Control and Tuning Methods in Switched Mode Power Converters

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Module - 06
Small-signal Performance Analysis
Lecture - 29
Derivation of Small-Signal Transfer Functions

Welcome this is lecture number 29. In this lecture, we are going to talk about derivation of small-signal transfer functions.

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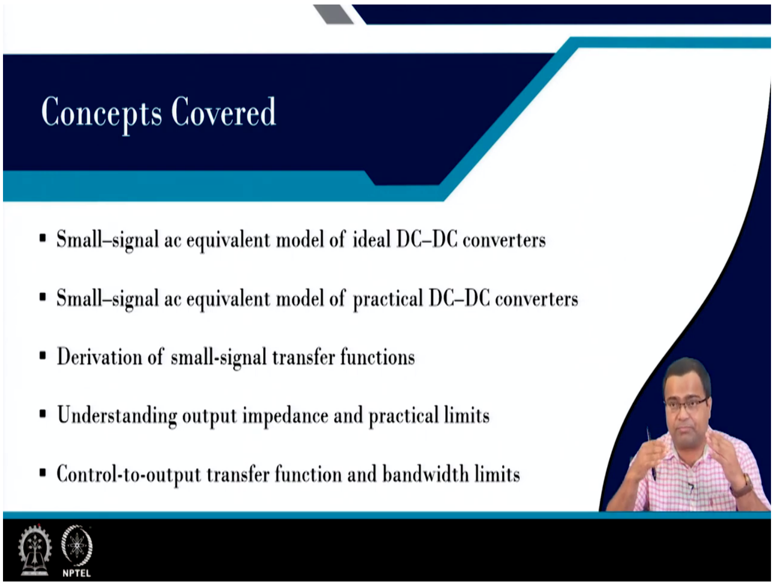

So, in this lecture we are going to first recapitulate our AC small-signal model of ideal as well as practical DC-DC converters, then we will derive various small signal transfer functions. And, then we try to understand you know, what is output impedance, what are the practical limit, then what are there, what is the control to output transfer function, then you know what are the bandwidth limit? So, we will try to understand some of the aspect in this lecture.

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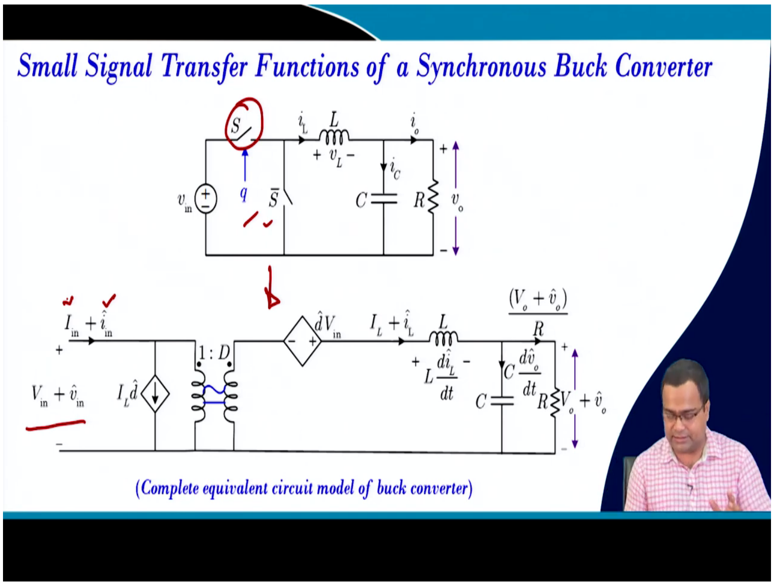

So, first we will consider small signal transfer functions of a synchronous buck converter. So, we have already discussed multiple times in the past that, if we take a synchronous buck converter, this is the ideal synchronous buck converter. In which you know this is a controllable switch, and this is a complementary switch, and this is a gate signal, the q is a gate signal.

So, if we you know if we recall that our large signal is equivalent, sorry large signal that equivalent circuit model, this buck converter can be ideal, buck converter can be represented by this ok. And, this is the input voltage and why it is large-signal, because we are considering both the perturb term as well as the DC term right.

So, it consists of both AC and DC and we have discussed in the past that this circuit can be used to, you know, we can carry out DC analysis as well as AC analysis.

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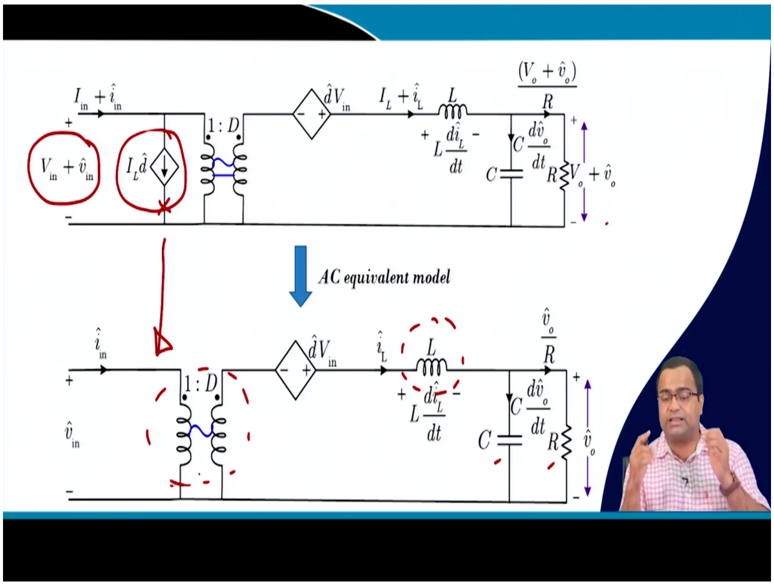

So, in the previous lecture, we have already completed the DC analysis. So, in this lecture we are taking this model, and we are considering the AC equivalent circuit and in order to give the AC equivalent circuit. The first thing will replace because you know this voltage shows if you look at this, this is a sum of a DC component plus AC.

Now, we can substitute all the DC components to be 0. So; that means, the voltage source will be shorted and current source will be opened. Another thing we will notice that if we consider this current source, this is this are dependent source, this is in parallel with the voltage source. And, since it is parallel with the voltage that is an input voltage, this has no meaning; that means it has no role so, we can simply drop it. So, we are getting this is the AC equivalent circuit.

So, in this circuit, we are all talking about the perturb model, where this is the inductor perturb model, like you know, as these are the inductor, capacitor, resistance; everything is there. And, this is the DC transformer, the step-down ratio 1 is to D, and in this circuit, we are going to consider the circuit to derive various transfer functions.

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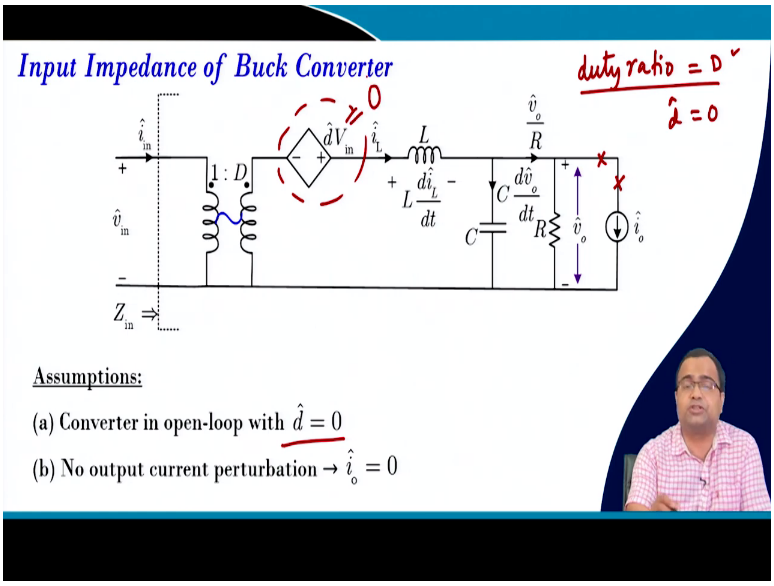

The first thing we want to do we want to, you know, derive input impedance. Now, we are all talking about the open loop buck converter, where you know there are few things; that means the duty ratio, if we consider duty ratio. So, we are giving a fixed duty ratio; that means, in this case, we are talking about duty ratio equal to D.

So, the D tilde in this case should be 0; that means, in order to derive input impedance, we have to set the duty ratio perturbation to be 0, but it does not mean there is no duty ratio. Because it has a fix duty ratio D but the perturbation is 0, because it is under fixed duty ratio operation.

We are also not considering this perturbation with this output current, because this is used it is an external sinking current. It is used to derive output impedance, but here we are considering input impudence. So, we can disconnect this particular path ok. So, there is no current and the current source should be open. So; that means we have removed this path and we have another thing since d perturbation d tilde is 0. So, this term will be 0 and; that means, we will short this path. And, this is exactly what we have shorted this path, and we have opened this path ok.

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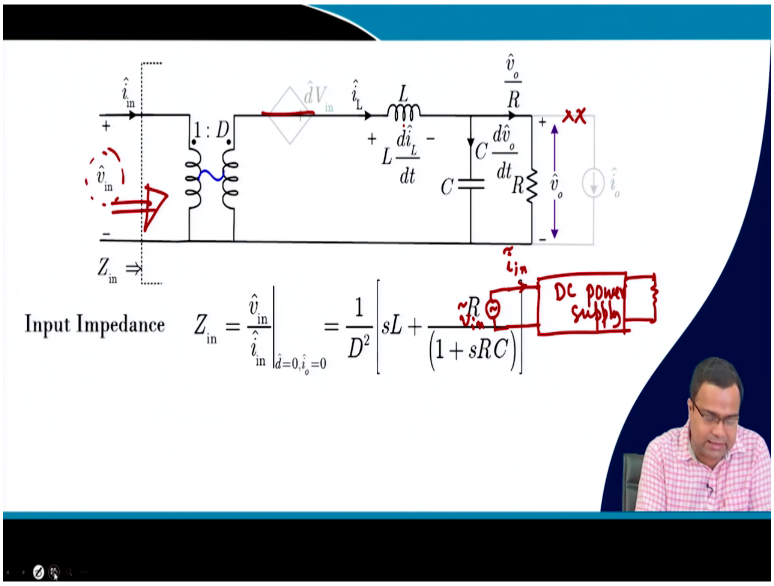

So, now if you look at the circuit from this perspective right as if you we are viewing from here, so as if we are giving an input excitation. So, you can think of, that means, if we have a laboratory power supply ok, you want to check the input impedance. So, this is like your DC power supply, your DC power supply. And this is connected to a resistive load. Let us say it is a resistive load. And, what you are trying to do, you are actually applying an AC excitation in the input voltage right.

Again, it consists of both DC and AC, but we are only considering the AC excitation. So, this AC excitation input perturbation if we apply and, of course, input current also, if you measure. So, if you measure AC input voltage, the AC term and that and the AC that input current component. And, then if we you know carry out that frequency response analysis; that means, if we sweep the frequency of excitation, then we can derive this input impedance right.

And, what does the input impedance indicate here? It is like L in series with the RC parallel branch; that means, it is nothing, but if we take this particular branch.

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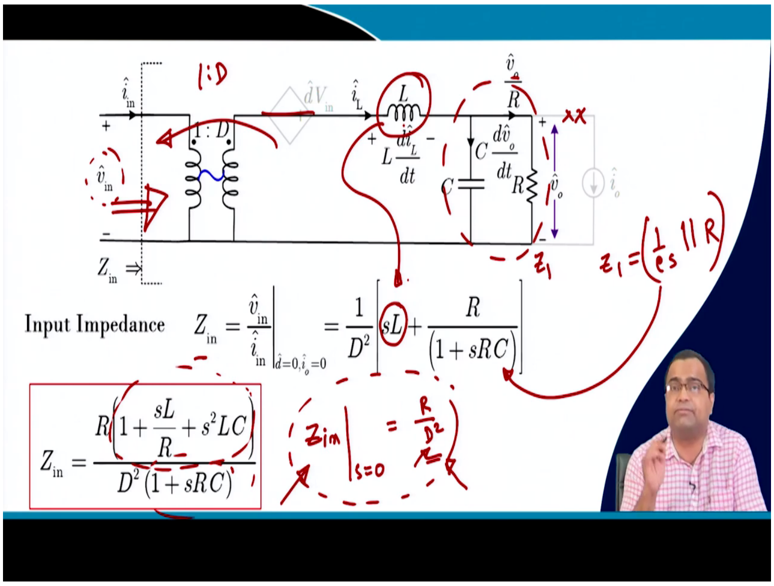

So; that means, here you can see s L is there, and it is in parallel. So, what is this branch? If you take these 2 be let us say Z 1. So, Z 1 is nothing, but 1 by c s that is the capacitor in parallel to the resistance. And, if you derive then you will get this part ok. And, this is added with the inductor impedance, which is this path, which has s L term. And, since this impedance if we take it to this primary site and there is a turn ratio 1 is to D, then we have to divide 1 by D square right, because it is an impedance right.

So, it is a turn's ratio square. And that will give you the input impedance; that means, the input impedance of a buck converter is nothing, but if you take the frequency dependent term right. So, it has it is an ideal buck converter, the input impedance has 2 0s right here, the impedance and 1 pole, because it is an impedance. So, do not worry about this improper transfer function, because it is an impedance right.

So, this input impedance, if you talk about the low frequency behaviour; that means, if you take s equal to 0, then what you will get; that means, you will get simply R by D square you will get R by D square. So, this is the input impedance of an ideal buck converter at low frequency, very low frequency.

In fact, this input impedance plays a significant role when we want to let us say it can be buck or boost any other converter. Suppose we want to connect to a solar panel, because we want to do maximum power point tracking or we want to extract maximum power. So, we need some kind of impedance matching right. So, then we have to adjust the duty ratio in order to match the impedance right.

So, in this case you can derive the same thing for a boost converter and because boost converter is generally used for you know extracting maximum power from a solar panel, because the boost converter input side you have inductor and the current is continuous right. So, I am just saying that the how to derive the input impedance. And, this input impedance plays a significant role in terms of impedance matching on the source side.

When you connect another source to this converter? It plays a significant role in the design of the input filter. Because you know we have already I think we are going to we have already discussed in the in the previous lecture, that you know in past lecture, I forgot exactly lecture number where we talk about the impudence analysis.

So, we have used the Middlebrook's extra element theorem 2 to identify the stability criteria for C is connected converter right, or when you put on filter then how do justify stability? In such cases, the input impedance plays a significant role in the converter's stability, right.

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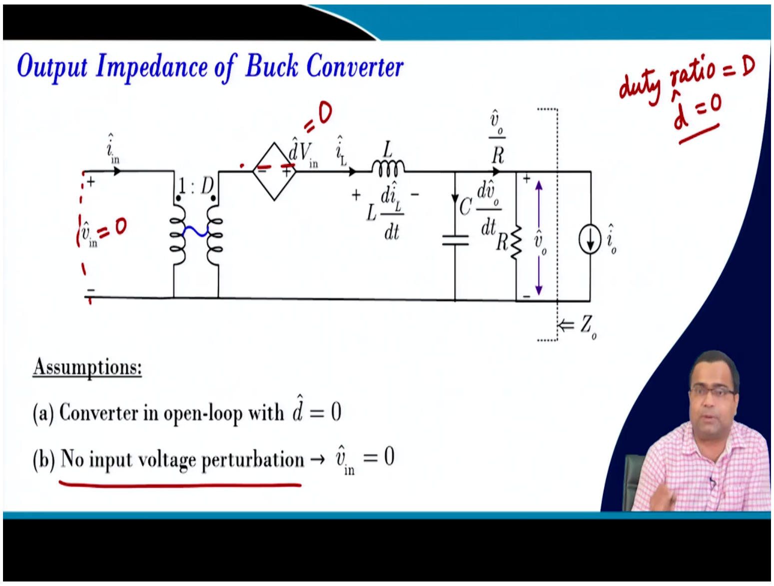

Now, we want to talk about output impedance. So, if you talk about output impedance now, again we are talking about the open loop output impedance where, we are applying a fixed duty ratio gate pulse. That means, our duty ratio, our duty ratio again will be set to duty ratio set to D capital D steady state and d tilde equal to 0 right.

So, again, this term will be 0 and we will short this path and here we are not talking about any supply perturbation; that means, there is no supply perturbation. So, if there is no supply perturbation, this term will also be 0 and if it is 0 this is a voltage. So, you have to short this path.

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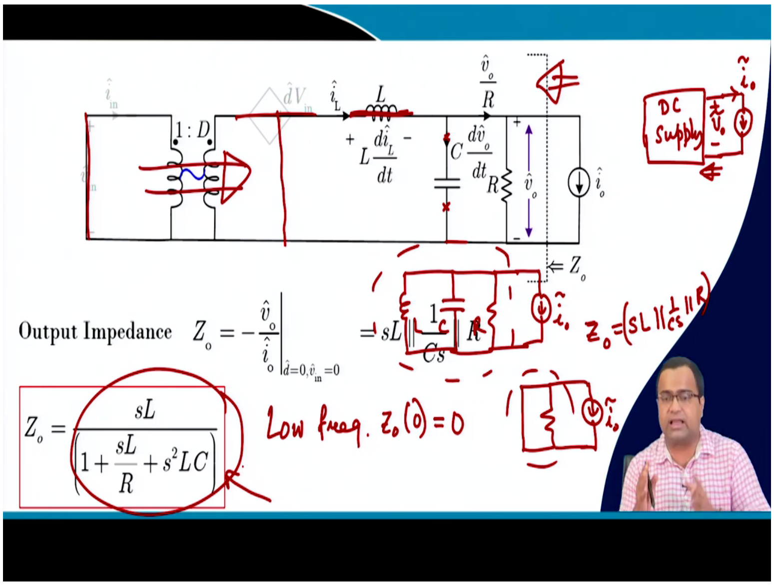

So, in order to find the output impedance, we have to short this path, because we are taking perturbation in the input voltages to be 0 and any 0 voltage is nothing, but the shorted path right. And, similarly since the duty ratio perturbation is 0 this is also shorted. So, now, we want to find out the output impedance as it we are looking from this side. Again, as if we have taken a DC power supply, a DC supply ok. And, this DC supply is connected to an external load right.

So, it is connected to an external load, and this is my i 0 sinking load and again I am talking about the AC excitation; that means, it will have DC term, but we have to separate out the DC term. Now, we are trying to measure the output impedance by looking from this point; that means, basically it is v 0 by i 0. Since it is sinking current it is minus v 0 by i 0, that is the output impedance right.

So, in order to derive the output impedance, you will find since it is shorted. So, we this will resemble like if you short this path simply. That means this will represent something like an inductor, in parallel to a capacitor and a resistance. And this is connected to your external current source sink.

So, this branch is actually representing your output impedance right. So, L C R, but if you do not consider resistance to be the part of the converter, then it is just L C, but we are talking about an ideal converter. In a practical converter, we will have a parasitic of the inductor, parasitic of the capacitor; they will play a significant role in the output impedance.

So, ah; that means it is a parallel combination; that means it is written as the output impedance is written as S L parallel 1 by C S parallel R. So, this is the branch. And, this can be derived like this right. That means, at very low frequency, if you said S equal to 0; that means, the low frequency behavior.

If you say the low frequency behavior, that means Z 0 0 close to 0 this will be simply 0; that means you can view from this circuit; that means, if you take these R L C branches at very low frequency, this inductor will behave like a short circuit right, very low frequency DC. Capacitor will behave like an open circuit right. You will get something like a shorted path in parallel to a resistance and that is your external current source right.

So; that means, this will like a 0 output impedance right. At very high frequency, the inductor will become like an open circuit and capacitor becomes a short circuit, again it will be 0. So; that means, in an ideal buck converter at very low frequency and at very high frequency, you have almost 0 output impedance. But in the mid range you will get this dynamic effect of the frequency dependent term, but in a practical converter we will see it will not be 0 very high frequency, neither it is 0 at low frequency. So, it has some finite value.

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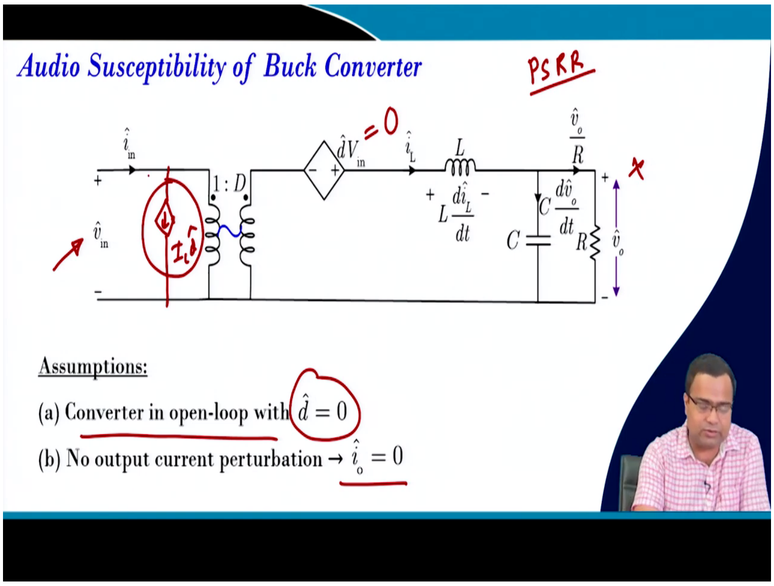

Audio susceptibility; so, audio susceptibility means we are talking about if we apply some perturbation in the supply, input side, we want to see what is the effect in the output? And, this is particularly important because you know any power supply. We want to see we know about something called the power supply rejection ratio, power supply rejection ratio.

That means, if there is any disturbance coming from the supply side, whether our output converter is able to completely attenuate that effect in the output or not, or is it going to get propagated to the output side. Here we are talking about the open loop DC-DC converter. Definitely this open loop converter this will be propagated, but once you close the loop, we want to make sure that, it is one of the control objective that; we want to reject the disturbance due to the supply voltage, those that should not appear in the output voltage. So, the first is that the converter in an open loop.

So, the d perturbation to be 0 that we have discussed, here we are not talking about output impedance. So, external current sink is also 0; that means that branch is open. That means, this will be 0.

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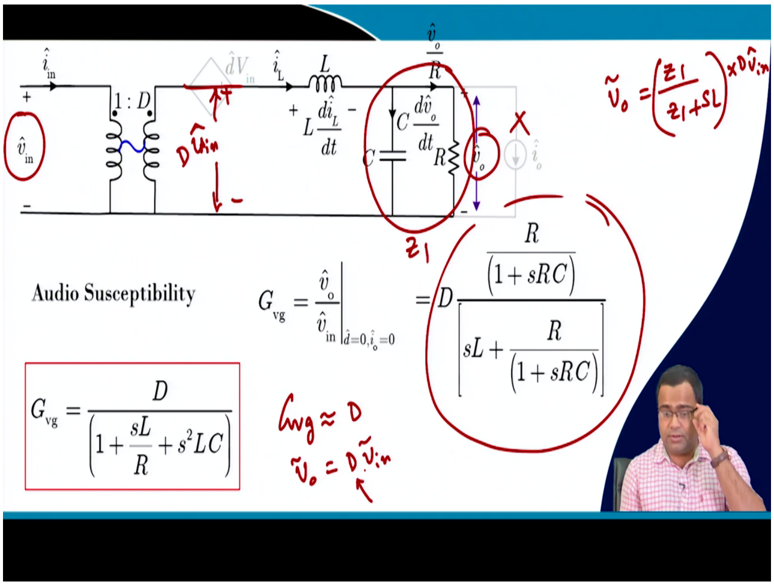

And this circuit will look; that means we have a supply disturbance and we want to see the effect in the output voltage and this path is the eliminated. This is also shortened. And, in all cases, we have eliminated this branch, which was there earlier, which was like a I l d term, because we discuss this will have no meaning, because it is already connected across a voltage source. That is why you have eliminated.

So; that means we have removed this path, because this has no meaning here, ok. Now, we want to derive this audio susceptibility and this can be derived by taking again this branch impedance and then the output voltage; that means, if this is Z 1. So, output voltage will be Z 1 by Z 1 plus S L this whole into what is the voltage across this path. So, this will be v in by into D sorry v in into D.

That means it is v in into d right. So, v in into d this term ok and this impedance so, it will be D into v in hat. So, from here you can find out this expression ok. So, this is the expression of the control to and you see at very low frequency, the audio susceptibility is simply equal to D; that means it is nothing but D into v in and which is consistent with the buck converter relationship. That means, if there is any disturbance slow frequency disturbance at the input supply side, it will be propagated to the output side by multiplying a scaling factor.

That means that, very low duty ratio operation, this effect will be small or it will be you know the effect will be negligible, not negligible. I think it will be low. But, if the duty ratio is high then, that effect will be propagated to the output side like a substantially it will be propagated right.

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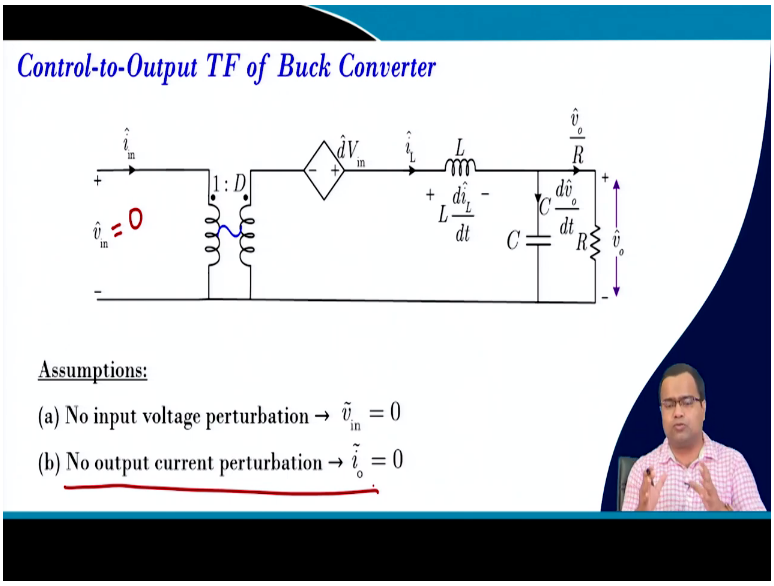

Another part, which is one of the most crucial part, because what we have discussed so, far audio susceptibility input impedance output impedance of an ideal buck converter, but in reality we are going to control the converter. So, for controlling the converter, the most important transfer function is the control to output transfer function.

Because, if we understand this control to output transfer function, then by means of close loop control, we want to achieve some desired output impedance, some desired input impedance, some desire audio susceptibility. So, this is one of the most important. So, here in order to achieve control to output transfer function we will simply say this to be 0, there is no supply disturbance and we will also not consider any output perturbation, during the process of derivation of control to output transfer function.

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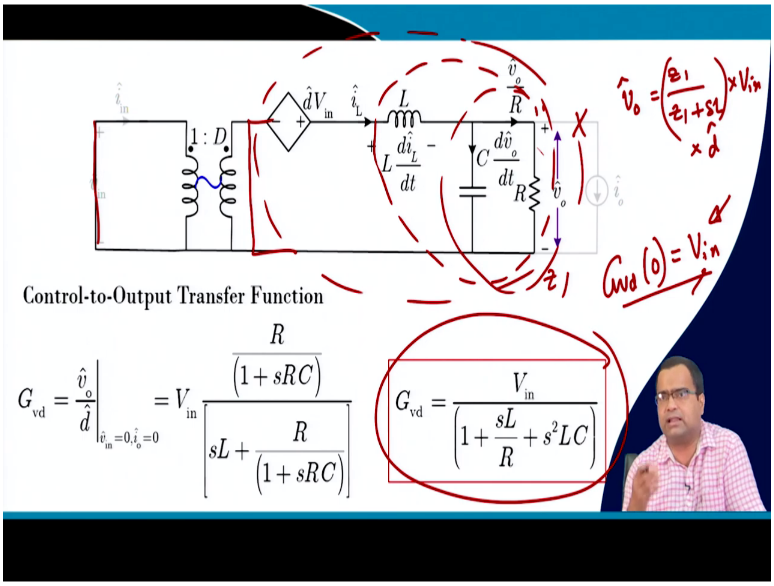

That means the circuit it will be shorted, and this path is already open. Now, we need to find out the transfer function of this and since it is already shorted so, this you can short here right. So, you are talking about this particular circuit; that means, if you want to find out the control to output transfer function again, if this is my Z 1. So, my v 0 hat will be Z 1 plus Z 1 S L this whole thing into V in into d hat right.

So, we want to get v 0 by d hat and if you do that you will get this transfer function like this. And, you will find here in the control to output transfer function in the like a DC gain; that means, if you take G v d DC gain, this is simply input voltage. That means, in this case, the DC gain is nothing but the input voltage; that means, if there is any variation in the input voltage, your DC gain is going to vary drastically vary, it is going to vary, because it is input voltage dependence ok.

So, if you want to control this converter my means of direct duty ratio control, then this is one of the problematic like input voltage dependency the DC gain. And, in order to reduce this effect, you know you can consider input voltage feed forward in order to do that and that we have already discussed.

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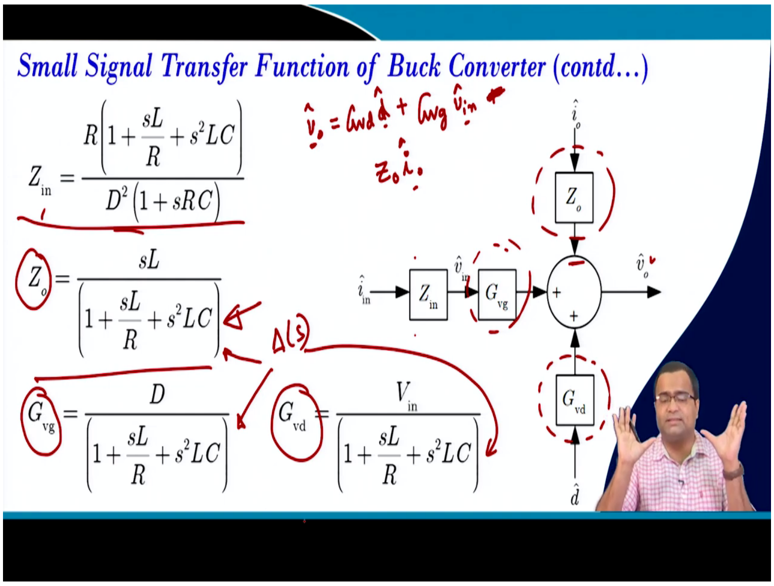

So; that means, small signal transfer function of a buck converter, we already talked about output impedance of the open loop buck converter, we already talked about the control to output transfer function, we already talked about audio susceptibility right. If we talk about from v in to v 0 and if we talk about input impedance also we can consider right.

Now, you will see the output voltage perturbation here. It is the sum up all this; that means, if you write down ok. If you write down v 0 tilde it is nothing, but G v d into d tilde plus G v g into v in tilde plus sorry minus, because there is a negative sign here, here minus output impedance into i 0 tilde. What does it indicate? That means, if you do not consider any change in the input voltage or load current; that means, a positive change in step in duty ratio in the output voltage should increase.

Similarly, a positive change in the input voltage your output voltage should increase, but a positive change in load current, the output voltage, is going to decrease. And, in an ideal converter under steady state again it will come back, but this negative effect will come like a undershoot behaviour. But, in a practical DC-DC converter you will see, the output voltage will get shifted and it will slowly reduce, if we increase the load current.

We have discussed in I think lecture number 19, that a practical voltage source if you take a series resistance in internal impedance. That means, as you draw more and more current, then the drop across the resistance increases as a result of voltage also decreases.

So; that means, this is the input impedance we have discussed, this is all ideal converter output impedance also we have discussed and audio susceptibility and then the last one is the control to output transfer function. You will see in output impedance audio susceptibility and control to output transfer function, the numerator polynomial; that means, the poles are common they are same. That means, it is like a delta S, which is common for all here, here as well as you can see it is here.

Input impedance is different because it is dimensionally different. There is something like this. Poles will have a significant role in all these three cases. Like and of course, it will also affect the input impedance, but we generally do not talk about input impedance, unless we talk about input filter or more converter connected to the like an input side, otherwise for a single DC-DC converter.

If we are only talking about load transient performance you know for; I mean if we assume that your control converter bandwidth is not affected by the input filter. Then, we can simply analyze the transient recovery and everything by simply this three transfer function, where the poles are common. So, the poles play a significant role right.

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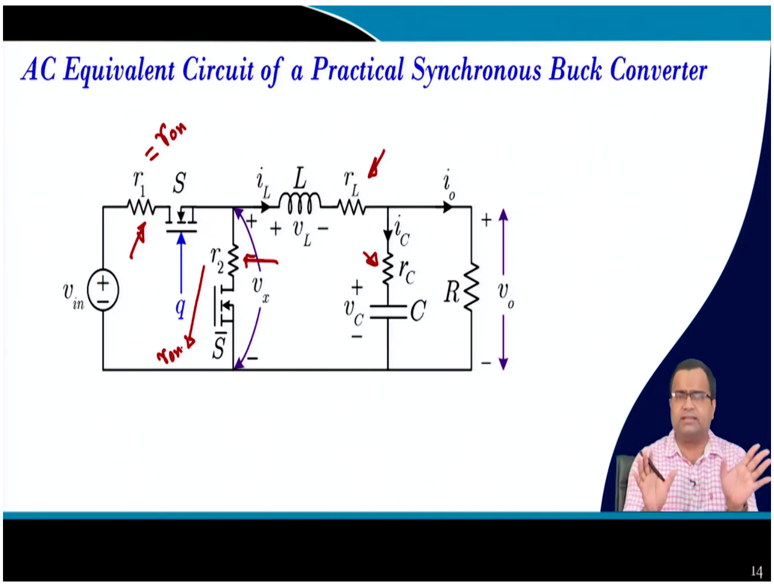

Now, if we talk about a practical converter where we are considering the on time resistance of the switches, the two switch the DCR of the inductor ESR of the capacitor. Then, we already know about the equivalent circuit.

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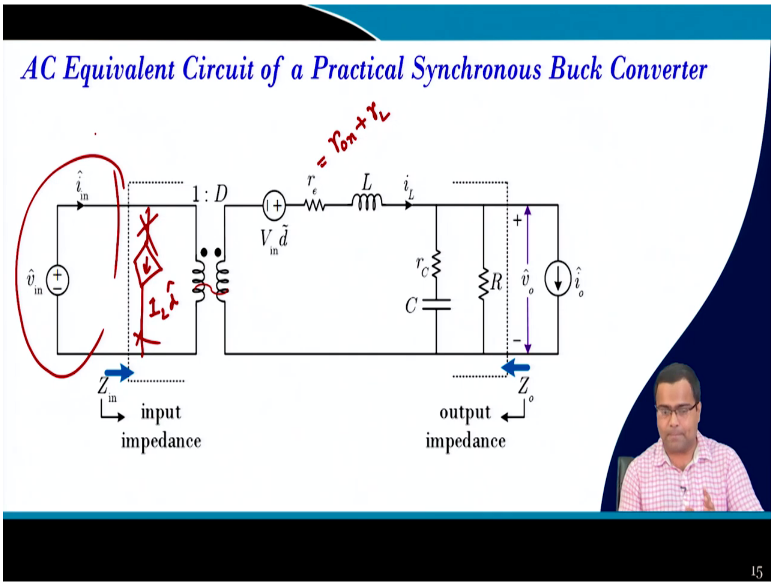

So, we know what is r e equivalent it is nothing, but the on state resistance of the switch plus r L. And, you are assuming the on state resistance of these two switches, they equal to r on. That means they equal to this. We are taking more or less same although there can be slight difference, but still this is applicable.

So, this is r_eq equivalent and this ESR. We have already discussed this equivalent circuit model. So, we are only taking the AC equivalent circuit. We have dropped this term there as one extra term was there which was I L d hat we have dropped it, because we have to ignore this, because it is connected across the voltage source.

So, it does not make any sense, because if you connect a current source in parallel to a voltage source, then that whole network behaviour will not be affected; that means, in analysis, we can simply remove the current source, ok. For the AC analysis ok so, all this case.

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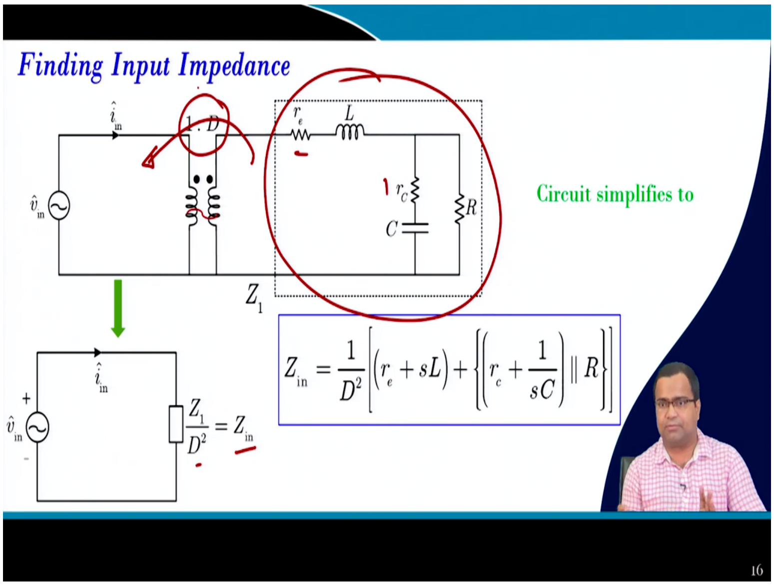

Now, input impedance the same method that we discussed only with parasitics right and what we will see the circuit simplifies to this. So, this is the input impedance, where this is Z in. And, this term if we take it to this side; that means, I am telling the whole thing like a Z 1, this whole branch as Z 1. And, if you take it will be 1 by D square, because there is a step-down ratio right and then you can obtain the input impedance expression from here.

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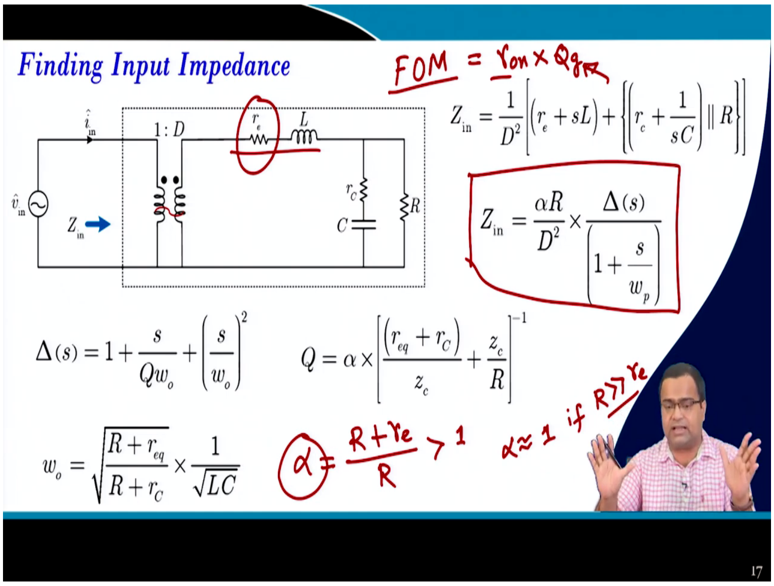

And, this input impedance can be shown here it is nothing, but this expression, where the alpha is nothing, but R plus r e, that is your r equivalent divided by R. So, generally it is greater than 1, if r e is greater than 0, but it can be approximated to 1, if R is very very greater than r e right.

And, generally, that is true because your load resistance should be higher than the equivalent resistance. But, suppose we are talking about a very high current application, low voltage high current where the load resistance itself is very low, because your load current is high. And, you have chosen an inductor as well as switch where the equivalent resistance can be comparable with the load resistance, then this alpha will have a significant effect ok.

But, generally the design thumb rule is that a load r e should be small, but that poses a constant because if you want to find an inductor with a very low DCR right. If you want to reduce the DCR significantly, then you have to pay more price right for a low DCR inductor. Similarly, if you are talking about low RDS on switches, then you need to figure out that you know because the transistor there is a figure of merit it FOM. In a MOSFET, this is generally r d s on multiplied by Q g; that means, on state resistance multiplied by the gate charge.

So, generally if you want to select a MOSFET with low r d s on then it may. So, happen that you will end up with more Q g. So, you if you operate at higher stitching frequency, then a switching loss will increase due to this Q g ok. So; that means, we have to select this component very precisely about how to select MOSFET, inductor and so on.

But, what I am trying to show here, even if you select some MOSFET or inductor and if you can extract the value of you know DCR on state resistance from the data state, then you can obtain the output impedance input impedance from this expression right. So, all these parameters the pole 0s are given here ok.

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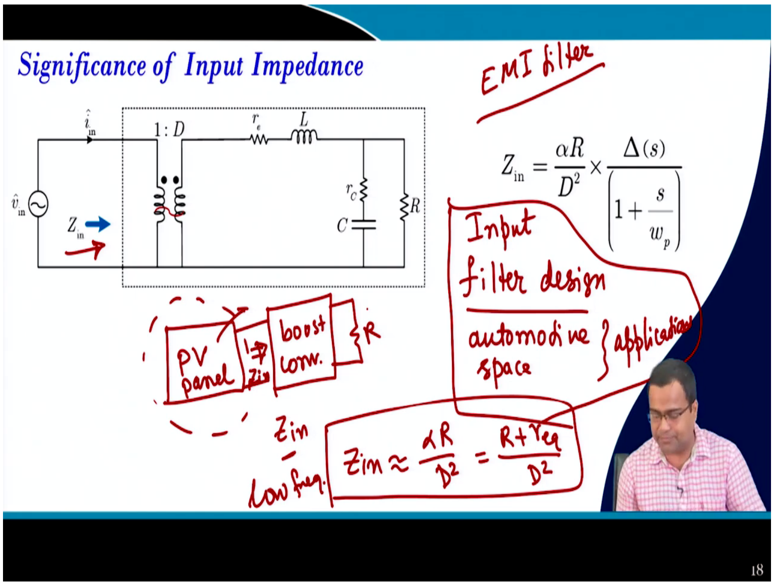

Now, if we what is the significance of the input impedance. As I told the input impedance plays a significant role when you want to talk about input filter design? Because most of the DC-DC converter, suppose if you are if you go for automotive applications. If you go for automotive application, space application, we have discussed these applications.

So, in this application, you need to meet certain stringent EMI constraint that means; you need to consider one input filter which is we call it as a EMI filter. So, when such EMI filter is considered, then it becomes very important to justify the stability of the converter with the EMI filter.

And, then in order to justify stability we need to get the transfer function of the EMI filter, and we need to get the input impedance of the converter, and then using Middlebrook extra element theorem, you can justify the stability of the converter along with the input filter ok. So, the input filter is play a significant role. The input impedance plays a significant role if you want to design a stable power converter with an input filter.

But, suppose if your input filter bandwidth is low, then your control bandwidth will also get reduced, because your you have a high inertia input filter at your input side. So, you cannot slew of the inductor can fast. So, as a result you have to slow down the converter right. Otherwise, what will happen if you want to speed up it will become unstable, because your input filter has a low bandwidth and your converter high bandwidth. So, it cannot because in a cascaded system, the overall bandwidth will be limited by the worst case bandwidth right. So, that thing also we have discussed.

Another aspect is that if we want to consider. So, it is a buck converter. You can do the same thing for a boost converter right. So, suppose if you talk about a PV panel; that means, you are talking about a PV panel solar panel and you have connected a boost converter right, a boost converter ok. And, maybe you have a resistive load at the output side.

So, then it is very important to find out what is the input impedance of the boost converter right? That means, it is important to find input impudence and, since the PV panel the changes. The dynamics of the PV panel changes very slowly because you know it is in the order of second or even minutes or in the hours.

So, we will consider the low frequency behaviour of the input impedance, and which can be shown at low frequency, the input impedance will be simply at low frequency alpha R by D square. And we know what is alpha? It is; that means, it will be R plus r equivalent divided by D square.

So, this is a low frequency behaviour. This is particularly important for that maximum power point tracking and such kind of application where we need to take into account the impedance of the source as well. Because, you need to match, because you know you can apply the maximum power transfer theorem. So, if you can derive the input impedance and then if you consider considering the resistance, then you need to find out at what point should be operated to extract the maximum power.

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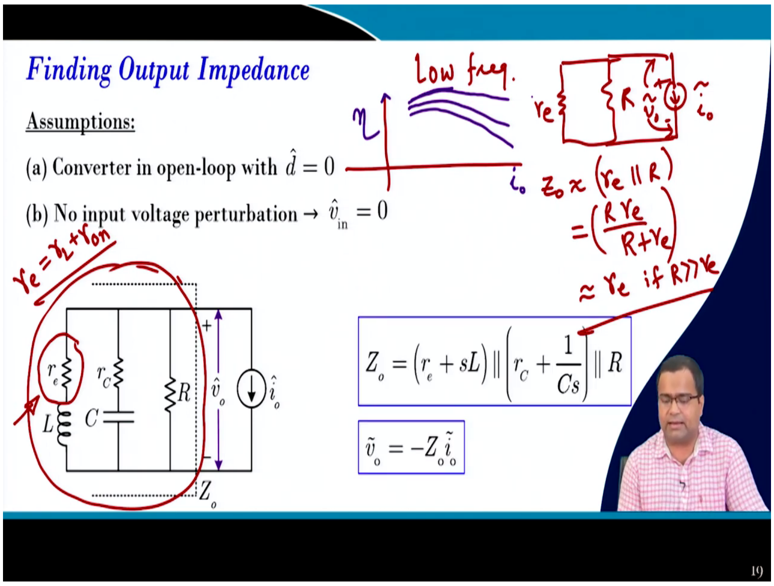

Now, output impedance we have already discussed output impedance of a of an ideal buck converter. If you consider a practical buck converter right so, we know that output impedance will look like the inductor, capacitor resistance will be in parallel. And, I do discuss at very low frequency this inductor will behave like a short circuit; that means, at low frequency.

So, if you say the low frequency behaviour, it will be like r e in parallel with resistance R right and that will be connected to your external current ok so, external current ok. That means, in this case, the output impedance can be simply r e parallel R, which is nothing, but r into r e by R plus r e. And, this will be equal to r e r e is very very small. And, this can be shown that it is almost equal to r e if R is greater than r e right.

So; that means, this is a limit of the output impedance at low frequency, which is coming due to the D C R; that means, r e is nothing but what or r e is nothing, but r e is nothing, but D C R plus r on. So, you have to suitably select the switches with low RDS on low DCR.

So, that the DC impedance low frequency can be smaller, but again, these are open loop characteristics. Even if we have a finite the low frequency resistance like effect, by means of close loop control if you put an integral action, you can achieve very tightly regulated output voltage. So, this will not cause problem.

But, this will cause a problem in your efficiency, because we have discussed the efficiency strongly depends on this r e. If r e is high, then efficiency will degrade, particularly when the load current increases. And, you will you end up with something like this kind of profile; that means, you will get something like this kind of profile as you increase load current, when your efficiency you will fall right. And, if your r e is higher and higher so, this profile will tend to even sharper ok.

So, you need to choose; that means, and if you have a very good r e you may get something like this kind of curve, almost flat curve ok.

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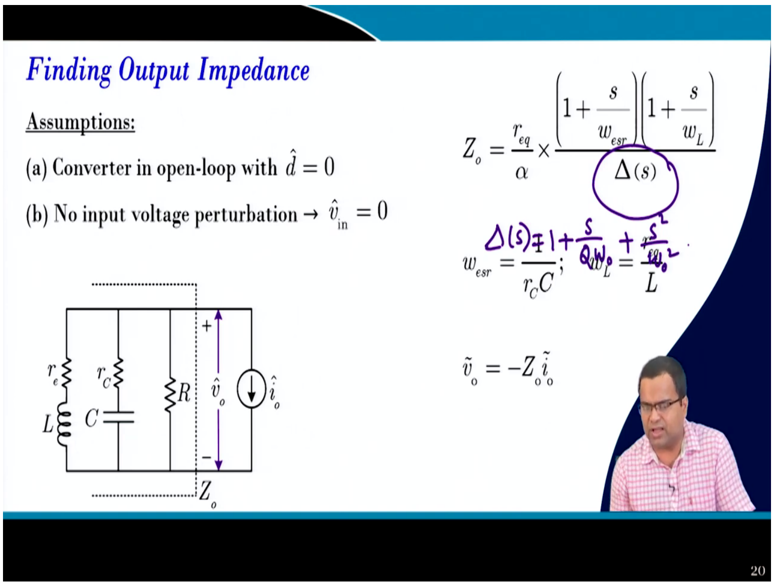

So, this r e can be calculated and this will look like that means, as I said, that your poles are common and we have already discussed the pole. What as the pole? So, here delta s 1 plus s by omega 0 plus s square by this is natural frequency right yes. So, those we have discussed and you have two additional you know you have 2 0, 1 0 is due to the e s r. So, the ESR 0 is due to the capacitor and the other 0 is coming due to the inductor that equivalent ok.

Even if you take an ideal like if there is ESR is very low. So, this 0 will come into the picture.

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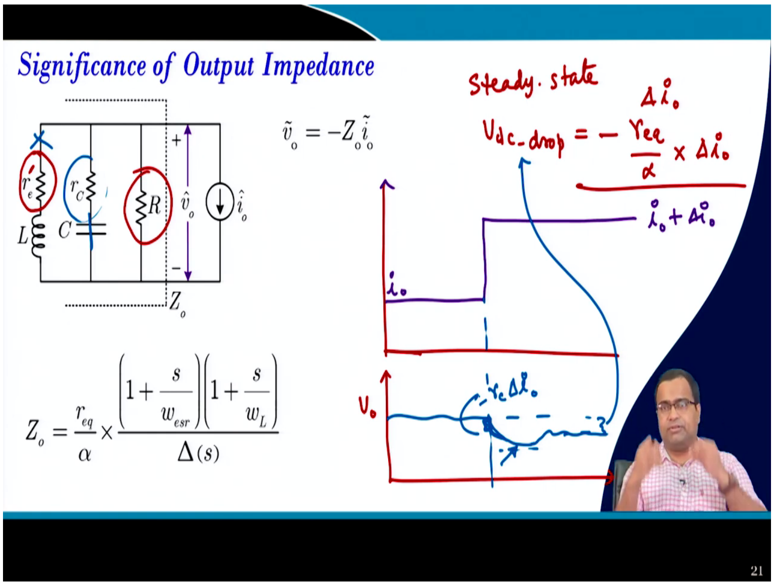

So, what is the significance of output impedance? Output impedance plays a significant role in your load transient response. So, if you take you know DC-DC converter idea like an open loop DC-DC converter, you will see, if you want to get at steady state, at steady state. Suppose we apply a load state transient of delta i 0, then you will find there is a DC drop. You can say a drop voltage, which will be minus of r e equivalent by alpha into delta i 0.

That means, this drop is coming due to this particular branch and along with this branch ok. So, this drop if there is no parasitic resistance, then there will be no drop; that means, if you take two steady state current; that means, if I consider, let us say your load current was here. There is I am talking about a DC-DC converter. The load current is here; there is a load transient and it has gone up.

Now, if you take the profile of the output voltage, output voltage was like this ok. So, sorry I am talking about the output voltage. So, if we take so, this is my load current i 0 plus delta i 0 and this is your i 0. Now, I want to draw another graph. So, I want to draw below another graph here and this is my v 0 ok. So, what I want to do here if I extend this. So, my output voltage like this suddenly there is a transient effect, but it will get settled somewhere here.

So, this difference is you called as a drop delta or I will say. So, this is nothing but. So, this quantity is this drop is nothing, but this drop this is a load state transient ok. But, if there is no r e then it will come back again to this point, if there is no r e, but this is not the case for a practical converter. So, there will be some drop right. Again, the transient behaviour; that means the frequency dependent term will shape the undershoot, like if there is an overshoot undershoot or such kind of behaviour.

How far like? What will be the maximum, like you know, undershoot that can arise under load step input voltage. So, those will be decided by this transfer function frequency dependent term. And, the high frequency there will be jump, there will be jump here and this jump will be esr. Because, that very high frequency this will be open circuited. This is a short-circuited and this r c will come.

So, r c will create a drop so, very high frequency. So, you will get a jump; that means, every load transient you would have a jump and this jump will be r c into delta i 0 that state and this is generally negative; positive load step to a negative charge.

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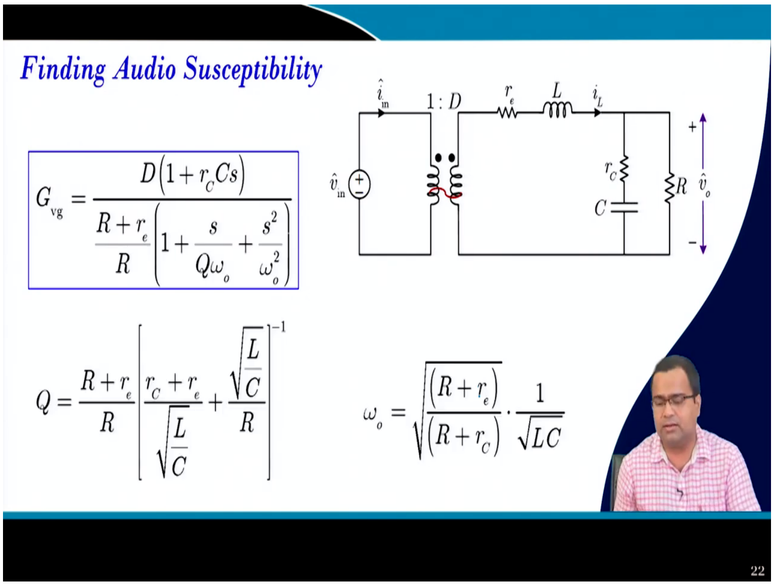

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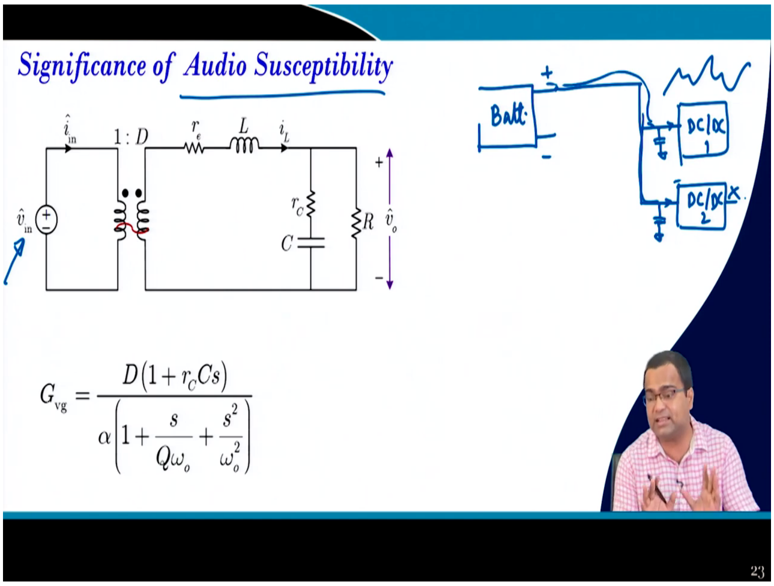

Similarly, we can find out audio susceptibility for a practical converter and we can get all the transfer functions and what is a significance I told you? That audio susceptibility if there is any disturbance, particularly where the term audio susceptibility comes. Because, if there is any audible noise present in the supply source, which can be because it may. So, happen that suppose you are talking about a battery of a let us say your mobile phone where the multiple converters are connected; that means, this is your battery, you know I am talking about a battery.

So, battery has a positive terminal, negative terminal. Now with this positive terminal you have like a DC DC 1, another like a DC DC 2 like this and though there is a small small input filter. But, you know in if you go to PMIC like a power management circuit, it will be very hard to put large capacitor.

So, there will be a small small input cap will there for each converter, because of power density is a concern. Now, suppose if there is there is there will be input current like, it is drawn by this converter. Suppose, for some reason, one of the converter undergoes sub harmonic in stability kind of behaviour.

So, that will be reflected in the battery, and this battery is connected to the other converter. So, this can propagate through the converter. So, if you have a very good audio susceptibility, it will be rejected at the output end, ok. So, that is why it plays a significant role.

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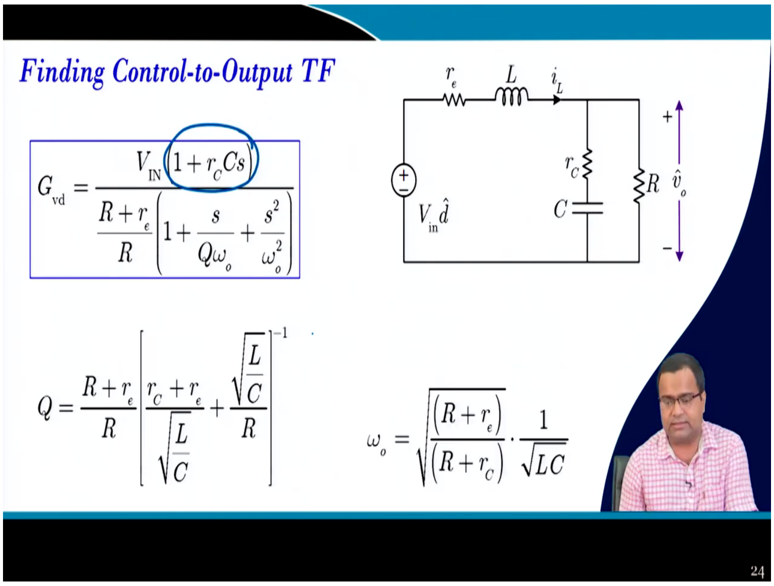

Output impedance we know how to find out and we all already have discussed sorry this is a control to output transfer function. We have already discussed in the control to output transfer function we have 1 0 here, this is due to the e s r.

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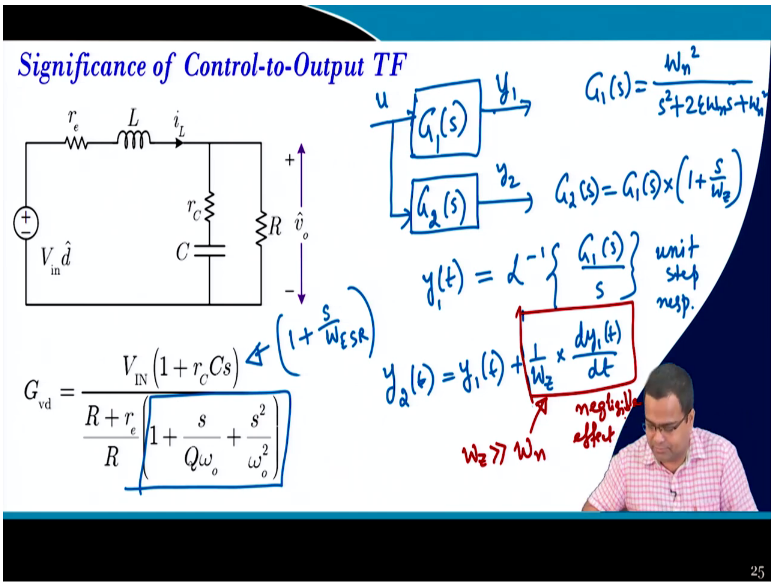

So, we generally write this like this like a ESR 0. So, this 0 is basically this can be also written as 1 plus s by omega ESR. So, ESR is 0 and this ESR 0 if the ESR is low it will go very high frequency. So, that effective negligible, but it will be primarily dominated by this two poles right.

And, we will discuss in the next lecture you know effect of poles and 0s. Suppose, if you incorporate to 1 0 into a system, then how is it going to affect the response? That also will going we are going to discuss. But, I can tell you if you take a transfer function G of S which likes say you know and you are giving 1 input u and this has an output y 1. And, I am taking another transfer function, which is G, this is G 1, this is G 2, suppose this is y 2 and you are using the same input right.

If, G 1 s is something like whatever you have studied in control system, a square plus 2 zeta omega n s plus omega n square. And, if we take G 2 s to be G 1 s multiplied by some s plus omega Z 1 0. Then, if you take the time domain response of y 1 t, we can obtain because the time domain response will be what. The time domain response if we take the initial condition to be 0, it is the Laplace inverse of G 1 s it is unit step response. So, we are talking about unit step response right.

Then you can find out; that means, for a given unit step, then you will find out y of 2 of t is nothing, but y of t plus 1 y omega Z into d y 1 of t by d t. So, this is the extra term that is coming due to the 0. So, this is extra term coming, this extra term is coming due to the 0 and if this 0; that means, if omega z is very very high; that means, 1 far away, or it is very very you know I would say omega n, rather I will say omega n because is a relative term right.

So, if you take omega n then this has negligible effect, it is negligible effect negligible effect. But, if it is coming close to omega n then this will have a significant effect and this will add some derivative action ok. So, this can affect the response of the system.

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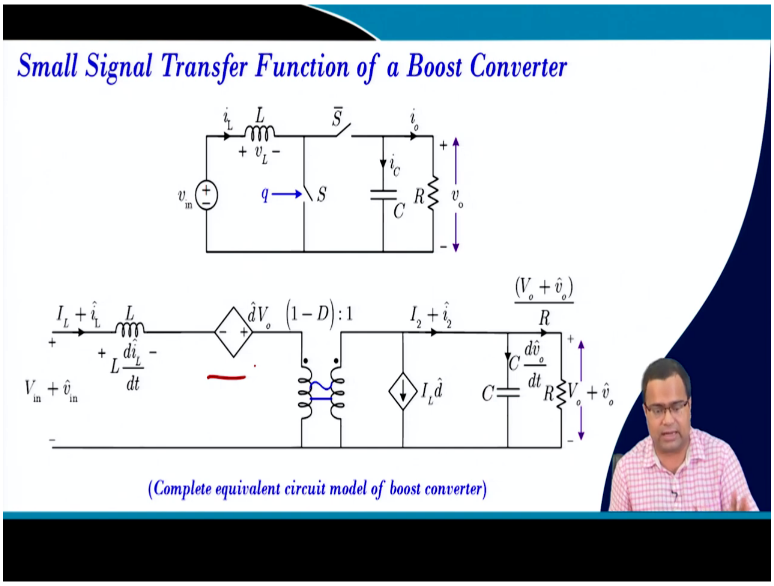

Now, if you go for a boost converter, we have already discussed the AC equivalent like an equivalent circuit model.

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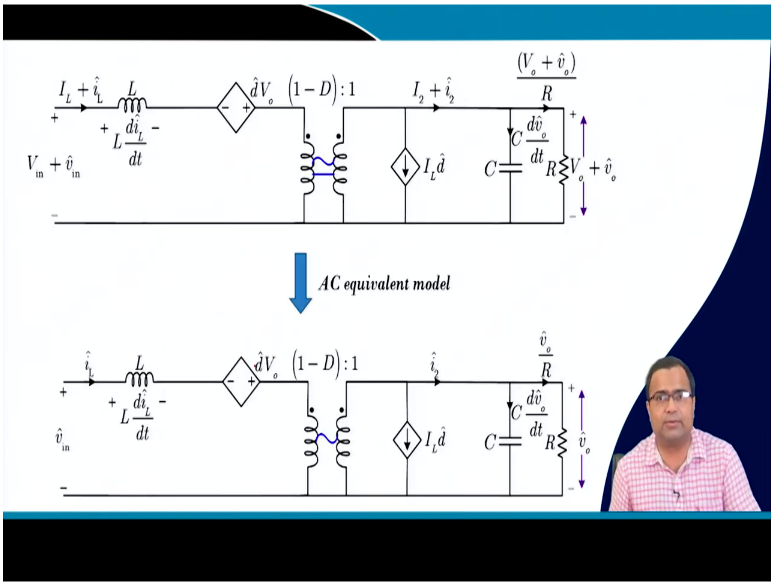

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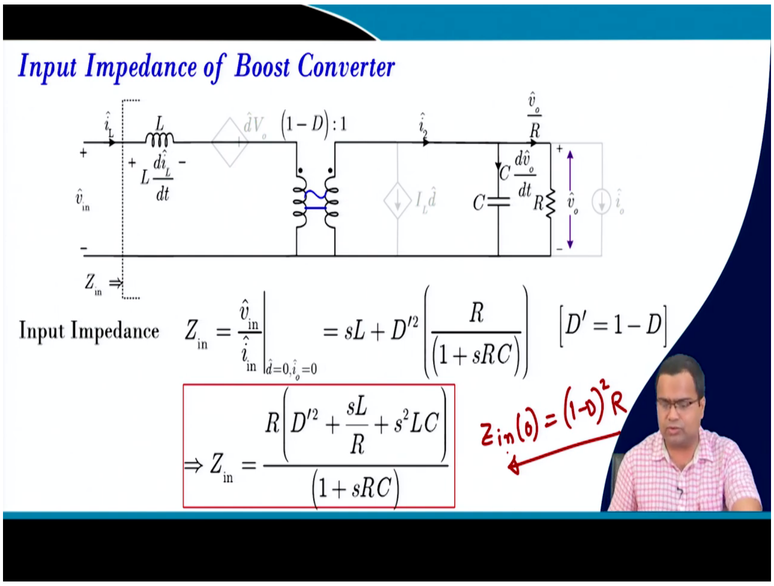

So, you can get the AC equivalent circuit model of a boost converter and we can find out the input impedance a same method and as I told this input impedance of a boost converter is very important. Particularly when we are talking about like a (Refer Time: 44:18) like a solar panel where at low frequency this input impedance will simply become 1 minus D whole square by R. In this case, it looks like right.

Because this is now connected to a solar panel so, you have to match the impedance right for maximum power extraction, ok. So, this also plays a significant role when you want to consider an input filter and so on.

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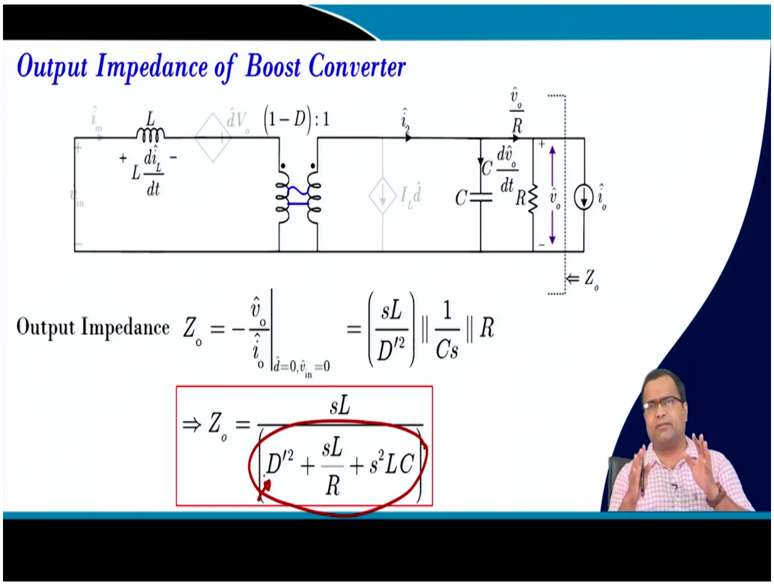

So, output impedance the same method you can apply and you will find the output impedance something similar, but in this case the poles here will have a duty ratio dependence, which was not there earlier. In buck converter, the poles do not depend on the duty ratio, but in boost converter, it also depend on; that means, the Q factor, the natural frequency Q factor, they also depend on duty ratio in a boost converter.

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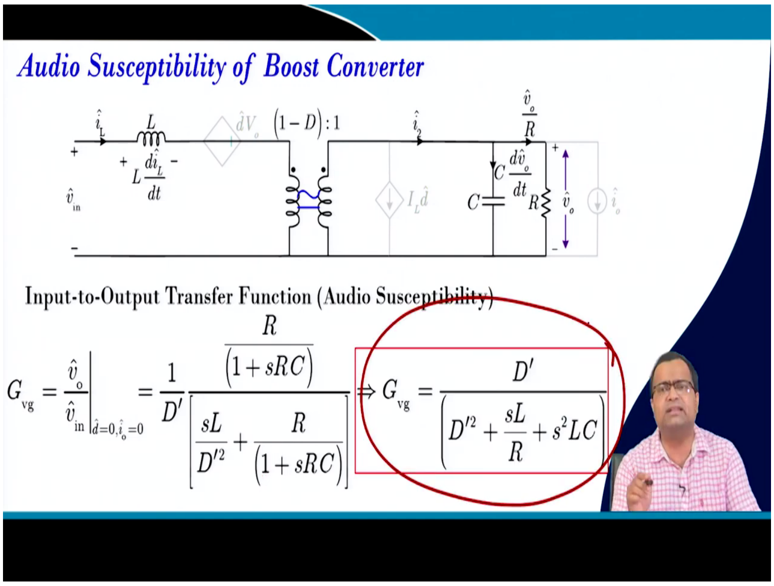

That is why if you change the duty ratio, the whole pole location will also get shifted and it becomes very difficult to compensate for a wide range of duty ratio. You can also derive the audio susceptibility by the same method and you will get the audio susceptibility expression. So, I am not going to separately discuss, because this is all standard we have already discussed.

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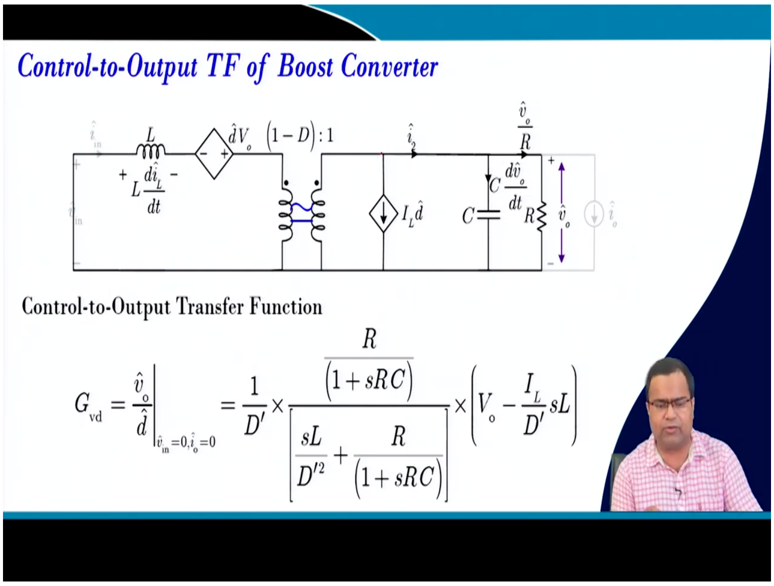

And, if you want to you know to derive the control to output transfer function for a boost converter, then if you go step by step. That how to V 0 and v it requires few more steps. It is not very state forward like a buck converter.

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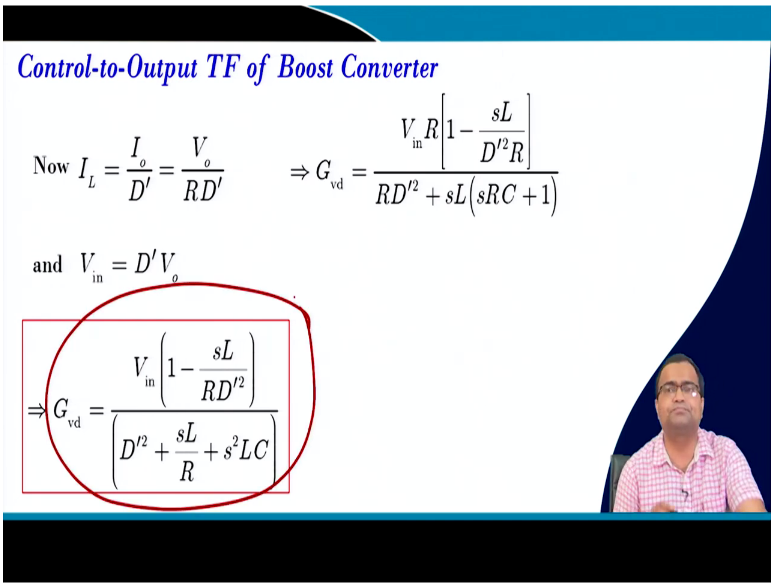

And, if you follow those steps and you know substitute the intermediate variable, then you will get the control to output transfer function a boost converter like this.

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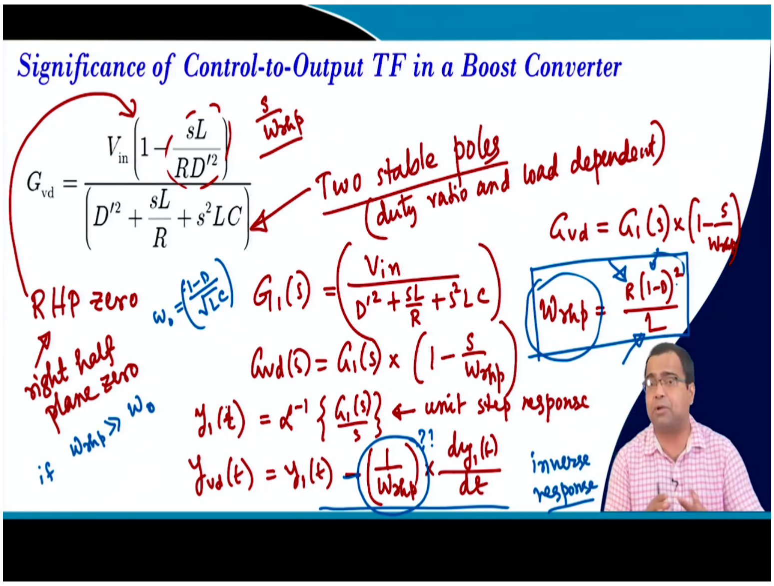

And, as I said, it has two poles; that means, it has two poles, two stable poles, but duty ratio dependent duty ratio and load dependent. In case of buck converter, it was low dependent, but it was not duty ratio dependent on the pole location, but here it is. And, it has 1 right half plane 0, that is called RHP zero; that means it is nothing, but right half plane 0, the right half plane zero and this actually makes.

Because, we have discussed just the previous case if we take a G 1 of s which is let us say in this case particularly. Suppose this G v d let us say this G v d is equal to G 1 s into 1 minus s by omega RHP suppose. So, G 1 is nothing, but let us say V in D dash square S L by R plus S square by into L C. So, this is the transfer function, where we have only 2 stable pole there is no 0, but G v d s is nothing, but G 1 s into 1 minus s by omega r h p. And, what is omega r h p it is nothing, but R 1 minus D whole square by l. Because, it is coming from this expression we are writing s by omega r h p right in this form.

That means, if we write y of 1 s y of t in time domain, it is the inverse Laplace of you can get G 1 of s by s. So, we are talking about unit step response and y G v d t is nothing, but y 1 of t minus 1 by omega r h p, this factor into d y 1 of t by d t. And, you see, we have discussed that, if this quantity is large; that means, if omega r h p is in 1 case is very very greater than omega you know natural frequency, where the sorry natural frequency, where the natural frequency can be represented, because we can take the duty ratio is common.

So, natural frequency let us say omega 0 is 1 minus d y square root of L C. If we take this is the natural frequency is very large that this effect is negligible. But, if it comes close, if it comes close to the axis, in is severe and because of a negative sign, it introduces an inverse response. And, in the next presentation, we will discuss with MATLAB case study that how it is going to affect? Ok.

But, in a boost converter, this right half plane 0 location varies with the load current. It varies with the duty ratio. And, of course, in a boost converter, it is better to choose a smaller inductor. Our objective is to push this r h p 0 as for right-hand side as possible so that this effect can be reduced ok.

But, load and duty ratio is something not in our hand. It will change. So, that is why it is very difficult to design a boost converter for a wide range of load current and duty ratio using a fixed frequency for a fixed gain controller. So, if you go for digital control and if you can adjust the gain, then you can optimize the performance. Otherwise, if you design based on you know worst-case scenario, then you cannot design a converter at compensator for a wide load current a duty ratio range, because that will simply kill your performance significantly and nobody will buy it.

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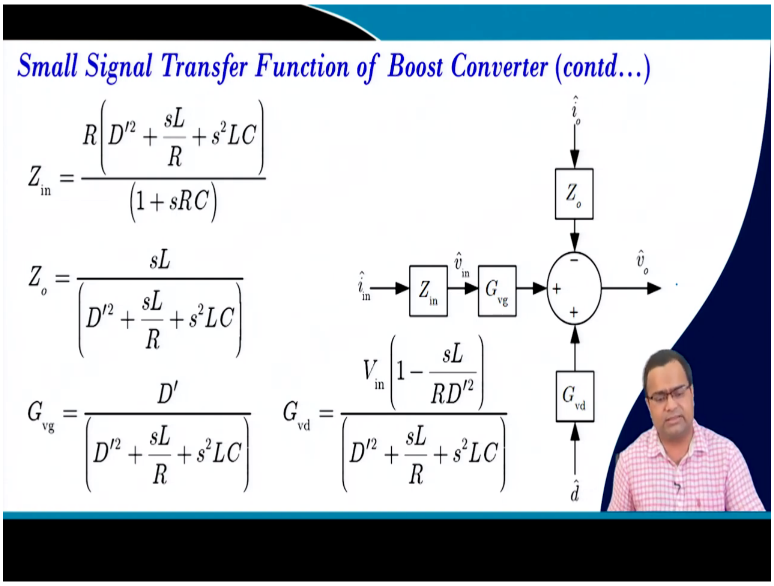

So, small signal transfer function of a boost converter we have discussed different input impedance, output impedance, audio susceptibility control to output transfer function, and we have discussed their significance same thing is applicable for a boost converter.

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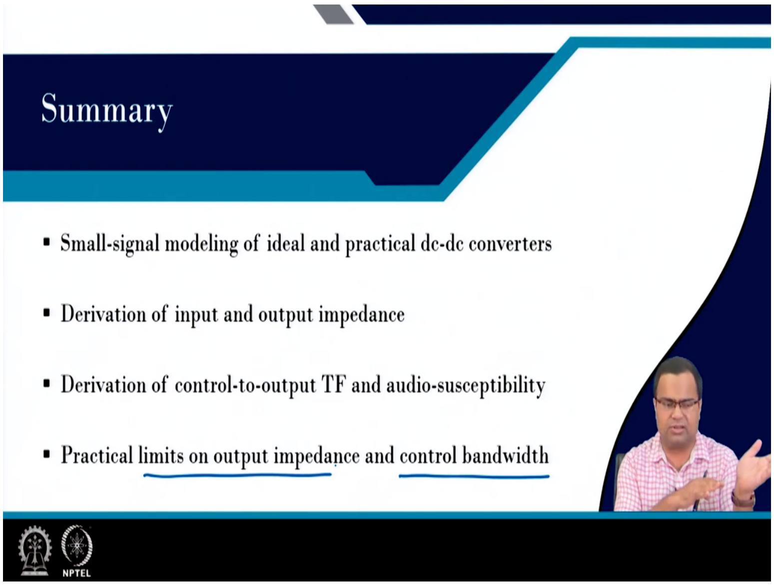

So, in summary we have discussed small-signal model of an ideal and practical DC-DC converter, we have derived input output impedances of DC-DC converter with practical parasitic; we have derived control to output transfer function audio susceptibility.

We have understood some practical limit on output impedance and control bandwidth, because in case of right half plane 0, your control bandwidth can be limited because of the inverse response of the right half plane 0. And, in case of output impedance the practical parasitic can put a limit both for ESR drop and also your that DC value, which can be anticipated by means of closed loop control, but still efficiency will degrade, with this I have to finish it here.

Thank you very much.