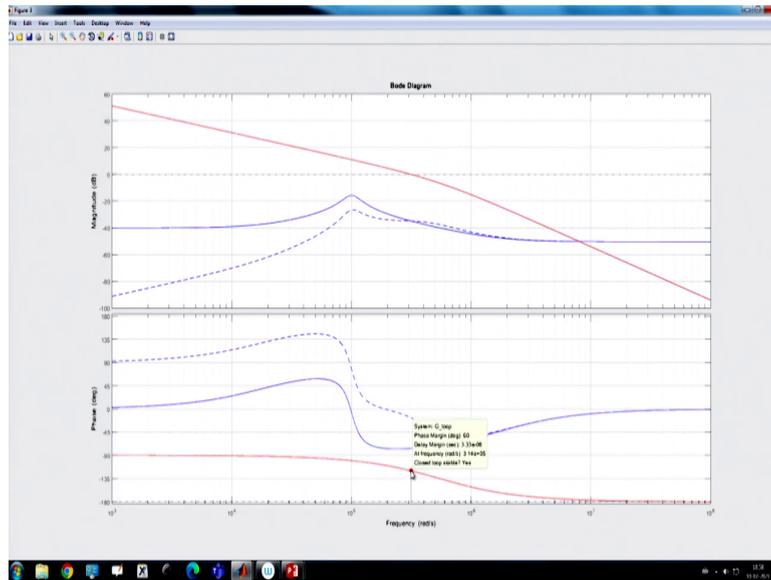


And you see that they are matching quite accurately that loop step transient; that means your actual switch simulation and the small signal model they are matching, but the response is not very satisfactory.

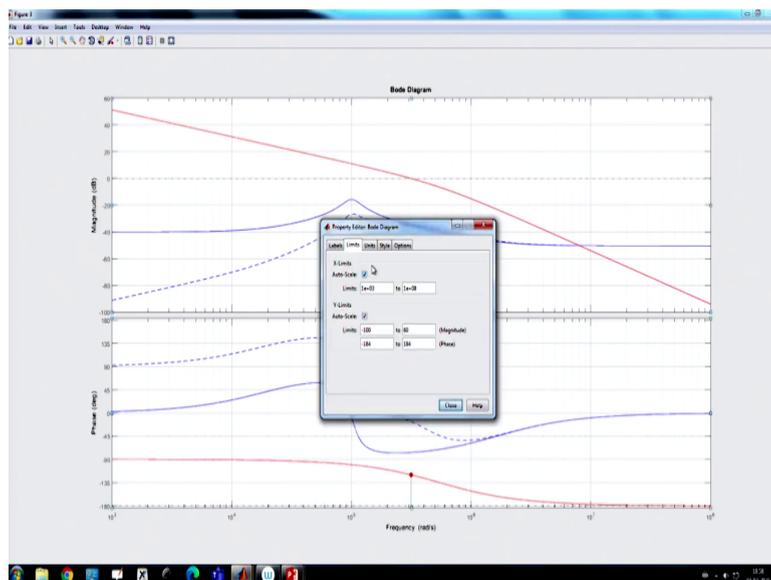
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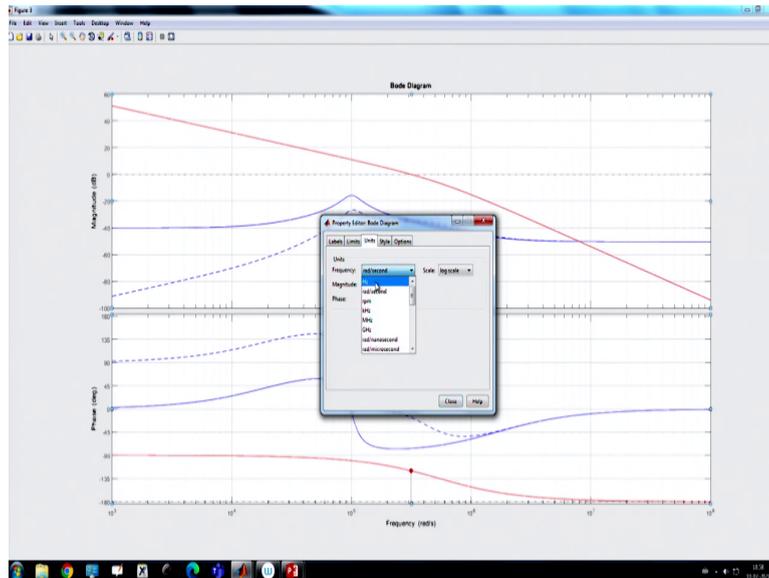
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(Refer Slide Time: 38:27)



(Refer Slide Time: 38:29)



Why? Because if you go back to the Bode plot. So, this is the Bode plot; the red one is the loop transfer function ok and if we see all the stability characteristics. So, we are getting you know we have set 60 degree phase margin, and it is in radian per second.

So, we should change it unit 2 Hz ok and then we are setting 50 kilohertz bandwidth. But open output impedance you see at low frequency it is high it is much higher than the closed loop 1 and then there is a picking and then it is getting this point at high frequency it will be decided by the ESR.

That we have discussed output impedance and high frequency, your inductor behaves like an open circuit. So, it is RC capacitor will behave like a short circuit, then RC and R_r parallel combination will give you the output impedance at very high frequency. And for open loop low frequency, it is RL and R will come in parallel that you have discussed. But in close loop, we can reduce the low frequency output impedance, so that we can make the converter that means, if it is a voltage source, we are trying to achieve a very low DC output impedance.

So, that it has almost nearly perfect voltage regulation for the entire load range or the nominal load range. But at high frequency that ESR that effect cannot be avoided that will create a jump, in order to but this ringing (Refer Time: 39:39) effect is coming. So, what can you do because you know the expression, if we increase the loop gain then this output impedance can be reduced. The effect can be reduced if you increase the loop gain ok.

So, now what we are going to do, we want to increase the loop gain; that means, here our transient response shows that you know almost 15 percent overshoot undershoot and also the settling time is quite large, because it takes you know sort of 0.25 millisecond which is not fast enough. Then what we have to do? We have to increase the bandwidth. Let us take increase the bandwidth.

(Refer Slide Time: 40:18)

```

1  clc; close all; clear;
2  clc;
3  %% Parameters
4  buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5  R=1; r_eq=r_L+r_l; alpha=(R+r_eq)/R;
6  Io_max=20; R_min=Vref/Io_max;
7
8  f_sw=1T; w_sw=2*pi*f_sw;
9  z_c=sqrt(L/C); w_o_ideal=1/sqrt(L*C);
10 w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
11 Q=alpha/((r_C+r_eq)/z_c)+(z_c/r/R);
12
13 %% Define zeros
14 w_x=1/(r_C*C); w_x2=1/((R+r_C)*C); w_x2=r_eq/L;
15
16 %% Control-to-output TF Gvd
17 num_c=(Vin/alpha)*(1/w_x2);
18 den_c=[1/(w_o^2) 1/(Q*w_o) 1];
19 Gvd=tf(num_c,den_c);
20
21 %% Open-loop Output Impedance
22 num_o=(r_eq/alpha)*(1/(w_x2*(1/w_x2)+(1/w_x2)+1));
23 den_o=[1/(w_o^2) 1/(Q*w_o) 1];
24 z_o=tf(num_o,den_o);

```

(Refer Slide Time: 40:36)

```

1  % clc; clear; close all;
2
3  DCM_En=0;
4  I_L_int=1; V_c_int=1;
5
6  sim('buck_converter_VMC.slx'); clc;
7  t=buck_result.time; t_scale=*1e3;
8  x=buck_result.data;
9  i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3); Vcon=x(:,4);
10
11 Plot_buck_simulation;

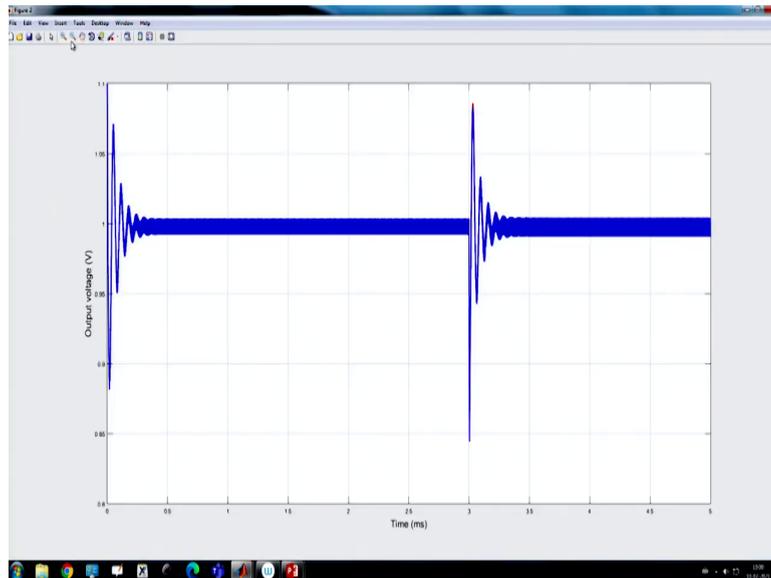
```

Warning: 'buck_converter_VMC' contains 1 algebraic loop(s). To see more details about the loops use the command Simulink.BlockDiagram.getAlgebraicLoops or the command line Simulink.debugger by typing 'sidebug'.

'buck_converter_VMC' in the MATLAB command window. To eliminate this message, set the Algebraic loop option in the Diagnostics page of the Simulation Parameters Dialog to "None"

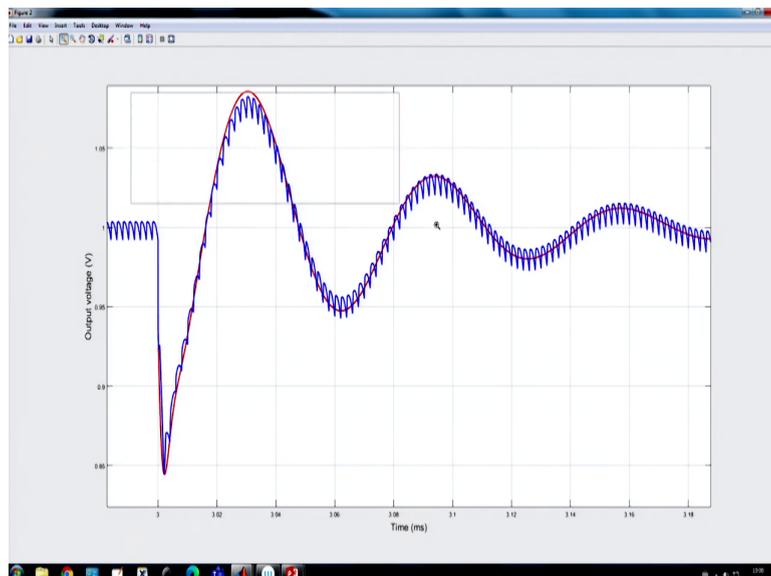
> In buck_converter_simulation (line 6) Found algebraic loop containing: 'buck_converter_VMC/Buck converter', 'buck_converter_VMC/Buck converter', 'buck_converter_VMC/Buck converter', 'buck_converter_VMC/load', 'buck_converter_VMC/Sum' (algebraic)

(Refer Slide Time: 40:39)



Suppose we want to achieve the bandwidth. Let us say you know instead of 50 kilohertz, 100 kilohertz, though we have to check the model validity 100 kilohertz bandwidth that we are going to achieve. Now, we run the actual switch simulation and check what happens. So, earlier it was a 15 percent overshoot undershoot.

(Refer Slide Time: 40:43)



Now, you will see first of all the model mismatch start coming into the picture. So that means, models are not matching nicely the predicted behavior and the switching behavior are varying. But we could reduce the overshoot/undershoot within 10 percent right and we can

also increase the settling time. So, it is more or less same not very much improved, but we can reduce the overshoot undershoot. That means, essentially, what we are trying to do by shifting up the loop gain by increasing the loop gain, because we are trying to increase that crossover frequency we are trying to reduce this peaking effect.

Because the closed loop output impedance is the open loop output impedance divided by the 1 plus loop gain, but high loop gain around this peaking frequency can reduce the impact. But this is not a solution because we cannot set the loop gain to be very high, say. What is the alternative solution?

(Refer Slide Time: 41:44)

Under High Load Resistance

- For very high resistive load, i.e., R is very high

$$\underline{R \gg r_e, r_c} \quad R_d \approx r_e$$
$$\omega_o \approx \frac{1}{\sqrt{LC}} \quad Q \approx \left(\frac{Z_c}{r_c + r_e} \right)$$

20

(Refer Slide Time: 41:56)

Under High Load Resistance

- Open-loop output impedance
- Control to output TF

$$Z_o(s) \approx R_d \times \frac{\left(1 + \frac{s}{\omega_z}\right) \left(1 + \frac{s}{\omega_{z2}}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

$$G_{vd}(s) \approx \frac{V_{in} \left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}\right)}$$

where $R_d \approx r_e$ Thus, $Z_o(s) = \frac{R_d}{V_{in}} \times G_{vd}(s) \times \left(1 + \frac{s}{\omega_z}\right)$

So, alternative solution is that we want to shape the loop gain at very light load R is much larger than this high sorry, very high resistive load, then we can simply take the effect of this R_d to be equal to r_e ok.

This is equal to r_e and the control to output transfer function is G_{vd} all this we know. So, the closed loop output impedance if you see the expression of this output impedance and the control to output transfer function ESR zero is common. ESR zero is common and the poles are common right common. So, we can write the output impedance in terms of the control to output transformation into that additional zero ok this additional zero ok.

(Refer Slide Time: 42:35)

Closed Loop Output Impedance

▪ Loop gain $K(s) = G_{vd}(s)F_m G_c(s)$ $K \triangleq G_{vd}F_m G_c$

(Refer Slide Time: 42:45)

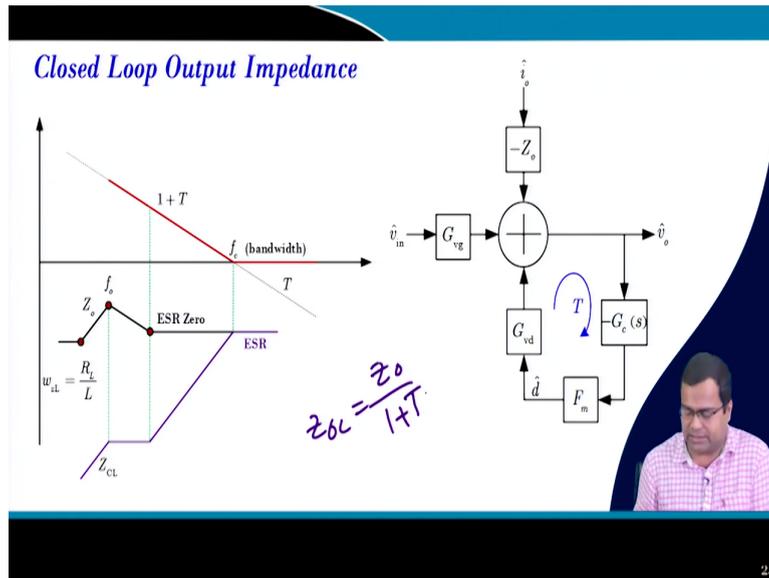
Closed Loop Output Impedance

▪ Closed loop output impedance

$$Z_{oc}(s) = \frac{Z_o(s)}{1 + K(s)} = \frac{\left(\frac{R_d}{V_m}\right) G_{vd} \left(1 + \frac{s}{w_p}\right)}{1 + G_{vd} F_m G_c}$$

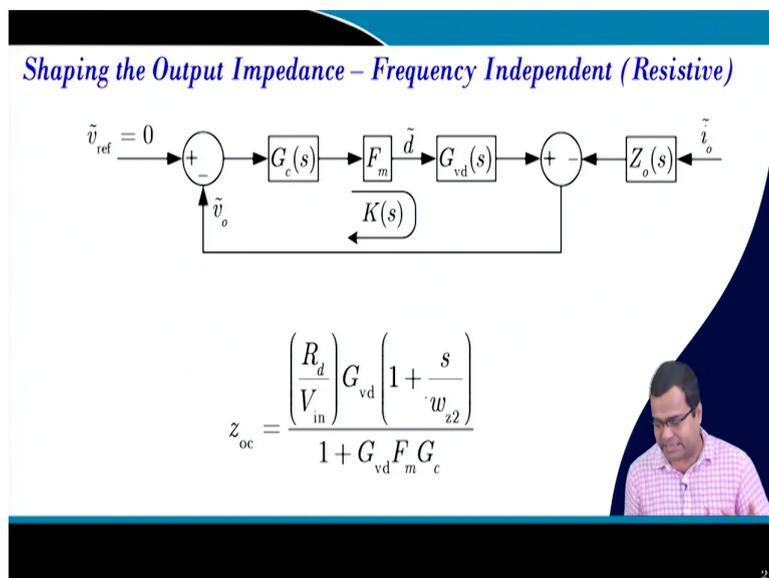
So, now, next so the closed loop output impedance can be written as that the loop gain this product and the loop gain can be written like this and then closed loop output impedance expression. If you write the open loop output impedance divided by 1 plus loop transfer function and we have already obtained the output impedance in terms of control to output transfer function and loop gain also has the control to output transfer function.

(Refer Slide Time: 43:01)



So, if you want to now, this is the expression of the closed loop. So, the closed loop output impedance can be written as open loop divided by 1 plus this loop. Here it is expression t.

(Refer Slide Time: 43:13)



(Refer Slide Time: 43:25)

Shaping the Output Impedance – Frequency Independent (Resistive)

Objective: Design compensator $G_c(s)$ such that z_{oc} resembles a resistance r

$$z_{oc} = \frac{\left(\frac{R_d}{V_{in}}\right) G_{vd} \left(1 + \frac{s}{w_{22}}\right)}{1 + G_{vd} F_m G_c} = r \quad ?$$

So, we have now we want to make the close loop output impedance nearly independent of frequency, that means frequency independent is it possible or not. That means, we have to set the resemble like a resistance. If we can do that, then our initial voltage source discussion if we have just a resistance in series know you know infinite bandwidth. That means it can respond to any frequency can we do it or not.

So, can we make the whole close loop output impedance to be simply a resistive output impedance and what should be the value of resistance. So, that is our objective now. So, we need to make this r and we need to find out what is the value r .

(Refer Slide Time: 44:00)

Shaping the Output Impedance – Frequency Independent (Resistive)

$$\left(\frac{R_d}{V_{in}}\right) G_{vd} \left(1 + \frac{s}{w_{z2}}\right) = r \left(1 + G_{vd} F_m G_c\right)$$

Why?

Constraint:

Resistive closed-loop output impedance $r \geq r_c$

At high frequency, z_{oc} is limited by the ESR

Thus, $z_{oc} = r$ (simply resistive drop)

$r_c || R \approx r_c$

27

So, first of all that means this is just you know this term I am taking this side. So, next expression now there is a constraint. What is the constraint? The resistive closed loop output impedance should be larger than or equal to ESR. Why?

Because if you take the output impedance then it is a parallel combination of R LC at very high frequency this will be open circuited this will be shorted and that means your r c and R will be in parallel, so which can be written as roughly r c if r c is very small. So, you cannot achieve output impedance smaller than this because this is the limiting factor ok.

(Refer Slide Time: 44:41)

Shaping the Output Impedance – Frequency Independent (Resistive)

Find G_c

$$G_c = \frac{1}{F_m G_{vd}} \left[\left(\frac{R_d}{r V_{in}} \right) G_{vd} \left(1 + \frac{s}{w_{z2}} \right) - 1 \right] = \frac{1}{F_m} \left[\frac{R_d}{r V_{in}} \left(1 + \frac{s}{w_{z2}} \right) - \frac{1}{G_{vd}} \right]$$

where

$$G_{vd} = \frac{V_{in} \left(1 + \frac{s}{w_z} \right)}{1 + \frac{s}{Q w_0} + \frac{s^2}{w_0^2}}$$

28

So, it has to be smaller than this and that is that means now G_c from this expression we need to find out what is G_c ok. So, $G_v d$ is known. Everything is known only G_c has to be found in terms of r ok.

So, now G_c is written like this and if you substitute $G_v d$, so that means this r that means this is our close loop output impedance that we want to obtain and if you find substitute this expression this expression into this one, then you will get a numerator polynomial and the denominator polynomial ok.

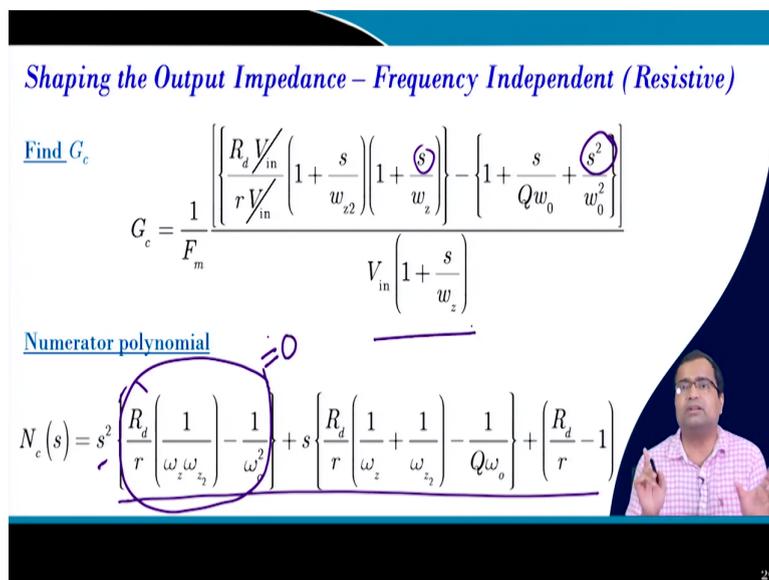
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Shaping the Output Impedance – Frequency Independent (Resistive)

Find G_c

$$G_c = \frac{1}{F_m} \frac{\left[\frac{R_d V_{in}}{r V_{in}} \left(1 + \frac{s}{w_{z2}} \right) \left(1 + \frac{s}{w_z} \right) \right] - \left[1 + \frac{s}{Q w_0} + \frac{s^2}{w_0^2} \right]}{V_{in} \left(1 + \frac{s}{w_z} \right)}$$

Numerator polynomial = 0

$$N_c(s) = s^2 \left[\frac{R_d}{r} \left(\frac{1}{\omega_z \omega_{z2}} \right) - \frac{1}{\omega_0^2} \right] + s \left[\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z2}} \right) - \frac{1}{Q \omega_0} \right] + \left(\frac{R_d}{r} - 1 \right)$$


Now, since we want a proper transfer function. So, s square term coefficient must be 0 whatever term square; that means, for numerator polynomial s square term, coefficient is related to this. That means, we can separate out the term coefficient associated with the s square is nothing but this term ok. So, we want to make this 0 because we want a proper transfer function because your denominator is a first order system.

(Refer Slide Time: 45:54)

Shaping the Output Impedance – Frequency Independent (Resistive)

Numerator polynomial

$$N_c(s) = s^2 \left[\frac{R_d}{r} \left(\frac{1}{\omega_z \omega_{z_2}} \right) - \frac{1}{\omega_o^2} \right] + s \left[\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} \right] + \left(\frac{R_d}{r} - 1 \right)$$

For proper TF G_c , the co-efficient associated with s^2 must be zero.

$$\frac{R_d}{r} \times \frac{1}{\omega_z \omega_{z_2}} - \frac{1}{\omega_o^2} = \frac{r_e}{r} \times \frac{r_c C L}{r_e} - \frac{L C}{1} = 0 \quad \rightarrow \quad r = r_c$$


30

And we need to set to must be 0 and if we do it, then it turns out to be you know by substituting this and this will get cancelled and you will get something like from this equation you will get r equal to r c. That means, I can set the closed loop output impedance look like a resistive ESR. It is the ESR.

(Refer Slide Time: 46:19)

Shaping the Output Impedance – Frequency Independent (Resistive)

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - \frac{(r_c + r_e)}{\sqrt{\frac{L}{C}}} \times \sqrt{L C}$$

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - (r_c + r_e) \times C$$

$$= \left(\frac{L}{r_e} - r_c C \right) > 0$$


31

(Refer Slide Time: 46:24)

Shaping the Output Impedance – Frequency Independent (Resistive)

Also $\frac{R_e}{r} - 1 = \frac{r_e}{r_c} - 1 > 0$

Since $r_e > r_c$ Simply a first-order lead compensator

$$G_c = K_c \times \frac{\left(1 + \frac{s}{\omega_{cz}}\right)}{\left(1 + \frac{s}{\omega_z}\right)}$$

$$G_c = \frac{1}{F_m V_{in}} \times \frac{\left[s \left(\frac{L}{r_c} - r_c C \right) + \left(\frac{r_e}{r_c} - 1 \right) \right]}{(1 + r_c C s)} = \frac{\left(\frac{r_e}{r_c} - 1 \right)}{F_m V_{in}} \times \frac{\left(1 + \frac{s}{\omega_{cz}} \right)}{\left(1 + \frac{s}{\omega_z} \right)}$$

So, now if you substitute ESR, then you can find out the expression, the numerator and denominator polynomial of the original expression.

Because we have this original controller transfer function. This is original expression. So, we have to substitute this r then we these things are known. So, then we substitution we will find out the controller transfer function, it can be shown that the time constant of the inductor will be larger than you know so yeah then the capacitor time constant, because ESR is much smaller. I am sorry. So, it should be r e by ok it should be L e yeah sorry, now here we are getting one expression which is r e by yeah, L by r e ok. So, yes I think we are getting a term r c common. It should be fine. It is like it is fine.

So, this term so we need to we can find out that this term should be greater than 0 ok.

(Refer Slide Time: 47:38)

Shaping the Output Impedance – Frequency Independent (Resistive)

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - \frac{(r_c + r_e)}{\sqrt{\frac{L}{C}}} \times \sqrt{LC}$$

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - (r_c + r_e) \times C$$

$$= \left(\frac{L}{r_c} - r_c C \right) > 0$$

0.5 / 3



31

(Refer Slide Time: 47:50)

Shaping the Output Impedance – Frequency Independent (Resistive)

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - \frac{(r_c + r_e)}{\sqrt{\frac{L}{C}}} \times \sqrt{LC}$$

$$\frac{R_d}{r} \left(\frac{1}{\omega_z} + \frac{1}{\omega_{z_2}} \right) - \frac{1}{Q\omega_o} = \frac{r_e}{r_c} \left(r_c C + \frac{L}{r_e} \right) - (r_c + r_e) \times C$$

$$= \left(\frac{L}{r_c} - r_c C \right) > 0$$

$r_c = 3 \text{ m}\Omega$
 $L = 0.5 \mu\text{H}$
 $\frac{L}{r_c} = \frac{0.5}{3} = \frac{1}{6} \times 10^{-3}$
 $3 \times 10^{-3} \times 200 \times 10^{-6}$
 $= 600 \times 10^{-9}$
 $=$



31

And because this 2 are in micro you know because what is L, L is our 0.5 micro divided by r c which is 3 which is 3 milli right. So, if we if you substitute r c equal to 3 milli ohm and L equal to 0.5 micro Henry. So, L by r c will be 0.5 by 3 that is 1 by 6 and it will have a dimension of 10 to the power 10 to the power I mean minus 3 right. I will check that and what will be 3 to 10 to the power minus 3 into we have taken 200 into 10 to the power minus 6.

So, we are getting 600 into 10 to the power minus 9; that means, if we write in terms of this 1. So, this term; that means, this is positive ok so this is correct. So, if you now substitute the R_d we have taken approximately equal to r_e , then since r_e equal greater than r_c because the summation of DC r and the $R_d s$ 1 should be larger than ESR, because you have to take the ESR small. Then the transfer function of the controller looks like this.

So, this resembles a first order lead compensator, where it can be shown that the 0 will be left side and the pole will be right side.

(Refer Slide Time: 49:21)

```

34 Q_max=alpha*((r_c+r_e*exp(z_c)));
35 den_c_max=1/(w_c^2)*1/(Q_max*w_o);
36
37
38 %% Type - III compensator
39 f_c=input('Select BW in kHz');
40 %f_c=50; PM=60;
41 %w_c=2*pi*f_c*1e3;
42 %PM=input('Select phase margin in degree');
43 %theta=deg2rad(90-PM);
44 %k_x=tan(theta); w_cp=w_c/k_x;
45 %K_c=((alpha*w_c)/(Fm*Vin))*(sqrt(1+(k_x^2)));
46 %num_con=K_c*den_c;
47 %den_con=1/(w_c*w_cp)*(1/w_c+1/w_cp);
48 %Gc=tf(num_con,den_con);
49
50 %%
51 %%
52 %%
53 %%
54 %%
55 %%
56 %%
57 %K_c=1*((r_e*exp(z_c)-1)/(Fm*Vin);

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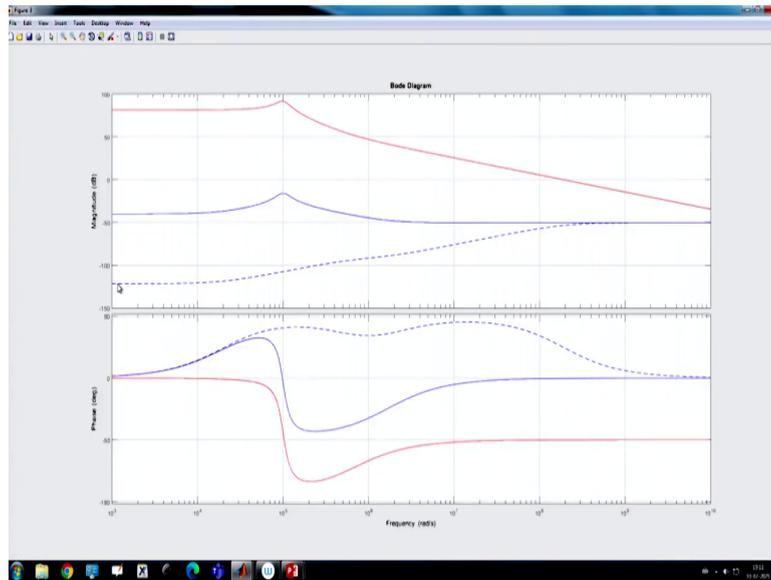
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```

38 %% Type - III compensator
39 % f_c=input('Select BW in kHz');
40 % %f_c=50; PM=60;
41 % %w_c=2*pi*f_c*1e3;
42 % %PM=input('Select phase margin in degree');
43 % %theta=deg2rad(90-PM);
44 % %k_x=tan(theta); w_cp=w_c/k_x;
45 % %K_c=((alpha*w_c)/(Fm*Vin))*(sqrt(1+(k_x^2)));
46 % %num_con=K_c*den_c;
47 % %den_con=1/(w_c*w_cp)*(1/w_c+1/w_cp);
48 % %Gc=tf(num_con,den_con);
49
50 %%
51 %% Lead compensator
52 % %K_c=10000; w_cz=(2*pi*f_c*w)/5;
53 % %num_con=K_c*(1/w_cz);
54 % %den_con=1/(w_cz);
55 % %Gc=tf(num_con,den_con);
56
57 %% Lead compensator - analytical
58 % %K_c=1*((r_e*exp(z_c)-1)/(Fm*Vin);
59 % %tau_L=L/r; C_tau=C*r;
60 % %w_cz=((r_e*exp(z_c)-1)/(tau_L*tau_C));
61 % %num_con=K_c*(1/w_cz);

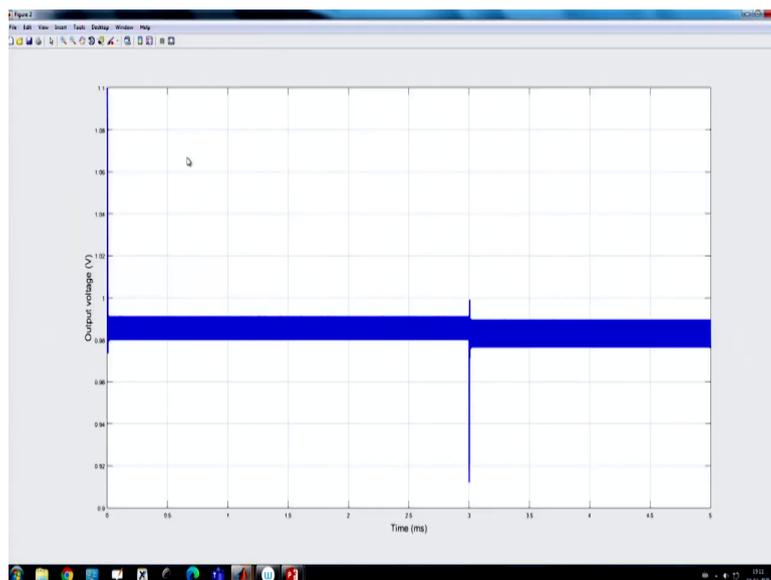
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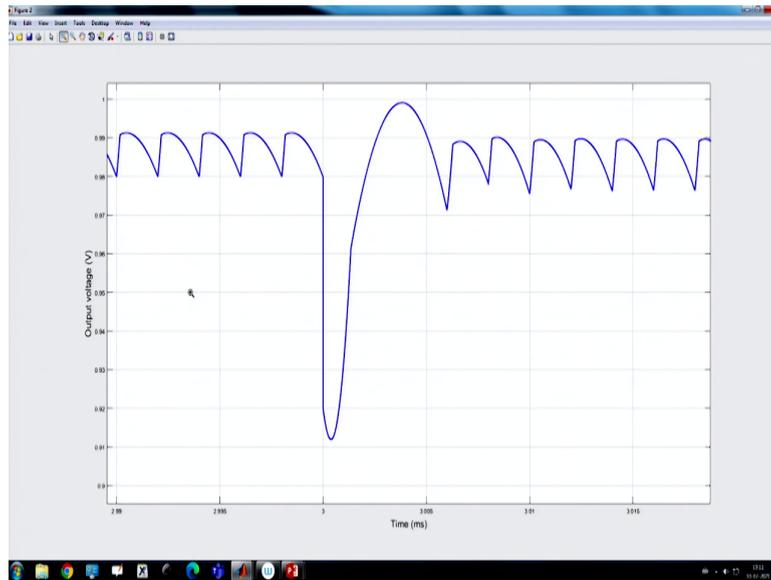


So, now we want to consider a lag-lead compensator then our earlier type 3 compensator. So, we will comment on this one and now call a lead compensator ok, uncommon and let us run it. So, we have run and now, this is the if you see the close loop output impedance it has no peaking effect, it is very much reduced here and going here ok and if you see the response of the converter.

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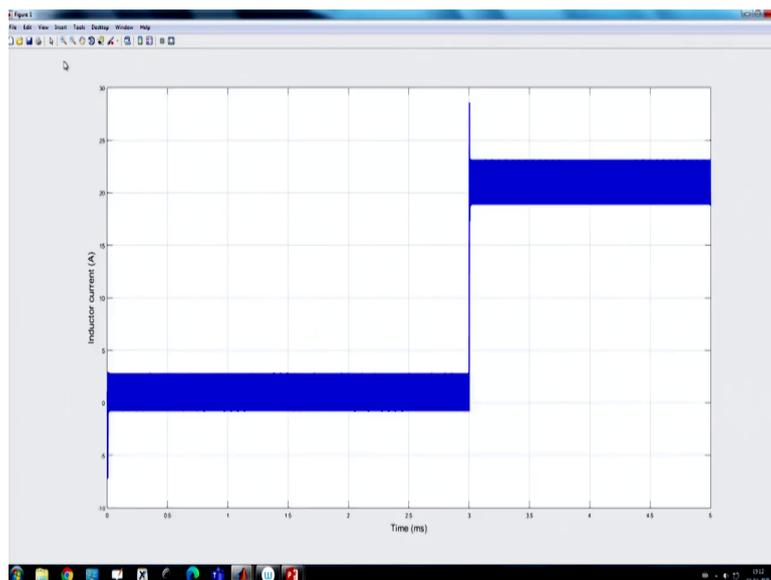


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So, the response will transient performance load transient performance, it looks like extremely fast load transient performance very, very fast and it responds but there will be slight steady state error because we have not used any integral action. Because we have used a resistive droop naturally, the resistive droop will slightly vary the operating point because this is a droop control.

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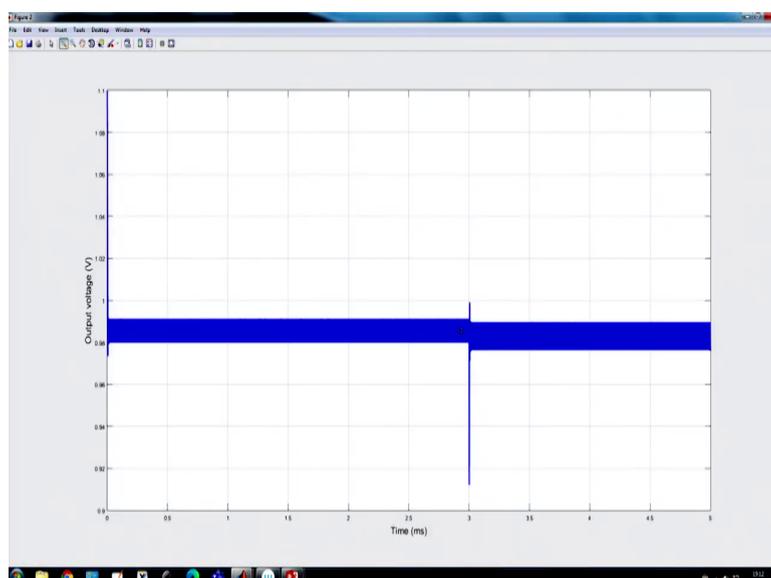
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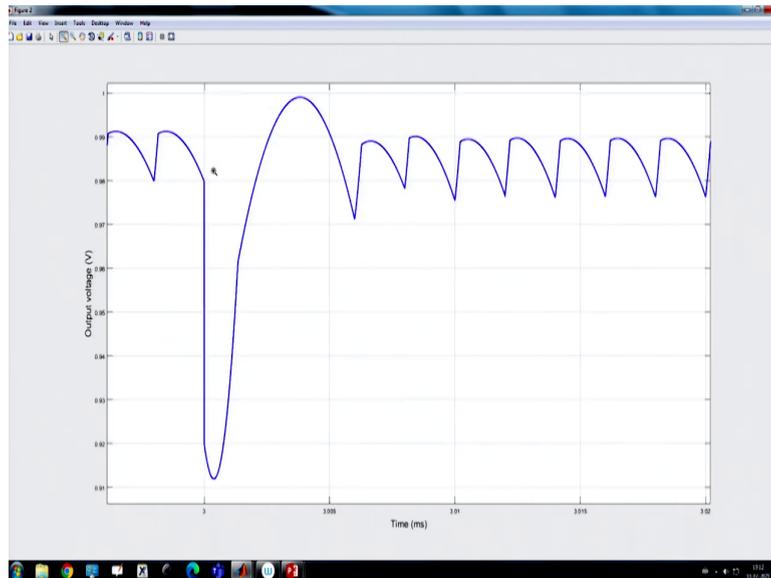
And that we have discussed you know earlier lecture in droop control. So, it is like a but it is a voltage mode control. And if you see the inductor current waveform, so the inductor current waveform looks like you know extremely fast, very fast transient response.

So, which means the lag-lead compensator will be a very you know suitable choice for load transient point of view, but you may not get zero steady state error. But by increasing the gain you can reduce the steady state error, but it can almost behave like an immediate response to the transient and that we got from the response.

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Like it almost responds immediately. I mean almost very small transient maybe one cycle or so, yeah because of this finite slew rate of the inductor and capacitor ok.

(Refer Slide Time: 51:05)

Summary

- Type-III compensator design in a VMC buck converter
- Robust compensation within admissible control bandwidth
- Design method for shaping output impedance
- MATLAB case studies for design of a VMC buck converter



So, with this I summarize type 3 compensator design was discussed. Then robust compensation within the admissible control bandwidth was discussed, design method for shaping output impedance was discussed and then mat lab case study for design of a voltage mode buck converter was discussed in detail on different design we have considered a mat lab case study. So, with this I want to finish here.

Thank you very much.