Control and Tuning Methods in Switched Mode Power Converters
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Module - 06
Small-signal Performance Analysis
Lecture - 30
Small-Signal Model Validation Using MATLAB and Time Domain Correlation

Welcome this is lecture number 30. In this lecture, we are going to talk about Small-Signal Model Validation Using MATLAB and Time Domain Correlation.

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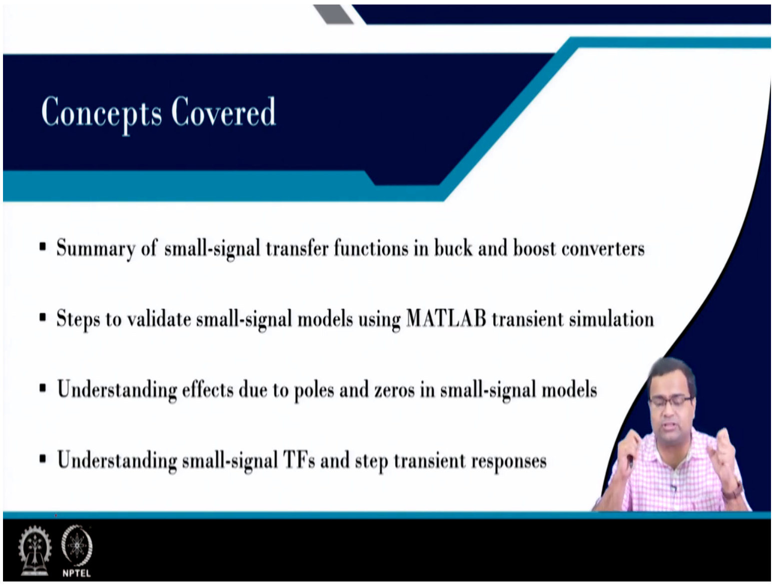

In today's lecture, we are going to talk about first we want to summarize various small-signal transfer functions in buck and boost converters. Then I want to show you step-by-step procedure how to validate ACs small-signal model using MATLAB transient simulation.

Suppose you have linear transfer functions that you have derived in the previous class and we want to see how accurate those models right, we want to verify and here we want to use a transient simulation, ok. Then understanding effect due to poles and zero in small-signal model and finally, understanding small-signal transfer function I am state step transient response.

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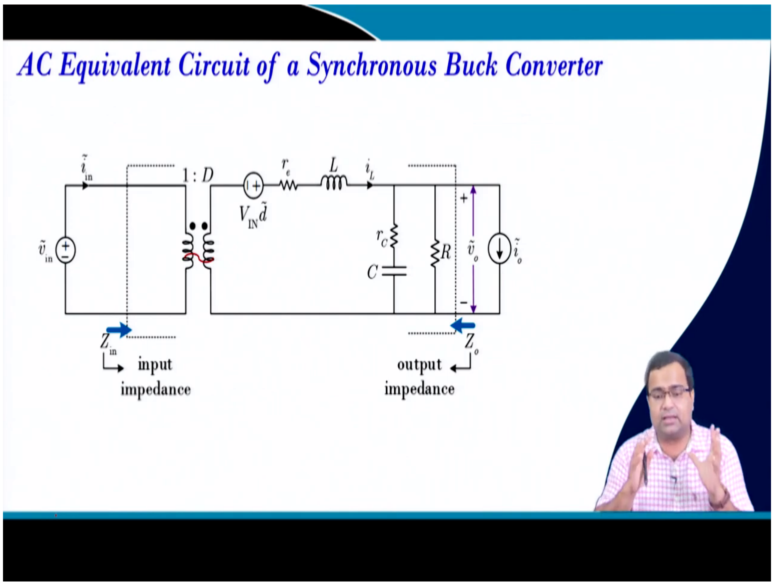

So, let us you know if we recall in our previous you know maybe past two or three lectures we have derived the AC equivalent circuit right, AC equivalent circuit of a practical buck converter where in this circuit we can derive various transfer functions. We can derive input impedance. We can derive output impedance. We can derive audio susceptibility. We can derive control to output transfer function.

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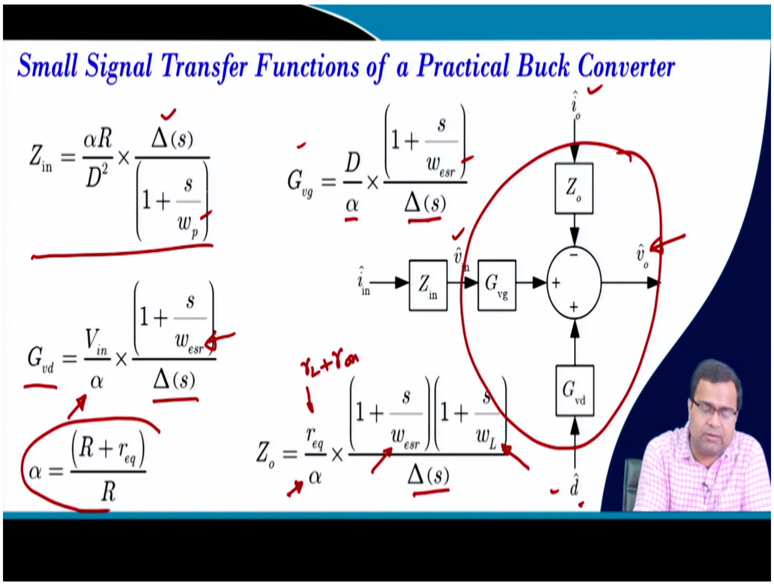

So, this is the small-signal block diagram and here we are interested in this particular terms right that means we have a supplied perturbation. You can have load perturbation and the duty ratio perturbation in case of open loop is an intentional perturbation; that means, we have not closed the loop. So, in that case, it is also a perturbation right.

And how this all perturbation or individual perturbation can inference or affect the output how it is going to change the behaviour of the output voltage ok that we are going to see. We have derived input impedance like this ok and we will see delta s which is a polynomial; it is a second order polynomial.

We have derive audio susceptibility. Again, this polynomial is common and we will see what are the alpha, what is esr, what is omega p. We have also derived control to output transfer function. Again, this polynomial is common. We will talk about alpha soon. We will also talk about ESR soon.

We have also derived output impedance. Here alpha we will talk about, r equivalent is a r L plus r on of the switch and we will also discuss what is a esr 0 and this is another 0 coming into picture and this denominator which is a pole second order pole. And this alpha is nothing but r plus r equivalent by R ok, where R is a load resistance.

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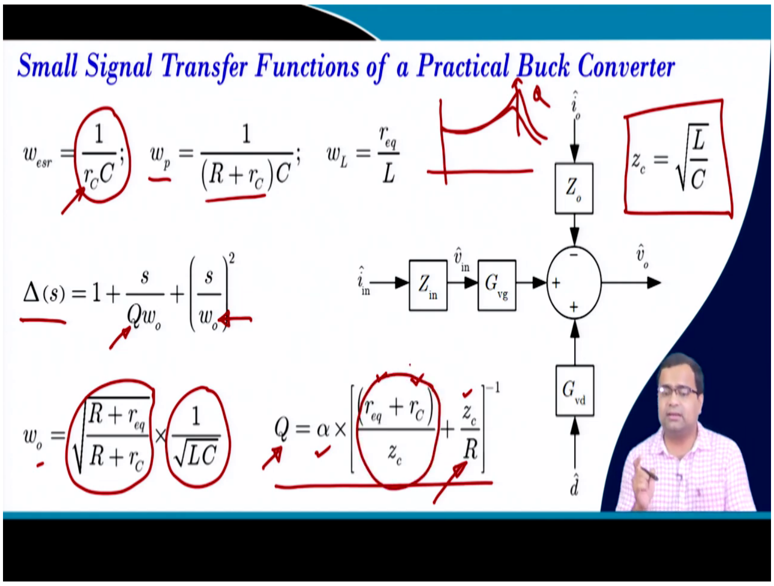

Now, esr zero is very simple. It is 1 by r c into C, where r c is the esr ok. Omega p is 1 by R plus r c into C ok and then omega L r equivalent by L. These are all well known. The polynomial which is the pole is a second order polynomial, which consist of the Q factor and the natural frequency omega 0. And what is omega 0?.

Omega 0 for ideal buck converter is 1 by square root of LC. For a practical buck converter it should be scaled by this term and this term will be roughly equal to 1, if r equivalent and r c are close to each other ok, but more or less this omega 0 is mainly 1 by square root of LC.

Then the Q factor, Q factor is this alpha already we have discussed, r equivalent we discuss, r c is the esr, z c is the characteristic impedance which is nothing but square root of L by C ok. And this is the Q factor and this Q factor is the one which causes the change in the peaking; that means the phase margin. If the Q factor is high; that means, the peaking will be high right you know.

Suppose if we draw the gain plot in one case, it will be like this. If you go further, increase it will go like this ok. So, this high Q factor actually if the Q factor increases. And when we want to evaluate which will who decide the Q factor. So, this is primarily the load resistance ok. In case of a constant current load, if there is no resistive load, then Q factor are dependent are decided by only the parasitic and the characteristic impedance ok.

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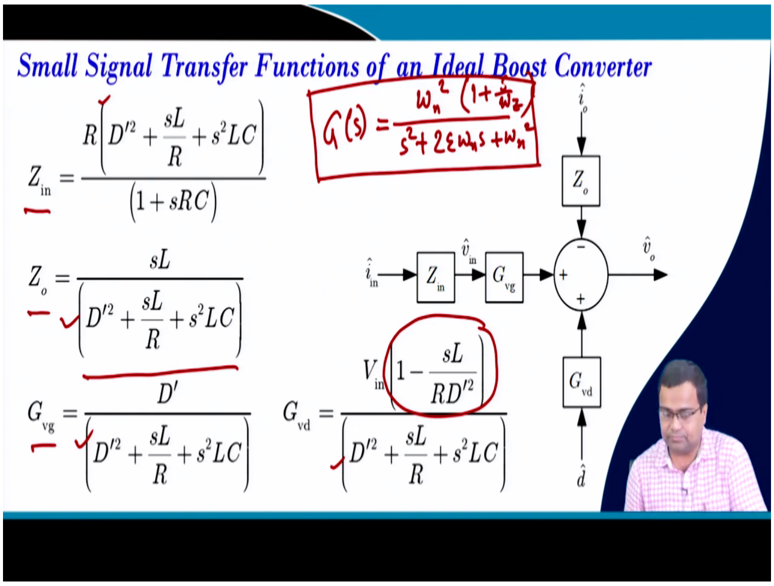

So, in this transfer function, if you take a boost converter again, we can derive the input impedance. We can derive the output impedance of a book boost converter. It is an ideal boost converter. Again, it has a two pole second order system. Then, if you take the audio susceptibility, we can derive and these polynomials are common and control to output transfer. This also has two pole.

But, in addition there is a right half plane zero ok. So, right half plane zero. That means, that is one of the major problem in a boost converter. The right half plane zero will and in this presentation we are going to take we are going to talk about if a transfer function has a right half plane zero or a s zero, then how is it going to affect the response of the system.

Because in our second order you know undergraduate control system course we have studied like you know a normal transfer function starting point was omega n square s square plus 2 zeta omega n s plus omega n square. So, this transfer function we have studied and you can write this transfer function even earlier transfer function as well. So, the in this transfer function our zeta is the damping ratio ok, and it is linked with the Q factor in the earlier in a buck converter we will talk about Q factor right.

Now, if we obtain the step response, you know we have studied in enough detail. Maybe if we recall that the response will be under damp if the zeta is less than 1. But of course it should be greater than 0 and it will be over damp. It is greater than 1 and it is critically damped, and zeta equal to 1. So, we can get the transient response for a step input.

Now, if we add another 0 with this transfer function, suppose we had a 0 1 by s omega z then how is it going to change the response of the system that we want to see.

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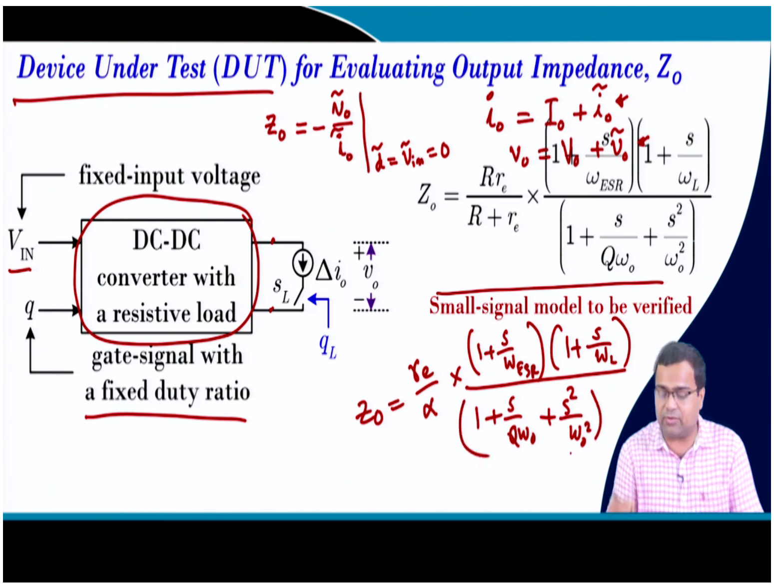

Now, before we move forward because we want to validate our model. Whatever we have derived, whether those models are accurate or not first we have to check and that is why we are talking about the device under test. In fact, this test you can also carry in the experimental prototype also.

So, how to check output impedance? So, we have to take a DC-DC converter ok. And we know in the derivation of output impedance what we have considered? It is the perturbation of the output voltage when we perturb an external current source. Provided that for input open loop output impedance, the duty ratio perturbation input voltage perturbation should be set to 0; that means, we should use a fixed input voltage and we should use a fixed duty ratio gate pass.

Then in this converter; that means, if the power supply. In fact, you can you know in lecture number 13, we talked about the output impedance of a practical voltage source right. So, if you take a power supply from the laboratory and you want to check or measure the frequency or basically the transient response, or you want to measure the frequency response of the output impedance. What you can do?.

So, input voltage; that means, for the power supply here because only the terminal voltage this terminal voltage will be terminal points will be available. So, we can connect an external sinking load and we can slowly change the load either by a sinusoidal excitation and we can do a frequency sweeping and then measure what is the output voltage peak-to-peak AC-AC and the sinusoidal.

That means here in the load current if we take load current, it will have a constant load current and on top of that you will have an excitation perturbation load current, which will be a sinusoidal excitation. And similarly output voltage will have two components; a constant output voltage and an excitation.

So, due to this excitation of this current, you will get an excitation in the output voltage. And by measuring peak-to-peak output voltage and measuring the peak to peak of the current excitation, then you can get the gain and phase plot you know and you should also measure the peak to peak you know the time ship; that means, to measure the phase ship.

So, here we are talking about output impedance and we have already discussed the output impedance expression. So, what is the output impedance? We know that z 0 in more compact form alpha when there are two 0; 1 by S omega ESR. And there is one more 0 S by omega L and the whole thing divided by 1 plus S by Q omega 0 plus S square by omega 0 square. This thing we have already discussed.

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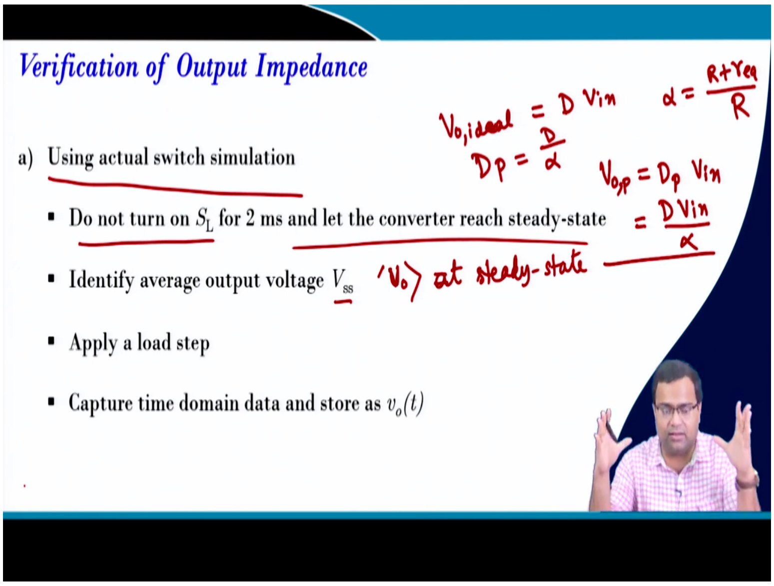

Now, you want to verify output impedance. So, in order to verify, we want to verify with whom because we derive the small-signal model from actual switching converter. So, the response of the small-signal model must be matched with the response of the actual switching converter. So; that means, using actual switch simulation.

So, we should not turn on the load; that means, there is a load switch ok. So, there is a load switch, this is a load switch right. So, you should not turn on for 2 millisecond let the converter reach steady state because if you turn on the converter even a practical converter or a simulated.

So, initially there will initial point for a practical converter will it is totally discharge and in the simulation you can set some customs in initial condition but you need to provide some 2 millisecond time. So, that the output slowly reaches to steady state. After 2 millisecond, then at the end of 2 millisecond we need to identify what is the steady state voltage; that means, what is the average voltage. So, this is the average voltage. I will say the average voltage at steady state. This is the average voltage at steady state.

Now, the question is whether we measure this in case of open loop because it is a practical converter. Because we have discussed that practical output voltage in case of ideal, we know that output voltage for an ideal converter will be simply D into V in. But in case of a practical converter, our D practical is nothing but what?.

It is nothing but D by alpha. What is alpha? Alpha is R plus r equivalent to R right. So, the practical duty ratio is smaller than the actual duty ratio that we have discussed in the dc equivalent circuit. So; that means, our V 0 practical should be D practical into V in which is nothing but D into V in by alpha; that means, we can find out without searching that ok. So, you can find out.

So, now, we will apply a load step of transient in 2 millisecond, ok. Then we need to capture the time domain data because this has the full data of the output voltage and we have to store at a v 0 t.

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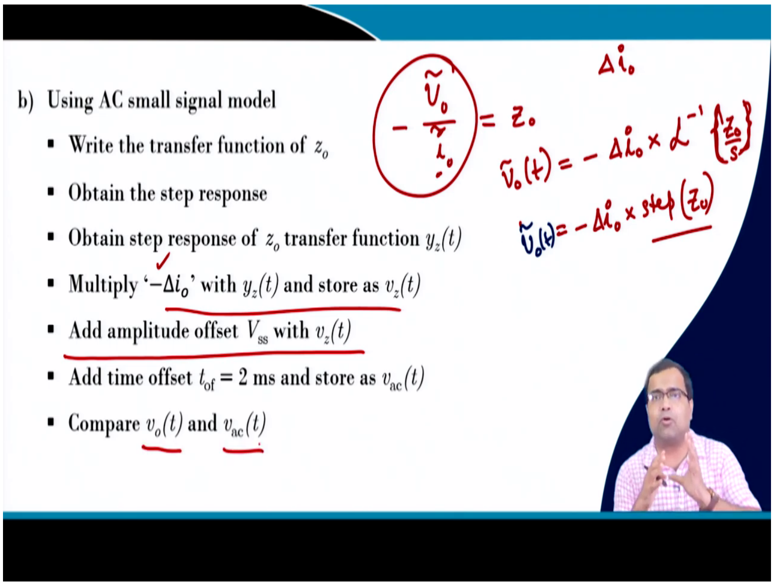

Next, because we want to verify the actual switch simulation with our AC small-signal model. So, we have to first write down the transfer function we need to obtain the step response and generally step response. If you write in MATLAB it will give a unit step response. So, you need to multiply scale.

So, you have to obtain the step response for output of the output impedance. Then have to multiply minus delta i 0 because we have discussed that delta v 0 by delta i 0 minus is our output impedance right so; that means, if we want to write delta v 0 t it will be suppose we apply a load step of delta i 0 this term ok. So, then it should be minus delta i 0 which is the step size of Laplace inverse of Z 0 t; so, Laplace inverse ok.

That means whatever time we will get by inverse Laplace if you take. And this Laplace inverse you know because we need to do the step response, but if I want to do Laplace, we need to put s because this step if you take the step response the Laplace transform it will be delta i 0 by s.

And in MATLAB we will see that it is nothing but minus delta i 0 in two step response of Z 0 because it is the unit step response; that means, as if we apply unit load state multiply. So, because this will have because of this negative sign this negative is coming ok.

Next after getting this, it is just the AC equivalent circuit. So, it will give the AC excitation right, it does not have any DC. So, we need to add amplitude offset with the step response that we obtain from an from AC simulation right. Then we also need to add offset time and we will slowly come to this point and then we want to compare actual switch simulation and the result obtained from small-signal model.

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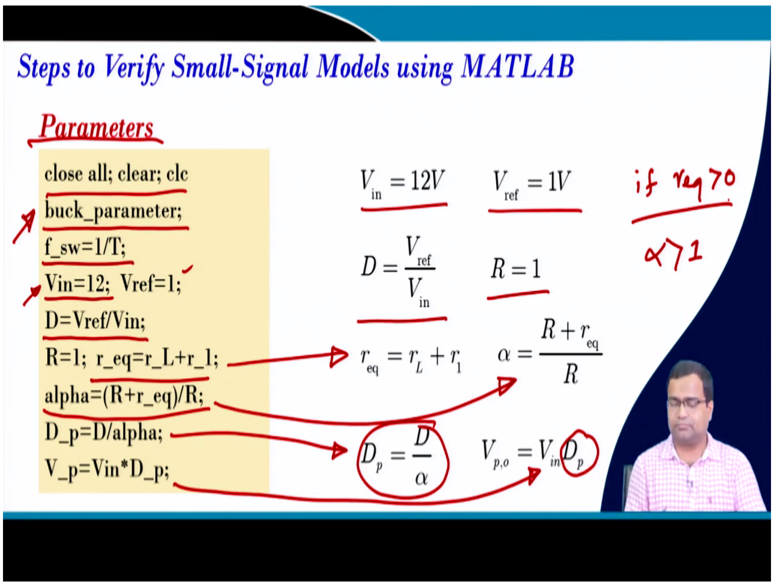

So, let us go how in MATLAB how can we verify small-signal model step by step using transient MATLAB transient simulation. So, first you know I will go to the code. Before I go to the code, I am taking the MATLAB code. I have broken into multiple like there are multiple line, but I have broken into multiple pieces. So, here we are setting the parameter initially. We are clearing up the screen everything closing all the plot ok.

Now we are calling buck parameter. So, this parameter will load all the parameter like esr, you know input voltage output voltage, inductor output capacitor everything will be stored time period and switching frequency is 1 by T. Now, this input voltage is we are setting 12 volt although we have to define here, but if we want to customize with a different input voltage.

You can call redefine here and reference voltage 1 volt. This is our desired value, and it is we are going to check the open loop simulation. So, it is expected the output voltage will not be desired one in case of a practical converter. So, the ideal duty ratio will be D equal to V ref by V in.

So, if we check 12 volt is the input, 1 volt is the output and our ideal duty ratio is D into V ref by V in D equal to V ref by V in and R equal to 1. Then this r equivalent you can see this r equivalent is here. It is nothing but the sum of DC R plus R D s on. And this alpha where it is written here it is here, this alpha here ok.

Next, what is the practical duty ratio? The practical duty ratio is D by alpha and the practical steady state voltage will be because idea of steady state voltage should be D in to V in, but since there is parasitic drop due to the DC R of the inductor and the r D s 1 of the switch, so, the practical output voltage will be V in into practical duty ratio which is here.

And here alpha is greater than 1, if r equivalent is greater than 0 and that is true if we in case of ideal r equivalent is set equal to 0 in that case alpha equal to 1, but in practical alpha greater than 1.

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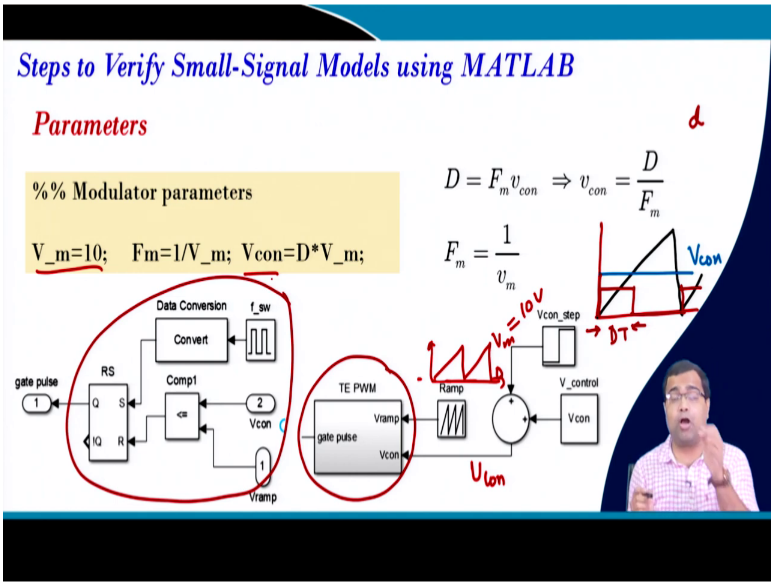

The next because here, what we want to show here the modulator. That means, what is modulator? Even though you are using an open loop, but in order to generate duty ratio we are using a control voltage and which is compared with the sawtooth waveform. That means this is my control voltage. This is my control voltage which is here and it is compared with this is my sawtooth waveform right.

This is my sawtooth waveform and sawtooth waveform has the value this is my V m and this is 0 and this is my V m is set to 10 volt ok. So, in this case i set V m equal to 10 volt. Now, this control voltage, once you compare with a sawtooth; that means, if I draw here and if I use a different colour. So, this is my sawtooth wave form ok and if I use my control voltage here, this is my V con then this will generate my duty ratio.

That means, this will be my duty ratio will be here. So, this will be my D into T. You can say instantiate duty ratio then it will be d if it is steady state duty ratio, it will be D d ok. So, this is implemented. The logic inside the trailing edge PWM is implemented here ok. So, this is the block which is implemented.

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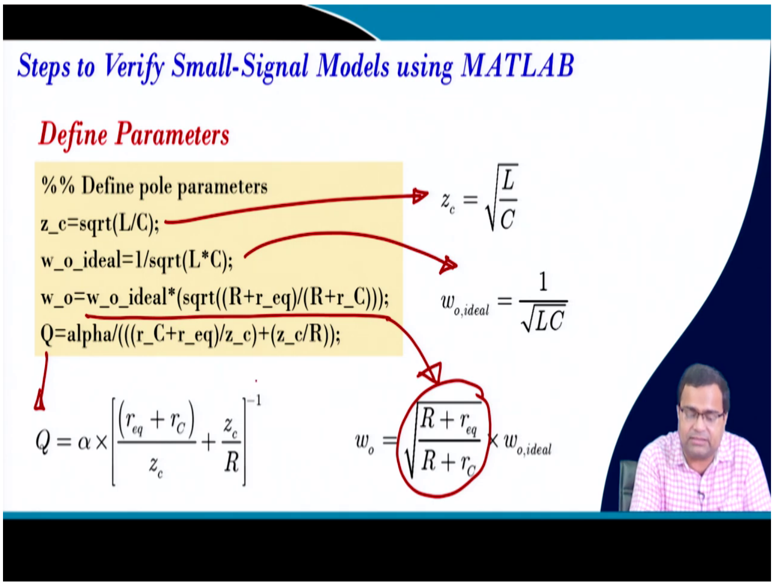

Next steps to verify small-signal the same thing were going on. Define the pole parameter because we have studied that what is our z c is the square root of L by C. It is here. Then we have studied the ideal one. If it is an ideal converter the natural frequency is 1 by square root of L C, but in case of practical we need to multiply by this factor that also we have discussed.

So, this whole line is here. This line is here and finally, you have to obtain the Q and which is nothing but here ok. So, Q. Now, so, it is alpha 1 divided by; that means, there is a minus 1 sign ok.

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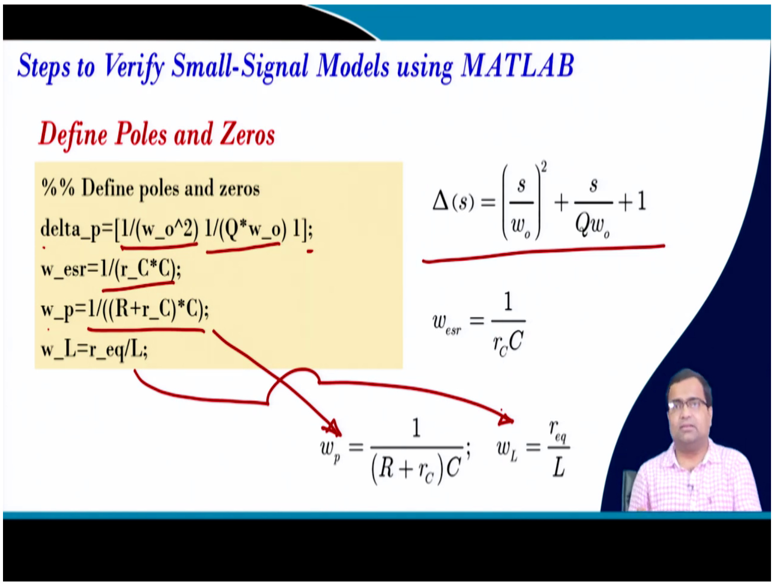

Next, define poles and zeros. So, we have already talked delta p which is delta p which is nothing but you know s square by omega 0 square. So, it is 1 by omega 0 square then 1 by Q into omega 0 and 1. Then esr 1 by r c. So, it is simply set here esr ok. So, esr 0 is just a constant. I mean you do not have to bother about this esr 0. And then pole is 1 by this pole; it is here and the other 0 which is here it is here ok r equivalent by L ok.

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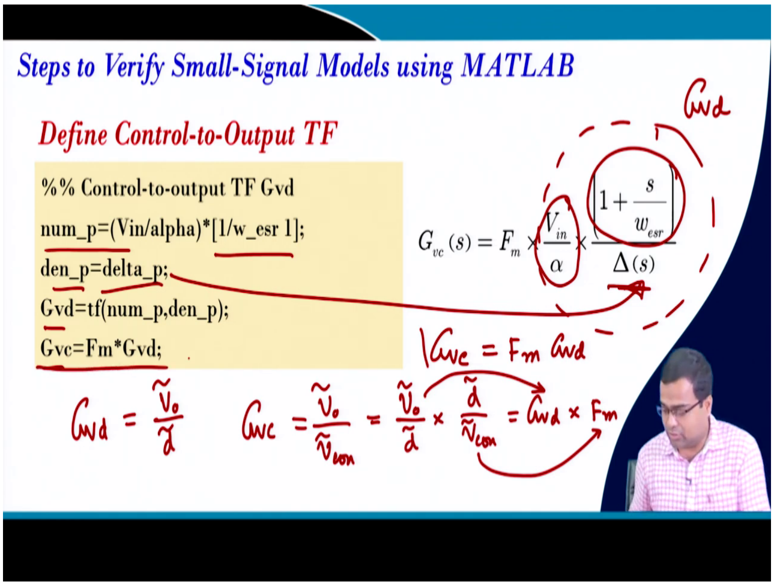

Now, next we want to verify. Suppose if you want to write the control to output transfer function. So, numerator is common right. So, sorry what is numerator in the so, numerator is what? It is V in by alpha in this esr 0. So, this is the 0. And what is the denominator?.

Delta p s, it is there in the denominator delta. So, this delta is nothing but this. Then G vd is this transfer function is my G vd, this transfer function is my G vd. And what is my G vs G vc? That is the controlled to output transfer function. So, this is my F m into G vd ok.

Now, remember what is G vd? G vd is nothing but V 0 perturbation by d perturbation and what is G vc is nothing but V 0 perturbation by V controlled voltage perturbation and this is equal to V 0 by you know d perturbation into d perturbation by V con perturbation and this is nothing but G vd in to F m ok. So, this term is F m, and this term is our G vd. So, this is written here ok.

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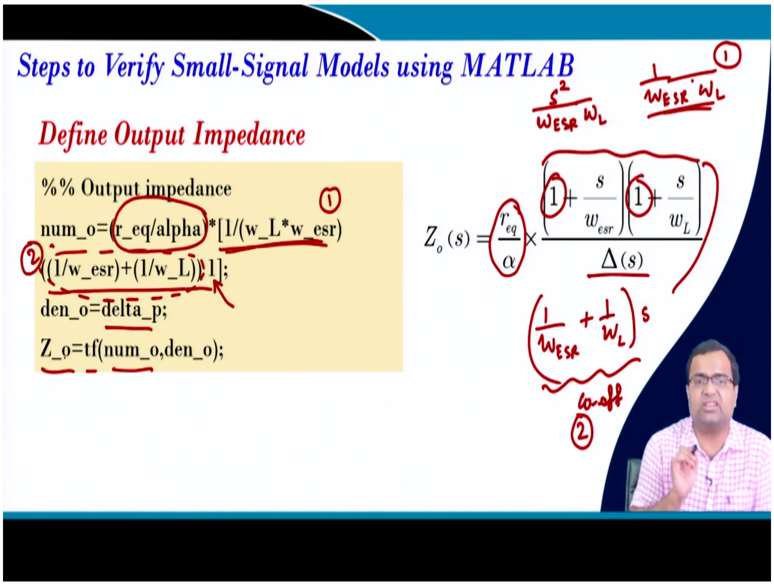

Next, define output impedance. What is the output impedance expression? It is r equivalent by alpha which is written here. Then it has it is a continuation this line is the continuation. So, the first thing if you multiply this quantity, what will get? If you write down this, it will be S square by omega ESR into omega L; that means, the coefficient will be 1 by omega ESR into omega L and this coefficient is the 1st coefficient. So, this is 1st coefficient.

Then what is the second coefficient? The second coefficient will be here. If you take the second coefficient 1 by omega ESR plus 1 by omega L the polynomial will be S and this is our coefficient, the second coefficient and this second coefficient is this one. So, this one is our second coefficient.

And third coefficient is 1 because if you multiply these two terms, it will be 1 then denominator is delta p which is nothing but this. And the transfer function is in the transformation between numerator and denominator. So, there is output impedance.

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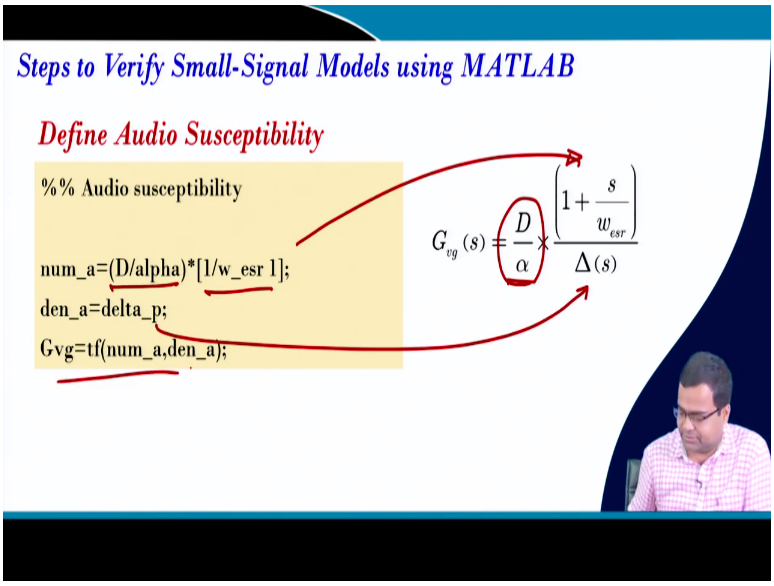

Then you can also write audio susceptibility. This term here, it is here, then esr 0 it is here, then delta p it is here and then transfer function.

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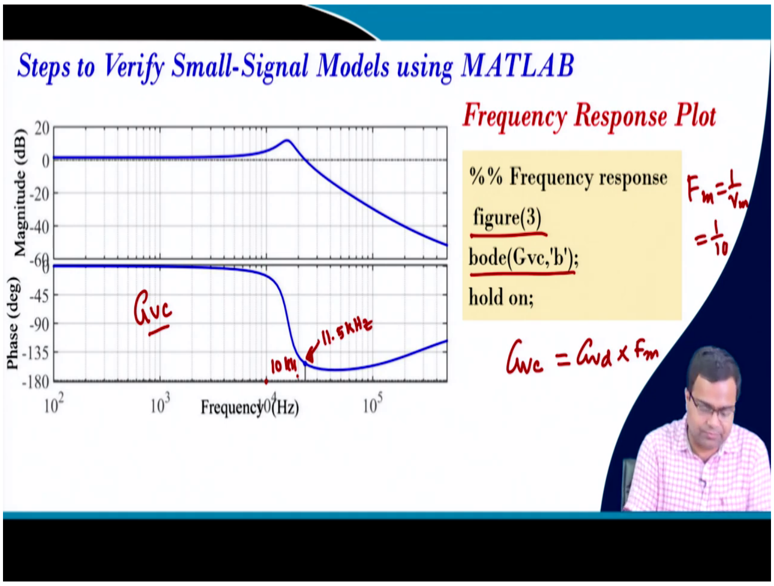

Now, if we want to plot frequency response; that means we are putting some figure number because there are many figures. First, we want to plot Bode plot of control to output transfer function which is G vc. So, G vc please remember, it is G vd into F m. Since F m we have taken what is F m? We have taken 1 by V m and this is 1 by 10.

So, this will reduce the gain. As a result, you can see the DC gain of this transfer function, the low frequency gain is very low. It is almost close to 0 Db. this is a control to output transfer function which is G vc ok, and at this point if you see what is this point?.

This is our 10 kHz and here it will be roughly 11 and around 11.5 kilohertz. So, it will be around 11.5 kilohertz ok; sorry; 10 10 per 20. So, this will be sorry it is not ah. So, it will be 20 that means we can take 20.

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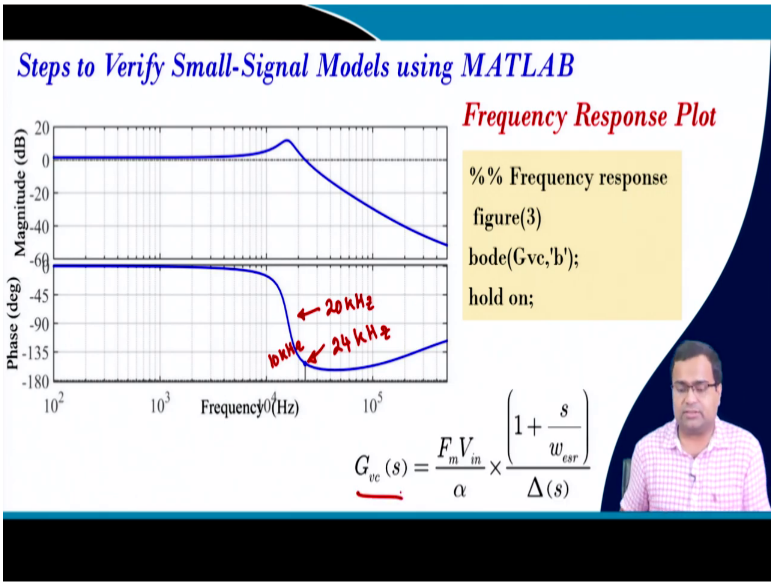

So, this is our 10 kilohertz, this point is our 20 kHz and this point will be roughly like a 24 kilohertz or so, and this is a transfer function between this ok.

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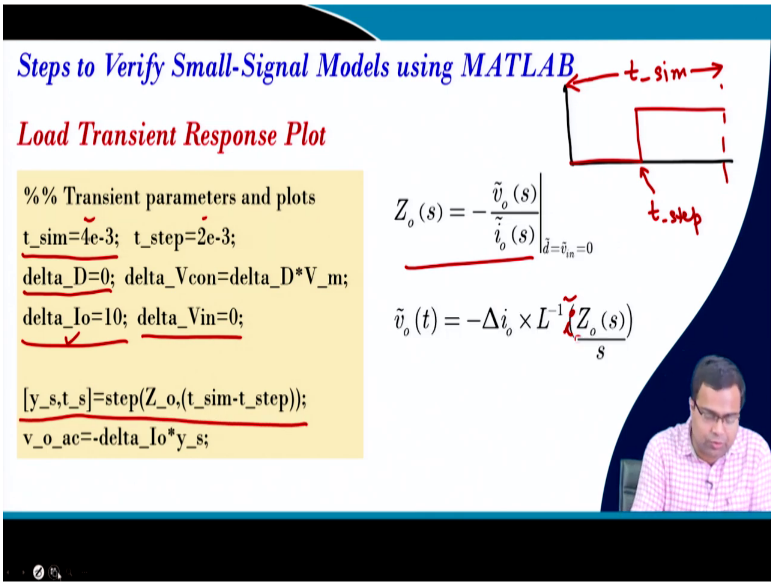

Next, we need to check now load transient response, because we need to verify the load transient response. So, here is a total simulation time ok; that means, if I want to simulate let us say if we want to simulate I will apply the load step at this time. So, this is my t step time, and this is my total simulation time, which I am calling as a t sim, total simulation time. So, total simulation 4 millisecond and applying step at 2 millisecond, ok.

Now you can take any kind of transient. Suppose you want to take a perturbation the duty ratio or you can take the perturbation in the load current; that means, step change in the load current or step change in the input voltage right. So, whatever you do? Since we are changing the load current, and we want to check the load transient response. So, it is the output impedance we decide because if you recall the load current, this is the output impedance expression.

And in the small single transfer function if you operate under fix duty ratio and fix input voltage the output voltage will be affected if you excite from an external current source ok and that will be that can be model using output impedance, the effect due to can be model using output impedance.

So, now, here it shows the step response of the output impedance; that means this will show the step response of the output impedance. So, this will show the step response of the output impedance, Laplace transform, then we have to multiply with the step size ok.

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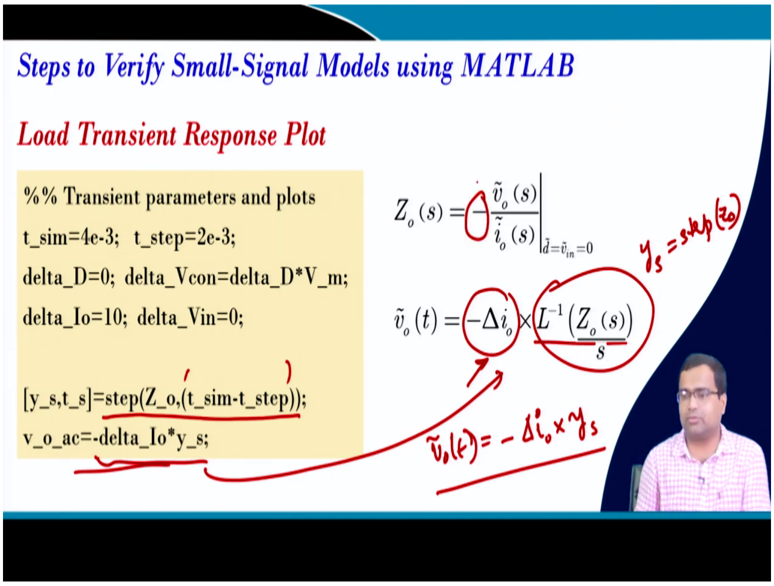

And this is here the step response and we are extending the time from you know we have drawn; that means, from the step applied till the final time. And since it has a negative sign right, you see this is this line is exactly here. So, as it this we have obtained using MATLAB which is y of s which is obtained like a step of z 0 in MATLAB.

So; that means, our v 0 perturbation it is the AC excitation is minus delta i 0 into y of s. So, this is here ok. If we apply a positive duty ratio step, this quantity need not to be positive because it is coming positive due to the symbol ok.

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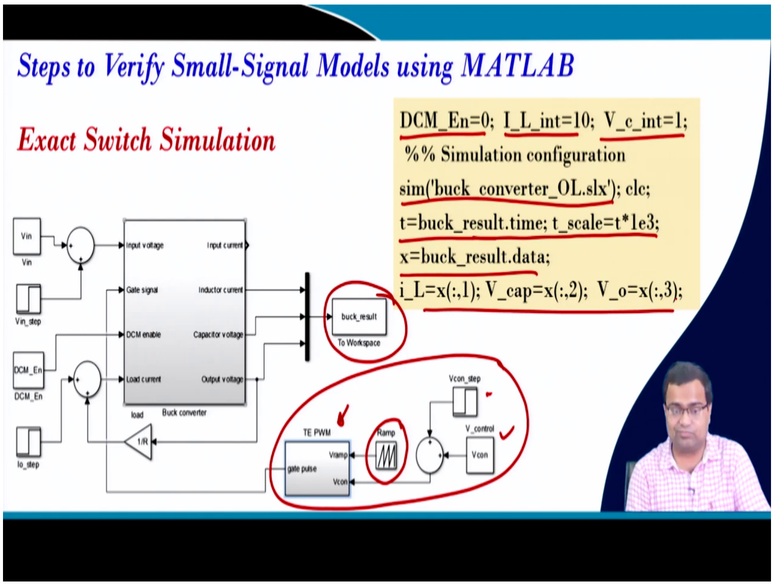

Similarly, now we want to verify with exact switch simulation. So, this is our block. Here we are using a modulator here. This is our control voltage, which you can perturb and this is your ramp signal and if you have already seen the inside picture. Now, here we are setting the DC m enable 0; that means, you are setting synchronous DC-DC converter because we have derive the model for synchronous DC-DC converter.

You can take any arbitrary initial condition of the inductor and the capacitor voltage. Then we are calling the simulation file from the dot m file. We are running the simuling file name as buck convertor underscore open loop. Then we are storing the result because this result is stored in the workspace. The time scaling we are storing this data all these things were explained at the very beginning, ok.

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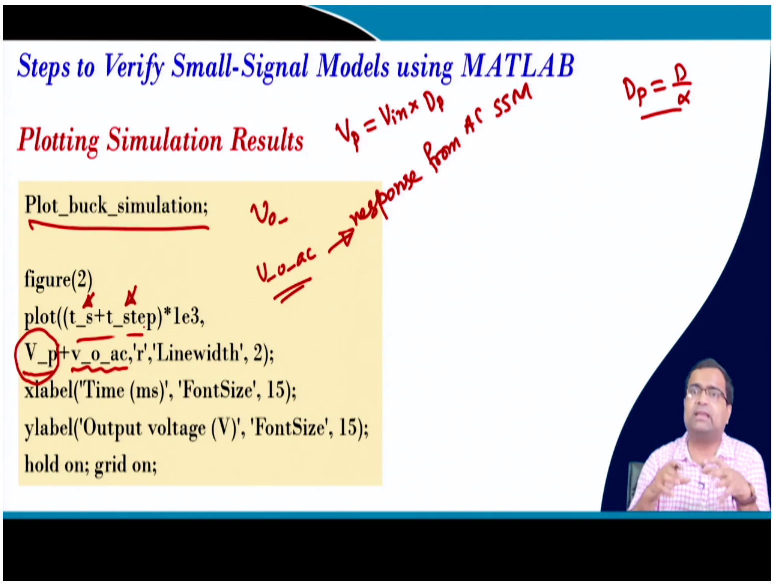

Now, we want to verify and then plot. After you finish this simulation; that means, these data are stored right. Now, we can use any of this data with the scaling time to plot it. So, how to plot? So, you want to plot by using this is our a plot common. I will explain what is this. But this response we obtain this AC simulation. You see this v 0; that means v 0. Sorry, here it is v underscore 0 underscore AC. So, this is the response; this is the response from AC's small-signal model.

We have to add this response with the DC, so, the to get the complete picture. And what is the DC? Because it is under open loop. So, the DC operating point before the step is applied will be V p which is a practical voltage. And what is V p? It is V in into practical duty ratio. And what is the practical duty ratio? This is nothing but D by alpha that we have discussed, ok add it.

And you also need to add offset time. This is the step time, and this simulation was from zero. So, you have to add. So, this is the offset time; that means our AC model response will start from the step time before that there will no response because you are only it will only give the excitation when there is a step input right.

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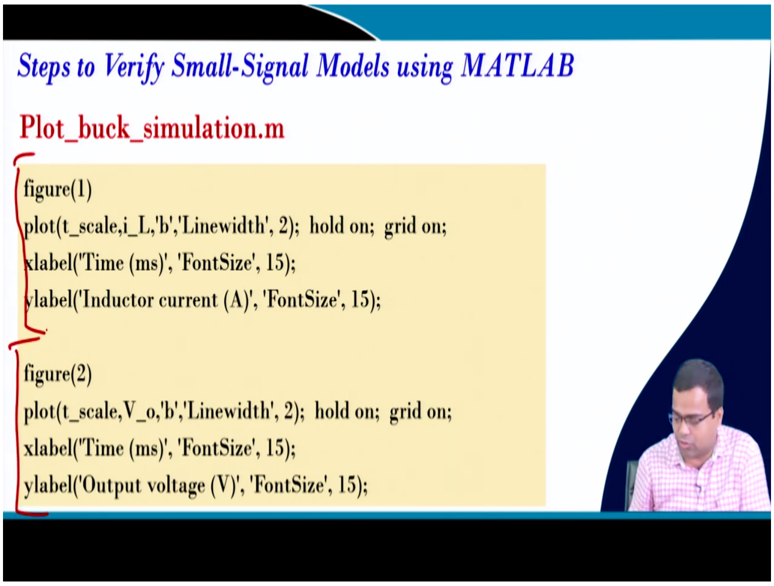

So, now simulation file from the actual switch simulation whatever we have stored. In figure 1, we are plotting inductor current and figure 2 we are potting actual output voltage.

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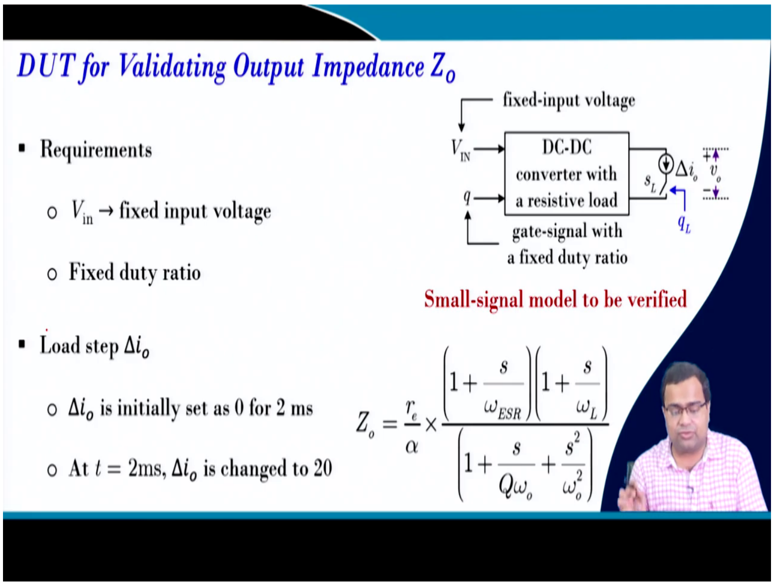

So, now we want to check this device under test. So, fix duty ratio and actual. So, let us go to the simulation.

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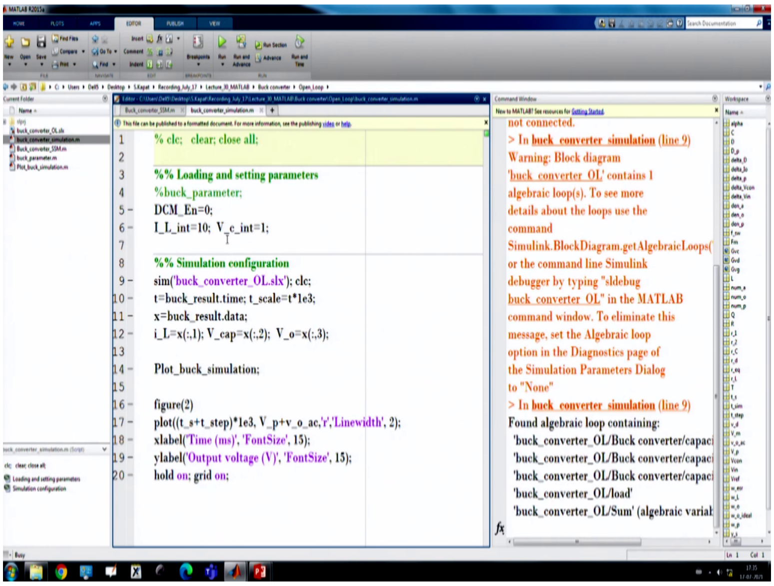

So, here we want to first set.

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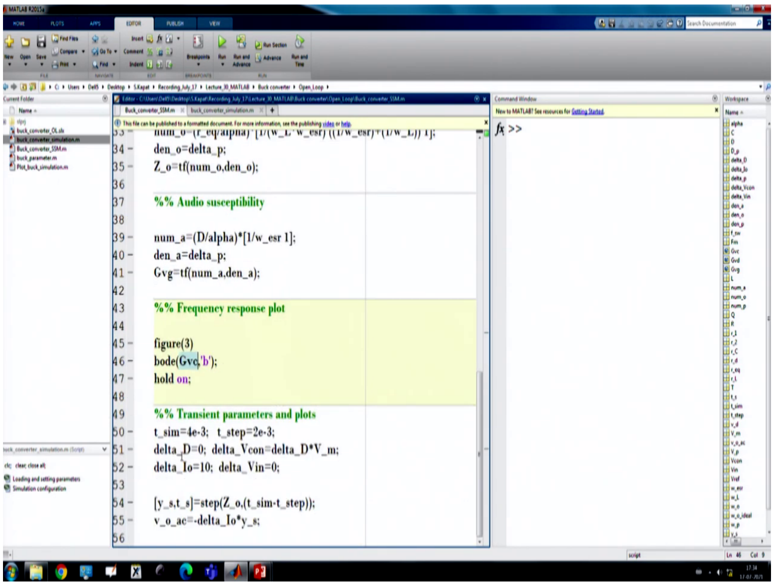

I have explained step-by-step everything. So, 10 ampere load step I am applying. An initial well of the load resistance is 1 ohm ok.

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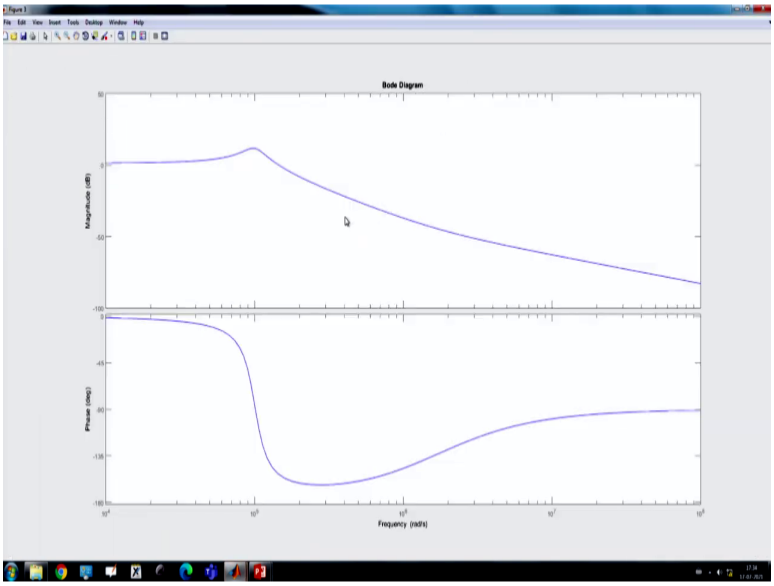

So, let us run it. This is the Bode plot that I have already shown. It is now if you grid on.

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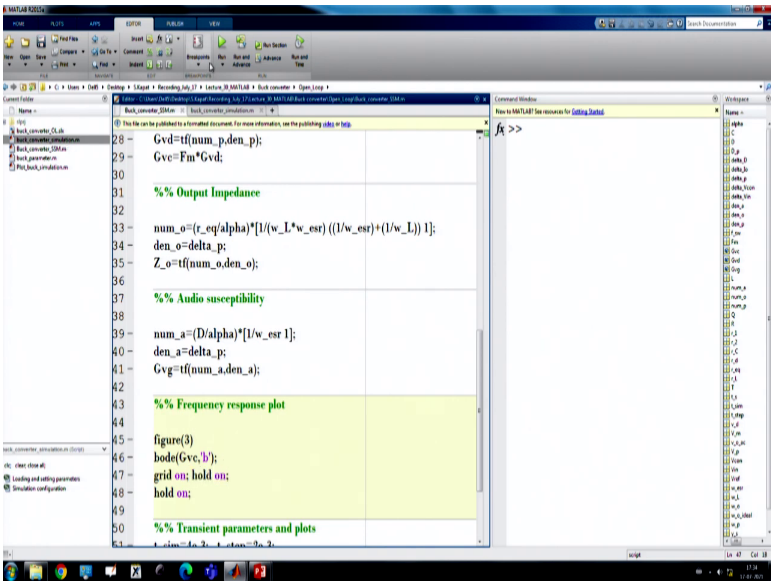

So, let me write down here the bode plot. We can write grid on hold on ok. So, it will grid on.

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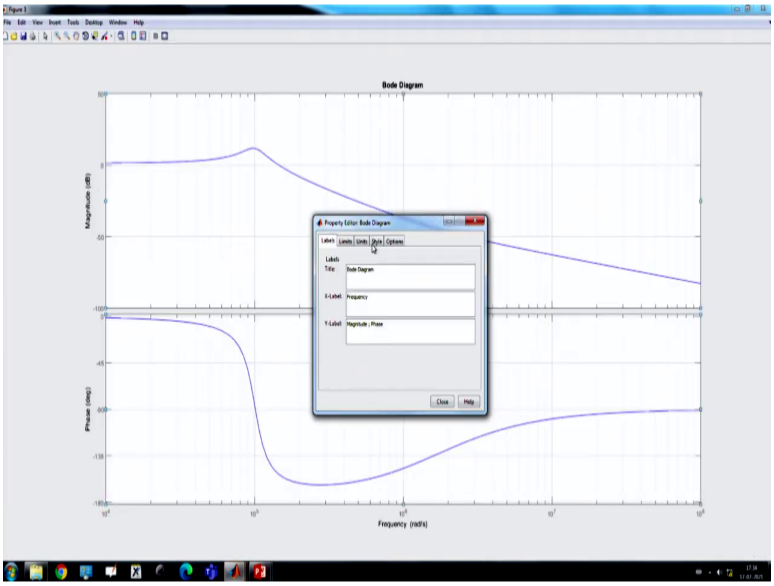

Now, we need to plot using hertz.

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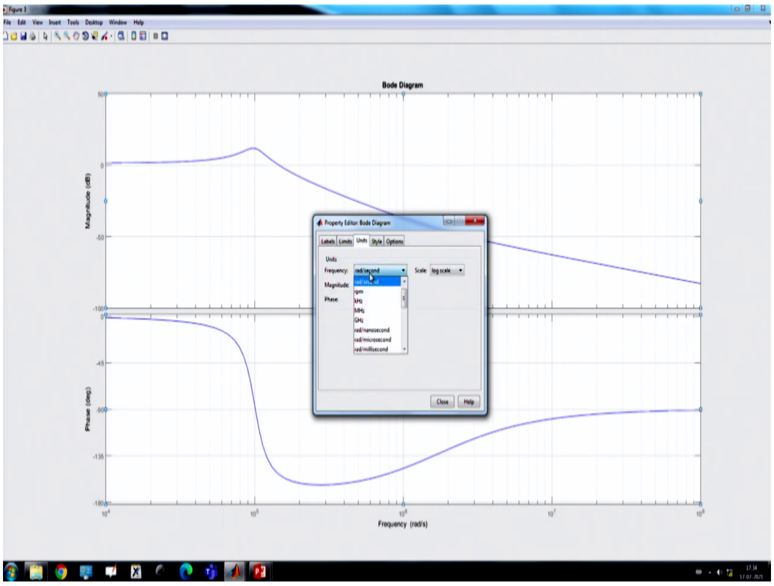

Not in radian per second.

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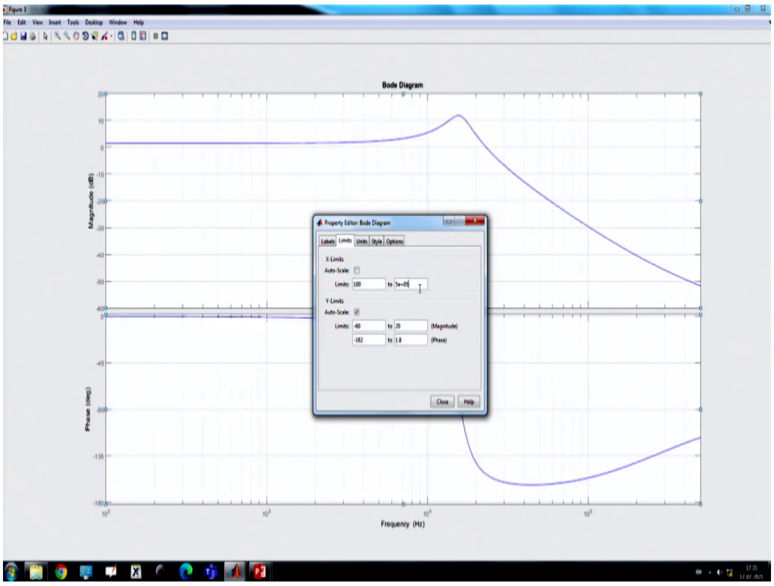

And we should limit to because our switching frequencies 500 kilohertz and we want to plot from let us say 100 ok.

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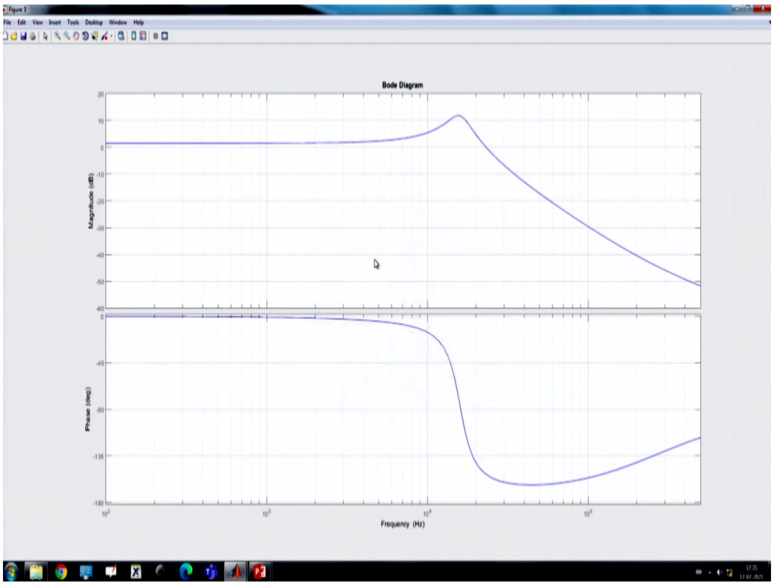

So, this we have already seen. This is the v G vc not G vd G vc ok. So, we need to increase the gain we need to provide high v c gain in order to boost up otherwise you have a very poor bandwidth ok. This is an uncompensated loop transfer function ok. We will see that. Next, what is next?.

Now, we want to check the actual switch simulation; that means, now if we run the actual switch simulation and I have explained step-by-step everything. So, where we have to capture the AC model, then we have to add with the DC offset. So, everything you see, I am taking the V practical that I have explained everything. Now, this is the simulation.

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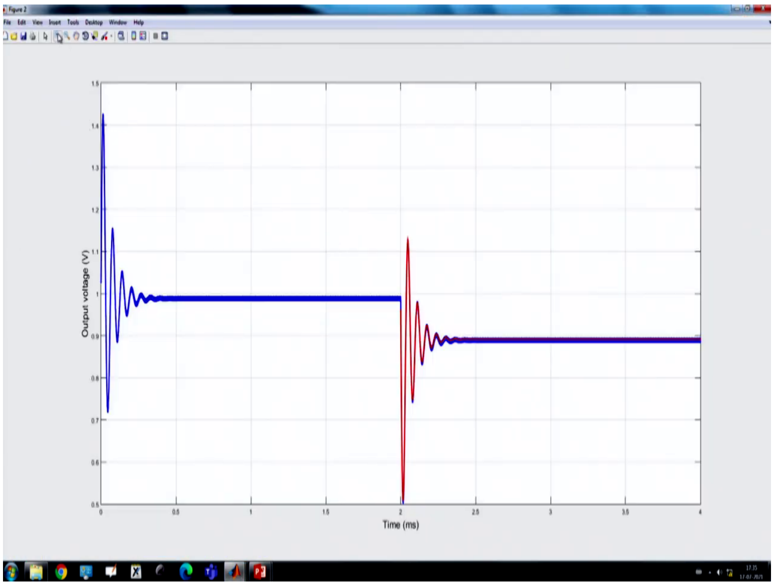

So, the red colour trace is the waveform which is obtained from AC model, AC small-signal model.

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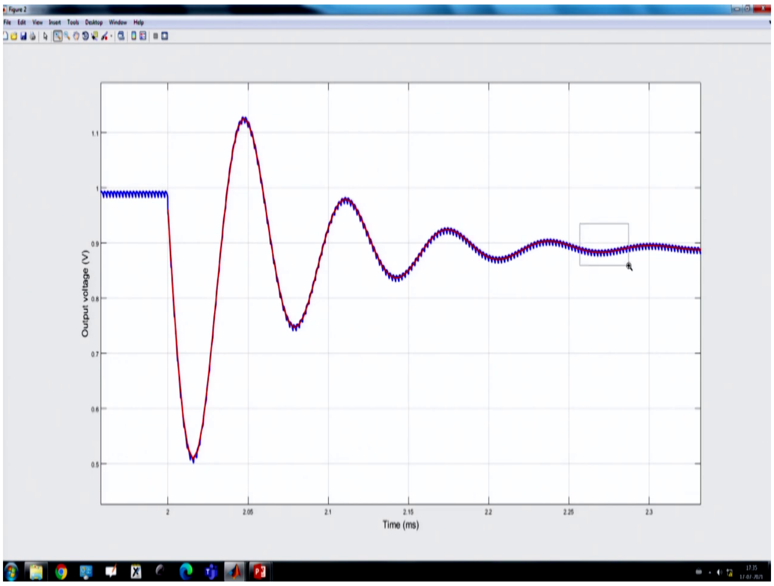

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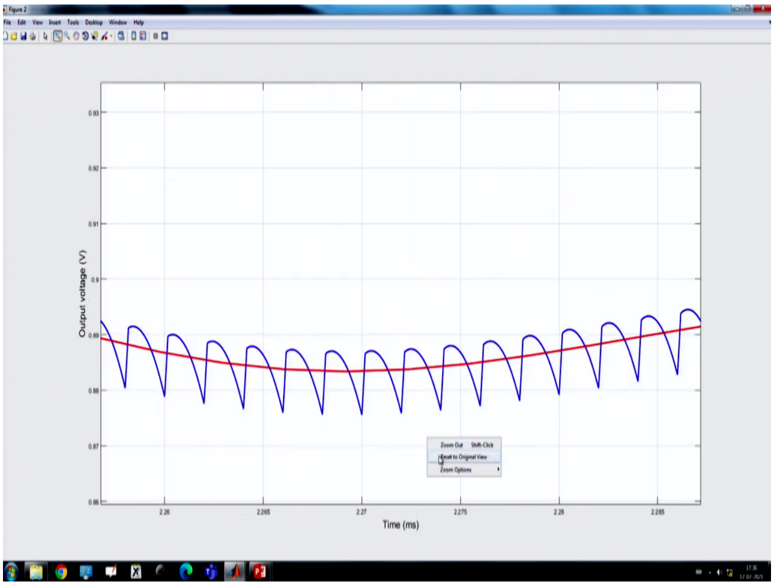

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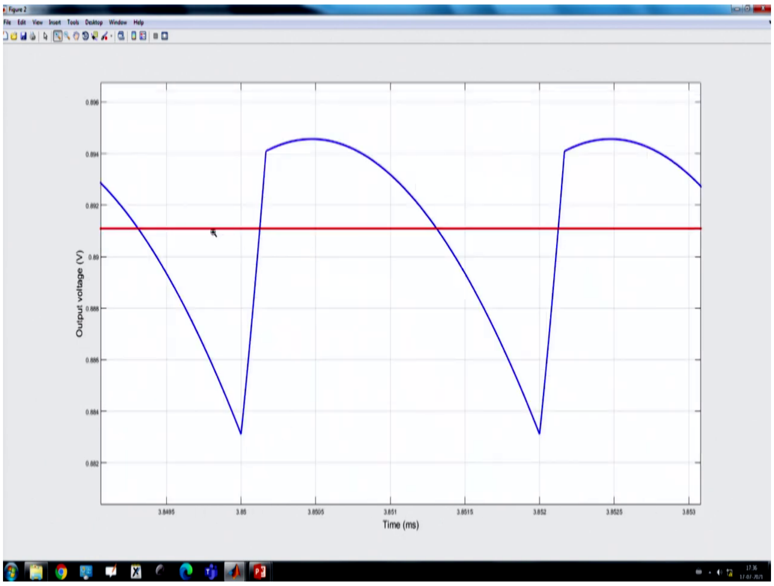

And you can see if you keep on zooming that represent the average voltage correctly; that means, if you take the average a more or less capturing the average voltage it is going almost from the average value ok.

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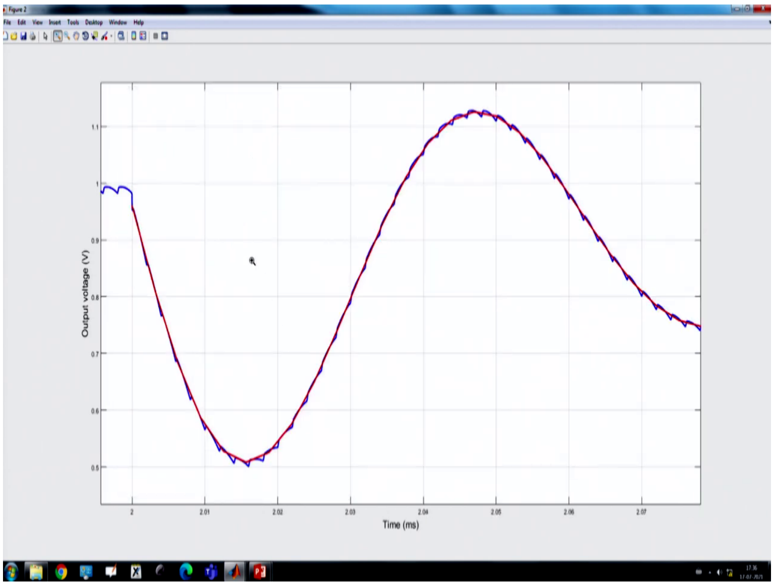

And if you check the chant transient response; that means, load transient response for an open loop convertor. The AC small-signal model is perfectly capturing the actual switch simulation. That means you can see the actual switch simulation. It is perfectly capturing. All these models are perfectly captured ok; that means, for load transient response, our model is perfect. Now, you want to check the same thing for if we apply a supply transient rather than load transient.

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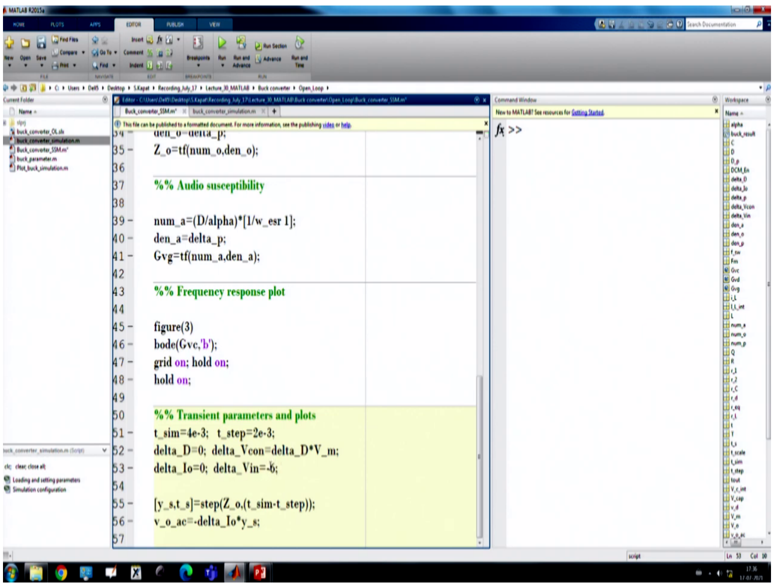

So, if we apply a supply transient, let us say we want to apply minus 6 volt it is originally 12 volt and it will change from 12 volt to 6 volt. So, naturally it is an open loop the voltage will dip. And we need to change here. What is our audio susceptibility? G vg. So, this should be changed here G v g.

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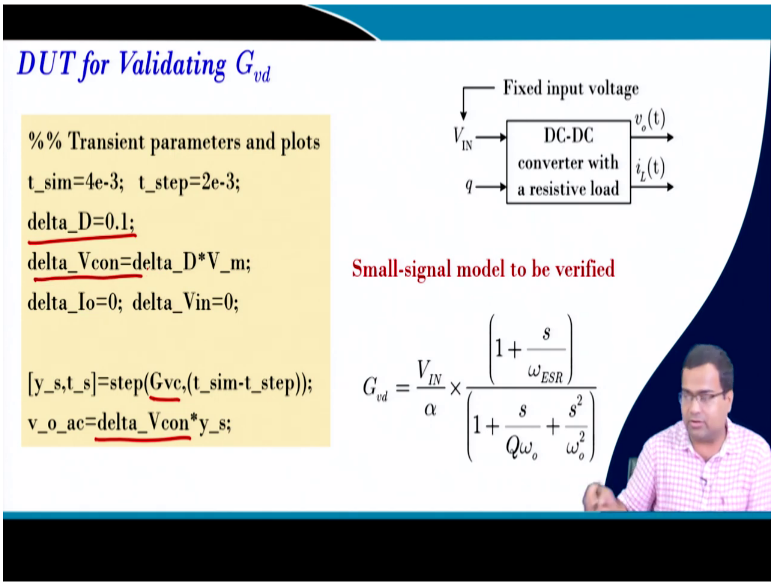

Because we need to check if you go back if you want to check the output impedance that we have discussed. So, this part I have discussed, then I want to check G vd ok. Let us check G vd first. So, G vd what you have to do? I need to check this change is; that means, I need to take a step change in the duty ratio ok and then this should be G vc am you need to delta Vcon because ultimately this delta V delta D will reflect a change in the Vcon ok.

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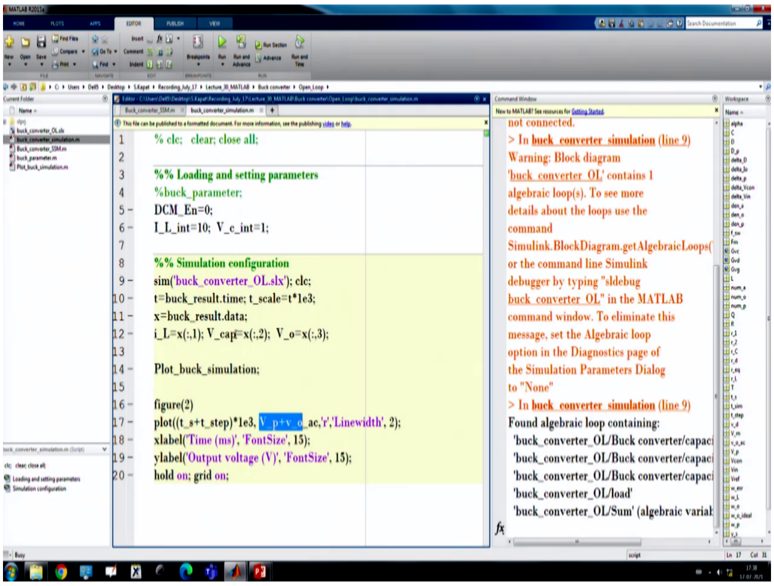

So that means, we want to first see G vc because we want to make apply a step transient to the duty ratio 0.1. And similarly we have to take delta Vcon since you are taking G vc and this is our delta Vcon. We have to multiply and just run the simulation. Now, we are running.

We are actually repeating the same process, but now this time we are trying to match with the control to output transfer function that means if there is a change in the converter voltage or maybe a 0.1 duty ratio. So, how is it going to change?

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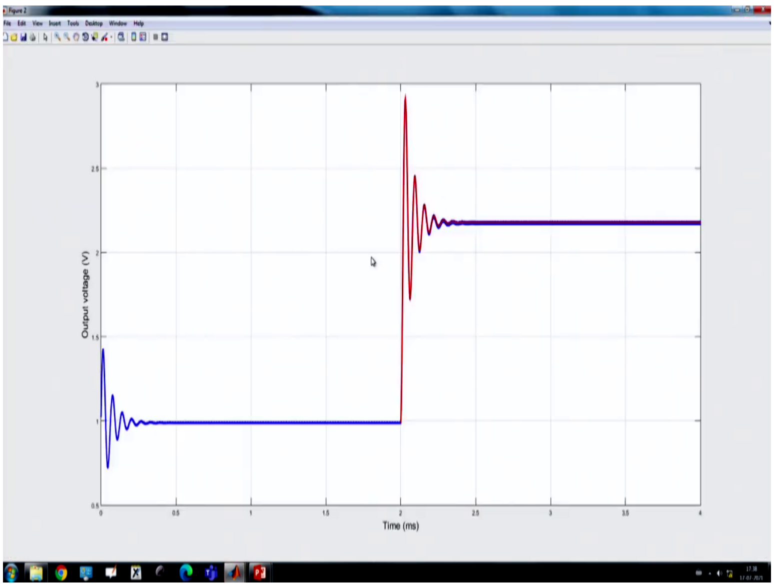

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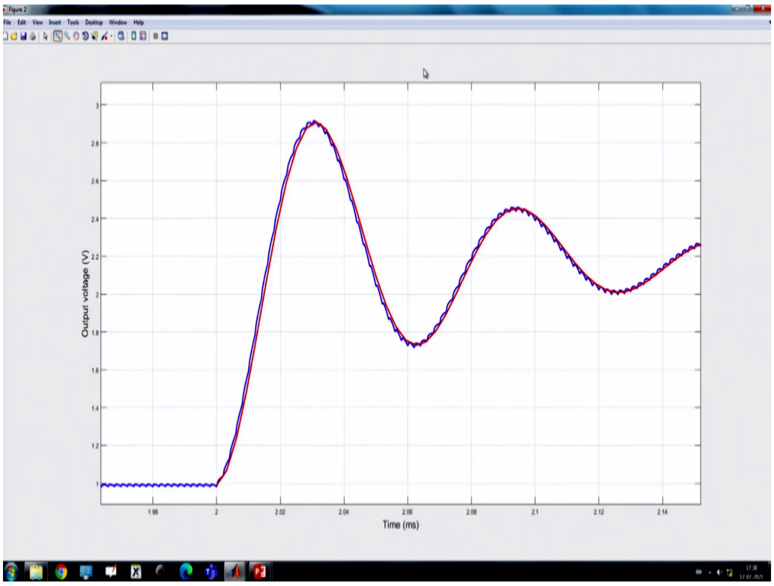

So, you can see there is a slight deviation, but it is acceptable, you know.

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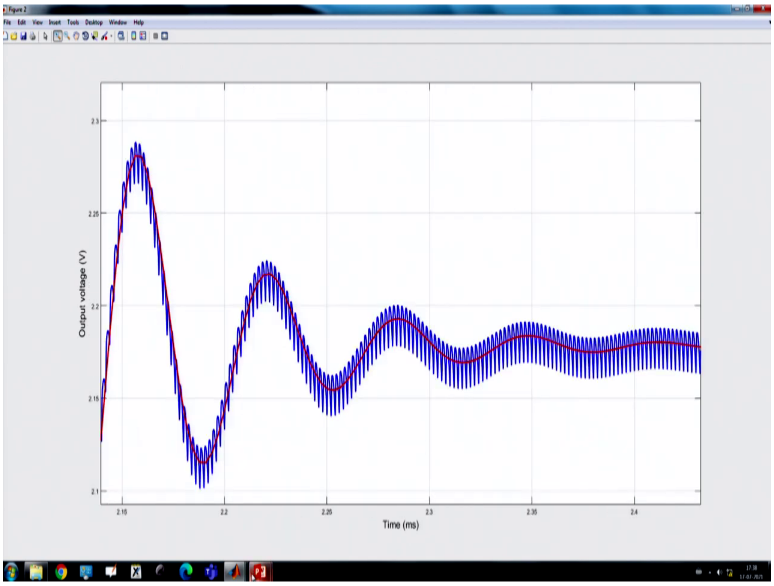

In the acceptable range otherwise the red traces is going.

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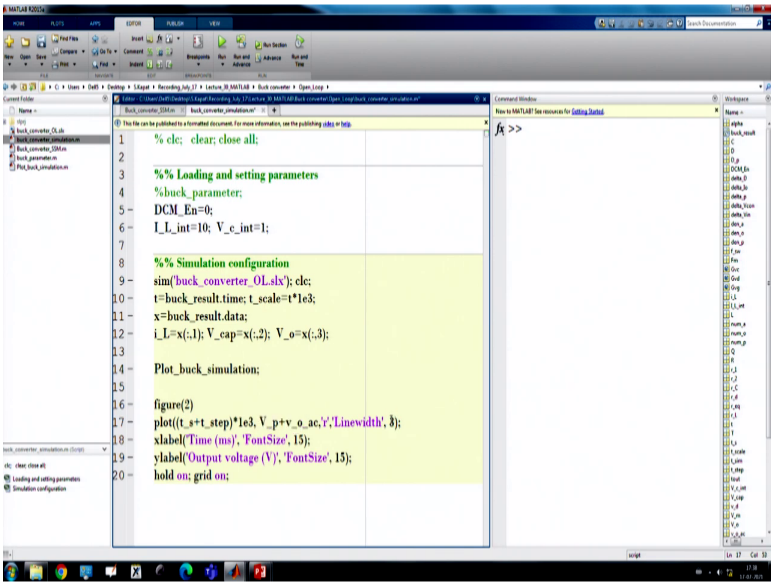

So, I think it would be better if we just you know increase this width. So, maybe we can use 3.

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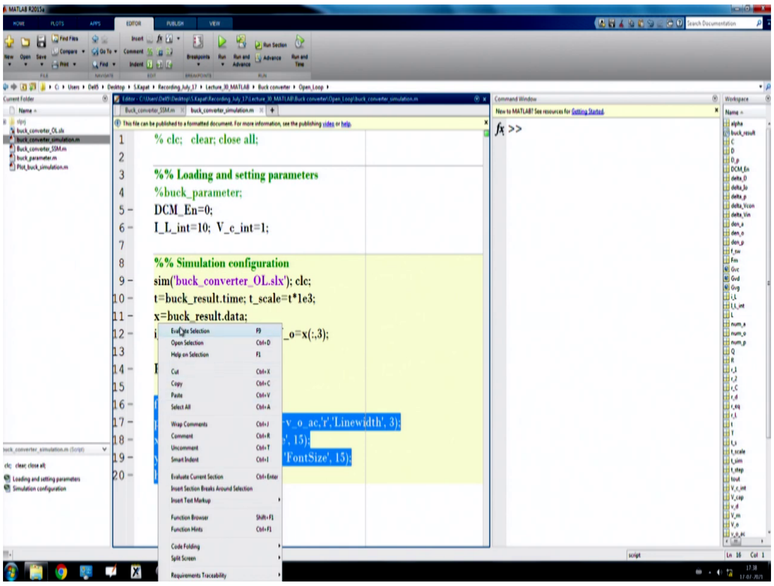

So, if we just you know rerun again yeah.

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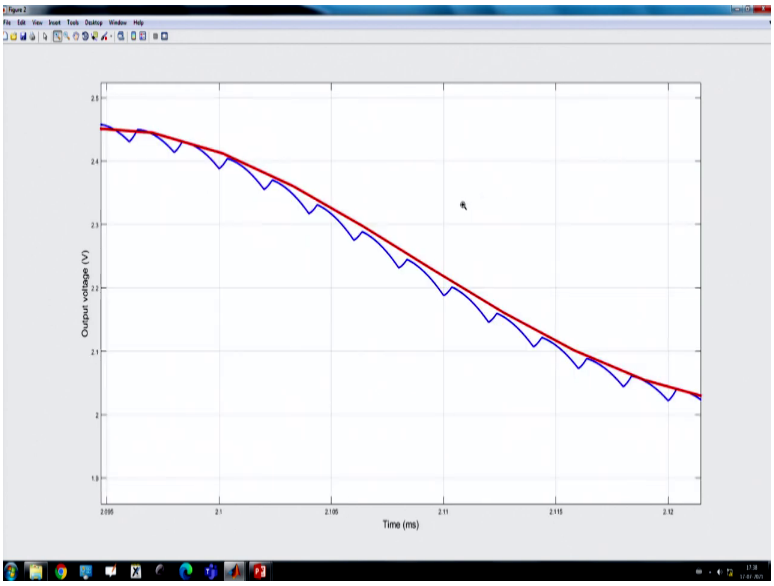

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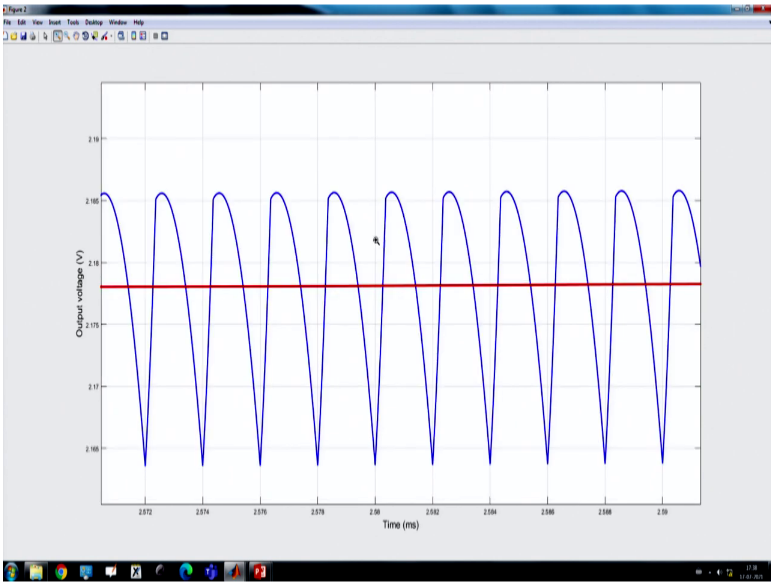

So, it will be better. So, if we check go. So, there is a slide deviation other is there matching quite nicely.

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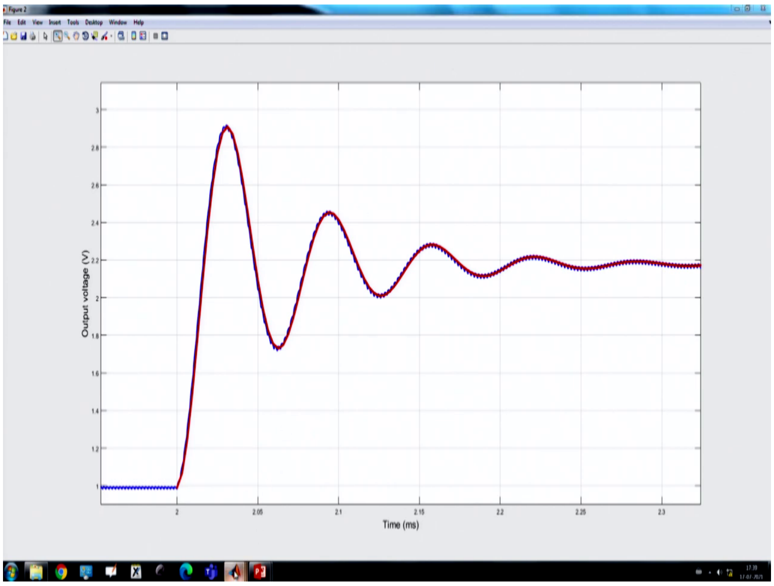

That means this control to output transforms are looks it is perfectly fine.

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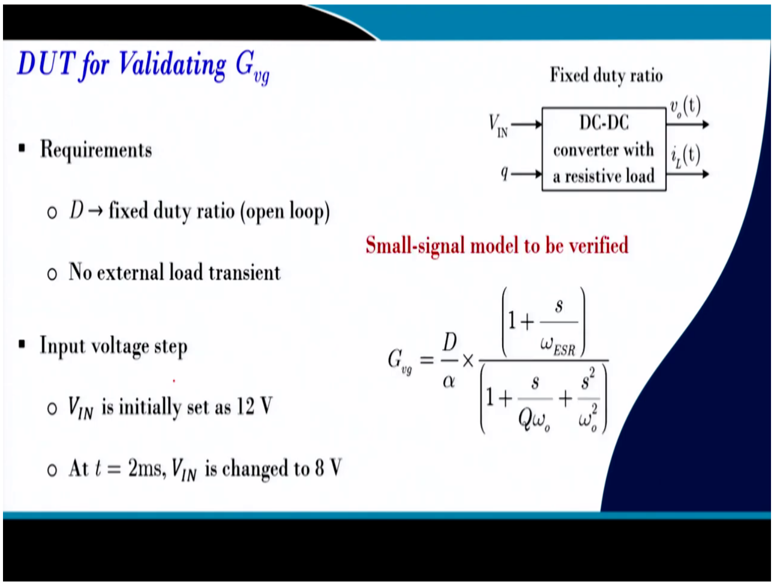

Now, we go for audio susceptibility. In audio susceptibility, we need to keep the duty ratio fixed because you are talking about open loop audio susceptibility. No external load transient response, input voltage step we have to apply. So, initially it was 12 volt now, at 2 millisecond will change from 12 volt to 8 volt; that means it will come to and this is a transfer function ok.

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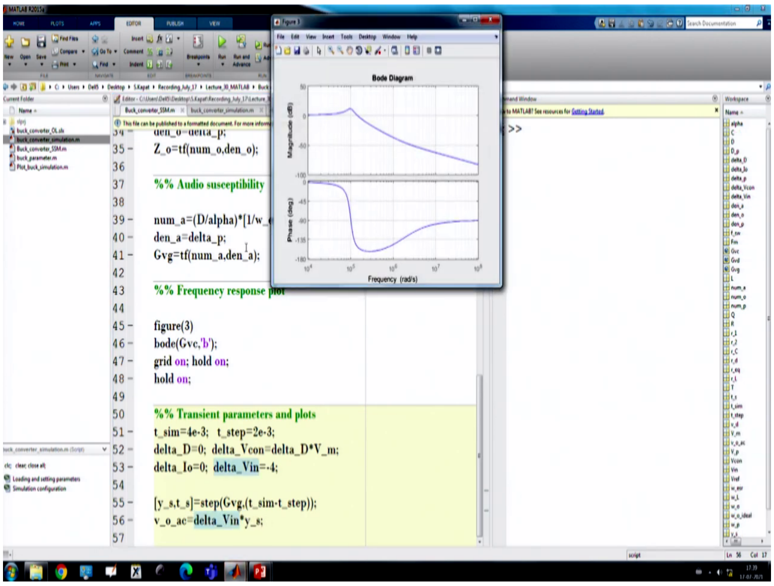

So, audio susceptibility, what we will do? Now, we will go back. We will make a change in minus 4 volt because now no duty ratio step is applied and here it will be G vg because audio susceptibility and we have to multiply with delta V in, so, delta V in ok. Now, we are all set. Let us run.

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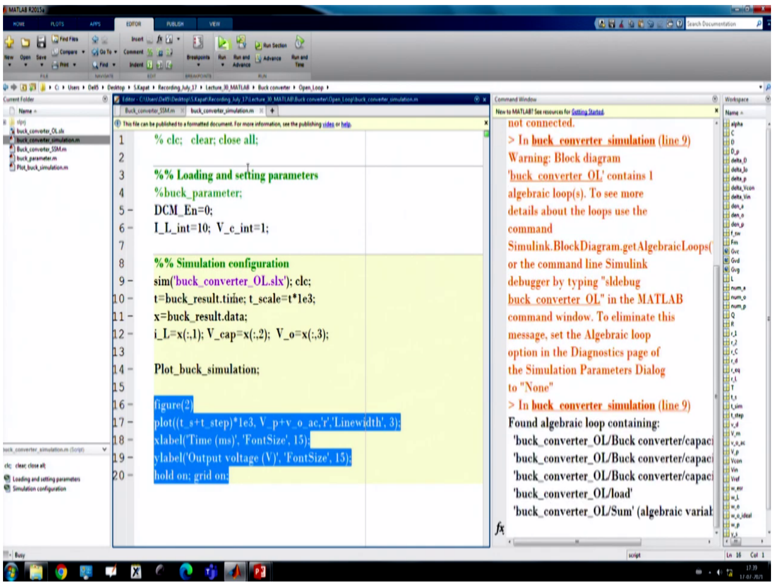

And if we check now, we are checking the result with audio susceptibility where we are applied a step change in the input voltage. Originally it was 12 volt and now we want to change to 8 volt. So, minus 4 volt step is applied.

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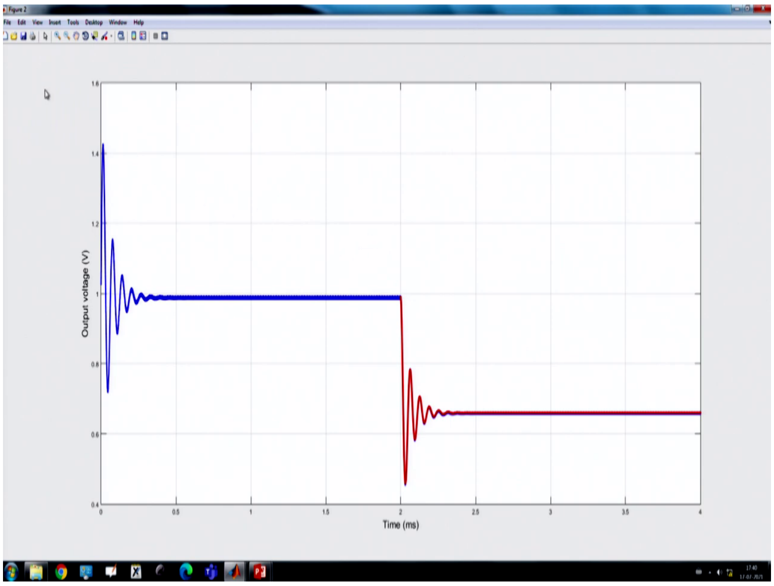

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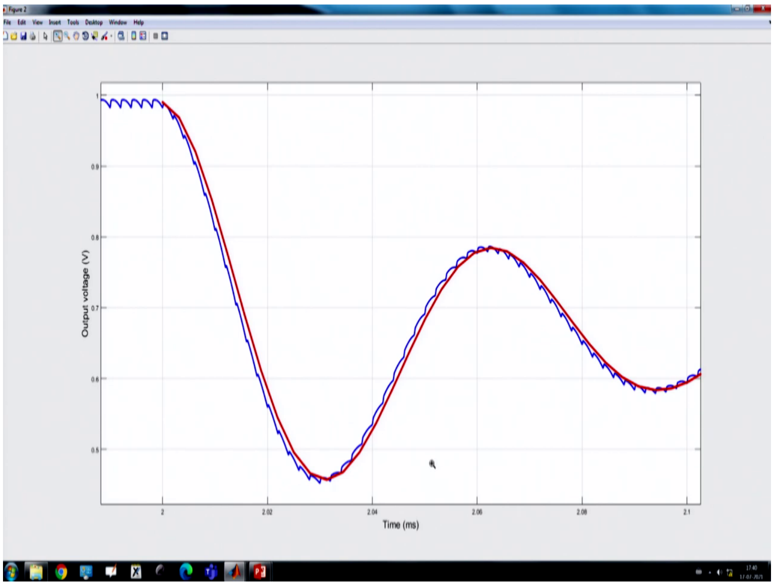

Now, if you go and zoom this waveform, it is perfectly valid. I mean, almost nearly I they are matching quite nicely ok. So, we can assume that you know this model works fine ok. There is a small division because you know in the audio susceptible expression there is a duty ratio D. So, we have to change the duty ratio update the duty ratio ok in order to match this that is why the slight shift is happening ok.

Because you know this duty ratio because we have applied sorry here we have changed the input voltage, but we have kept the duty ratio con. So, in this case sorry duty ratio is fine because we have kept the same duty ratio, but input voltage has changed.

When the input voltage has changed in the actual converter, what will happen? So, we have matched the all three response fine, and this is what I have already explained minus 4. So, it is perfectly fine.

That means now we are we have learn how to verify small-signal model using MATLAB transient simulation and that we have done for open loop converter and we will see in subsequent lecture we can actually match the close loop response using transient simulation as well.

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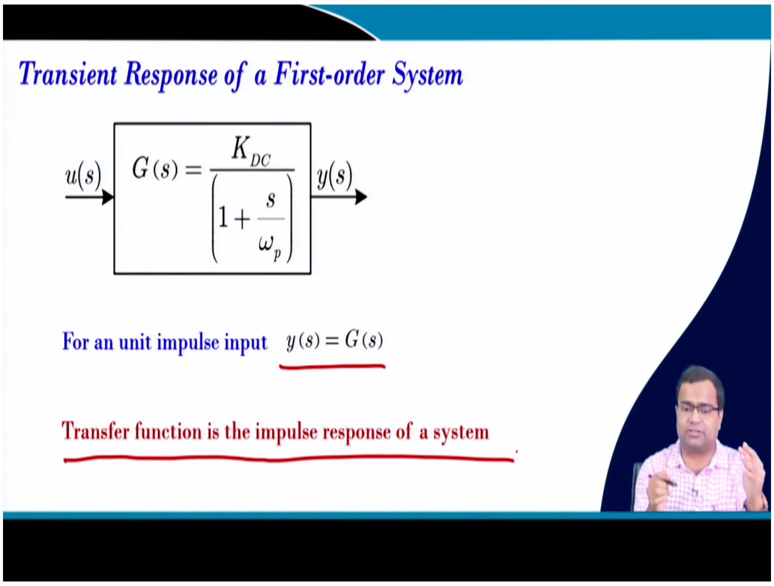

The next question will come to our mind. Now we know that our model is good. We can move forward now. We want to see what is the effect of poles and zero. Because in the subsequent lecture when we want to design the compensator, we should keep in mind what should be our desired transfer function or what should be our desired step response right. So, for that we need to understand what is the response due to poles and zeros.

If you take a first order pole, this is a first order system right. Then, if we apply an impulse response, then G of s y of s is equal to G of s and this is the proper definition of a transfer function. So, the transfer function because you know you know in many cases people define the transfer function is a Laplace input of Laplace transform of you know output by Laplace transform of the input. But this may not be a very precise definition.

So, more precise definition of the transfer function is that the transfer function is the impulse response of a system, ok, here it is same.

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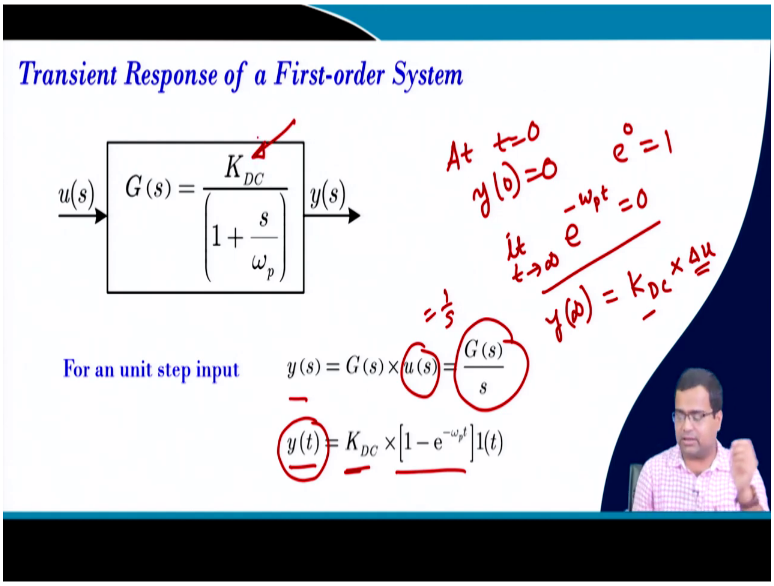

Next, if you want to see the transient response step transient response, now we applied an unit step input. If you apply unit step input, then output will be G of s by s because this u of s will be equal to 1 by s if is unit step. Then we can obtain by means of inverse Laplace the DC gain multiplied by this term.

That means, at t equal to 0, the output will be 0 right; that means, I would say at t equal to 0 or y of 0 is 0 because e to the power 0 is 1 right. At t equal to for limit, t tends to infinity e to the power minus omega p t will be 0 ok. Then what is our final value? Our final value is simply k DC or in the sense that this will be k DC into the input step size.

That means, what is the input step size? In this case, the input step size was unity. So, the output will be simply k DC in to 1. So; that means, if the DC gain is 1 then output steady state value will be 1. If the DC gain is 2, the output steady state value will be 2. And if it is 0.5, that means this is a scaling factor. So, this is a DC gain, and that is determined by this particular gain.

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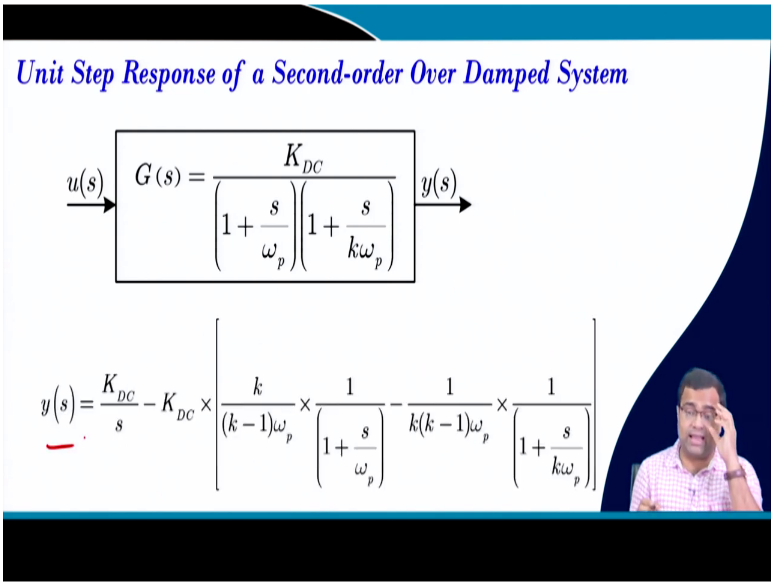

Now, we want to see what is the response of an over damp second order system because we often encounter this question. Like you know we phase this question and we often interchangeably use that if two poles are widely separated, then the response of an over damp second order system is primarily dominated by the dominant pole and the dominant pole the pole which is close to the imaginary axis.

And remember that we are talking about two stable poles. And if it is dominated by the pole which is close to the imaginary axis, that means it will behave like a first order system but, whether this transfer function response will be exactly identical to a first order system.

Because if we want to characterize experimentally a system, which is an over damp second order and a system with a first order then how do you check by means of a step response in experimentally how do you check their performance whether the system is second order first order. Definitely, they will not be exactly identical. They will be close, there will be some difference.

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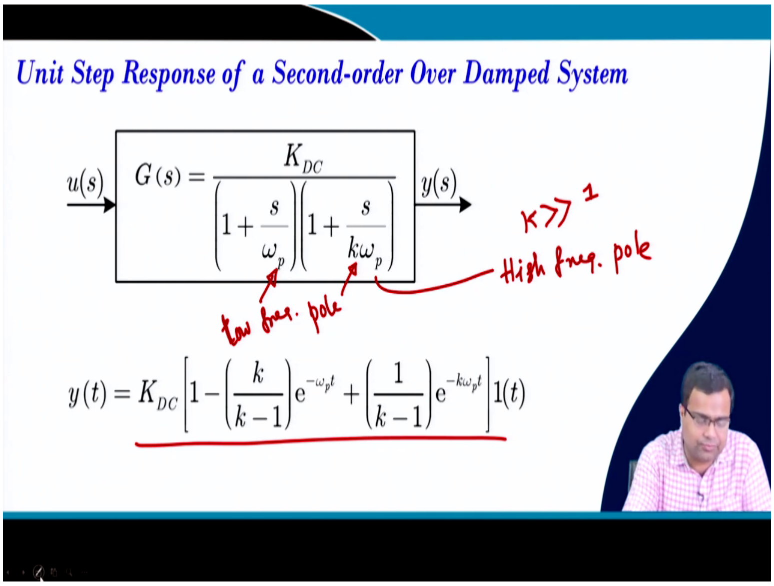

If you take the full y of s again, we can write down the full expression. And if you take the inverse Laplace transformation because we are just keeping the intermediate state, it can be written like this ok. Now, you see, we have taken 1 pole k times omega p another pole omega p. And if we set k equal to be very high; that means, this pole is a high frequency pole right pole and this pole will be low frequency pole, this is our low frequency pole right. So, not you know ok.

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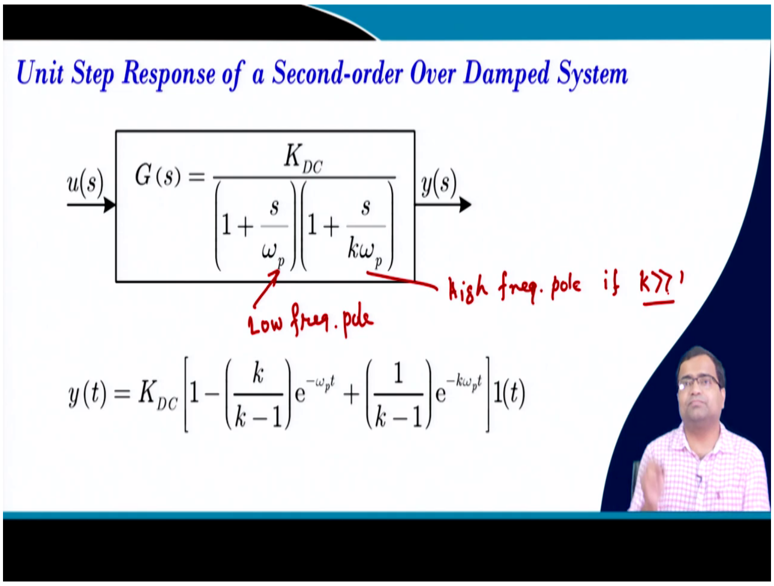

So, this is our low frequency pole, and this is our high frequency pole if k is very very greater than 1.

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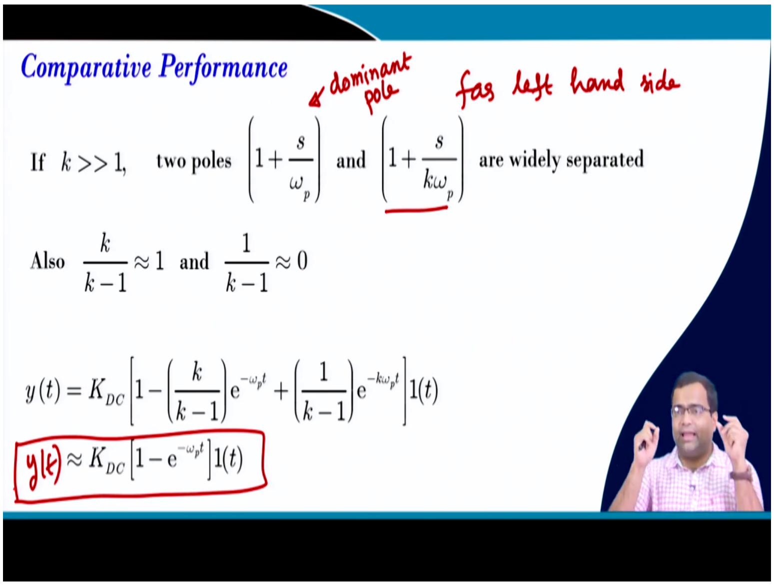

Now, if k is very greater than 1, the two poles are widely separated. This pole is the far left-hand side and this will be our dominant pole that we have studied. That means the response of the system be primarily dominated by this dominant pole. Then we can write k by k minus 1 is almost equal to 1 and 1 by k minus 1, 0. So, the response of the second order system can be approximated to be y of t and you see this is the same response of the first order system.

That means the approximate response of the second order system is almost identical to a first order system. Now, the question is whether they will be really identical or not because if they are identical, then we cannot differentiate between an overdamped second order system and a first order system. There has to be something.

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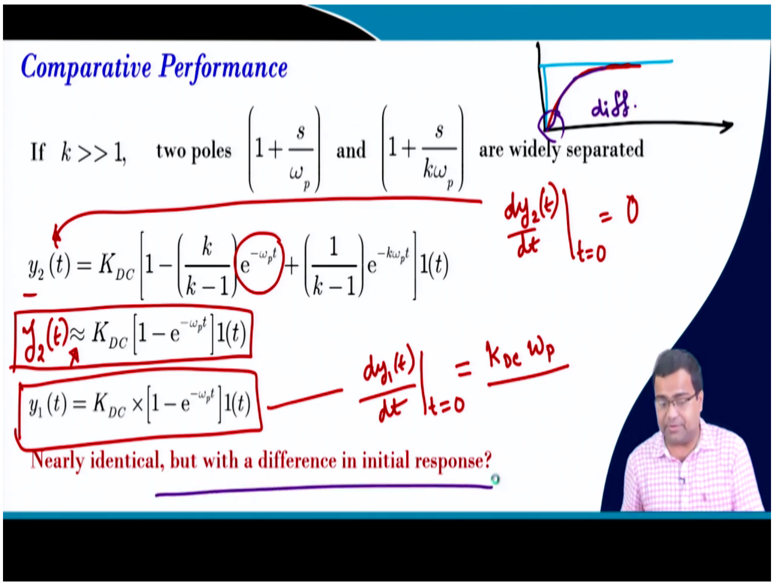

Next again we are rewriting this equation. So, this is a response y of 2 output of the over damp second order system and this is a response of the first order system. See, this is approximate sense, not exact sense right. Now, the next question if you take d 2 y dt ok and if you find out; that means, the original expression I am talking about this expression from this expression and you set at t equal to 0.

If you find out; that means, a difference is this response. So, this term will minus omega t come minus k omega t will come, you will find this will be simply 0. But if you take this response and if you write d y 1 t dt and which is computed at t equal to 0, what you will find?

You will find it is nothing but k DC into omega p that is it. That means, at the beginning of the response, there will be a difference that means the derivative. That means, if you draw the response; that means, if I draw you know the response if I apply a step input let us say I am applied a step input here then the response of the first order system will be like this, it will be like this ok. It will not touch, it will just go close, it will just go close.

And the response due to the second order system initially it will go and then try to match try to match they will match because they are identical. So, there is a difference. If you zoom out, there is a difference in their initial condition and this is the initial response. It will be different.

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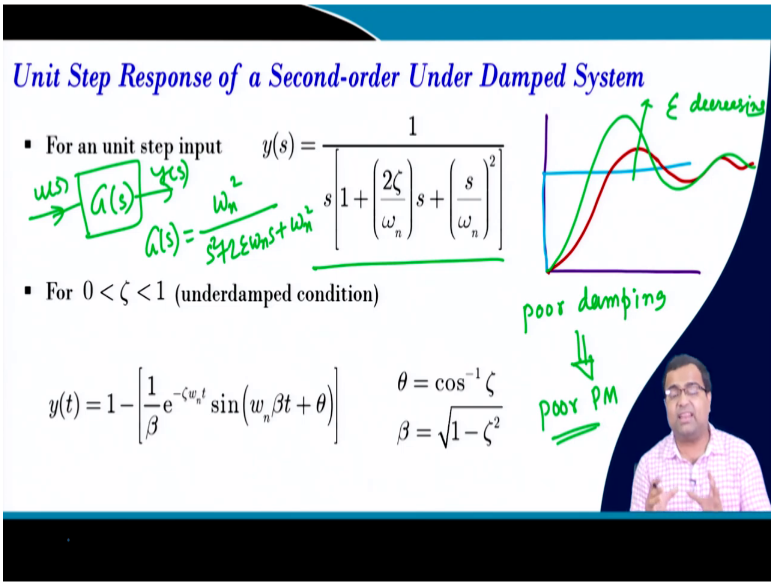

Similarly, if you take the second order system response, then you can find out depending upon the damping ratio. So, you can draw the plot, you know. We have studied this in the control system context if we apply a step response. If the k DC equal to 1, here it is a unity gain, then you will find that ok like this.

And if it is even overdamped then you know even over damp like this, so, depending upon the zeta. So, this way your zeta is decreasing ok; that means it will be more poorly damp ok. And this will be you can relate this: the poor damping poor damping will eventually lead to poor phase margin.

That means, if you want to shape the response of the system in close loop control, we need to provide sufficient phase margin so that we can cut down the overshoot undershoot because those overshoot understood may not be acceptable ok. And we have discussed in lecture number 13 that open loop converter has a high overshoot undershoot. In fact, we saw just in the simulation result. But that is simply not acceptable.

So, in case of close loop we have to reduce that and we have to keep within a very transient limit because this power supply will be delivering you know current to the digital processor ok, where we mix to maintain the output voltage of the DC-DC converter within the noise margin of the digital system ok.

Now we have studied all this control system because this is a transfer function for original transfer function was G of s where input was u of s. And this was a box of output where G of s we consider like an omega n square a square plus 2 zeta omega n s plus omega n square. At this we have studied in our control system later right. Now, my question is what will happen in this system if you add a 0.

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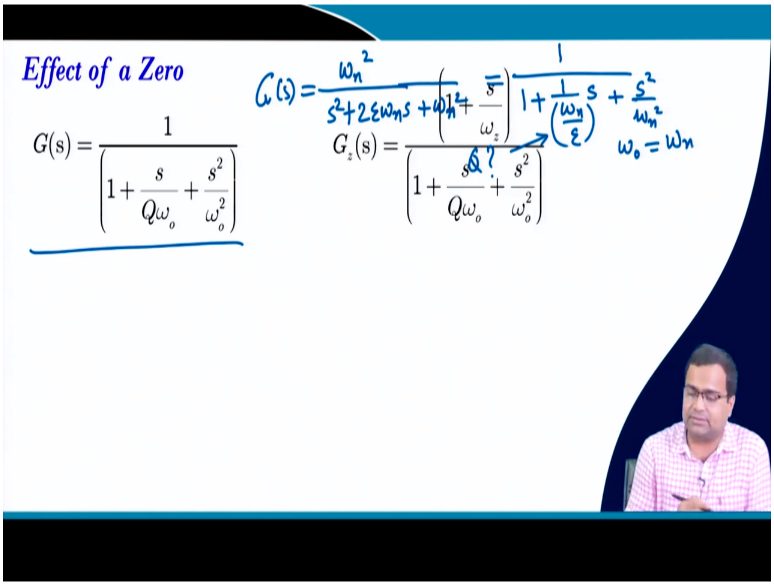

Let me this is my original system. Now, it looks like in the form of our DC-DC converter. In case of control system, we have studied G of s in terms of omega n square s square plus 2 zeta omega n s plus omega n square. You can simply divide omega n square in numerator and denominator. So, you will get 1 divided by 1 plus you know 1 by what we will get? You will get omega n by zeta into s plus s square by omega n square ok.

So, from here you can relate what is my Q. What is my Q? Our omega 0 in this case, is nothing but omega n. So, you can rewrite all this ok.

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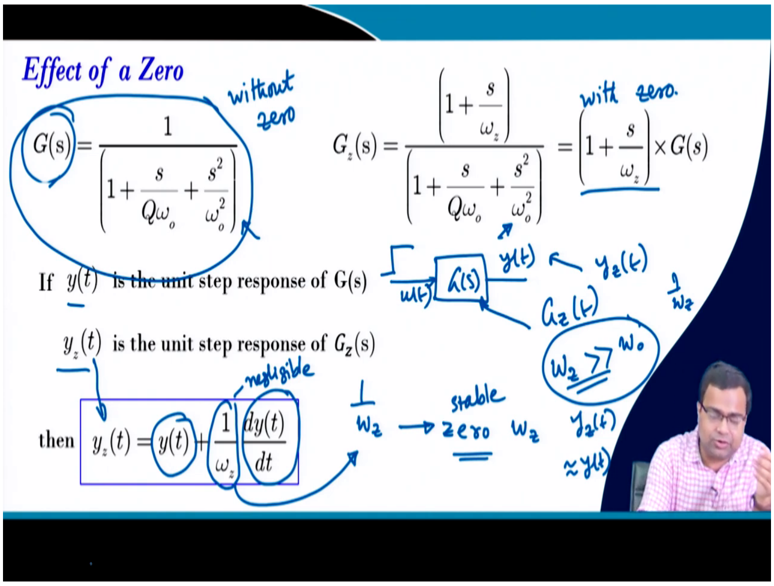


So, in this case, you know we have found that. Now, we have another transfer function. So, this is a transfer function without zero without zero and this is with zero. Now, it can be written as this zero into the original transfer function, this one right.

If y of t is the unit response of this; that means, we have applied a unit step to this system; that means, this is our system G of s and we have applied a (Refer Time: 51:47) step here and we want to check what is the response of the system. This is my input ok.

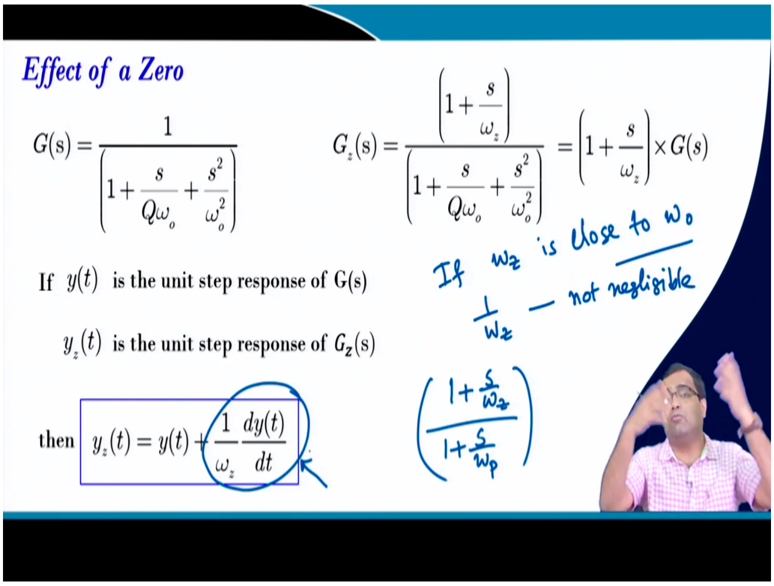
So, then this is a unit step response ok and y of z is unit step response of if you replace this with omega 0 t then it will be replaced by y z of t we want to check this. And it can be shown that the response with zero is nothing but the sum of the response without zero plus 1 by omega z into derivative of the response without zero. And this 1 by omega z term which is coming from here, where this is our zero location; that means, our zero, which is nothing but omega z.

And you see that if the zero is, but this is a stable zero. It is a stable, you know; that means, it is located in the left-hand side. If the 0 is very far away in the left hand; that means, if the omega z is very very greater than omega 0. That means, this 0 if it is a complex conjugate pole, then the frequency will be omega 0, right. And the omega z is far left-hand side of the imaginary axis then or basically in the pole means in a far left-hand side; that means, the frequency is very high.

Then this 1 by omega z in this case, then 1 by omega z here it becomes negligible. That means, in that case, your omega z t will be approximately equal to omega t. That is why most of the control system text book we study only this transfer function without 0 because we assume even though there is a 0 that 0 is located far left-hand side by means of close loop controller or it originally far left-hand side ok.

So, then the response will be almost identical. The next task, if the zero comes close to the imaginary axis if this is not the case; that means, you know we are talking about another case.

(Refer Slide Time: 54:21)

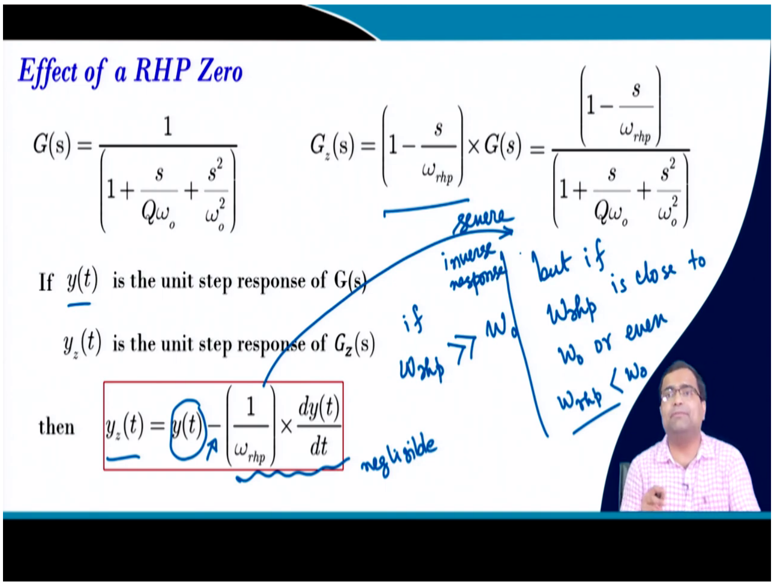


If omega z is less than equal to omega 0; that means it is coming close, it is close, or others I will say in the language I should say if omega z is close to omega 0 it may be close. It will be slightly higher, lower, close. Then 1 by omega z is not negligible. In fact, it can be quite large and if the 0 comes close to the imaginary axis, this response will be very, very far.

Even if you take a first order system, you can have overshoot in the response if there is a 0 ok. You know the first order system. A single pole system there is no overshoot because it is an overdamped response. But in the single pole system; that means, if you take one transfer function 1 by S plus omega z by 1 by S plus omega P such system you may you can have overshoot depending upon the location of the 0. So, you can have overshoot and then it will decay down ok.

So; that means, we have understood that if the 0 comes closer, this 0 can inject you know this that derivative action and that can create overshoot.

(Refer Slide Time: 55:52)



But what happened if the 0 in the left right half plane zero; that means, 0 on the other side? That means this is my right half plane zero. Same analogy, if y of t is the unit step response of G of s without zero, y 0 of t the unit step response with zero then the total response of the system with zero will be the response without zero minus 1 by rhp into this.

Now, there is a negative sign. Again, the same analogy. If omega rhp is very very far, then this effect is negligible. It has no zero, no effect. But, this is case 1. If, but if omega rhp is close to omega 0 or even omega 0 omega rhp is smaller than omega; that means it is now coming close to the imaginary axis. Then this can be severe; that means, this can be in that case it can be severe. So, this will lead to an inverse response, severe inverse response.

And if you take a boost converter, we will see this inverse response is reflected in the transfer function ok and we will see that.

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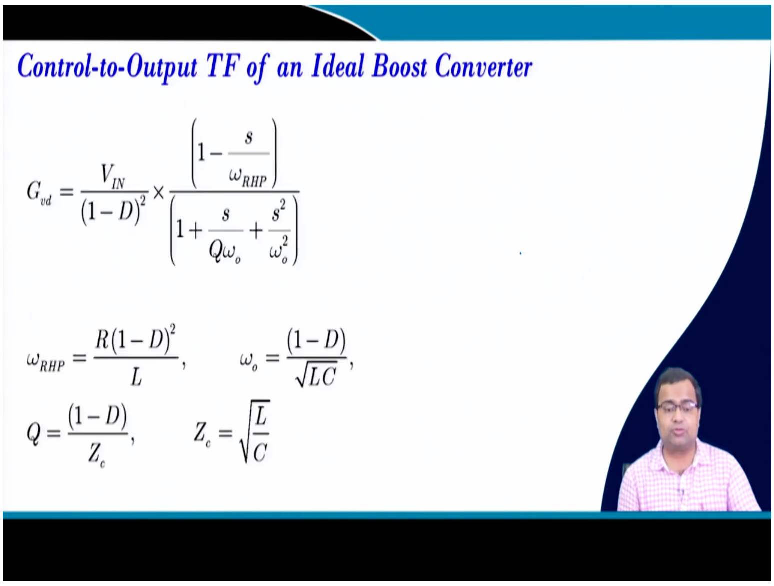

So, we will take an ideal boost converter and we want to run the simulation and show that what happens if we go to a boost converter ok.

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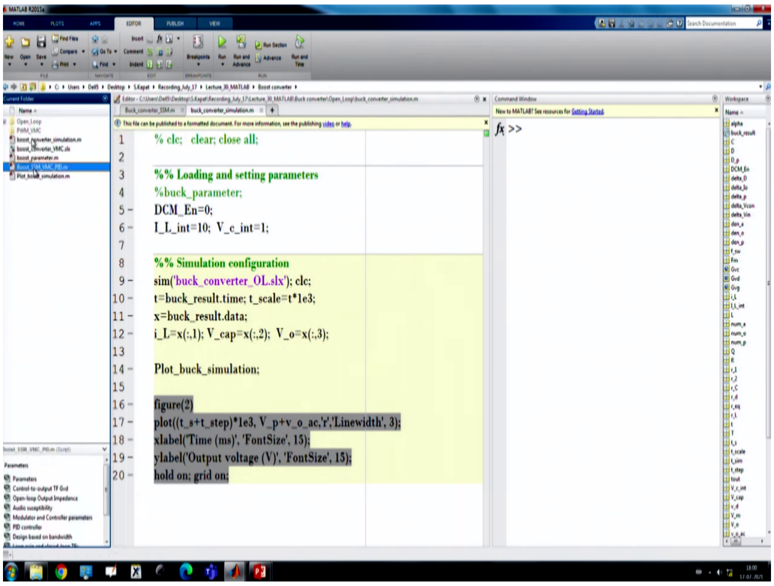

So, if you go to a boost converter, ok. So, let us go to the boost converter. So, we can check that the boost converter ok just hold on ok.

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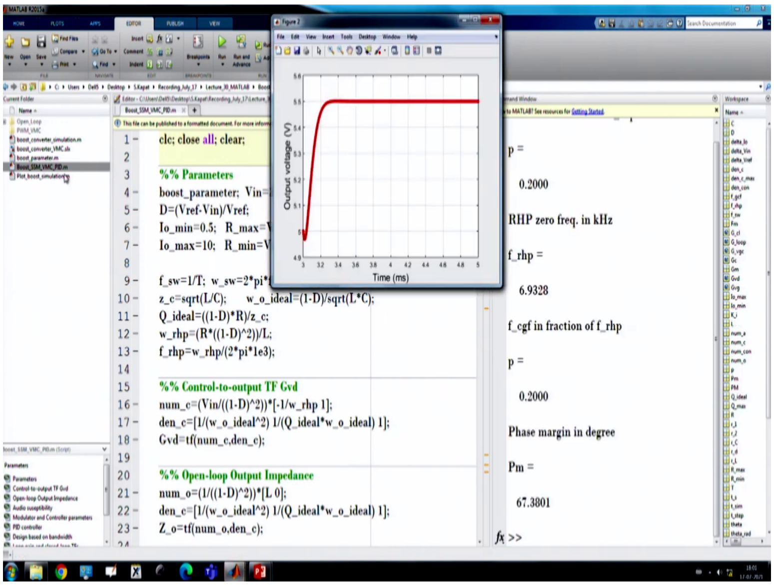

So, we are just showing one example of a boost converter, ok.

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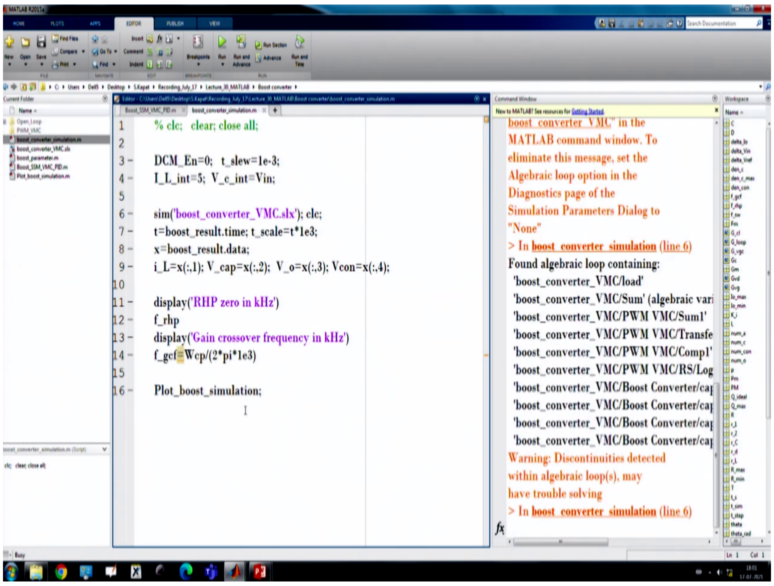

And this is the simulation. I am just showing the response of the boost converter.

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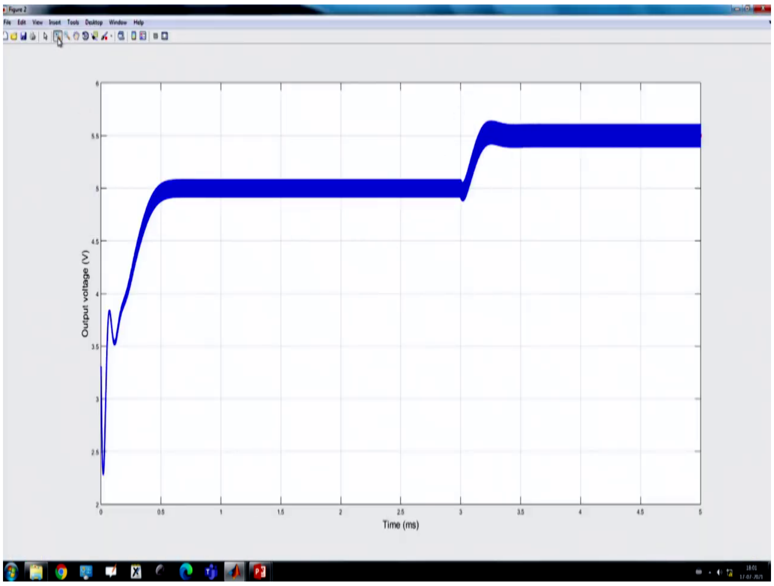

So, here I am just showing one example.

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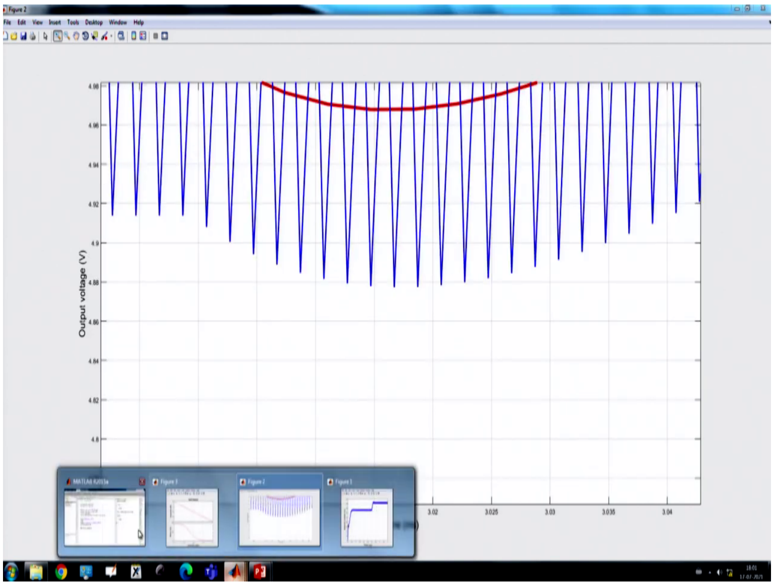

There is a reference voltage transient ok and you can see just a minute.

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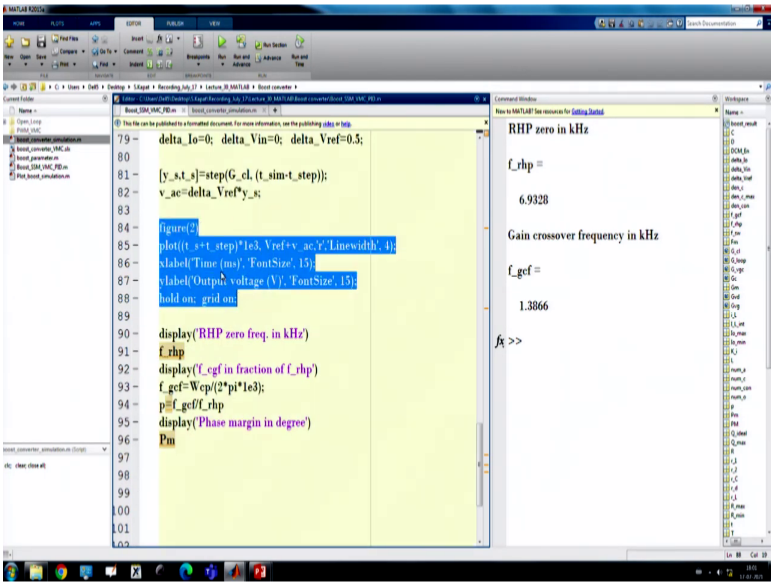

If I just redraw this one, that means this figure.

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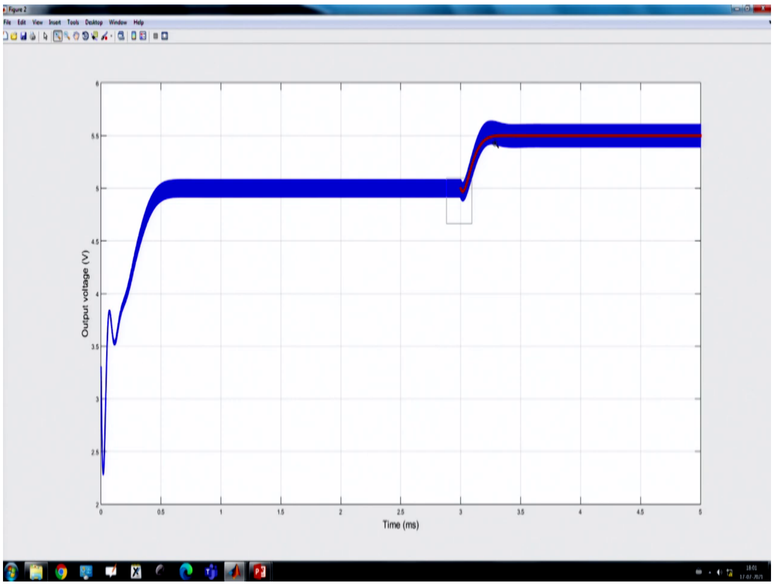

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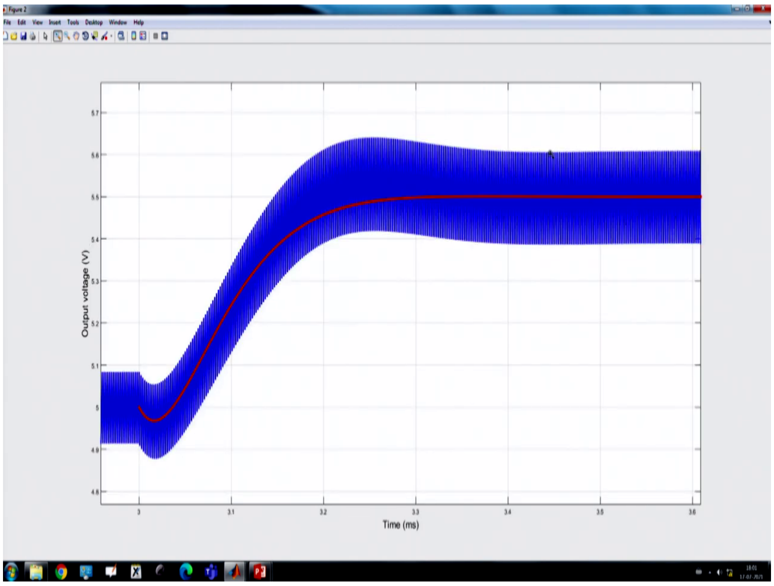
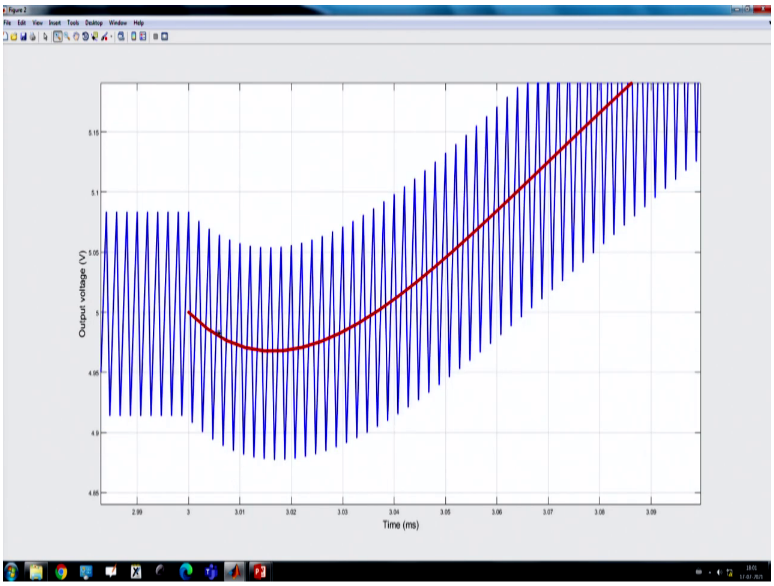


Yes. So, you can see that this is a small-signal response, and this is the actual switch simulation. So, initially I mean we have we want to change the reference voltage. We have applied a positive reference step change.

(Refer Slide Time: 59:10)



So, it should have increased from the beginning, but because of this right half plane zero, you see there is an initial undershoot. It is first coming down and then going up ok and this initial undershoot, depending upon the rhp zero location it can be severe.

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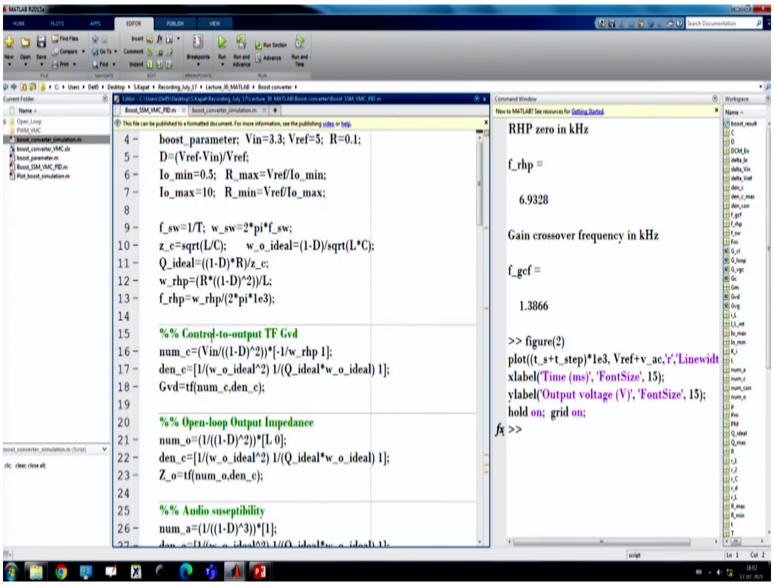

Suppose if I want to plot the same thing, and I change the load current. That means, I want to now change the load current; I am further increasing and I am not stopping it.

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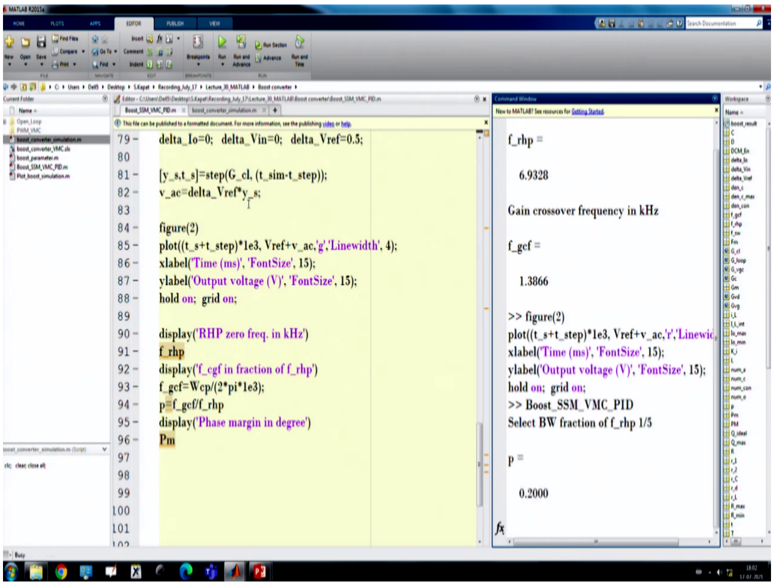

Suppose, let us use another colour. Let us say green colour and running. So, again, 1 by 5.

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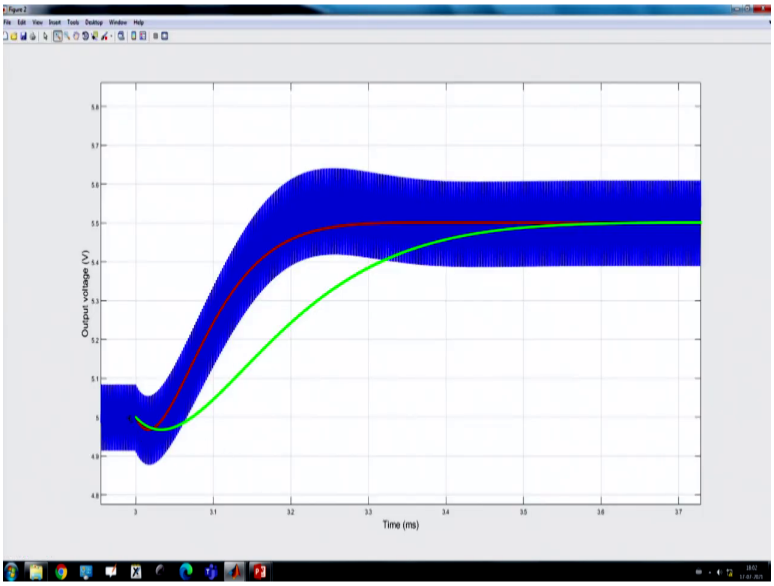

See now it becomes even slower sluggish because we are designing the converter based on this. I am showing an example of a closed loop.

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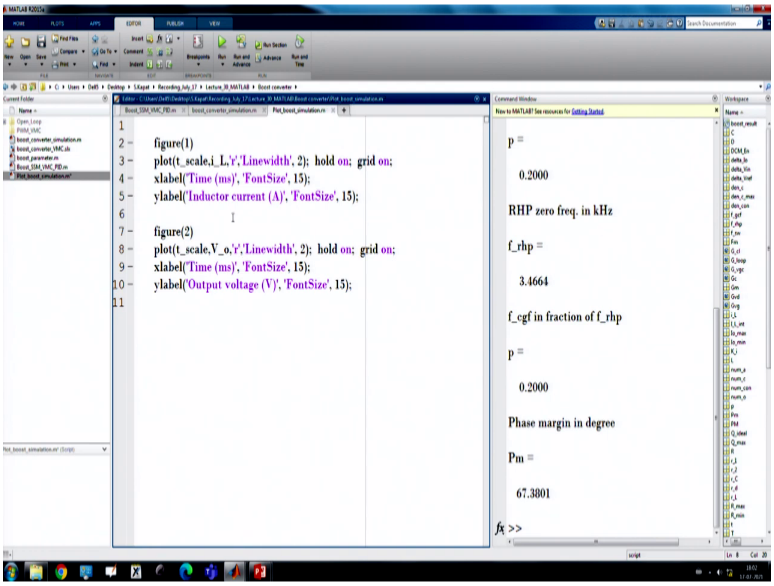
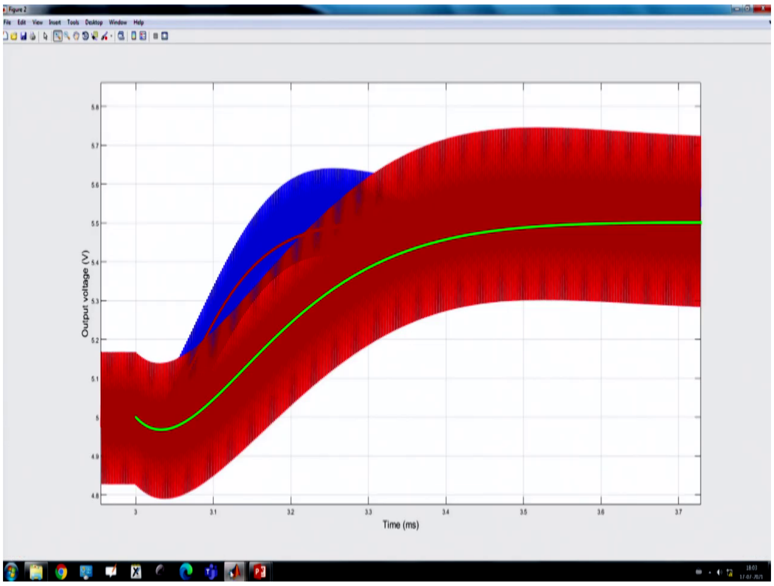


And here I want to rerun, but in the plot comment i want to use another colour. Let us say I am using you know what do I say earlier, red colour ok. So, red colour corresponding to this and I want to show you.

(Refer Slide Time: 60:15)



Yes.

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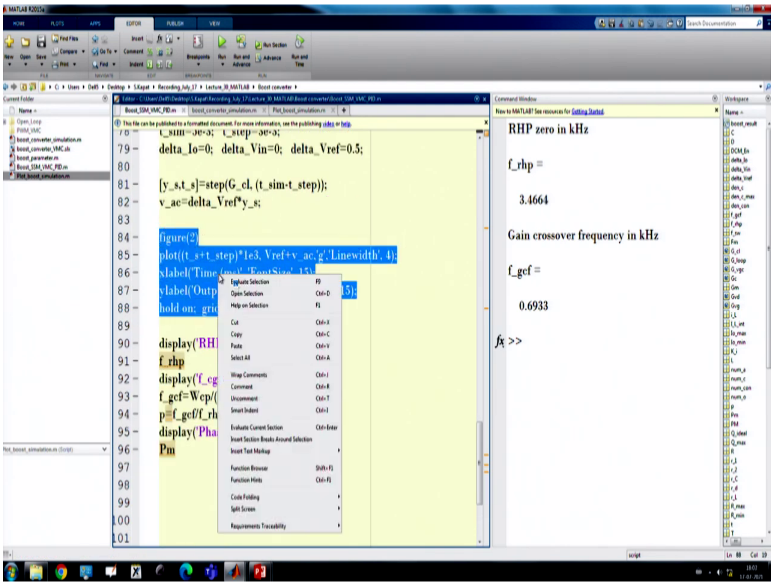

So, I want to run one more once more where this colours green colour should come evaluate.

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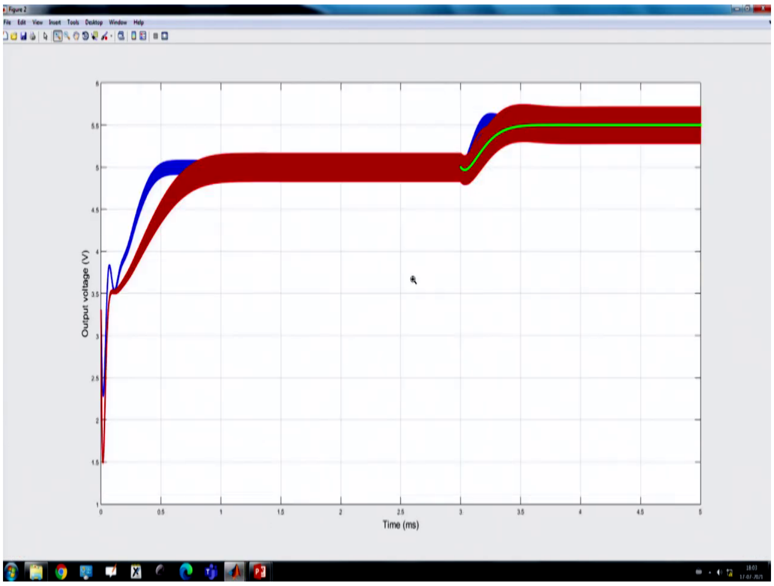

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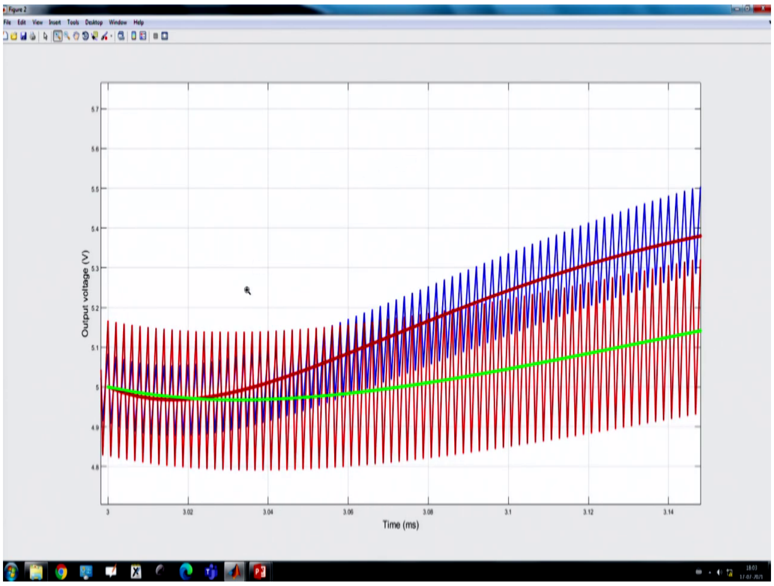

Yes. So, these are the two cases. The first one if we take the blue one where the right half plane zero. Now, the right half plan zero location become further, you know. So, it is even further. So, it becomes much slower and this bandwidth becomes slower because of the; so that means, we can simply compare without switch simulation. From this plot, we can simply compare this plot.

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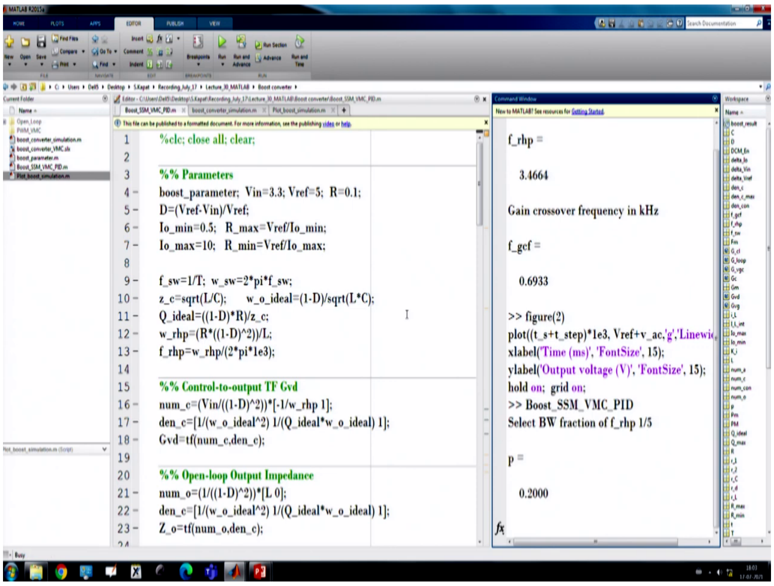

That means if we want to do R, this is the case.

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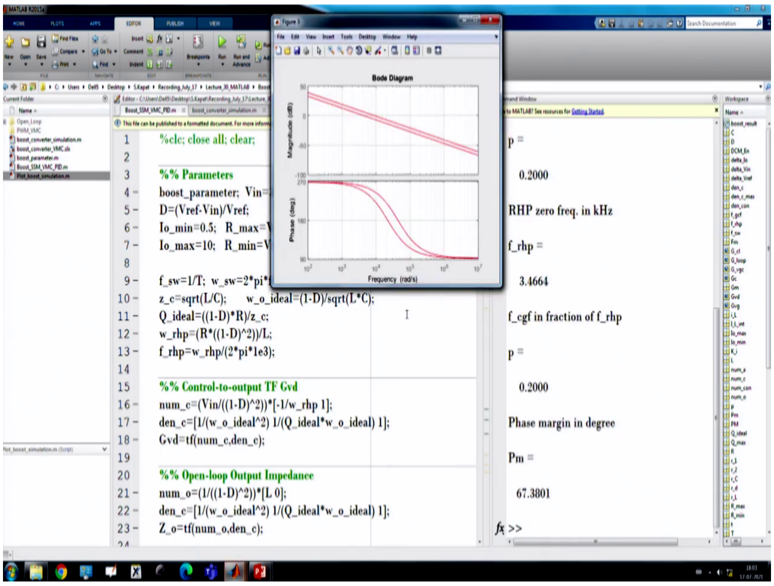

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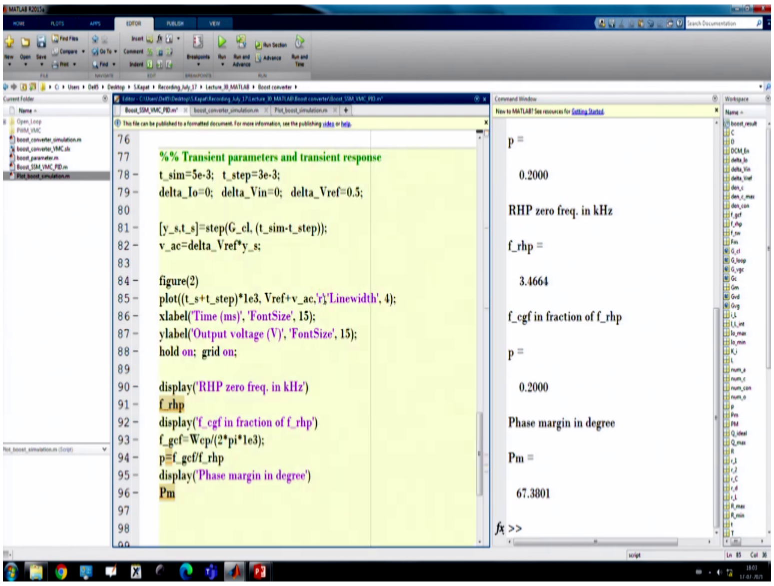

1 by 5 and if we hold this same value with a red colour.

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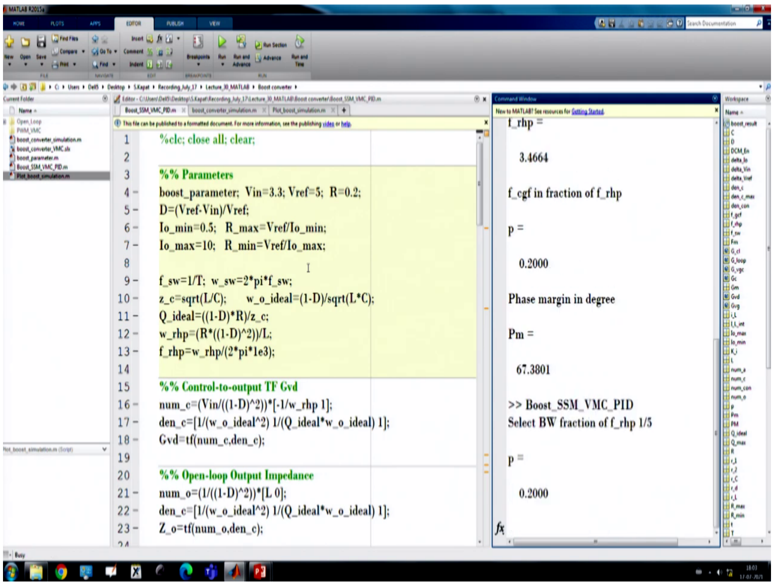

And if we change the load resistance to 0.2 and if we do 1 by 5.

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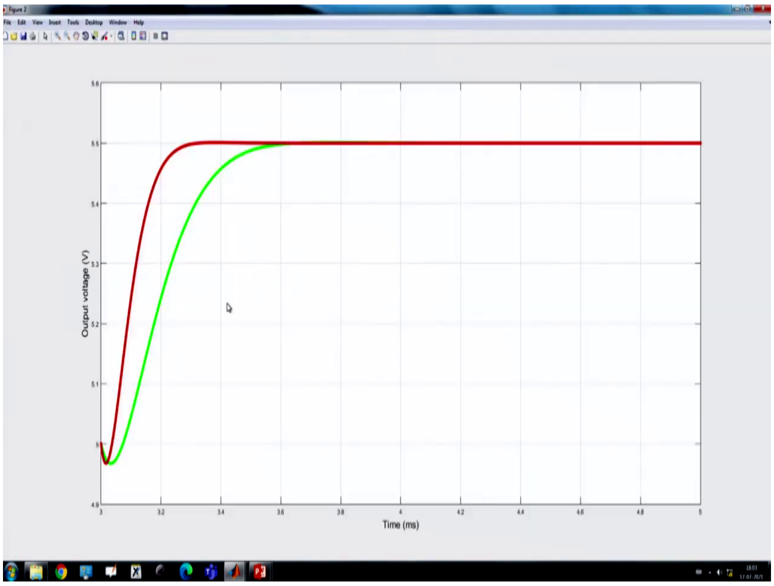

So, I am to show that because the second case right half plane zero is coming close to the imaginary axis. So, I am using you know the bandwidth of the controller is smaller.

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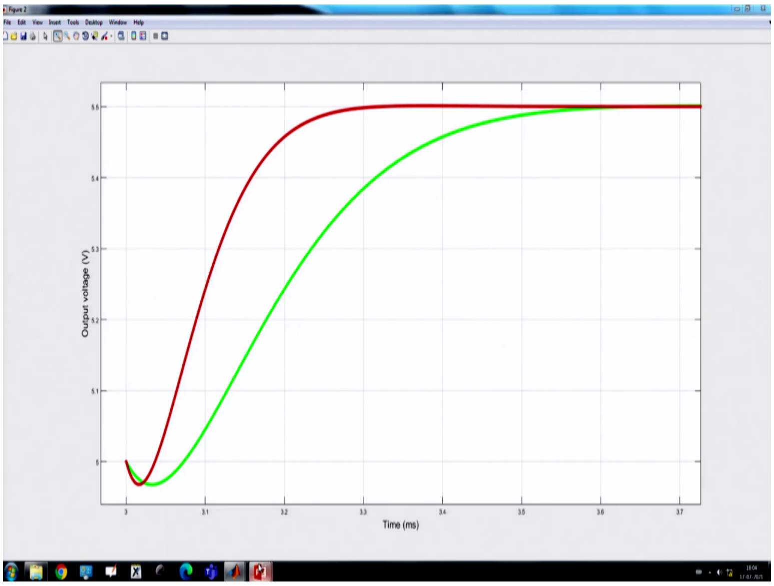
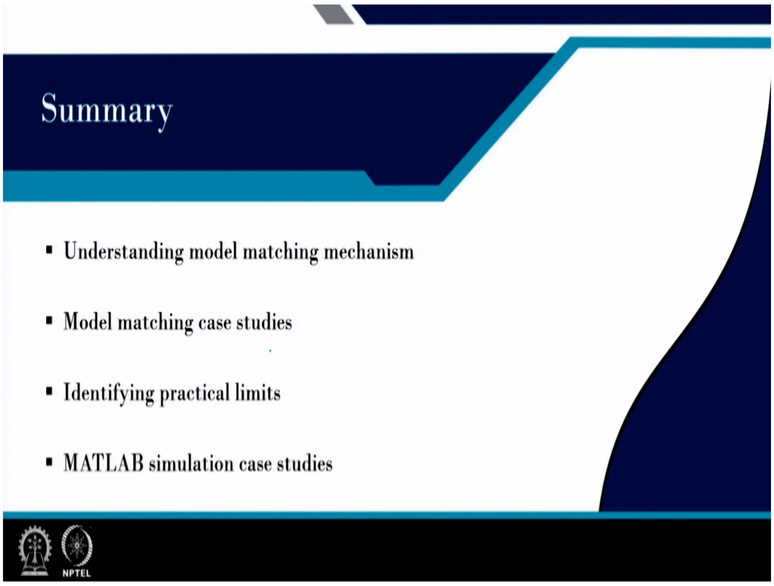


So, as a result; that means, if you want to design a closed loop control, if the right half plane zero comes close to the imaginary axis, then it put a constraint on your controller bandwidth. And in the green case, the right half plane zero is coming close to the imaginary axis. So, we have to slow down the system, otherwise it may be unstable. So, that is why the green colour is much slower than the red one and this is the effect due to the right half plane zero.

So, we will say more detail because we will design voltage mode control boost convertor. And we will show all these case studies there how to design the compensator so that we can get the maximum performance using voltage mode or current mode control in the subsequent lecture. But one thing I can say that I have shown you the response due to the small-signal model and the response due to the actual switch simulation they are closely matching.

(Refer Slide Time: 62:14)



So, with this I want to summarize that we have discussed like a model matching mechanism. We have shown some case study for transient AC simulation, then we have identified some practical limit and MATLAB simulation case studies were presented. So, with this I want to finish it here.

Thank you very much.