

Control and Tuning Methods in Switched Mode Power Converters
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Module - 05
Modeling Techniques in SMPC
Lecture - 26
State Space Averaging Technique

Welcome, this is lecture number 26 in this lecture we are going to talk about State Space Averaging Technique.

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The slide is titled "Concepts Covered" and lists four bullet points: "State space modeling of a simple RLC circuit", "Dynamics of switched mode power converters", "State-space averaging technique", and "Finding linearized small-signal model and DC operating points". The slide features a blue and white color scheme with a small inset video of the professor in the bottom right corner. Logos for IIT Kharagpur and NPTEL are visible at the bottom left.

First I will consider a State space modeling of a simple RLC circuit, then I will show the Dynamics of switched mode power converters, state space averaging technique and then finding linearized small signal model and DC operating points.

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Important Definitions

- State variables
 - Inductor: Current through L
$$v_L = L \frac{di_L}{dt} \triangleq L\dot{x}_1$$
 - Capacitor: Voltage across C
$$i_C = C \frac{dv_C}{dt} \triangleq C\dot{x}_2$$

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So, first the important definition state variables in if we consider like you know any RLC circuit; that means, if there is any energy storing element in electrical circuit like inductor and capacitor. We generally consider the current through the inductor to be one state variable, which is i_L . The voltage across the capacitor is another state variable, which is v_C . So, here i_L is considered to be x_1 and v_C is considered to be x_2 .

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Important Definitions (contd...)

- State vector
$$\underline{\dot{x}} \triangleq \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dots \end{bmatrix}^T$$
- State-space equations
$$\underline{\dot{x}} = \underline{A}\underline{x} + \underline{B}\underline{u}$$
$$\underline{y} = \underline{C}\underline{x} + \underline{E}\underline{u}$$

where, $\underline{u} \rightarrow$ input
 $\underline{y} \rightarrow$ output

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Next the state vector, if there is more than one state are there, then we consider them in a vector format and that you call state vector. Then we can write down the state space equations

for a given dynamical system, where we can represent a first order as well as higher order dynamical system. And the state space representations for a higher order dynamical system we write say suppose if the system order of the dynamical system is n , then we write a first-order differential equation and that we write in terms of state space representation.

So, we like in terms of A matrix and B matrix, where x is the state vector and u is the input vector, similarly we can write also output vector in terms of state vector and the input vector. Where u is the input vector and y is the output vector.

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Example

$v_L = L \frac{di_L}{dt}$

polarity shown is instantaneous

RLC circuit

- KVL Equations

$$i_L(t) = C_o \frac{dv_c(t)}{dt} \quad \checkmark \quad \dots(1)$$

$$v_{in}(t) - ri_L(t) - L \frac{di_L(t)}{dt} - v_c(t) = 0 \quad \checkmark \quad \dots(2)$$

If we take a RLC circuit, it is a series RLC circuit where we are considering here by inductor current to be one state and the capacitor voltage to be another state and we have taken the polarity of this voltage accordingly.

Now, if we write KVL equations, then the first thing I'd like what is the inductor current? The inductor current is passing through this capacitor. So, which is $C_o \frac{dv_c(t)}{dt}$ this is one equation. Similarly, what are the inductor current dynamics? That means, if you take the voltage across the inductor, this is my v_L . What is v_L ? v_L is nothing but $L \frac{di_L}{dt}$.

And if you write the KVL the full equation, so v_{in} then this drop across this resistance r into i_L then $L \frac{di_L}{dt}$ is the v_L that is a drop and minus v_c which is the capacitor voltage and we can add up to 0 the whole.

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Example (contd...)

- Rewriting (1) and (2) in state-space form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{dv_c(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{C_o} \\ \frac{1}{L} & -\frac{r}{L} \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_x + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_B v_m(t)$$

$$\underbrace{v_o(t)}_y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_x$$

$\dot{x} = Ax + Bv_{in}$

Taking $x_1(t)$ as the output \rightarrow capacitor voltage

RLC circuit
polarity shown is instantaneous

Then this series RLC circuit if we write the state space equation, where we have considered the first state to be capacitor voltage here. Here, we have taken state 1 to be capacitor voltage and the state 2 to be inductor current.

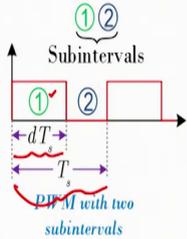
So, we can write in terms of A and B matrix, where A matrix consists of four elements 0, then second element, then this third and fourth element. So, these equations we already wrote in the previous slide and we can just plug in to write in terms of, so that means we can write something like \dot{x} equal to $Ax + Bv_{in}$ in this case it is v_{in} which is the supply; that mean it is the input to this particular RLC circuit.

Similarly, we can write output equation, so here output we have taken output voltage, which is same as the capacitor voltage. So, it is simply 1,0 of the state vector right. So, we are taking x_2 as the output sorry x_1 because x_1 is nothing but the state variable 1 which is the capacitor voltage; that means, it is nothing but the capacitor voltage.

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State-space of Pulse Width Modulated (PWM) converters

- Consider a PWM converter with 2 subintervals



- Each interval represent a linear RLC circuit

PWM with two subintervals

The next part, the state space for Pulse Width Modulated converter or pulse width modulated converter, so consider a PWM converter with 2 sub interval. That means, in this switching converter, whether you can take a buck, a boost or any other converter. In continuous conduction mode, generally you know for hard switch converter like a buck and boost as well as buck boost. So, we will get 2 switch state switch is on and switch is off.

If there is only one controllable switch, then we are getting 1 state when the switch is on which is state 1 and the switch is off state 2. And we are talking about pulse width modulation where the total time period is constant and the on time is nothing but d into T_s . Each interval represents a RLC circuit and we will take it.

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State-space in Pulse Width Modulated (PWM) converters

- Computing the state-space equations for the two subintervals

<i>During Subinterval 1</i>	<i>During Subinterval 2</i>
$\dot{\underline{x}} = A_1 \underline{x} + B_1 v_{in}$	$\dot{\underline{x}} = A_2 \underline{x} + B_2 v_{in}$
$\underline{y} = C_1 \underline{x}$	$\underline{y} = C_2 \underline{x}$

Different state-space equations for the two subintervals



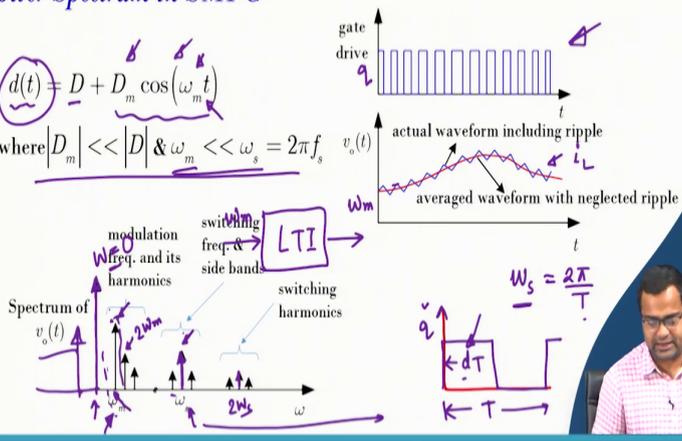
Now, we want to compute the total overall state space equations of two subintervals. So, we have to write down the sub interval 1, sub interval 2, for each subinterval we have to write the dynamical equation and then we need to arrange in the state space format ok. Then, we will find the state space model should be different for two sub intervals, because if they are identical then in there is no switching action right.

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Power Spectrum in SMPC

$d(t) = D + D_m \cos(\omega_m t)$

where $|D_m| \ll |D|$ & $\omega_m \ll \omega_s = 2\pi f_s$



gate drive

actual waveform including ripple

averaged waveform with neglected ripple

Spectrum of $v_o(t)$

modulation freq. and its harmonics

switching freq. & side bands

switching harmonics

$\omega_s = \frac{2\pi}{T}$



Now, first of all, before we move forward for state space averaging we need to understand the power spectrum in switch mode power converter. So, switch mode power converter: we

provide a gate signal q in under pulse width modulation, we generally consider a duty ratio d and you know we have already considered. That means, if we take the power converter, this is our duty ratio, this is our d into T and this we take total time period to be constant and we are talking about this.

Now, this time period is constant. That means it is a periodic signal. But, this duty ratio also we can apply a small perturbation, that means the duty ratio can also have a sinusoidal excitation along with the steady state value D . We can also inject a small amplitude sinusoidal excitation with this duty ratio. Now, where the amplitude of D_m ; that means, excitation amplitude is much smaller than its steady state value ok.

And ω_m we are assuming that means the excitation signal the frequency we are basically trying to sweep this frequency from 0 frequency slowly we are increasing, but this frequency that is the modulating frequency is much smaller than the switching frequency.

Then, if we draw the waveform of this converter where this is the gate signal for our case, it is q and we can draw corresponding switching behaviour of the converter. So, this can be inductor current. Now, where if the ripple is small, then we can neglect the effect of the ripple and we can straight away take the average dynamics.

But, how far this average dynamics is valid? If you draw the power spectrum of this converter, where we have to consider a periodic signal q with time period T . That means, if we take ω_s it is $2\pi/T$, where T is the time period that is my switching frequency in radian per second.

And we also have a modulating frequency because the gate signal is modulated using a sinusoidal excitation. Then, what will be the resultant frequency spectrum of this gate signal? So, it has two things, one thing we have this gate signal is a periodic signal which actually periodically occurs in every time period T , but in this gate signal we have one duty ratio term and that duty ratio also we are varying.

That means it is a frequency modulation where we are modulating the duty ratio with the frequency, modulating frequency ω_m and we also have a switching frequency. And if we draw the power spectrum of the output voltage, first there will be a large component of the DC quantity right and along the DC quantity we will also have some component right side

and left side as a modulating. This is a DC component, which is ω that means equal to that means here ω equal to 0.

And since we have also injected one sinusoidal excitation. So, it will also have a side lobe and which is ω_m . Similarly, we can draw the ω minus ω_m left side, but I am not drawing in the left side then ω_m it will be twice ω_m and so on. So, side lobe will be created because it is a frequency modulation same thing is applicable.

So, here if you do not modulate this signal if you skip this modulating signal to be 0 frequency. That means, it is simply a periodic signal with a fixed duty ratio and there we will get the component as a fundamental like a DC component, switching which is a fundamental component, their harmonic which is $2\omega_s$ and so on.

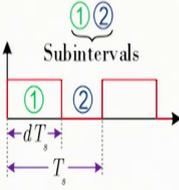
But, since we are also injecting another low frequency excitation, so on top of this switching that you know DC component, switching frequency and harmonics, we will also have side lobe which will be created by the modulating frequency. This will also have a side lobe. Now, if the ω_m is much smaller than ω_s and that we have consider, then this side lobes are almost close to it. This component, that means this switching frequency component, ok.

Now, if you can show the dynamics because we are interested in obtaining the behaviour of the system, if we excite with a modulating frequency ω_m . We want to see what is the effect of the output? That means, if we excite the duty ratio with ω_m sinusoidal excitation, we want to see what is the effect in the output voltage corresponding to that particular frequency. Because we want to generate or we want to derive a linear model, linear time invariant system that LTI model.

And we know for an LTI system if we excite with an ω_m frequency, then the output should also contain output should contain only that excitation frequency only, the other frequency should not present in a linear system. But, it is a switching converter and we want to get an approximate linear model; that means, we are interested in obtaining what is the power spectrum at the excitation frequency ok.

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State-space averaging



Subintervals

- Subinterval 1 is active for dT_s time
- Subinterval 2 is active for $(1-d)T_s$ time

Merging the two intervals \rightarrow *Averaging*



So, this is a side band we have discussed and the switching harmonics. Now, under state space averaging, what we are doing? So, we are assuming that our averaging is valid for low frequency and under this particular assumption. That means, the excitation signal amplitude is much smaller than the original DC quantity and the excitation frequency is also smaller. So, we can ignore the behaviour of the ripple information. So, we are only considering the average dynamics of this converter.

So, under this condition, we can move forward with the state space averaging. So, this requires the assumption the ripples are negligible compared to the average dynamics or the ripples are very fast or this is the assumption where the switching frequency is infinitely fast. So, if the switching frequency is infinitely fast; that means, this particular frequency will move further right side far away.

And then we can simply extract the information corresponding to ω_m we can extract ok. Now, in state space averaging, we will write the equation of subinterval 1 which is valid for the time period of d into T and sub interval 2 which is activated during $1 - d$ into T and then we can obtain this is the average quantity.

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State-space averaging

$\bar{x} \triangleq \langle x \rangle_T$

$$\langle \dot{x} \rangle_T = \left(\underbrace{dA_1}_{\text{subinterval 1}} + \underbrace{(1-d)A_2}_{\text{subinterval 2}} \right) \langle x \rangle_T + \left(\underbrace{dB_1}_{\text{subinterval 1}} + \underbrace{(1-d)B_2}_{\text{subinterval 2}} \right) \langle v_{in} \rangle_T \triangleq F_{av}$$

$$\dot{\bar{x}} = \left[dA_1 + (1-d)A_2 \right] \bar{x} + \left[dB_1 + (1-d)B_2 \right] \bar{v}_{in} \triangleq F_{av}$$

$$\langle y \rangle_T = \left(\underbrace{dC_1}_{\text{subinterval 1}} + \underbrace{(1-d)C_2}_{\text{subinterval 2}} \right) \langle x \rangle_T$$

$$\bar{y} = \left(dC_1 + (1-d)C_2 \right) \bar{x}$$


In fact, we will use this \bar{x} to be it is represented in terms of x , which is average over a cycle time period T . Then we can write \dot{x} ; that means, we can write $\dot{\bar{x}}$ should be equal to d into A_1 plus $(1-d)$ into A_2 \bar{x} plus d into B_1 plus $(1-d)$ into B_2 \bar{v}_{in} these are the average quantity ok. And that we are considering this to be denoted as F_{av} ok. Similarly, we can write \bar{y} is equal to d into C_1 plus $(1-d)$ into C_2 . This whole thing to be \bar{x} ok.

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State-space averaging (contd...)

- Concept of perturbation

Averaged quantity = Steady state quantity + Small AC variations

$$\langle x \rangle_T = \bar{x} + \hat{x}$$

$$\langle v_{in} \rangle_T = \bar{v}_{in} + \hat{v}_{in}$$

$$\langle y \rangle_T = \bar{y} + \hat{y}$$

$$d = D + \hat{d}$$

Assumptions

Steady state quantity \gg Small AC variations

$|\hat{x}| \ll x$



Now, state space averaging first we need to consider perturbation; that means, we talk about \bar{x} which can be written as capital X plus small perturbation. That means, we have seen earlier that if we take the original duty ratio, it consists of two parts; one part is capital D which is a steady state quantity or the fixed quantity plus some AC excitation and where which we are taking as a \hat{x} it can be AC excitation it can be state excitation.

So, we are talking about \hat{x} , because in frequency response we are assuming this \hat{x} will carry because it will be the injected sinusoidal signal and we want to get the approximate linear model, so this all this you know notation it is represented here. The assumption is that we told earlier that all this perturb quantity, if you take the magnitude it should be much smaller than its steady state quantity. That is the assumption; this is true for all this quantity. So that means their corresponding magnitude of this should be much smaller than the capital V.

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Taylor Series Approximation and Linearization

$$\langle \dot{\hat{x}} \rangle_{T_s} = \left(\frac{dA_1}{\text{subinterval 1}} + \frac{(1-d)A_2}{\text{subinterval 2}} \right) \langle \hat{x} \rangle_{T_s} + \left(\frac{dB_1}{\text{subinterval 1}} + \frac{(1-d)B_2}{\text{subinterval 2}} \right) \langle v_{in} \rangle_{T_s} \triangleq F_{av}$$

$$\dot{\bar{x}} = F_{av}(\bar{x}, \bar{v}_{in}, d)$$

$$\dot{\hat{x}} = F_{av}|_{SS} + \frac{\partial F_{av}}{\partial \hat{x}}|_{SS} \hat{x} + \frac{\partial F_{av}}{\partial d}|_{SS} \hat{d} + \frac{\partial F_{av}}{\partial v_{in}}|_{SS} \hat{v}_{in} + H.O.T$$

Handwritten notes:

- $X = \begin{bmatrix} I_L \\ V_C \end{bmatrix}$
- $X = -A^{-1} B V_{in}$ (with note: $A \rightarrow \text{non-singular}$)
- $\dot{X} = 0$
- $F_{av} = [dA_1 + (1-d)A_2] \bar{x} + [dB_1 + (1-d)B_2] \bar{v}_{in}$
- $d \rightarrow D, \bar{x} \rightarrow X, \bar{v}_{in} \rightarrow V_{in}$
- $F_{av}|_{SS} = \underbrace{[DA_1 + (1-D)A_2]}_A X + \underbrace{[DB_1 + (1-D)B_2]}_B V_{in} = 0$

Then, in order to obtain the linearized model we have to write the Taylor series expansion which means this is the original model and we wrote that \dot{x} is nothing but F into average which is a function of x dash sorry function of x dash comma v in dash comma d right. Then, we have to obtain the perturb model and which is by applying Jacobian. How can you find out F average under steady state?

So, what is our F average? Here our F average we know that it is d into A_1 minus sorry plus 1 minus d into A_2 x average plus d into B_1 plus 1 minus d into B_2 v in average ok, so this

quantity we know. Now, under steady state, we need to consider that \dot{d} should be under steady state; that means, if we take steady state, this should be replaced by capital D, \dot{x} should be replaced by capital X and \dot{v} in average should be replaced by capital V in.

That means, F average under steady state should be equal to what? Capital D A 1 plus 1 minus D A2 X plus D B1 1 minus D B 2 V in this is a steady state quantity. And it can be shown that this quantity will eventually become 0 vector, because under steady state this can be proved that this will be 0 because their derivative part is 0.

So, if you take the derivative of X dot is 0, and this gives us this equation. That means we can solve and if we take this to be A matrix and this to be B matrix, then we can solve X equal to minus A inverse B into capital V in this can be solved. Where the matrix A must be non-singular it must be non-singular and this is true because this A matrix average quantity can be shown to be nonsingular.

And then only we can find out uniquely the two steady state point that means, what is X? So, X consist of two parts; the average inductor current and the average capacitor voltage. So, these two can be uniquely characterized by this equation, if A matrix is nonsingular.

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Finding Operating Point

At steady-state equilibrium point

$$\underbrace{(DA_1 + (1-D)A_2)}_A \underline{X} + \underbrace{(DB_1 + (1-D)B_2)}_B V_{in} = 0$$

Equilibrium states

$$\Rightarrow \underline{X} = -A^{-1} B V_{in}$$

$$V_o = \underbrace{(DC_1 + (1-D)C_2)}_C \underline{X} \quad \underline{V_o}$$

Now, we have to find out that means; I told you so this quantity we can find out by this and also correspondingly we can find V_0 because we know D into C1 plus 1 minus D into C 2

into X is the average value. So, you can find out from here what is my average or steady state voltage.

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Taylor Series Approximation and Linearization

$$\dot{\hat{x}} = F_{av}|_{SS} + \left. \frac{\partial F_{av}}{\partial \bar{x}} \right|_{SS} \hat{x} + \left. \frac{\partial F_{av}}{\partial \bar{d}} \right|_{SS} \hat{d} + \left. \frac{\partial F_{av}}{\partial \bar{v}_{in}} \right|_{SS} \hat{v}_{in} + \text{HOT}$$

$$\hat{\dot{x}} \approx \left. \frac{\partial F_{av}}{\partial \bar{x}} \right|_{SS} \hat{x} + \left. \frac{\partial F_{av}}{\partial \bar{d}} \right|_{SS} \hat{d} + \left. \frac{\partial F_{av}}{\partial \bar{v}_{in}} \right|_{SS} \hat{v}_{in}$$

$$F_{av} = [d A_1 + (1-d) A_2] \bar{x} + [d B_1 + (1-d) B_2] \bar{v}_{in}$$

$$= (A_1 - A_2) \bar{x} d + A_2 \bar{x} + (B_1 - B_2) \bar{v}_{in} d + B_2 \bar{v}_{in}$$

$$\left. \frac{\partial F_{av}}{\partial \bar{x}} \right|_{SS} = [D A_1 + (1-D) A_2] \stackrel{\Delta}{=} A$$

$$\left. \frac{\partial F_{av}}{\partial \bar{d}} \right|_{SS} =$$

Handwritten notes in purple: $\hat{x}^2, \hat{d}^2, \dots, \hat{v}_{in}^2$

Now, the next part we need to do linearization and we discussed that this term will eventually become 0 ok. The next part of this part is zero and if you want to do Taylor series linearization, the higher order term consist of x hat square, it will also consist of d hat square and so on, it will also consist of v in hat square and other higher order term.

And we are ignoring we are assuming this perturbation square or the perturbation products are neglected or negligible. So, under that assumption this can be dropped or this quantity that means x hat dot can be approximated as dou F average dou x in this case into x hat plus dou F average into dou d this is computed at steady state this is computed at steady state into d hat plus dou F average into dou v in computed at steady state into v in hat.

So, this can be approximated when we ignore the higher order term; that means, the perturbation of the duty ratio, perturbation of the state variable as well as the perturbation input voltage should be small enough compared to their steady state quantity. And I will show you in the subsequent lecture that this assumption makes the small-signal model to be valid within a narrow bandwidth, beyond which this model is not valid because the perturbation will no longer remain small.

Now, the next part, how do you find out F average? So, what is F average? If we write F average, we know it was d into A_1 plus $1 - d$ into A_2 x plus d into B_1 plus $1 - d$ into B_2 this into v in. So, we can write further we can take it A_1 , that means we are taking this term and this term minus A_2 into x into d plus A_2 x 2 plus B_1 minus B_2 v in into d plus B_2 into v in because this term and these terms are accumulated to get this term ok.

Similarly, this term and this term is accumulated to get this term. Now, then if we want to find F_{av} $\text{d}F_{av} \text{d}x$ at steady state, from this equation we can straight away right that this is simply D into A_1 plus $1 - D$ because it is under steady state ok. Because this is we are differentiating with respect to this quantity and it is under steady state ok and this we represented by A matrix ok.

Now, what is $\text{d}F_{av} \text{d}d$ at steady state? Ok. So, here we can do one thing: let us ok because let us not take the steady state directly, because we can find out the steady state first let us derive. So, if we write down from this equation from this equation, if we write down from this particular equation, we are differentiating partially with respect to this quantity.

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Taylor Series Approximation and Linearization

$$\dot{\hat{x}} = F_{av}|_{ss} + \left. \frac{\partial F_{av}}{\partial x} \right|_{ss} \hat{x} + \left. \frac{\partial F_{av}}{\partial d} \right|_{ss} \hat{d} + \left. \frac{\partial F_{av}}{\partial v_{in}} \right|_{ss} \hat{v}_{in} + \text{HOT}$$

$$\hat{\dot{x}} \approx \left. \frac{\partial F_{av}}{\partial x} \right|_{ss} \hat{x} + \left. \frac{\partial F_{av}}{\partial d} \right|_{ss} \hat{d} + \left. \frac{\partial F_{av}}{\partial v_{in}} \right|_{ss} \hat{v}_{in}$$

$$F_{av} = [d A_1 + (1-d) A_2] x + [d B_1 + (1-d) B_2] v_{in}$$

$$= (A_1 - A_2) \bar{x} d + A_2 \bar{x} + (B_1 - B_2) \bar{v}_{in} d + B_2 v_{in}$$

$$\frac{\partial F_{av}}{\partial x} = d A_1 + (1-d) A_2$$

$$\left. \frac{\partial F_{av}}{\partial x} \right|_{ss} = D A_1 + (1-D) A_2$$

$$\stackrel{\Delta}{=} A$$

Handwritten notes in purple: $\hat{x}, \hat{d}, \dots, \hat{v}_{in}$

So, initially we are assuming that we are not talking about steady state; that means, what is this quantity we are partially differentiating. So, it will end up with $\text{d}A_1$ plus $1 - d$ into A_2 , because we are differentiating with respect to this quantity.

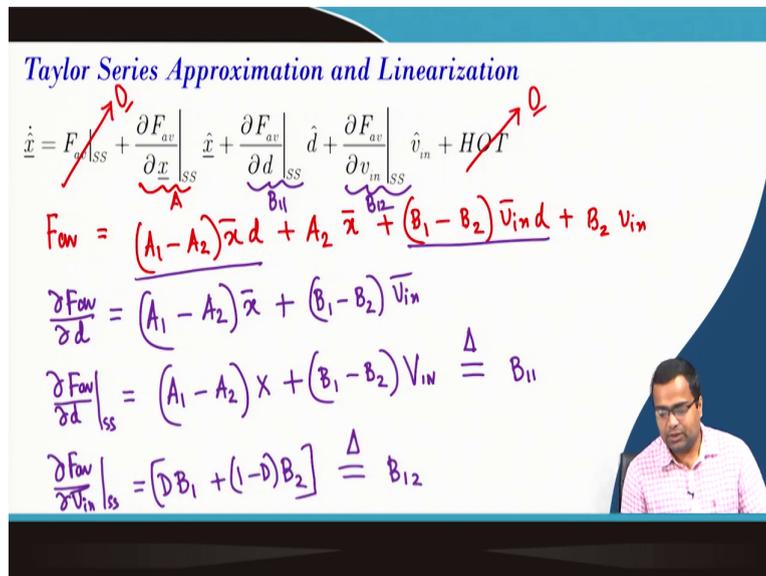
That means, if we want to write what is my dou F av by dou x at steady state, then we simply replace this duty ratio to be its steady state value. So, it will be D into A1 plus 1 minus D into A2 and this we represented as A matrix, that means this we represented by this matrix ok.

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Taylor Series Approximation and Linearization

$$\dot{\hat{x}} = F_{av}|_{ss} + \frac{\partial F_{av}}{\partial x}|_{ss} \hat{x} + \frac{\partial F_{av}}{\partial d}|_{ss} \hat{d} + \frac{\partial F_{av}}{\partial v_{in}}|_{ss} \hat{v}_{in} + HQT$$

$F_{av} = (A_1 - A_2)\bar{x}d + A_2\bar{x} + (B_1 - B_2)\bar{v}_{in}d + B_2v_{in}$
 $\frac{\partial F_{av}}{\partial d} = (A_1 - A_2)\bar{x} + (B_1 - B_2)\bar{v}_{in}$
 $\frac{\partial F_{av}}{\partial d}|_{ss} = (A_1 - A_2)\bar{x} + (B_1 - B_2)\bar{v}_{in} \stackrel{\Delta}{=} B_{11}$
 $\frac{\partial F_{av}}{\partial v_{in}}|_{ss} = [D B_1 + (1-D)B_2] \stackrel{\Delta}{=} B_{12}$



Now, next part again we are writing. So, what we are writing again; again we are writing. We wrote in the earlier equation F av was this part already we took 0 right, this we have ignored right. So, we have ignored this part. We found it is A matrix in the previous equation.

Now, what is F av? We saw it was A1 minus A2 into x average into d plus A2 into x average plus we separated B 1 minus B 2 into v in into d plus B 2 into v in. Now, we want to find out what is my dou F average with respect to dou duty ratio? So, we want to only consider these two terms which consist of d duty ratio other two terms are independent of d.

So, we will find out A1 minus A2 into x bar plus B 1 minus B 2 into v in bar; that means, if we take the steady state value of d under steady state, the states and the input voltage will be replaced by their steady state quantity. So, we will find out that A1 minus A2 into capital X plus B 1 minus B 2 into capital V in and this quantity we denote as B 11. So, this quantity we denote as B 11,

Now, how can you find out B 12? So, you can find out dou F av dou by the same method if we write down under steady state condition what will be this. So, from this expression, we

can clearly write that it will be only the last term; that means we will get D into B 1 plus 1 minus D into B 2. This will be my V in and this is represented by B 12 alright.

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Taylor Series Linearization

$$\dot{\hat{x}} \approx \underbrace{\frac{\partial F}{\partial x}}_{A} \Big|_{SS} \hat{x} + \underbrace{\frac{\partial F}{\partial d}}_{B_{11}} \hat{d} + \underbrace{\frac{\partial F}{\partial v}}_{B_{12}} \hat{v}_{in}$$

$$A = [D A_1 + (1-D) A_2] \quad B_{11} = (A_1 - A_2) X + (B_1 - B_2) V_{in}$$

$$B_{12} = [D B_1 + (1-D) B_2] \quad F_{av}|_{SS} = 0$$

$$\dot{\hat{x}} = A \hat{x} + \underbrace{\begin{bmatrix} B_{11} & B_{12} \end{bmatrix}}_B \begin{bmatrix} \hat{d} \\ \hat{v}_{in} \end{bmatrix} \Rightarrow \hat{x}(s) = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

Laplace transform

So; that means, we can find out A1, A11 this term. So, what is my A1? We found A equal to D into A1 plus 1 minus D into A2 that was our A1 correct. What was our B11? We found it is so if you go to the previous slide B11; A1 minus A2 into what X right yes, A1 into X plus B1 minus B 2 into V in. And what is B11?

So, if you find B11 sorry B11 is what B12 it is simply D into B1 plus 1 minus B into B 2. So, with this we can write all this and this B 2 it is a 2 into 1 matrix; 2 cross this is also 2 cross 1 matrix, this is 2 cross 2 matrix ok. Now, this whole thing which is a B matrix is a 2 cross 2 matrix, and this is also 2 cross 2 matrix this one.

So, now after writing this perturb model where we have ignored the higher order term, and the DC offset term was 0 set to, because under operating condition F av we wrote that F av steady state was a null matrix, then we got this linearize you know perturb linearized model and now we can apply Laplace transform. So, we can apply Laplace transform to find this transform function ok. So, x of s hat x s I minus A inverse into B into this.

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Small-Signal Model

$$\hat{x}(s) = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

$$\begin{bmatrix} \hat{i}_L(s) \\ \hat{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

$$\hat{v}_o(s) = C\hat{x}(s) = C(sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$


So, we can derive the small signal model \hat{x} of s in terms of $sI - A$ inverse of B to d , then we can see what is \hat{x} of s , it is \hat{i}_L of s , \hat{v}_c of s capacitor voltage. And the output voltage is nothing but C into \hat{x} of s , so it is C into $sI - A$ inverse B into this term.

Small-Signal Transfer Functions

$$\begin{bmatrix} \hat{i}_L(s) \\ \hat{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix} \quad \hat{v}_o(s) = C\hat{x}(s) = C(sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

Control-to-output TF $G_{vd}(s) = \left. \frac{\hat{v}_o(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = C(sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Audio-susceptibility $G_{vg}(s) = \left. \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} \right|_{\hat{d}=0} = C(sI - A)^{-1} B \begin{bmatrix} 0 & 1 \end{bmatrix}^T \hat{v}_{in}$

Control-to-inductor current TF $G_{id}(s) = \left. \frac{\hat{i}_L(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = \begin{bmatrix} 1 & 0 \end{bmatrix} (sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$



How can I obtain now using this model? These two models we have already discussed how can we obtain control to output transfer function, which is $G_{vd}(s)$. That means, it is the $v_0(s)$ all will be s by $\hat{d}(s)$ v_0 hat s by $\hat{d}(s)$, which is obtained without considering any perturbation in the input voltage, because it is control to output transfer function where we keep the input voltage constant.

Then, you say this matrix here there are two terms; one is \hat{d} and \hat{v} . But, we want only \hat{d} . So, we need to multiply a column vector which is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, because we want to extract only this term. Similarly, if we want to talk about audio susceptibility, which is \hat{v}_0 by \hat{s} \hat{v}_0 hat by \hat{s} by \hat{v} in hat by \hat{s} , and in this case, we are taking the duty ratio to be constant; that means, it is under fixed duty ratio operation.

So, you can straight away write $C \hat{s} A$ whatever it is there; that means, you know \hat{v}_0 this with this we have to multiply $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ because we want to extract this one. So, this one it is just the other. Here we extracted \hat{d} here we extracted \hat{v} in hat ok. So, these two things we have extracted. Control to inductor current transfer function $G_{id}(s)$ which is $i_L(s)$ by $\hat{d}(s)$ and where we are considering the input voltage perturbation to be 0.

Here it is the same model here, but we will take we want to take this one and this one; that means, the first case, we have to write a row matrix $\begin{bmatrix} 1 & 0 \end{bmatrix}$ and then a column matrix $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, then we can write down from this. That means, we can derive control to output transfer function, audio susceptibility, control to inductor current transfer function everything that we can derive from this transfer function.

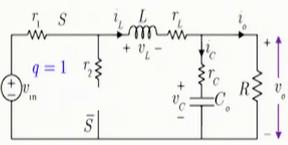
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Now, the state space averaging of a synchronous buck converter it was the method I have discussed how to start with a state space model of a DC DC converter using two sub interval dynamics and from there how can you obtain the average model and from the average model

we can derive we can apply Taylor series to extract the small signal model as well as we can find the equilibrium point.

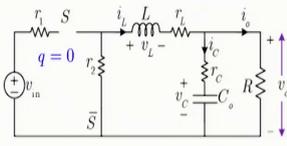
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State Space Modeling of a Synchronous Buck Converter



Circuit configuration during Subinterval 1

$S = \text{ON}; \bar{S} = \text{OFF}$



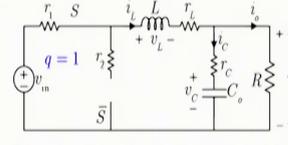
Circuit configuration during Subinterval 2

$S = \text{OFF}; \bar{S} = \text{ON}$

Now, in the practical DC-DC converter we have two sub interval switch ON and switch OFF, this we are talking about in continuous conduction mode because we are using synchronous configuration.

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Modeling – Sub-interval 1



Circuit configuration during Subinterval 1

$x_1 = i_L$
 $x_2 = v_c$

State Equations

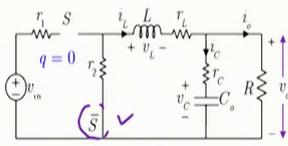
$$\begin{cases} \frac{di_L}{dt} = -\frac{1}{L}(r_1 + r_L + \alpha r_c)i_L - \frac{\alpha}{L}v_c + \frac{1}{L}v_{in} \\ \frac{dv_c}{dt} = \frac{\alpha}{C_o}i_L - \frac{\alpha}{RC_o}v_c + 0 \cdot v_{in} \end{cases}$$

Output Equation $v_o = \alpha v_c + \alpha r_c i_L$

Then, we can write down the equation under individual configuration; that means, when the switch is ON what are the state equation right and the output equation. And we are taking i_L as one state x_1 ; that means, our x_1 is i_L and our x_2 is v_C right.

(Refer Slide Time: 35:27)

Modeling – Sub-interval 2



Handwritten notes:
 $x_1 = i_L$
 $x_2 = v_C$

State Equation

$$\begin{cases} \frac{di_L}{dt} = -\frac{1}{L}(r_2 + r_L + \alpha r_C)i_L - \frac{\alpha}{L}v_C \\ \frac{dv_C}{dt} = \frac{\alpha}{C_o}i_L - \frac{\alpha}{RC_o}v_C + 0 \cdot v_{in} \end{cases}$$

Output Equation $v_o = \alpha v_C + \alpha r_C i_L$

Circuit configuration during Subinterval 2

Then we can consider sub interval 2 where the switch is OFF main switch, but this complimentary switch is ON, it is ON under this condition again. Our state variable is i_L and x_2 is equal to v_C . So, you can write the state space equation from these sub system dynamics.

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Overall State Space Model

$$\dot{x} = \underline{A}_j x + \underline{B}_j v_{in} \quad x = \begin{bmatrix} i_L & v_C \end{bmatrix}^T$$

Subinterval 1	Subinterval 2
$\underline{A}_1 = \begin{bmatrix} -\frac{1}{L}(r_1 + r_L + \alpha r_C) & -\frac{\alpha}{L} \\ \frac{\alpha}{C_o} & -\frac{\alpha}{RC_o} \end{bmatrix}$	$\underline{A}_2 = \begin{bmatrix} -\frac{1}{L}(r_2 + r_L + \alpha r_C) & -\frac{\alpha}{L} \\ \frac{\alpha}{C_o} & -\frac{\alpha}{RC_o} \end{bmatrix}$
$\underline{B}_1 = \begin{bmatrix} \frac{1}{L} & 0 \end{bmatrix}^T$	$\underline{B}_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$
$C = C_1 = C_2 = \begin{bmatrix} \alpha r_C & \alpha \end{bmatrix}$	

Then, the overall state space model that means, for two subsystems we will get two A1 A2 matrix and two B1 B 2 matrix and C matrix which is common, because our inductor if you go back to the circuit you will find this particular part is common right. So, our C matrix will be the same, C matrix will be same, because it is associated with the output side dynamics.

(Refer Slide Time: 36:22)

Applying State-space Averaging and Linearization

- State space average dynamics $\langle x \rangle = [dA_1 + (1-d)A_2]\langle x \rangle + [dB_1 + (1-d)B_2]\langle v_{in} \rangle$
- Considering perturbations $\langle x \rangle = X + \hat{x}$; $d = D + \hat{d}$; $\langle v_{in} \rangle = V_{in} + \hat{v}_{in}$
- Equilibrium point $\underbrace{(DA_1 + (1-D)A_2)}_A \underline{X} + \underbrace{(DB_1 + (1-D)B_2)}_B V_{in} = 0$ X = -A⁻¹B V_{in}
- Linearized small-signal model

$$\hat{\dot{x}} = A\hat{x} + B\hat{v}_{in} + \underbrace{\left[(A_1 - A_2)\underline{X} + (B_1 - B_2)V_{in} \right]}_{B_{11}} \hat{d}$$

$$\hat{y} = \begin{bmatrix} \alpha r_c & \alpha \end{bmatrix} \hat{x}$$

So, applying state space averaging technique that we discuss first we have to obtain the state space average dynamics, then we have to consider the perturbation that we discuss already. Then we need to find out the equilibrium point and from here we can find out X equal to minus A inverse B into V in and we discuss this is A B, and A is invertible, then we can find out the linearized small-signal model and this we have shown it can very easily obtained by Taylor series.

So, where we got A matrix to be is given us. I mean this all thing we can obtain A matrix we have already shown, B matrix also we have shown ok. So, we talk about this is B12 and this is this whole thing to be B11 that we have already derived earlier.

(Refer Slide Time: 37:27)

Small-Signal Transfer Functions

$$\begin{bmatrix} \hat{i}_L(s) \\ \hat{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix} \quad \hat{v}_o(s) = C\hat{x}(s) = C(sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

Control-to-output TF $G_{vd}(s) = \left. \frac{\hat{v}_o(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = C(sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$

Audio-susceptibility $G_{vg}(s) = \left. \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} \right|_{\hat{d}=0} = C(sI - A)^{-1} B \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

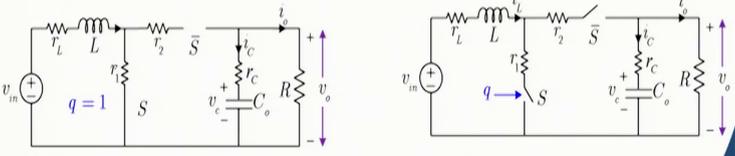
Control-to-inductor current TF $G_{id}(s) = \left. \frac{\hat{i}_L(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = \begin{bmatrix} 1 & 0 \end{bmatrix} (sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$



So, with this we can write down the full expression that we already wrote for a generic state space averaging approach, then we can obtain control to output transfer function, we can obtain audio susceptibility and as well as control to inductor current transfer function.

(Refer Slide Time: 37:47)

State-space Averaging in a Boost converter



Subinterval I

$$v_{in} - (r_1 + r_L)i_L - L \frac{di_L}{dt} = 0 \quad \dots(1)$$

$$C_o \frac{dv_c}{dt} = i_c = -i_o = -\frac{v_o}{R} \quad \dots(2)$$

$$v_o - i_c r_c - v_c = 0 \quad \dots(3)$$

Now, interestingly you will find that A matrix is common for a buck converter; that means, A; because if you take the individual subsystem except for this element. If we consider they are more or less same, then this A1 A2 both are same; that means your A matrix will be common. But, if we take a boost converter, it is something different where in a boost

converter again we can take two sub intervals; sub interval 1, sub interval 2. In this case, our switch main switch is on and the complementary switch is off and we can write down all the equations.

(Refer Slide Time: 38:35)

State-space Averaging in a Boost converter

▪ From (1), (2) and (3)

$x_1 = i_L$
 $x_2 = v_C$

State Equation

$$\frac{di_L}{dt} = -\frac{1}{L}(r_1 + r_L)i_L + \frac{1}{L}v_{in}$$

$$\frac{dv_C}{dt} = -\frac{\alpha}{RC_o}v_C$$

Output Equation $v_o = \alpha v_C$

Subinterval 1

And then we can plug in to make the state space equation where again we are taking x_1 to be i_L and x_2 to be v_C the capacitor voltage. This is for mode one equation and here it is the output voltage equation. In this case, the output voltage is not connected to the inductor current it is connected to the capacitor, output capacitor is output voltage is connected to the capacitor and capacitor is discharged by the load current.

(Refer Slide Time: 39:05)

State-space Averaging in a Boost converter

$$v_{in} - (r_2 + r_L)i_L - L \frac{di_L}{dt} - v_o = 0 \quad \dots(4)$$

$$C_o \frac{dv_C}{dt} = i_c = i_L - i_o = i_L - \frac{v_o}{R} \quad \dots(5)$$

$$v_o - i_c r_c - v_c = 0 \quad \dots(6)$$

Subinterval 2

But, if we take subinterval 2 when the main switch is off and this auxiliary switch is on that mean complementary switch is on for a synchronous boost converter, again we can write down all the equations.

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State-space Averaging in a Boost converter

- From (4), (5) and (6)

$x_1 = i_L$
 $x_2 = v_C$

$$\left\{ \begin{array}{l} \frac{di_L}{dt} = -\frac{1}{L}(r_2 + r_L + \alpha r_c)i_L - \frac{\alpha}{L}v_c + \frac{1}{L}v_{in} \\ \frac{dv_C}{dt} = \frac{\alpha}{C_o}i_L - \frac{\alpha}{RC_o}v_c \end{array} \right. \quad \text{Subinterval 2}$$

Output Equation $v_o = \alpha r_c i_L + \alpha v_C$

And then we can summarize the state space equation here using the state variable. In this equation, you will find the output equation is different, because here you have an additional inductor current term. Because in this case, when the switch is off the inductor current is connected to the capacitor side.

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Overall State Space Model

$$\dot{x} = A_j x + B_j v_m \quad x = \begin{bmatrix} i_L & v_c \end{bmatrix}^T$$

Subinterval 1

$$A_1 = \begin{bmatrix} -\frac{1}{L}(r_1 + r_L) & 0 \\ 0 & -\frac{\alpha}{RC_o} \end{bmatrix}$$

Subinterval 2

$$A_2 = \begin{bmatrix} -\frac{1}{L}(r_2 + r_L + \alpha r_c) & -\frac{\alpha}{L} \\ \frac{\alpha}{C_o} & -\frac{\alpha}{RC_o} \end{bmatrix}$$

$$B = B_1 = B_2 = \begin{bmatrix} 1/L & 0 \end{bmatrix}^T \quad C_1 = \begin{bmatrix} 0 & \alpha \end{bmatrix} \quad C_2 = \begin{bmatrix} \alpha r_c & \alpha \end{bmatrix}$$

$\alpha r_c r_L = \alpha r_c (L)$



So, as a result, the capacitor output voltage will have two if you see the output voltage dynamics C1 and C 2 are different. And as a result at each switching, whenever the switch changes its state, there will be a discrete jump due to this jump, because there will be a term which is alpha r c i x 1 and x 1 is nothing but alpha r c into iL. When the switch is on this will be 0. When the switch is off the iL is connected.

So, at every switching transition, there will be a discrete jump in the output voltage due to this you know and non-minimum phase behaviour, particularly the inductor current is connected and disconnected and this will create a direct jump in the output voltage. So, output voltage in a boost converter will have a discontinuous jump, whereas in case of buck converter, if you do not consider the effective series inductance ESL.

Then there will be no discrete jump because ESL will only increase the ripple magnitude, but it will not introduce a discrete jump. But, ESL in a boost converter will introduce a discrete jump.

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Applying State-space Averaging and Linearization

- State space average dynamics $\langle x \rangle = [dA_1 + (1-d)A_2]\langle x \rangle + [dB_1 + (1-d)B_2]\langle v_{in} \rangle$
- Considering perturbations $\langle x \rangle = X + \hat{x}$; $d = D + \hat{d}$; $\langle v_{in} \rangle = V_{in} + \hat{v}_{in}$
- Equilibrium point $\underbrace{(DA_1 + (1-D)A_2)}_A \underline{X} + \underbrace{(DB_1 + (1-D)B_2)}_B V_{in} = 0$
- Linearized small-signal model

$$\dot{\hat{x}} = A\hat{x} + B\hat{v}_{in} + \underbrace{\left[(A_1 - A_2)\underline{X} + (B_1 - B_2)V_{in} \right]}_{B_1} \hat{d}$$

$$\hat{v}_o = [DC_1 + (1-D)C_2] \hat{x} + (C_1 - C_2) X \hat{d}$$



$X = -A^{-1} B V_{in}$ $A \rightarrow$ invertible

Now, we can write down the state space equation again. Here you will notice that two system matrices are fundamentally different because these two terms are 0, but here it is nonzero right. It is fundamentally different. Even though you take an ideal buck converter, boost converter, these two terms will make this A1 and A2 matrix are fundamentally different.

But interestingly, here B1 and B2 matrix are common because in a boost converter, the input voltage is connected all the time. When the switch is ON or switch is OFF it is all the time it is connected. You see input voltage is never disconnected from the inductor side. So, B matrix will be always here in this case the same B matrix, but A matrix is different.

So, if you take the state space average model, then again you can write down the average dynamics, you can find out you can consider the perturbation and equilibrium point. And as I told here again, you will find out X equal to minus A inverse B V in, including this you know average this average model if you consider the steady state this A matrix will be in it is invertible.

But, if you take the individual A matrix for ideal case A1 is not invertible, because if all elements are 0 except for the last element it will be 0, if we go back to the equation under ideal condition this term will be 0. So, all three elements are 0 except for this one which is nonzero. But, if you take the average A matrix, which is invertible and again we can derive the linearized small-signal model from this expression using this expression ok.

So, this will be again for our case B12 and for our case it is this whole thing is B11 ok. And the average dynamics for the boost converter will have in this case this equation will be different, because we are talking about the output voltage, which is the output voltage average if we take the perturb quantity will be what will be the perturb quantity it will have two component one is because for a boost converter you have a duty ratio dependency C matrix are different.

So, we will get 1 D into C 1 plus 1 minus D into C 2. This whole thing into x hat plus C 1 minus C 2 that into capital X this whole quantity into d hat. So, output voltage will have two components; one is this term under steady state and another is this term and this will create a difference. So, this equation was actually it is just copied from a buck, but for boost it is the output voltage average dynamics.

(Refer Slide Time: 44:27)

Small-Signal Transfer Functions $C = (D+1-D)$

$$\begin{bmatrix} \hat{i}_L(s) \\ \hat{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix} \quad \hat{v}_o(s) = C\hat{x}(s) = C(sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

Control-to-output TF $G_{vd}(s) = \left. \frac{\hat{v}_o(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = C(sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$

Audio-susceptibility $G_{vg}(s) = \left. \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} \right|_{\hat{d}=0} = C(sI - A)^{-1} B \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

Control-to-inductor current TF $G_{id}(s) = \left. \frac{\hat{i}_L(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = \begin{bmatrix} 1 & 0 \end{bmatrix} (sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$

Small signal transfer function, so in the small signal transfer function again the same method can be used. We already found A matrix, B matrix, C matrix. So, we can obtain we can write down the expression of the control to output transfer function. So, in this case we also found that means, if we do not consider any ESR say these models are perfectly fine and if you do not consider you know. So, we can obtain the transfer function of a boost converter for different switch configuration.

So, here the C matrix is the equivalent C matrix which is D plus 1 minus D into what is this equivalent C matrix sorry.

(Refer Slide Time: 45:30)

Small-Signal Transfer Functions $C = Dc_1 + (1-D)c_2$

$$\begin{bmatrix} \hat{i}_L(s) \\ \hat{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix} \quad \hat{v}_o(s) = C\hat{x}(s) = C(sI - A)^{-1} B \begin{bmatrix} \hat{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

Control-to-output TF $G_{vd}(s) = \left. \frac{\hat{v}_o(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = C(sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$

Audio-susceptibility $G_{vg}(s) = \left. \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} \right|_{\hat{d}=0} = C(sI - A)^{-1} B \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

Control-to-inductor current TF $G_{id}(s) = \left. \frac{\hat{i}_L(s)}{\hat{d}(s)} \right|_{\hat{v}_{in}=0} = \begin{bmatrix} 1 & 0 \end{bmatrix} (sI - A)^{-1} B \begin{bmatrix} 1 & 0 \end{bmatrix}^T$

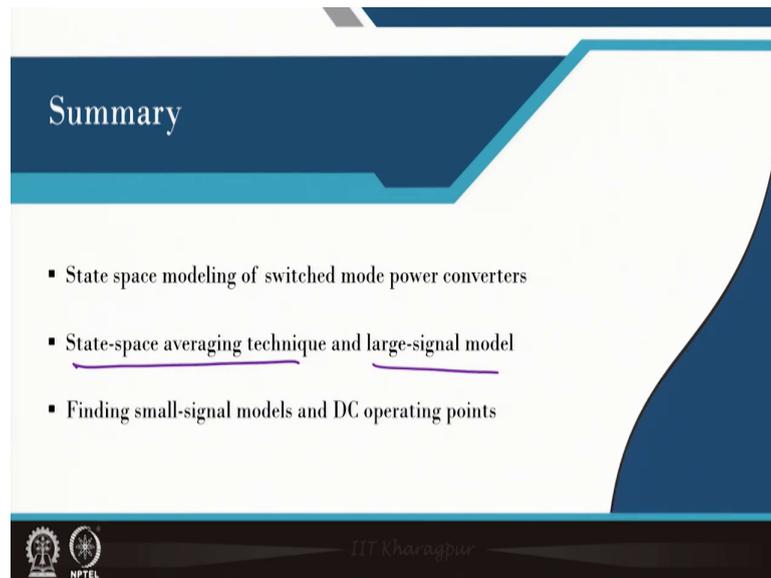


So, the C matrix that we obtain that C is equal to D into C 1 plus 1 minus D into C 2 that we can find out. So, we can derive audio susceptibility and control to inductor current transfer function.

So, we will see in subsequent lecture what are the transfer function, how does it look like and we also want to derive such transfer function using circuit averaging technique and we want to compare that which one is accurate whether they are same or they are different.

And then we want to show the validation of the small signal model, both large signal and small signal model and we want to match a response time domain response of the actual switch simulation and the same thing obtained from the linear small signal model and that we want to verify in subsequent lecture.

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The slide is titled "Summary" and contains three bullet points. The first bullet point is "State space modeling of switched mode power converters". The second bullet point is "State-space averaging technique and large-signal model", with "State-space averaging technique" underlined in purple. The third bullet point is "Finding small-signal models and DC operating points". At the bottom left, there are logos for IIT Kharagpur and NPTEL. At the bottom center, the text "IIT Kharagpur" is displayed.

Summary

- State space modeling of switched mode power converters
- State-space averaging technique and large-signal model
- Finding small-signal models and DC operating points

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So, with this I want to summarize that today we talked about state space modeling of switching power converter switch mode power converters. We also obtained state space. We have shown state space averaging technique and large signal model, which is the average model, then I have shown you how to find small signal model and DC operating points. So, with this we want to finish today lecture.

Thank you very much.