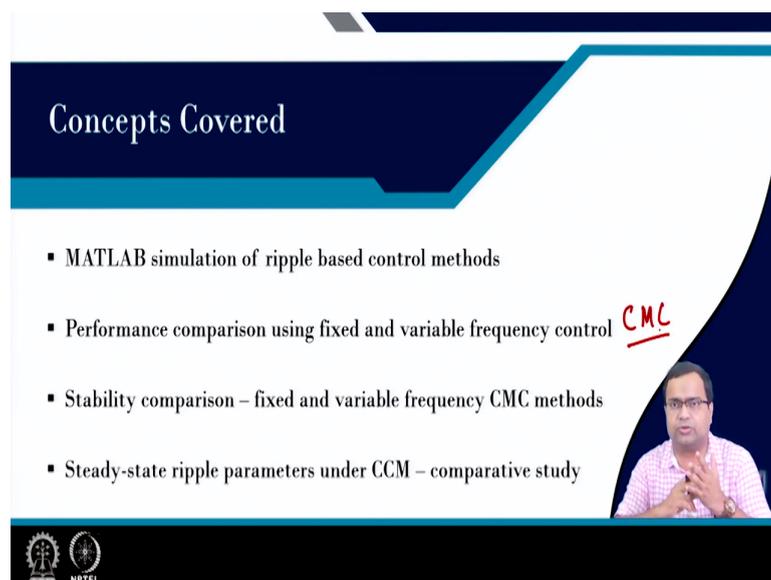


Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 04
Variable Frequency Control Methods
Lecture - 23
Stability and Performance Comparison Using MATLAB Simulation

This is lecture number 23. In this lecture we are going to talk about Stability and Performance Comparison using MATLAB Simulation. And this is you know we are going to consider here Variable Frequency control as well as fixed frequency control. And we want I also want to demonstrate some MATLAB simulation of different control technique, current mode control technique and their comparison.

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The slide is titled "Concepts Covered" and features a list of four bullet points. The text "CMC" is written in red above the third bullet point. A small video inset in the bottom right corner shows the professor speaking. The NPTEL logo is visible in the bottom left corner.

- MATLAB simulation of ripple based control methods
- Performance comparison using fixed and variable frequency control
- Stability comparison – fixed and variable frequency CMC methods
- Steady-state ripple parameters under CCM – comparative study

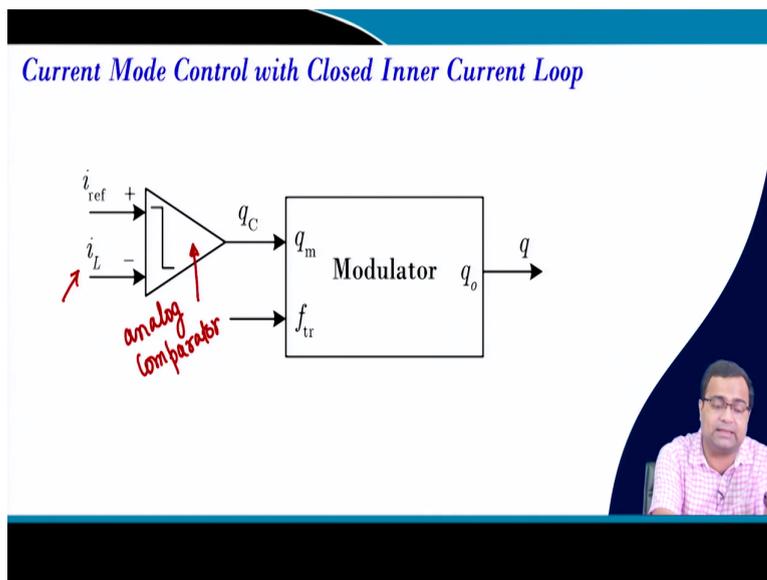
So, the main concept which is covered today's class is a MATLAB simulation of ripple based control, as well as you know, we have already simulated for fixed frequency control. Then we want to compare performance using fixed and variable frequency control. Particularly, we will talk about current mode control. So, current mode control context.

Then, we want to talk about stability comparison like a fixed and variable frequency current mode control method. And then steady state ripple parameter under current mode control we want to do a comparative study ok. And we are considering under continuous conduction

mode and we are talking about like a synchronous computation; that means, always under continuous conduction mode.

And we are going to talk about peak current mode control, valley current mode control under fixed frequency. Also, we are going to talk about constant on time, constant off time as well as hysteresis control, all are in under current mode control and then we want to see the comparison.

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So, I want to start with the basic modulator. This is a building block ok. So, if you want to implement a current mode control logic, the first thing that we require an analog comparator. So, this is my analog comparator so, in this analog comparator few; that means, this is our analog comparator and then this analog comparator is used to compare between the reference current and the inductor current ok. So, here this inductor current we are showing it is a sense inductor current.

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MATLAB Implementation of Ripple based CMC

- Peak current mode control
- Valley current mode control
- Constant on-time control
- Constant off-time control
- Hysteresis control

Handwritten notes:

Valley CMC: $i_v = i_{ref}$, $i_p = i_{ref} + \Delta i_H$

Peak CMC: $i_p = i_{ref}$, $i_v = i_{ref} - \Delta i_H$

Average CMC: $i_p = i_{ref} + \frac{\Delta i_H}{2}$, $i_v = i_{ref} - \frac{\Delta i_H}{2}$

Red annotations:

- ↑ Analogous - valley current control
- ↑ Analogous - peak current control

So, first we want to see MATLAB implementation of ripple based current mode control. So, in this context, we will talk about peak current mode control, then valley current mode control, then constant on time. So, which is analogous to, so this is analogous because both use valley current, valley current control ok. So, both controls vary.

So, the constant off time is analogous to peak current control. And finally, hysteresis control can be configured in peak version; that means, I will come to that point and it can be also configured to valley, so depending upon.

So, for example, if you are talking about it require a hysteresis band. Suppose we are talking about a hysteresis band like this and now we want to keep our inductor current within this band, ok. So, within this band, this is our peak reference, and this is our valley reference and this is our actual inductor current ok.

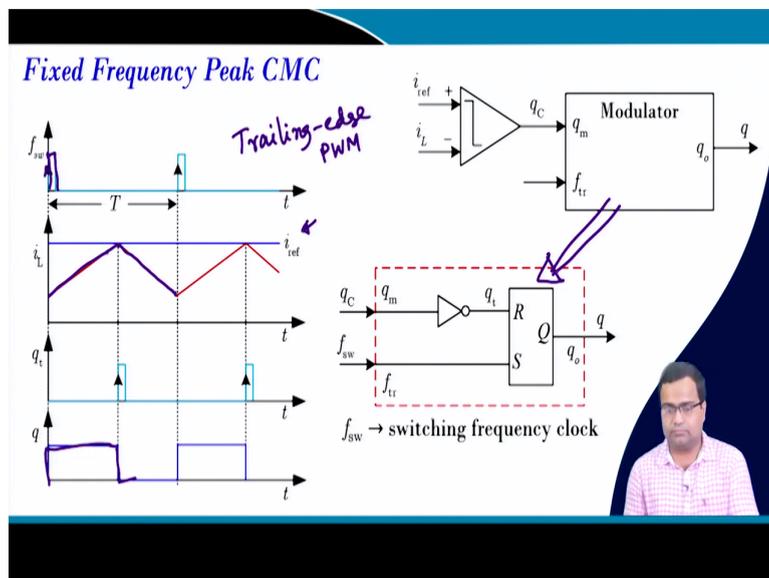
Now, if we set i_p to be equal to i_{ref} and then i_v is simply $i_{ref} - \Delta i_H$; that means, we are going to directly control the peak current. And the hysteresis band, which is nothing, but this band this is used to you know control the ripple of the current as well as this is used to you know control the switching frequency ok. So, this is here we are setting like a peak configuration, peak current mode configuration.

Now, if we want to do valley current mode configuration then, what we will do? We will simply set i_v to i_{ref} and i_p to $i_{ref} + \Delta i_H$. It has to just add the hysteresis band and if

we want to implement in average current mode control average current mode control, then you simply set i_{peak} to be $i_{ref} + \Delta i_H / 2$ and i_{valley} to be $i_{ref} - \Delta i_H / 2$.

So, this will implement average current mode control; that means, if we use a hysteretic controller, then you can very first average current mode control you can implement simply by using this configuration. But we will see this configuration suffer from variable frequency ok. So, sometime it is not possible and in fact, you need to sense the complete current. So, we will discuss some of this aspect.

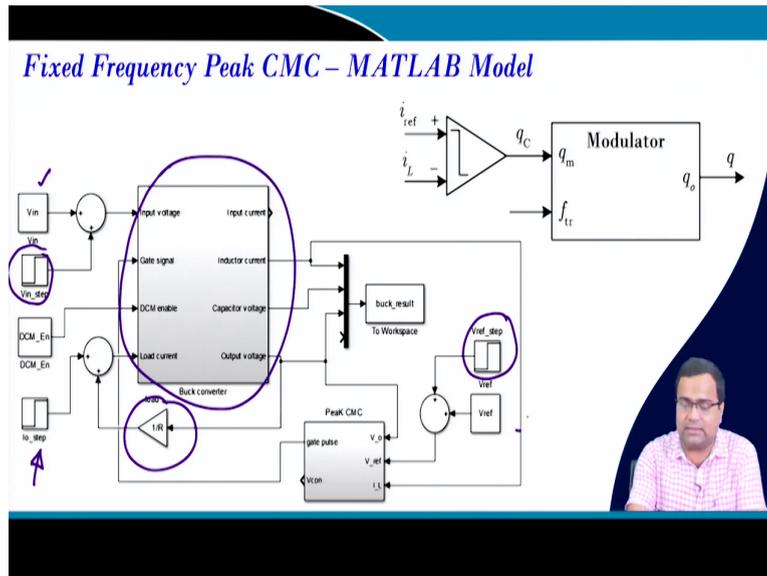
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So, in fixed frequency peak current mode control, we are talking about this modulator. Now if we draw the inductor current waveform let us say, so, in peak current mode control where reference current act as the peak reference current ok. And the inductor current will simply follow; that means, when it hit the upper limit it turns off and then it turns sorry it turns off.

So, it is under trailing edge modulation; that means trailing edge modulation, and we have discussed multiple time. So, it is trailing edge PWM where the switch turns on at the rising edge of the clock; at the rising edge of the clock the switch turns on, here the switch turns on ok. And the switch turns off whenever the comparator that means, inductor current hit the peak current limit ok. And the modulator structure will look like this in this peak current mode control.

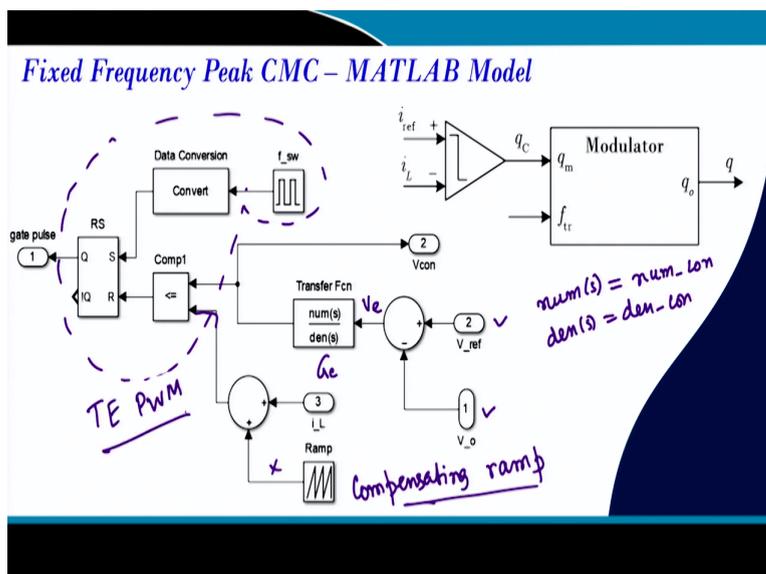
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So, if we want to implement MATLAB model. So, we have already this converter, so this is our converter model and we have already implemented. And here we are using resistive load and we can make a load transient by using a constant current load, external load. And this is our input voltage. Here we are talking about the ideal voltage source and we can add an input supply transient here.

Then we can also create a reference transient by applying a step transient here and this is a reference voltage. What is inside this block?

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So, this block inside you know this is our error voltage right, because this is our reference voltage, this is an output voltage and this is a controller ok. And this controller we are actually programming in the dot m file, so we are not separately putting a controller. So, if you click on that it will you know you will find the numerator which is here you will find if you go it is the numerator under controller that we will see soon. And if you go to denominator, then we will see denominator controller ok.

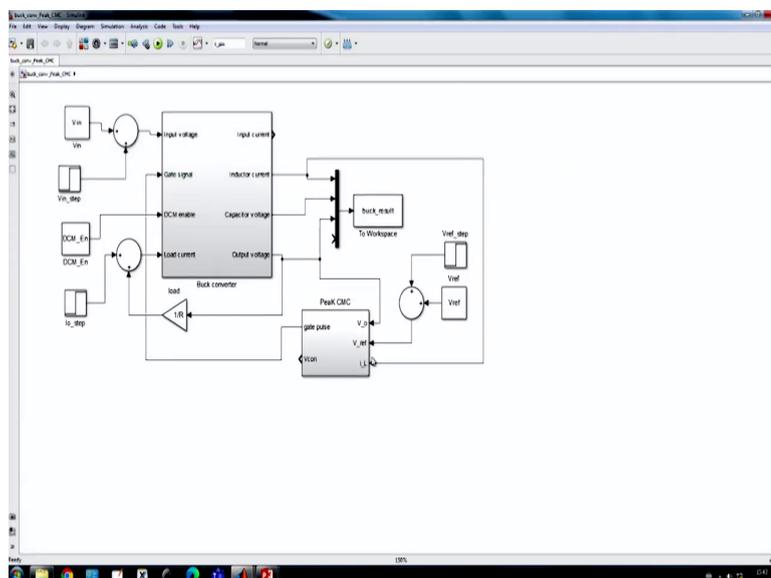
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```

1 - clc; close all; clear;
2
3 %% Parameters
4 - buck_parameter; Vin=12; Vref=1; D=Vref/Vin;
5 - R=1; r_eq=r_L+r_l; alpha=(R+r_eq)/R;
6 - Io_min=0.5; R_max=Vref/Io_min;
7
8 - V_m=1;
9
10 %% PI Controller
11 - K_p=100; K_i=300000;
12
13 - num_con=[K_p K_i];
14 - den_con=[1 0];
15 - Gc=tf(num_con,den_con);
16
17 %% Transient parameters and transient response
18 - t_sim=5e-3; t_step=2e-3; f_m=1e3;
19 - delta_Io=0; delta_Vin=0; delta_Vref=0.1;
  
```

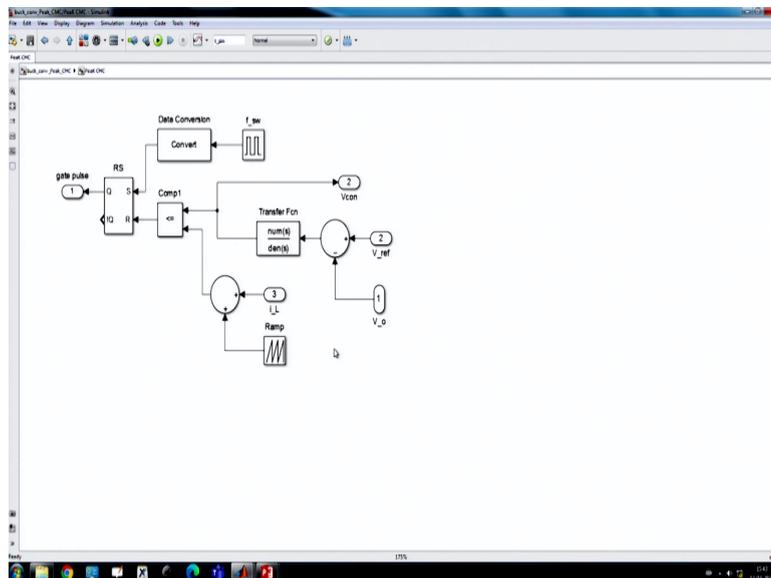
So, if you go back to a MATLAB file.

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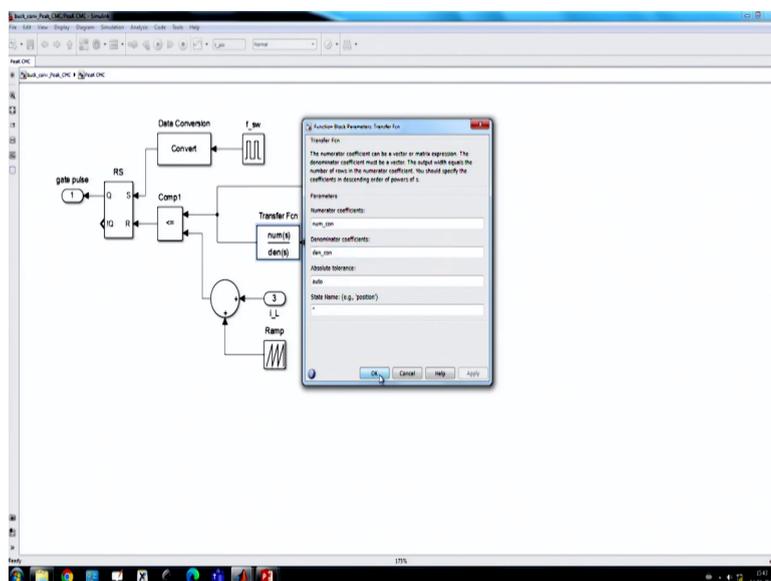


I will just show you the peak current mode control. This is what I am showing here.

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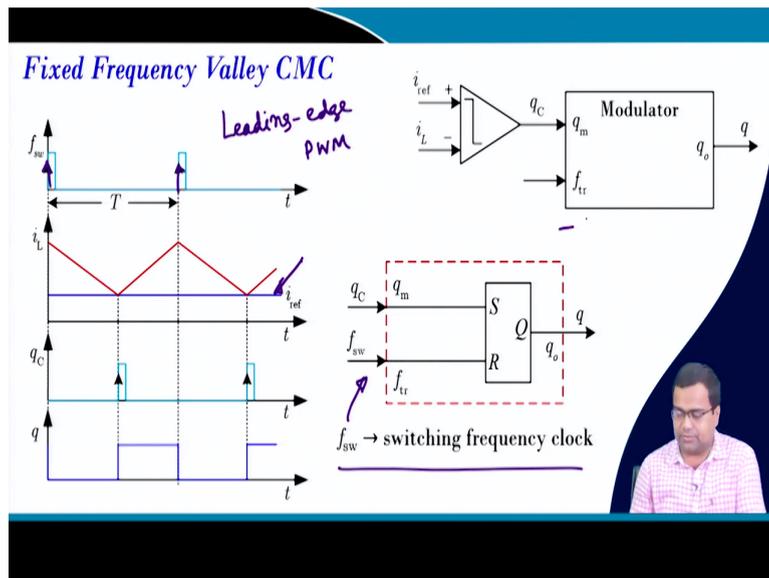
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So, if you go inside, then this is the model numerator control and denominator of the controller. Those we are programming from the MATLAB file ok, so we are not separately controlling. Now you can see there is a separate sawtooth waveform; that means, this is our ramp compensation or some compensating ramp. We may or may not need this because in generally if the duty ratio is much lower than 50 percent, we may not need this one, we may not need.

But as we go for you know higher duty ratio or even for higher gain also you may need to consider ramp in order to stabilize the current loop ok. So, this is an error voltage, this is an inductor current if the ramp is not there as if this is the inductor current is directly compared here ok. And this block, this particular block, if you see this including this clock it is our trailing edge PWM; trailing edge PWM block ok.

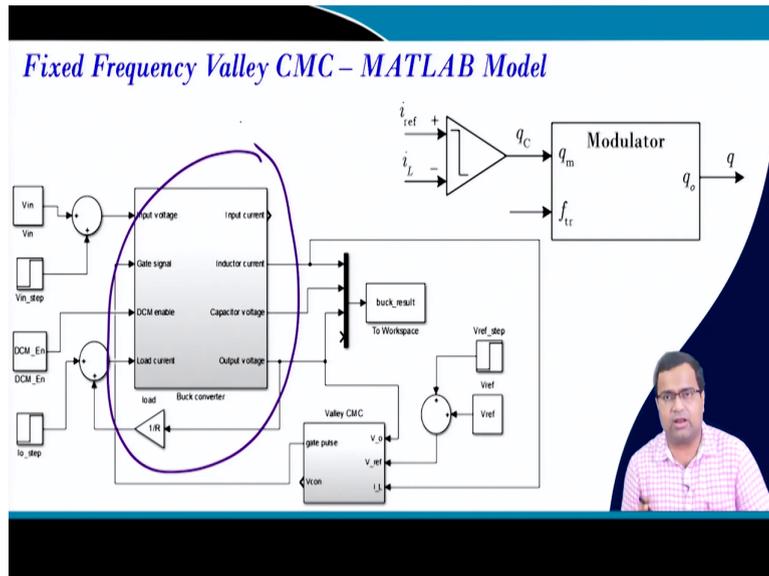
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Now, if you go for fixed frequency valley current mode control, now this architecture our reference current is our valley current and inductor current should be above that reference current and this is exactly shown here. So, here this is our valley current reference, this is our valley current reference ok and inductor current here it is leading edge modulation; leading edge modulation, leading edge PWM.

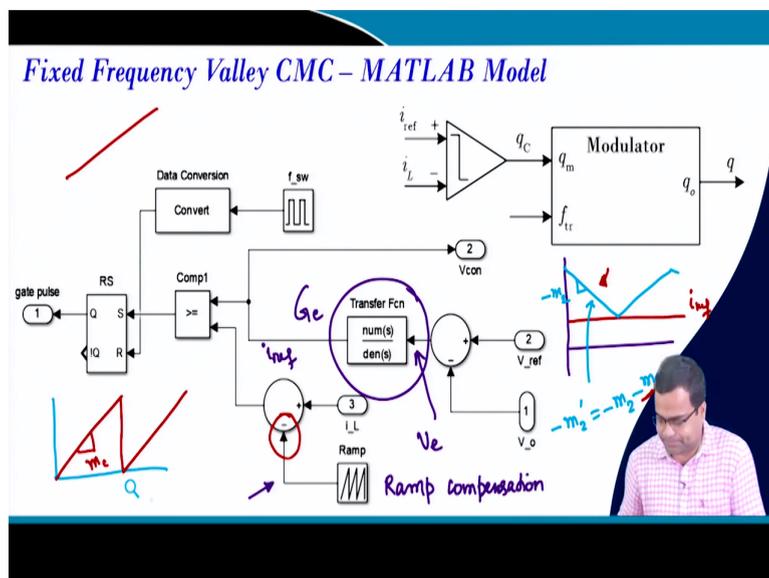
So, here at the beginning of the rising edge of the switching clock, the switch turns off fast and then current continues to fall. When the current hit the reference current, which is the valley current, then the switch turns on and switch will remain on until the next arrival of the next clock edge, ok. So, then again, the switch will turn off, so this is a process under valley current mode control. And if you want to and this is a switching frequency is used as a trigger pulse here in the generic block, ok.

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So, if you want to implement the MATLAB model, then again, this block is known. Everything is common. We will not change, what is inside the valley current mode?

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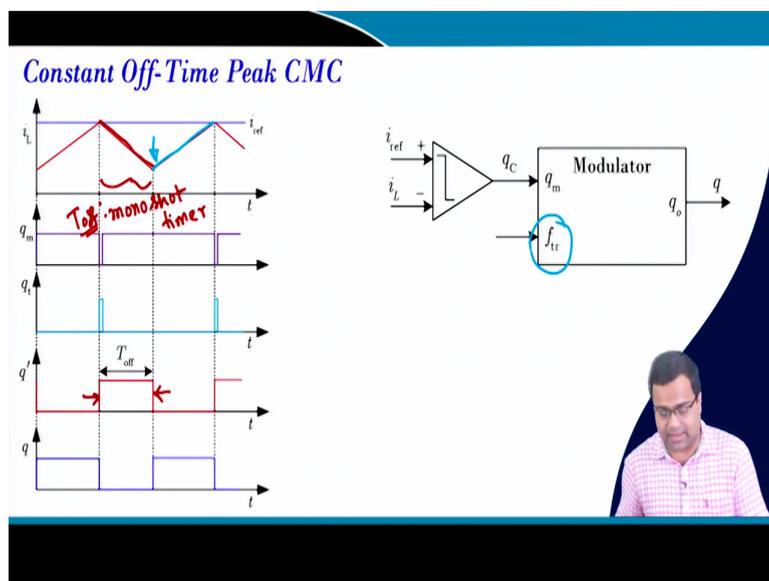
So, here again, this transfer function is the controller transfer function. So, this is our controller transfer function and again, this is our error voltage. So, this particular point is our error voltage, which is the reference voltage minus the output voltage. Then transfer function. Then this is our you know this is our refer valley current; that means, this is our i_{ref} that we discuss ok.

Now, in this case again you can use a ramp compensation ok, ramp compensation and you will find here our ramp is subtracted. Basically, we are talking about a positive ramp because, why? Here we are talking about valley current mode control. So, if you look at this valley current, reference the inductor current is actually like this and then it goes like.

So, here we have to add compensating ramp and, add means what? The falling slope, so if you want to add with the falling slope; that means your falling slope is minus m_2 . That means, our ramp should also be falling slope right then only we can add. That means we want to achieve minus m_2 dash which is nothing, but minus m_2 minus m_c where minus m_c is the slope of this. That means, if I take a ramp; if I take a ramp this ramp and if this ramp slope is m_c then if I want to subtract here, then I need to subtract here.

So, this point should be subtraction ok and this is added in case of peak current mode control because there we are talking about the rising slope and we have to add ramp. So, both will be added so that actual slope effective slope will be higher. So, here we are trying to increase the effective falling slope. So, we need to subtract the ramp where we are talking about the ramp to be a positive slope ramp, ok. If you take a negative slope ramp then it should be added ok.

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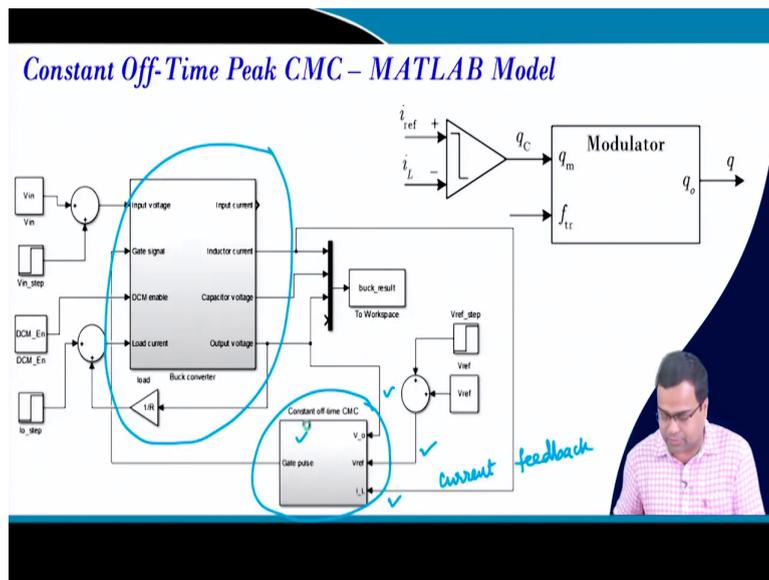
So, now we are talking about constant off time like a peak current mode control. So, this is analogous to our conventional peak current mode control, but earlier in conventional peak current mode control, our time period was constant. Here the off time is constant, which you can see here.

So here, how it works? Initially, if the switch turns on then when it hit the peak current limit, then switch turns off ok, so the switch turns off. Then when switch, how long switch will remain turn off? The duration is decided by the monoshot timer; it is decided by the monoshot timer ok.

So, this comes from the monoshot timer. This is our off time ok, it comes from monoshot timer. When the timer complete its counting then switch turns on ok and then when switch turns on, the current will rise and again hit it, so that is that is the logic.

So, what is the trigger pulse here? So, here the trigger pulse is that when the counter you know actually completes the counting; that means this is a time when the counter will stop, you know, finish counting and it would create a trigger pulse. So that means this will come from you can say when the counter will start. So, this is our you know trigger pulse will come from there.

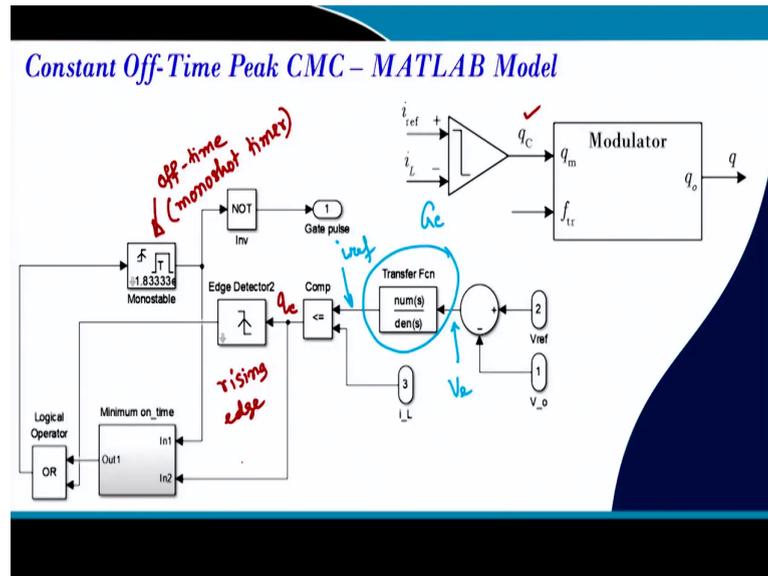
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Now, how to implement this constant off time? Again, this block is common, so there is no change. Now, we have only changed this block ok and we need a current feedback. So, all control techniques we are talking about a current based control. So, the current feedback is essential; that means, this is my current feedback ok, so this is essential.

And since we want to regulate the output voltage, reference voltage and the output voltage should be there. If you go inside this block because we want to see what is inside this block.

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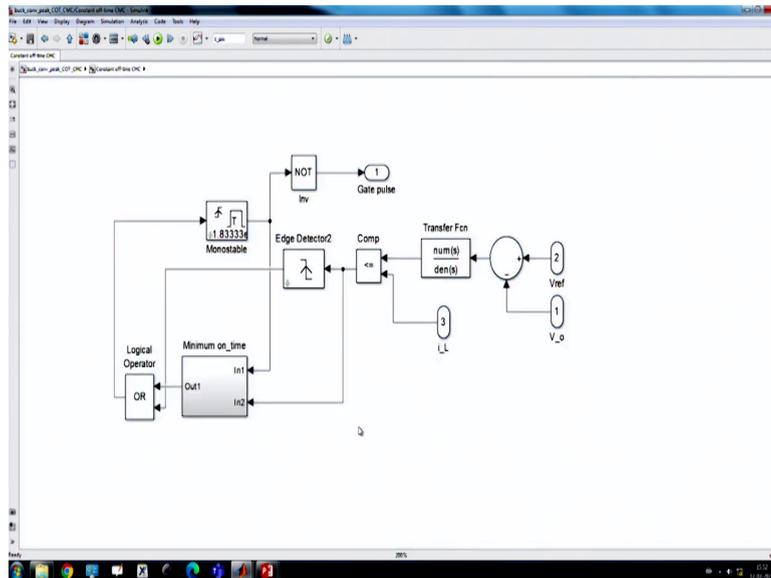


Again, the controller is common. This is our G_c controller and this you can think of reference current ok. And here this is our error voltage and we will see soon that such technique does not require a ramp compensation. Because they are inherently stable, the current loop is inherently stable for the entire duty ratio range. That means, unlike in peak current mode control that you are familiar with, if current loop become unstable when the duty ratio goes beyond 0.5.

In fact, it can be unstable even lower than 0.5 if the controller gain is high, then we need to add a ramp. In case of valley current mode control, if the duty ratio is lower than 0.5 the current loop is unstable and we need to add either negative slope ramp or subtract a positive slope ramp that we have discussed. But in this case, we do not need a ramp. So, then this current and reference they are compared. Then, what is a next task?

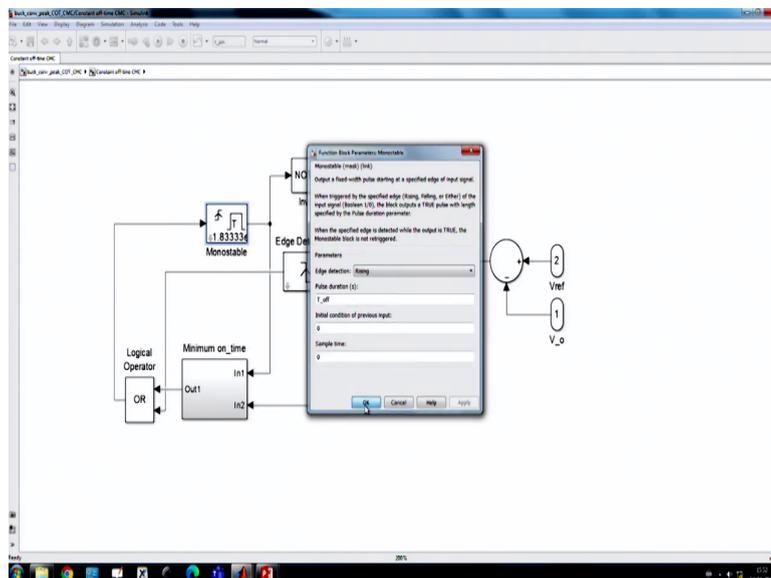
So, the next is this is our comparator output, like an output, comparator output. This output now is trigger circuit is used; that means we are rising. This is a rising edge, this is a rising edge ok. So, this trigger pulse is used as the input to the monoshot timer. So, this is the timer where we fix the off time, where we fix the off time, this is the monoshot timer, ok. So, this is a monoshot timer; a monoshot timer. In fact, why do not we go to the MATLAB, directly.

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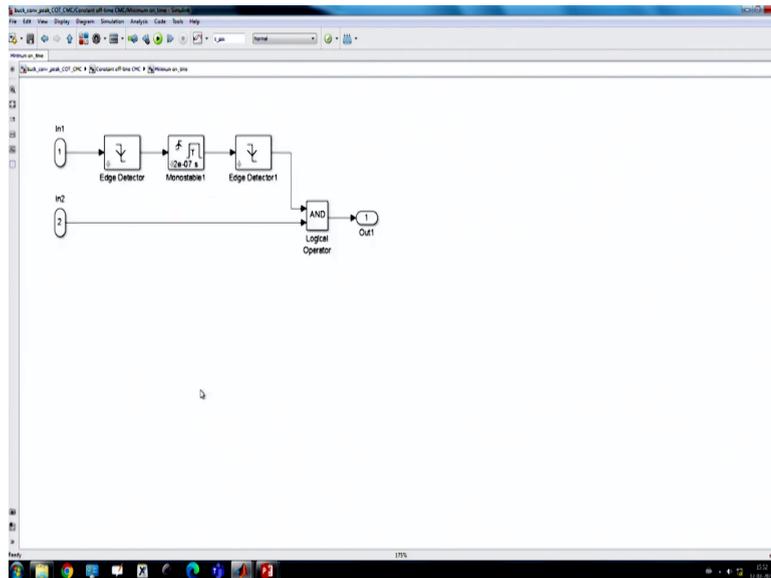
If you go inside this is what we have just discussed now ok.

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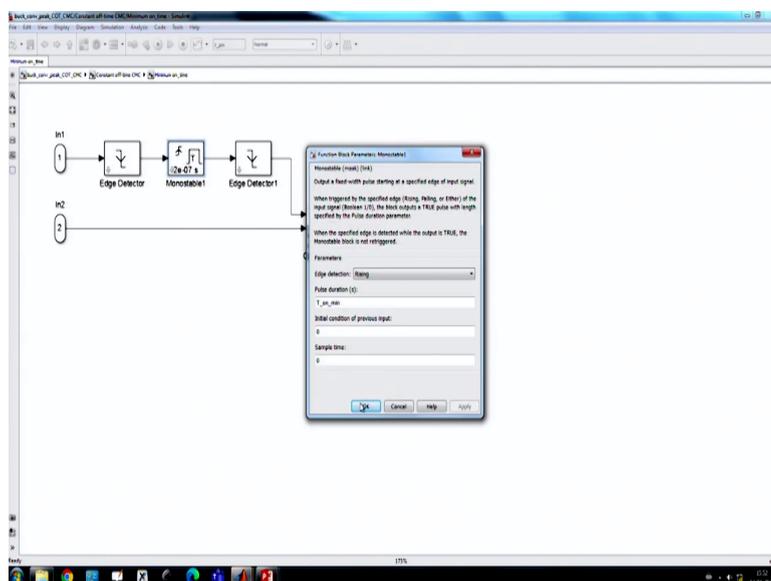
So, this is our monoshot timer where we are setting the constant off time ok. Now, what is this? This block is used as a minimum on time, because we have discussed earlier, if we do not incorporate this minimum on time, then the whole system can collapse.

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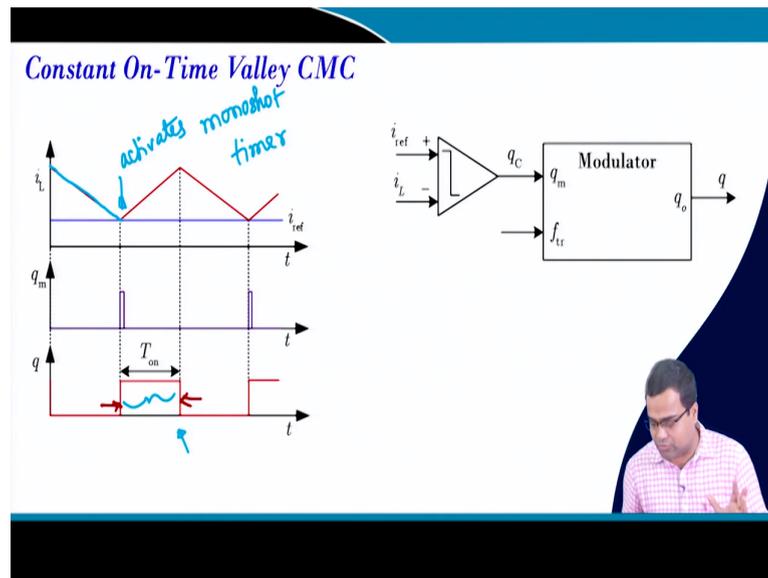
And it is nothing, but you need to detect an edge.

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And this is a delay block and then it is compared. So, this aspect we have discussed in the previous lecture when we discussed in detail about the constant off time. So, we are not going to spend time on that, so that means this part is clear.

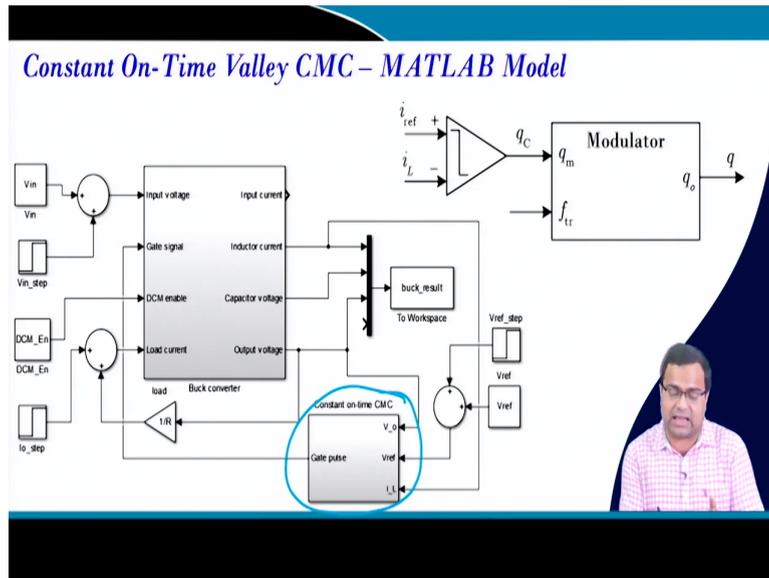
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Next we are talking about constant on time valley current mode control. So, this is similar to our fixed frequency valley current mode control. The only difference here we are fixing this on time rather than time period in the earlier case, ok. So, on time is fixed and here you can see it. Initially, the switch turns off and the inductor current start falling right. And whenever the inductor current actually you know hit the reference current, then the monoshot timer is activated. So, this is where it activates the monoshot timer.

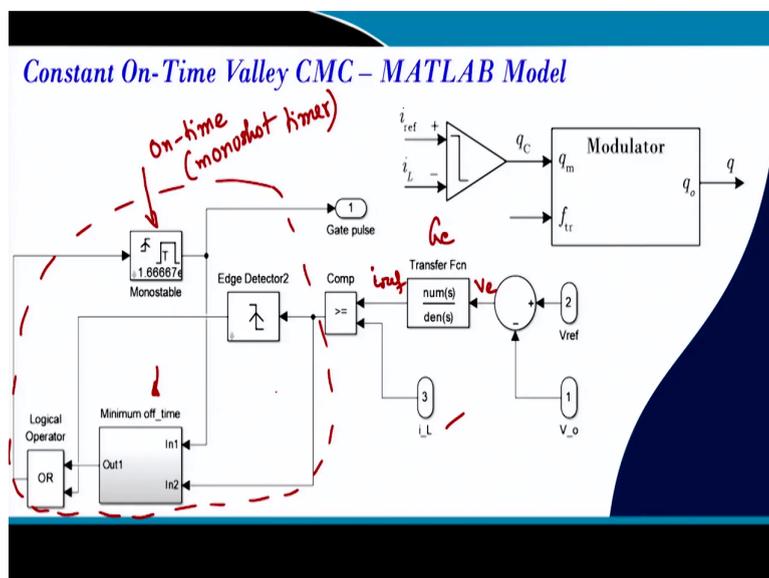
So, the monoshot timer will now this duration it is on and here it will finish the counting and then switch will turn off ok.

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And, again in this technique also if we want to implement in MATLAB, so this part is common. So, this is the constant on time controller that we are talking about. So, we want to see what is inside this block ok.

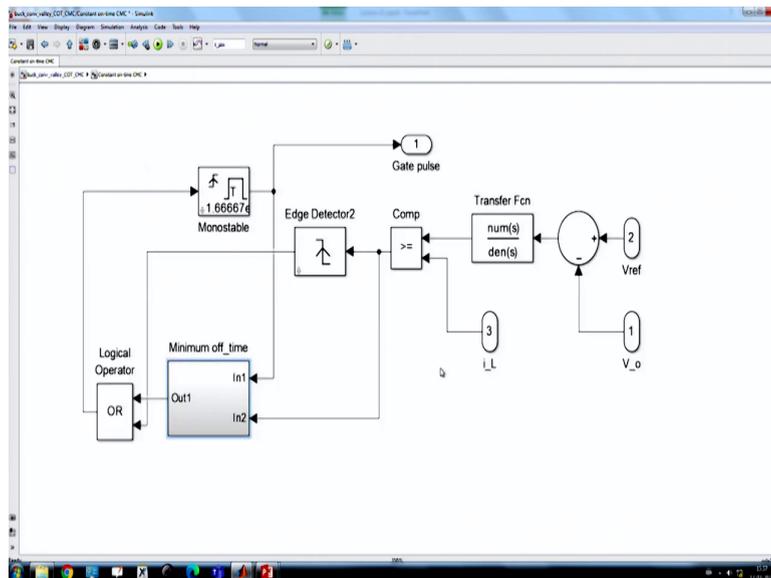
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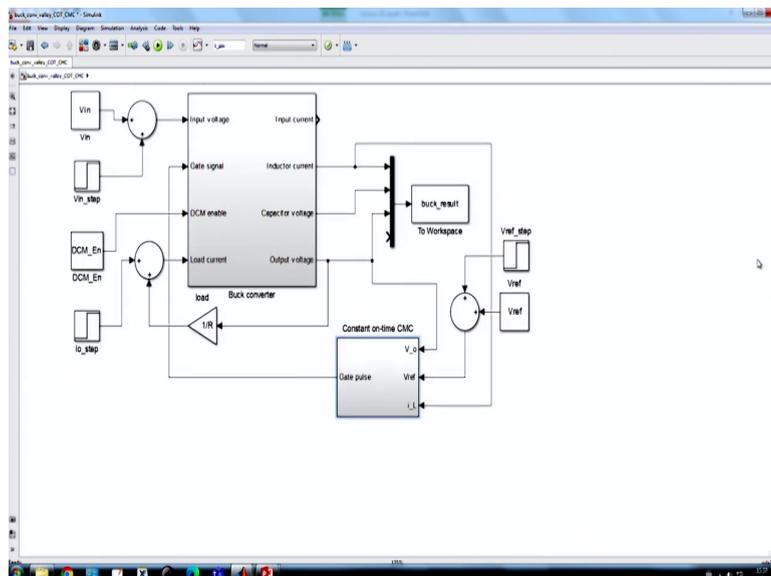
So, if you go inside in valley current mode control, you will find again this is our error voltage ok. So, this is our error voltage; this is our error voltage, this is our controller, this is our reference current, and this is an inductor current ok.

So, here this is the block, which is our modulator like constant on time modulator, and here we are setting the on time, we are setting the on time and using a monoshot timer, ok. And then here this block is used to set the minimum off time, ok. So, this is now if you go inside this MATLAB block, ok.

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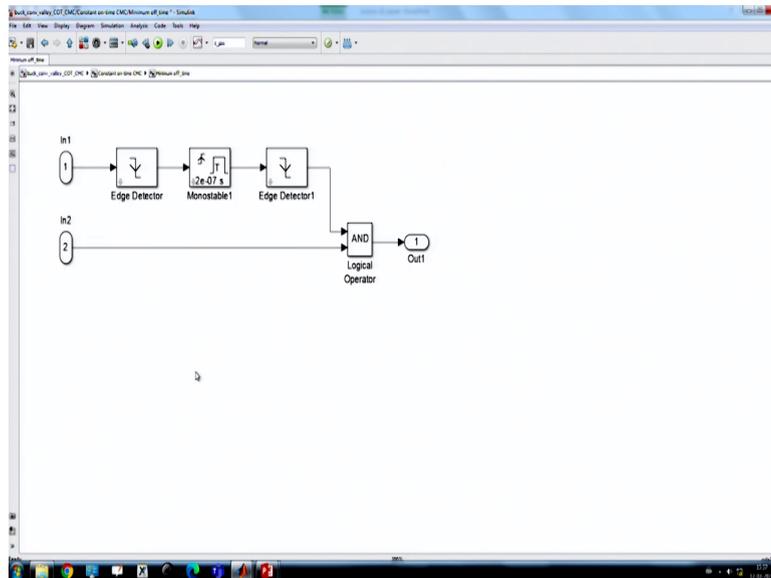


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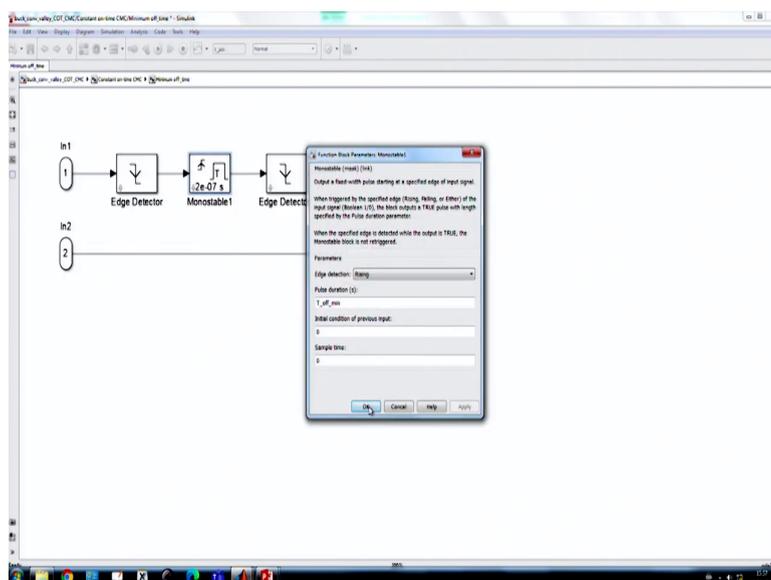


So, this is the monoshot timer. I mean, in a constant off time where if you go inside, this is what I have shown.

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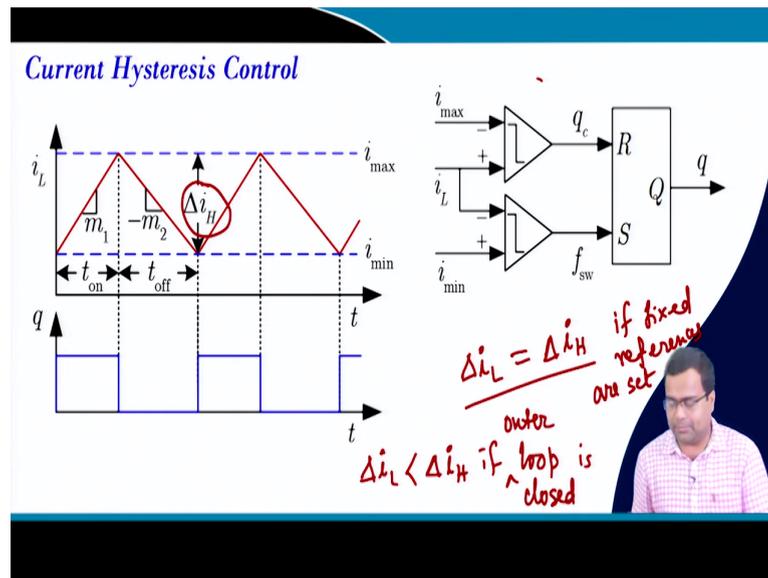


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And if you go inside, then you will find this is where we are setting the minimum off time, ok. So, this is the MATLAB realization minimum off time. This is a delay block ok. So, all these logics are discussed under the context of constant on time ok. So; that means, we are familiar with this all this control logic now let us go to our presentation.

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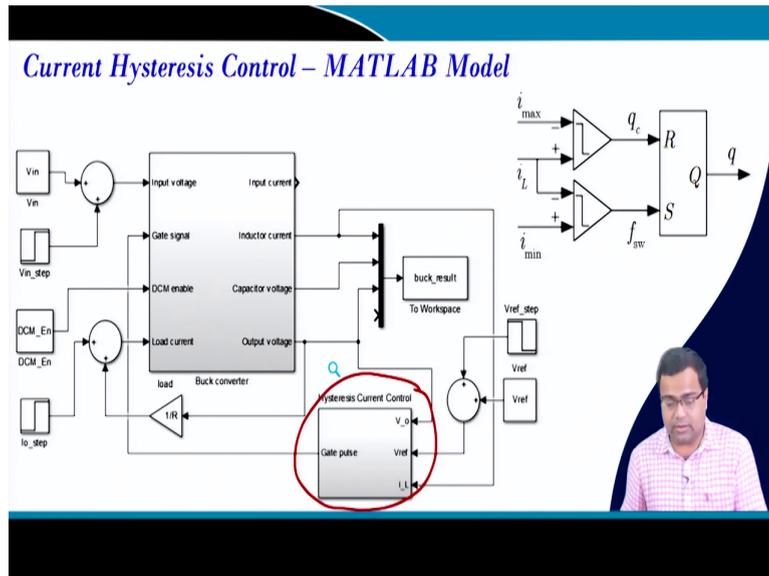
Now, current hysteresis control. So, we have discussed under the hysteresis control that actual current hysteresis control can be realized by using the sense inductor current and we can use a hysteretic comparator ok, and this thing. But now, for sake of simplicity, in the realization here we realize a current hysteresis control by using a two comparator structure. But this is for simulation in actual circuit, one comparator with hysteresis should be sufficient ok

So, here we are limiting the current in between the maximum and the minimum value. And this is our hysteresis band within the band we are trying to limit the inductor current. And we will see, in fact, that if you set the current reference just using constant current value, then you can find out because this is the inductor ripple.

The ripple inductor current will be equal to Δi_H that means both are equal if you set if you fixed a reference is set. But if the loop is closed; that means, if one of the reference is coming from the outer loop and then you add or subtract the hysteresis band.

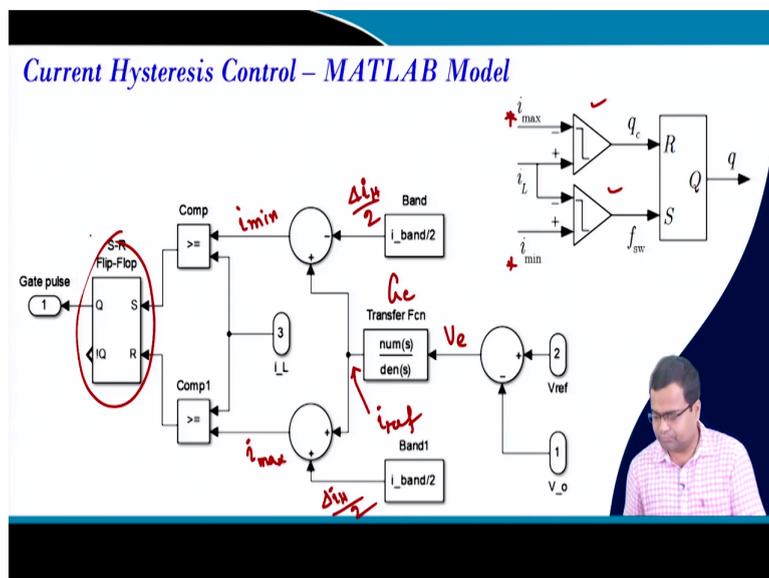
In that case, you will find the actual hysteresis band and the actual inductor current they will be different. In fact, the current ripple will be smaller than, so I will say that the current ripple will be smaller than Δi_H if loop, if outer loop is closed is closed ok.

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So, current hysteresis band can be realized, so this is where we are just changing our controller, all other things are the same.

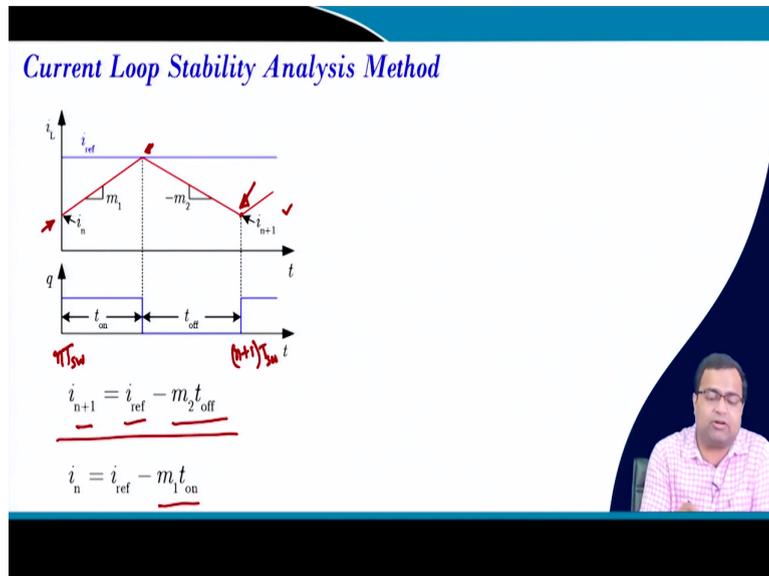
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And here this is our current reference, so here this is our reference current and this is our error voltage. This is our controller. And this is nothing, but $\Delta i_H / 2$ and this is also $\Delta i_H / 2$. So, this is our peak current is a max value, the upper limit, and this is the i_{min} . If we go back in order to make it consistent, yes. So, this is the i_{max} that we are talking

about, i_{min} so it is here. And then we are using comparator like this. Two comparators are used and the R S flip-flop ok.

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So, by that we can implement. Now we want to first see the current loop stability analysis ok. So, this is the current waveform you can think of. This is the peak current architecture, this technique you can use either using fixed frequency or you know constant off time or even you can take hysteresis. So, the first thing we want to obtain the sample value of the inductor current or the final value of the inductor current.

Let us say this is our m T a cycle, and this is our $m+1$ T s cycle; that means, at the end of the cycle, what is the value which is i_{n+1} and initial value is i_n during the cycle. So, I want to write the final value of the i_n of the final value of the inductor current in terms of the initial value of the inductor current, then we want to justify stability.

So, first thing from this equation you can write i_{ref} which this is i_{ref} minus m_2 is the falling slope and this is the off time. Where, what is i_n ? i_n will be equal to i_{ref} minus t_{on} this is our i_n ok.

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(a) Under fixed-frequency modulation

$$t_{\text{on}} = dT; \quad t_{\text{off}} = (1-d)T$$

$$i_{n+1} = i_{\text{ref}} - m_2(1-d)T$$

$$= i_{\text{ref}} + m_2dT - m_2T$$

$$i_n = i_{\text{ref}} - m_1dT \quad \text{Find } d$$

So, if you continue under fixed frequency control, our t_{on} is nothing, but d into T and t_{on} is 1 minus d into T . Then if you write earlier equation i_{n+1} we wrote $i_{\text{ref}} - m_2$ into t_{off} . So, you replace t_{off} here with this $1 - d$ then I want to get this dT . So, what was i_n equation? It was $i_{\text{ref}} - m_1$ into t_{on} , so t_{on} is nothing but dT . So, from this equation, I want to find out find d , find d from here.

(Refer Slide Time: 24:46)

(a) Under fixed-frequency modulation

$$t_{\text{on}} = dT; \quad t_{\text{off}} = (1-d)T$$

$$i_{n+1} = i_{\text{ref}} - m_2(1-d)T$$

$$= i_{\text{ref}} + m_2dT - m_2T$$

$$i_n = i_{\text{ref}} - m_1dT \Rightarrow dT = \left(\frac{i_{\text{ref}} - i_n}{m_1} \right)$$

So ok, so if you want to find d , dT basically, you have to find out dT , so I will say you find out dT . Then we substitute this one into this equation and then, what we will get here?

(Refer Slide Time: 25:06)

Thus,
$$\underline{i_{n+1}} = \underline{i_{ref}} + m_2 \left(\frac{i_{ref} - i_n}{m_1} \right) - m_2 T = - \left(\frac{m_2}{m_1} \right) i_n + \left(1 + \frac{m_2}{m_1} \right) i_{ref} - \underline{m_2 T}$$

Perturbed current dynamics becomes

$$\tilde{i}_{n+1} = - \left(\frac{m_2}{m_1} \right) \tilde{i}_n + \left(1 + \frac{m_2}{m_1} \right) \tilde{i}_{ref}$$

Handwritten notes:
 $i_n = I_n + \tilde{i}_n$
 $i_{ref} = I_{ref} + \tilde{i}_{ref}$

▪ **Assumptions:**

1. Perturbations in slopes are neglected
2. Nonlinear perturbed (product) terms are neglected

Now, we are writing the complete current equation, here this is the final current. And if you write the whole expression, it is a function of the initial current, reference current and the slope. Now, we are talking about the perturbed current dynamics. In order to obtain this perturbed current dynamics; that means, here we are replacing i_n equal to capital I_n plus i_n perturbed then we can write i_{ref} to be capital I_{ref} plus i_{ref} perturbed.

So, these two and we are not considering perturbation in the slope; that means, there are two assumptions: the perturbation of the slope is neglected, and the non-linear product which will come out from here of the perturbation those are also neglected. So; that means, the linear equation, perturb equation is like this.

(Refer Slide Time: 26:16)

Thus,
$$\underline{i_{n+1}} = i_{ref} + m_2 \left(\frac{i_{ref} - i_n}{m_1} \right) - m_2 T = - \left(\frac{m_2}{m_1} \right) i_n + \left(1 + \frac{m_2}{m_1} \right) i_{ref} - m_2 T$$

Perturbed current dynamics becomes

$$\tilde{i}_{n+1} = - \left(\frac{m_2}{m_1} \right) \tilde{i}_n + \left(1 + \frac{m_2}{m_1} \right) \tilde{i}_{ref}$$

- Assumptions:
 1. Perturbations in slopes are neglected
 2. Nonlinear perturbed (product) terms are neglected

Handwritten notes in red:
 Zero input stability
 $i_{ref} = 0$
 $i_{n+1} = - \left(\frac{m_2}{m_1} \right) i_n$ $\left| \frac{m_2}{m_1} \right| < 1$
 Zero state stability
 $i_n = 0$
 $i_{n+1} = \left(1 + \frac{m_2}{m_1} \right) i_{ref}$

Now, you will find two things, here the one thing ok. So, here there are two things we can write. One is called zero input stability. What is that? Zero input stability; that means, when there is a perturbation in the external input is set to 0. That means, we are using a fixed current reference the current reference is not perturbed and we are not exciting like you know there is no external excitation in the current reference.

So, it is a fixed current reference. Then this equation will simply become i_{ref} sorry, this equation will become that means we are talking about i_{n+1} perturbation is equal to minus m_2 by m_1 into i_n . That means, this we are going to justify stability for the inner current loop and that is a zero input stability.

If you can show that magnitude of m_1 by m_2 less than unity, then this will be stable. This can be shown it is stable if this quantity magnitude is smaller than 1. The other term that we are familiar with is a zero state stability; zero state stability. Here we are talking about the stability; that means, here we are setting the initial perturbation in the inductor current is set to 0. And we are trying to find the current perturbation as a function of that $1 + m_2$ by m_1 this perturbation into i_{ref} .

So, this will show that you know the current loop stability when we are exciting from the external current reference, but setting the initial current 0. But this will be only valid only for that particular cycle because a next cycle onward your current initial value will also give.

Because this perturbation will keep on accumulating, either if it is stable, it will slowly decay out and the current will come to steady-state or it will go somewhere else. And that we have discussed in the previous you know I think lecture number 19 that it can lead to subharmonic oscillation ok.

(Refer Slide Time: 28:43)

For constant reference, $\tilde{i}_{ref} = 0$; thus, $\tilde{i}_{n+1} = -\left(\frac{m_2}{m_1}\right)\tilde{i}_n$

For inner loop stability, $\left|\frac{m_2}{m_1}\right| < 1$

Slope	Buck Converter	Boost Converter
m_1	$\frac{V_{in} - V_o}{L}$	$\frac{V_{in}}{L}$
m_2	$\frac{V_o}{L}$	$\frac{V_o - V_{in}}{L}$



That means for constant current reference, where we call it about zero input stability for inner loop that actually gives us the inner loop stability. The magnitude of m_2 by m_1 must be smaller than 1 and for if you take a buck even boost or even buck boost and if you take m_1 slope for the buck and m_2 slope for the buck as well as boost.

(Refer Slide Time: 29:08)

▪ Buck Converter:

$$\left| \frac{m_2}{m_1} \right| = \left| \frac{V_o}{V_{in} - V_o} \right| = \frac{V_o}{V_{in} - V_o} \quad (\text{Since } 0 < V_o < V_{in})$$
$$\therefore \frac{V_o}{V_{in} - V_o} < 1$$
$$\Rightarrow V_o < \frac{V_{in}}{2}$$

$\Rightarrow D < 0.5$ *Peak CMC*
Inner loop stability

D < 0.45



And if you substitute this equation; that means, this magnitude then it can be shown that inner loop will be stable that when the duty ratio is less than 0.5. That means, under peak current mode control the inner loop stability, inner loop stability. But this is less than 0.5 is the it is not the sufficient condition it is the necessary condition.

The sufficient if you close the outer loop, it may so happen the actual current loop stability the whole overall stability maybe you know at a lower duty ratio. That means it can be even much it can be even less than 0.45 or so or less, depending upon how you are setting the outer loop voltage controller gain ok. But only for inner loop stability the requirement is that the duty ratio has to be less than 0.5 ok.

(Refer Slide Time: 30:09)

▪ **Boost Converter:**

$$\frac{m_2}{m_1} = \frac{V_o - V_{in}}{V_{in}}$$

$$\therefore \frac{V_o - V_{in}}{V_{in}} < 1$$

$$\Rightarrow V_o < 2V_{in}$$

$$\Rightarrow D < 0.5$$

Current loop stability by under Peak CMC

Now, for boost converter also you can substitute m_2 m_1 and then you can get the same result. You will get the current loop stability, for current loop stability under peak current mode control ok. Now the same thing if you go for constant off time control ok.

(Refer Slide Time: 30:32)

Current loop stability under Constant Off-Time CMC

$t_{on} \rightarrow$ available
 $t_{off} \triangleq T_c \rightarrow$ fixed

$$i_{n+1} = i_{ref} - m_2 T_c$$

$$\tilde{i}_{n+1} = \tilde{i}_{ref}$$

For constant current reference,
 $\tilde{i}_{ref} = 0$
 $\tilde{i}_{n+1} = 0$

Current loop is inherently stable irrespective of duty ratio

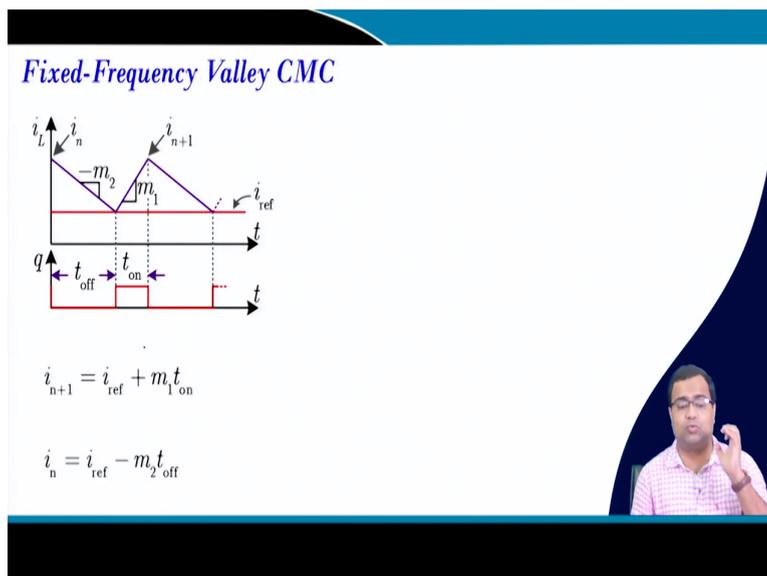
So, constant off time control we know the on time is coming from the outer loop. Because on time is decided by when the inductor current hit the peak current reference, but the off time for constant off time control is fixed. Now, we can refer to the same inductor current

waveform which we did earlier; that means, if we go back the same current waveform you can if you take this current waveform.

Now i_{n+1} in everything is known then we are writing this equation ok, so here $i_{n+1} = i_{ref} + m_1 t_{on}$. Now since this quantity is constant; that means, and this is an external current reference. So, the final current is simply a constant quantity, and it is a function of the reference current only.

So, it is independent of; that means, if you take the perturbation i_{n+1} is only a function of i_{ref} and this constant current reference it is 0. So, $i_{n+1} = 0$ this indicate the current loop is inherently stable irrespective of the duty ratio. That means, if you change the duty ratio very widely the current loop will never become unstable that is the beauty of this constant off time control. So, you do not need a compensating ramp.

(Refer Slide Time: 31:59)



Now, in fixed frequency valley current mode control.

(Refer Slide Time: 32:07)

a) Under PWM Control

$$t_{\text{on}} = dT, \quad t_{\text{off}} = (1-d)T$$

$$i_{n+1} = i_{\text{ref}} + m_1 dT$$

$$= i_{\text{ref}} - (1-d)Tm_1 + m_1 T$$

$$i_n = i_{\text{ref}} + m_2(1-d)T \Rightarrow (1-d)T = \frac{i_n - i_{\text{ref}}}{m_2}$$

Then we can show that the same method can be applied to derive the current i_{n+1} where here the i_{n+1} , we are talking about this current and this is my i_n . (Refer Slide Time: 32:19)

Fixed-Frequency Valley CMC

$$\therefore i_{n+1} = i_{\text{ref}} - \left(\frac{i_n - i_{\text{ref}}}{m_2} \right) m_1 + m_1 T$$

$$i_{n+1} = - \left(\frac{m_1}{m_2} \right) i_n + \left(1 + \frac{m_1}{m_2} \right) i_{\text{ref}} + m_1 T$$

Perturbed dynamics, $\tilde{i}_{n+1} = - \left(\frac{m_1}{m_2} \right) \tilde{i}_n + \left(1 + \frac{m_1}{m_2} \right) \tilde{i}_{\text{ref}}$

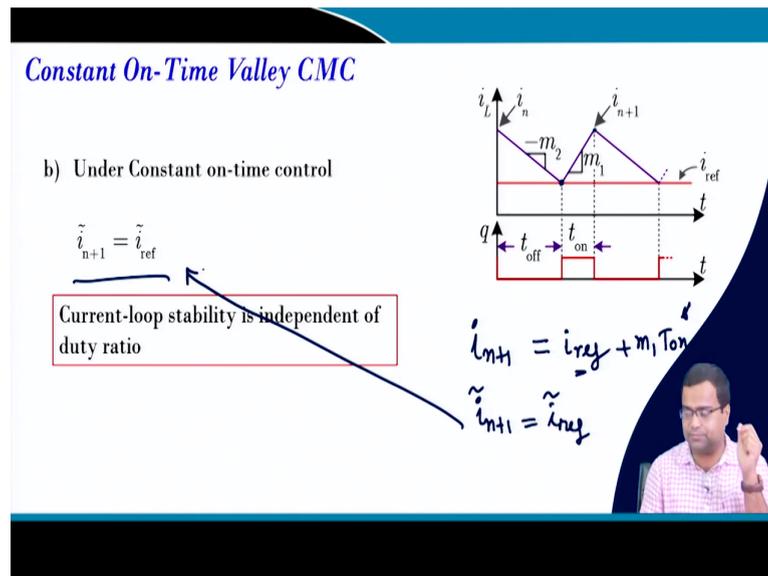
For fixed current reference, current-loop stability condition becomes, $D > 0.5$

Handwritten notes: $\left| \frac{m_1}{m_2} \right| < 1$ and $D > 0.5$

And if you write the whole equation again if you perturb if you know write one by one what are the equation i_{n+1} . Then what is an expression of i_n from there we are finding $1 - m_1 d T$ and then, if you substitute, this is the overall equation; that means, this is my i_{n+1} . Again there are two terms; one is an initial current another reference current and if we ignore the slope perturbation the perturbed current dynamics.

So, it can be shown that for inner loop stability you need m_1 by m_2 that magnitude should be smaller than 1 and this will give rise to the condition that d should be greater than 0.5. That means, for valley current mode control the current loop will be stable if the duty ratio is greater than 0.5 and it is unstable if the duty ratio is less than 0.5 and you can stabilize this current loop by using a compensating ramp ok.

(Refer Slide Time: 33:16)



So, constant on time if you consider the constant on time control it can be shown that, what is i_{n+1} here? i_{n+1} should be equal to what? It is, so this is the i_{n+1} it is nothing, but i_{ref} this value plus $m_1 T_{on}$, since T_{on} is constant i_{ref} is an external source. So, if you take the i_{n+1} perturbation, it is simply a function of the reference current perturbation ok and this is written here.

And that means, the current loop stability is independent of duty ratio; that means, it can be it will be stable for a wide range of load current ok. And that is why we discussed in lecture number 19 that, for low duty ratio voltage regulator module like mean low voltage high current application. Now the constant on time control is kind of as a very popular choice over you know peak fixed frequency current mode control.

(Refer Slide Time: 34:18)

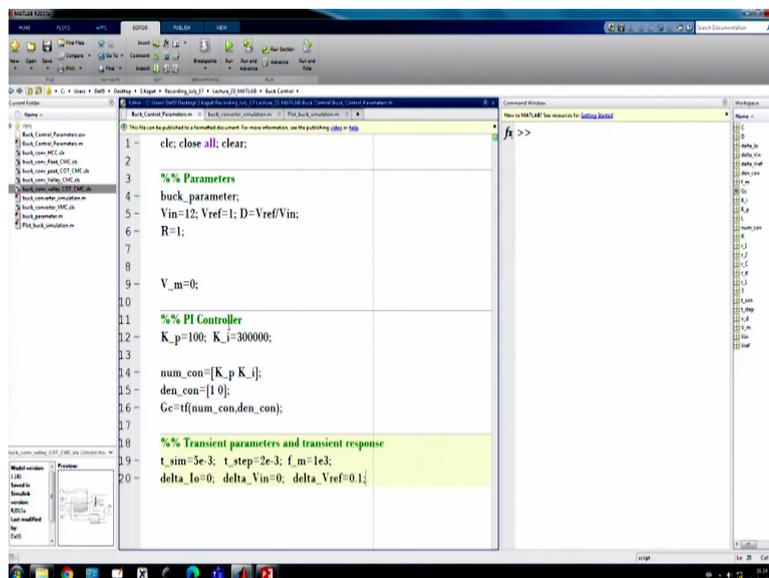
Performance and Stability Comparison – MATLAB Simulation

- Audio Susceptibility
- Transient response – low duty ratio operation
- Transient response – high duty ratio operation
- Current loop stability

$$\frac{1}{12} \Delta D = 0.083$$
$$\frac{8}{12} \Delta D = \frac{2}{3} = 0.67$$


So, the performance comparison, if you want to do MATLAB simulation of this audio susceptibility, transient response, for low duty ratio operation, high duty ratio operation, current loop stability. So, you need to check all this. Let us go to the MATLAB simulation ok.

(Refer Slide Time: 34:40)



```
1 clc; close all; clear;
2
3 %% Parameters
4 buck_parameter;
5 Vin=12; Vref=1; D=Vref/Vin;
6 R=1;
7
8
9 V_m=0;
10
11 %% PI Controller
12 K_p=100; K_i=300000;
13
14 num_con=[K_p K_i];
15 den_con=[1 0];
16 Ge=tf(num_con,den_con);
17
18 %% Transient parameters and transient response
19 t_sim=5e-3; t_step=2e-3; f_m=1e3;
20 delta_Io=0; delta_Vin=0; delta_Vref=0.1;
```

So, what are we going to do? We want to first use a ramp compensation. So, here in this code we are you know identifying the vref parameter in fact, this is not required all these things are not required. So, this only requirement is this ok and you have to set the input voltage, load current and so on and this is my modulator voltage.

So, if you use a ramp compensation then we have to set what is the maximum value. Unless simply taking a PI controller common for all techniques. And here we are applying a reference voltage transient of 0.1 volt and we want to compare the performance using different controller ok. Now we are going to this simulation and load current is 1 ohm.

(Refer Slide Time: 35:25)

```

1 % cdc; clear; close all;
2
3 DCM_En=0;
4 I_L_int=20; V_c_int=1.1;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_valley_COT_CMC.slx'); cdc;
10 t=buck_result.time; t_scale=t*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
  
```

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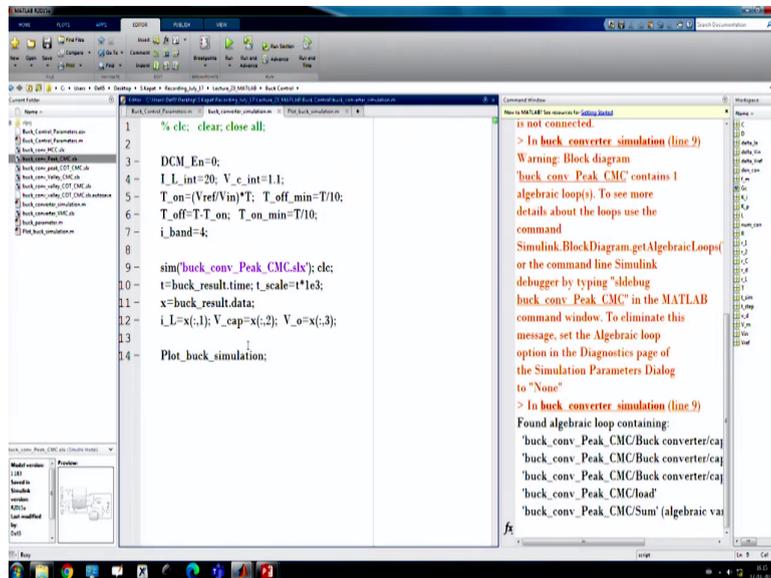
```

1
2 figure(1)
3 plot(t_scale,i_L,'LineWidth', 2); hold on; grid on;
4 xlabel('Time (ms)', 'FontSize', 15);
5 ylabel('Inductor current (A)', 'FontSize', 15);
6
7 figure(2)
8 plot(t_scale,V_o,'LineWidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11
  
```

So, let us first set and we are using the blue color waveform trace and initially we are going for peak current mode control. So, let us say we have different controllers here. You know we

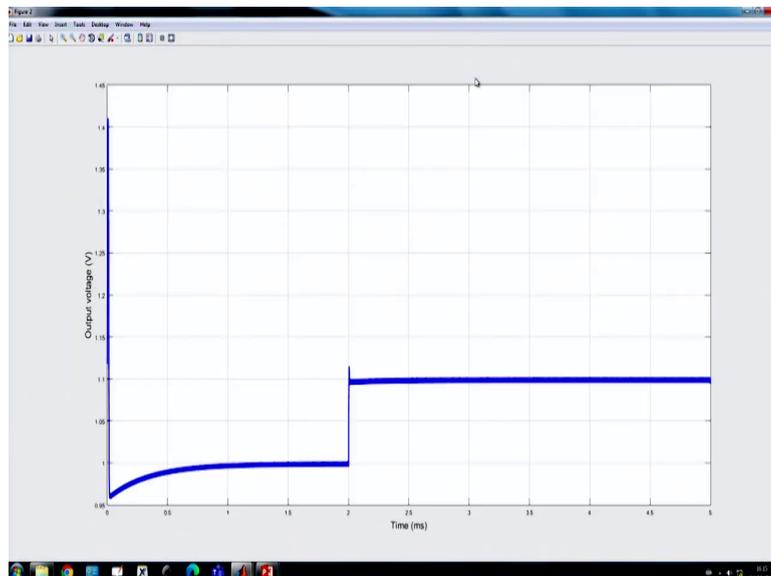
have stored here. So, let us say if we run the peak current mode control and I have shown all the diagrams here.

(Refer Slide Time: 35:43)



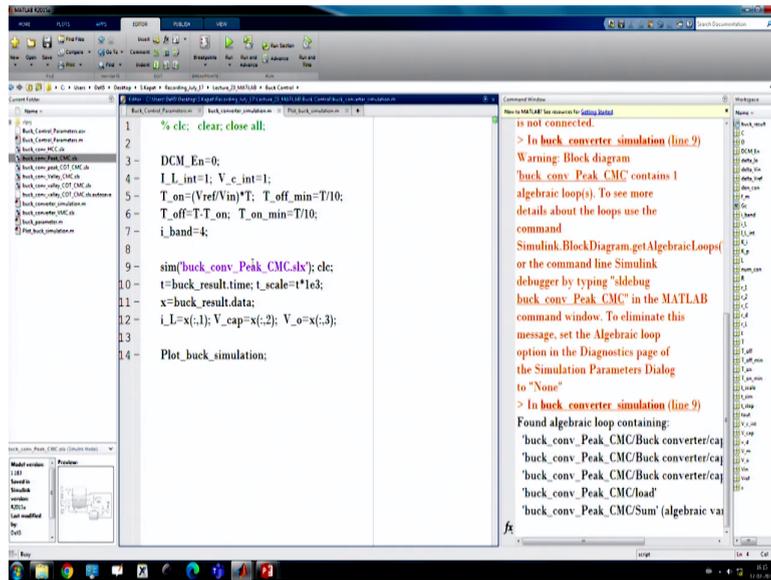
So, first I am drawing the peak current mode control ok.

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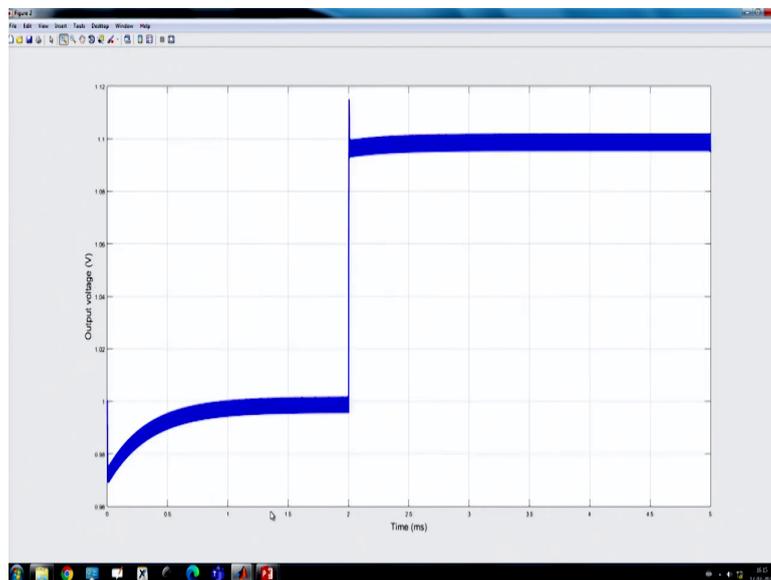
So, this is it shows, ok just hold on.

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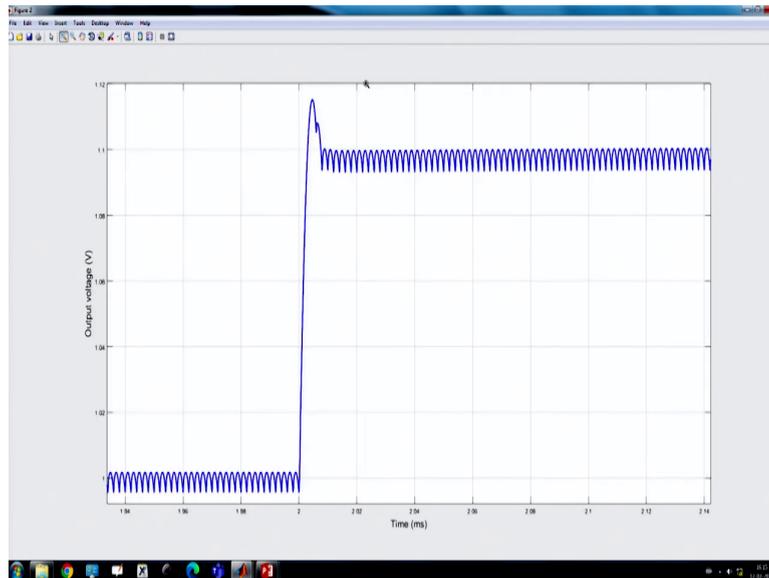
So, we want to change the initial condition to 1, 1 ok. So, we want to rerun.

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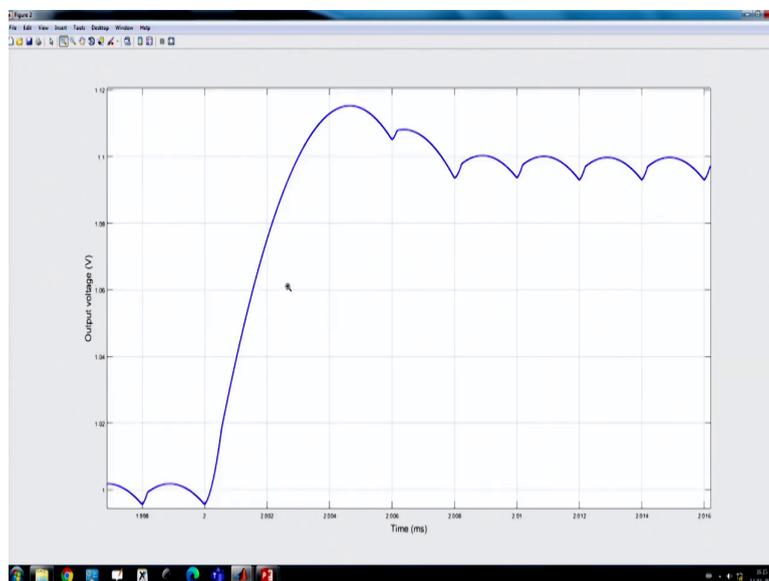
So, what I am showing here? This is my using peak current mode control, and it is my reference voltage transient.

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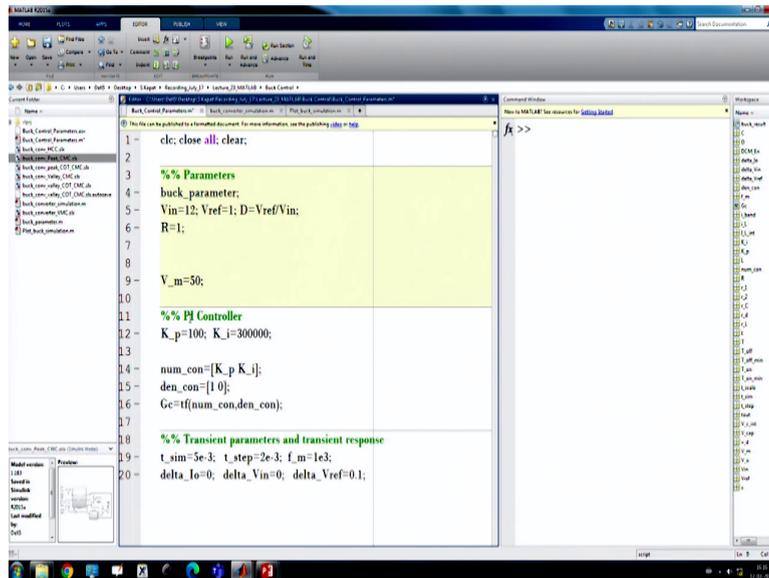
That means, I am setting some fixed gain and just comparing how does the controller you know respond to different control technique.

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So, this is a reference voltage transient using peak current mode control. Now, of course, this technique cannot be because the duty ratio is, here input is 12 volt, and the output is 1 volt. So, you should not use valley current mode control because it is completely unstable ok. And if you want to use valley current mode control, then you have to use a very high ramp compensation.

(Refer Slide Time: 36:48)



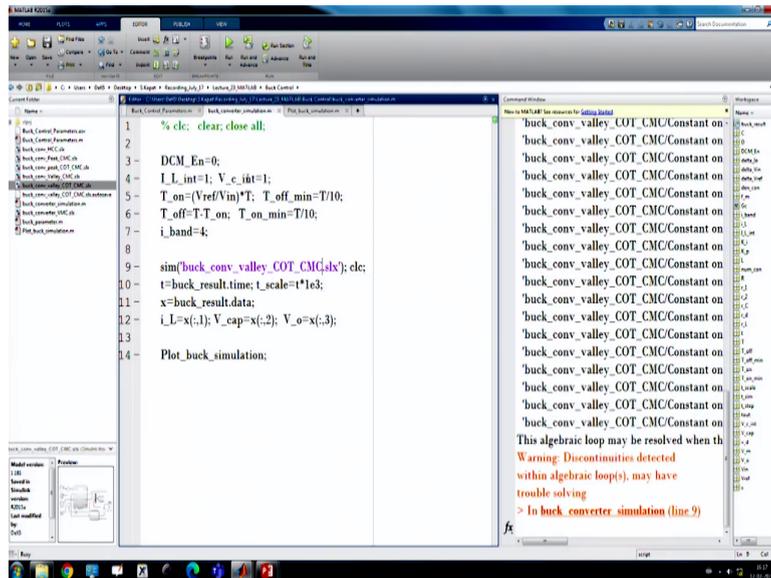
```
1 = clc; close all; clear;
2
3 %% Parameters
4 buck_parameter;
5 Vin=12; Vref=1; D=Vref/Vin;
6 R=1;
7
8
9 V_m=50;
10
11 %% PI Controller
12 K_p=100; K_i=300000;
13
14 num_con=[K_p K_i];
15 den_con=[1 0];
16 Ge=tf(num_con,den_con);
17
18 %% Transient parameters and transient response
19 t_sim=5e-3; t_step=2e-3; f_m=1e3;
20 delta_Io=0; delta_Vin=0; delta_Vref=0.1;
```

So, let us say if you use a valley current mode control with a voltage of you know maybe, so 50 because is a very low duty ratio operation. In fact, you should not use in this technique, for high duty ratio we will go for that.

Now, we said in lecture number 19 for low duty ratio operation. Since you have a very small on time you will get hardly any time to you know carry if you know continue the comparator operation and all this we cannot finish within that very small amount of on time when we are talking about a high switching frequency.

So, then we talk about the valley current mode control, but valley current mode control for low duty ratio is unstable. So, that we have discussed we will go for constant on time. So, if you do constant on time; that means, this is my constant on time. So, I want to compare the performance using constant on time which is the case here.

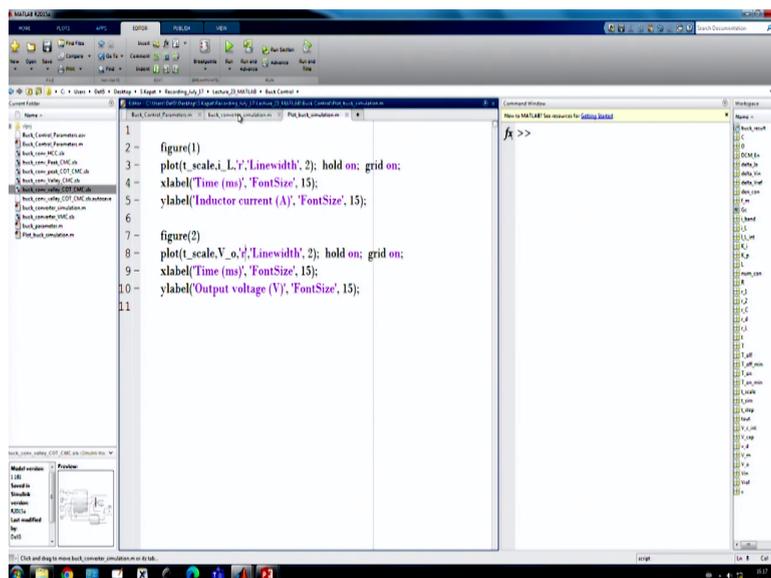
(Refer Slide Time: 37:35)



```
1 %clear; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=1;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_valley_COT_CMC.slx');
10 t=buck_result.time; t_scale=t*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Warning: Discontinuities detected within algebraic loop(s), may have trouble solving
> In buck_converter_simulation (line 9)

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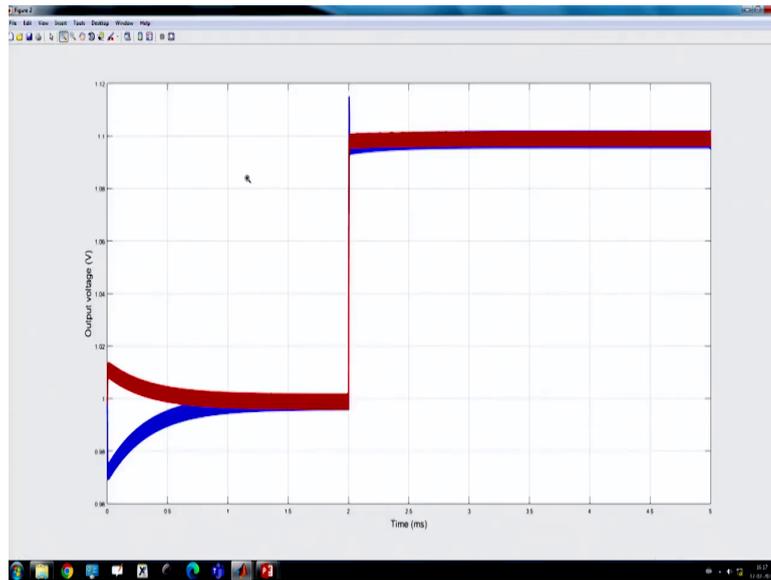


```
1
2 figure(1)
3 plot(t_scale,i_L,'Linewidth', 2); hold on; grid on;
4 xlabel('Time (ms)', 'FontSize', 15);
5 ylabel('Inductor current (A)', 'FontSize', 15);
6
7 figure(2)
8 plot(t_scale,V_o,'Linewidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11
```

fx >>

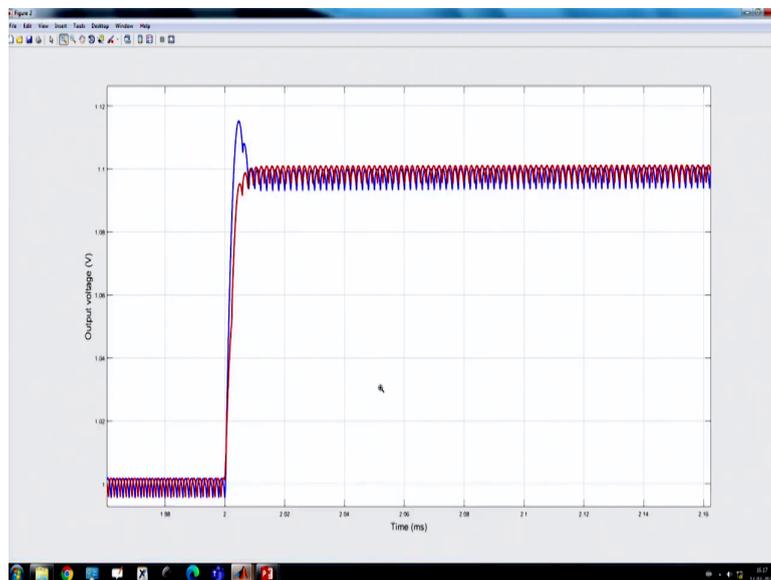
And now I am using another color which is a red color ok and let us run it and see what happen. So, here we are comparing with constant on time and I have already shown all this architecture ok.

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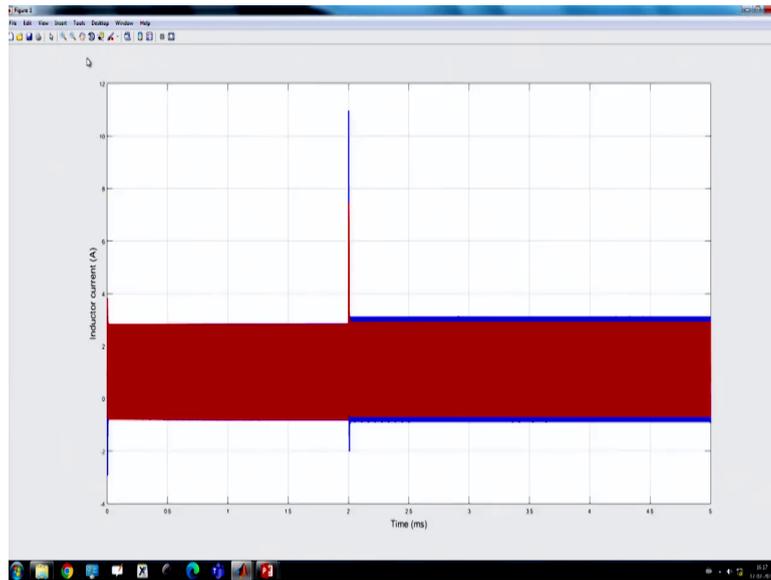


So, this is the performance under constant on time control ok.

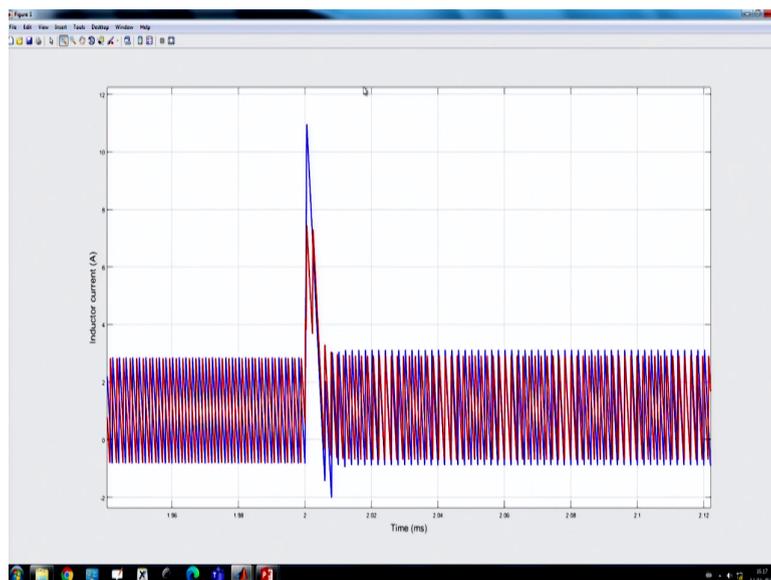
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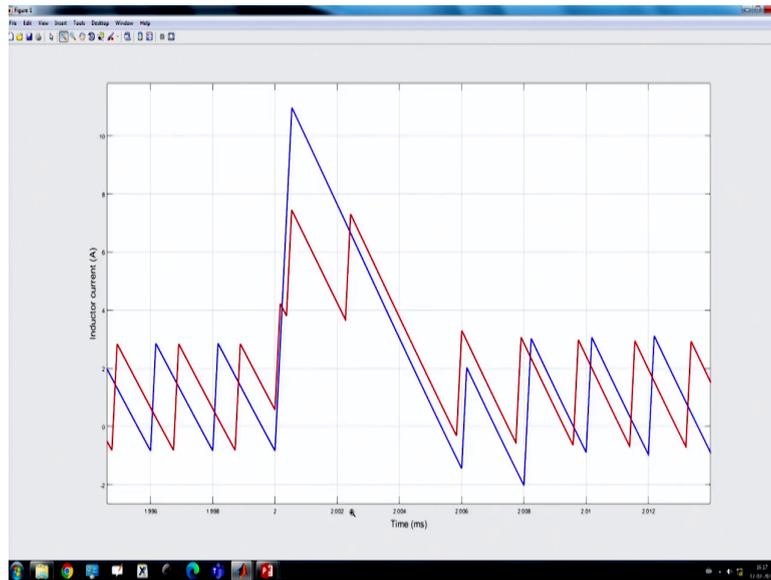
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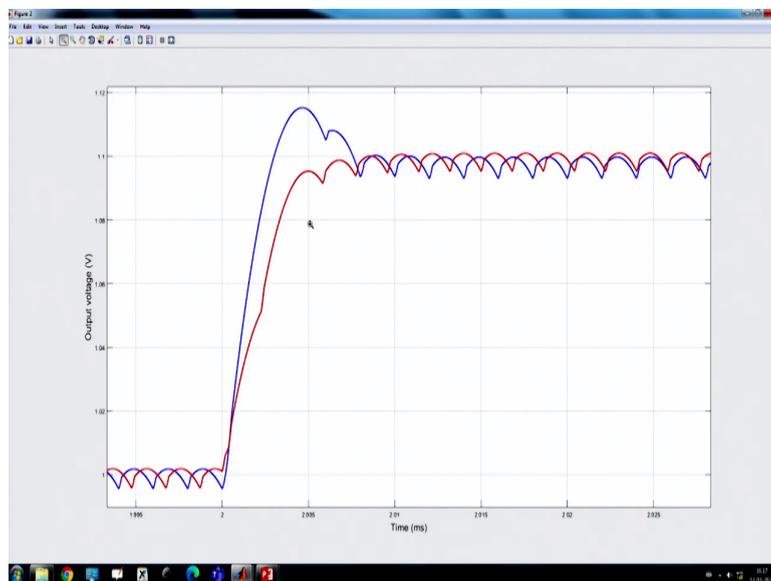


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And if you see the inductor current reference for the same controller gain, if we use one is our peak current mode control, which is the blue one and the red one is the constant on time control ok.

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And you will see in this particular case the constant on time control seems to be better because it has almost no overshoot ok for this controller.

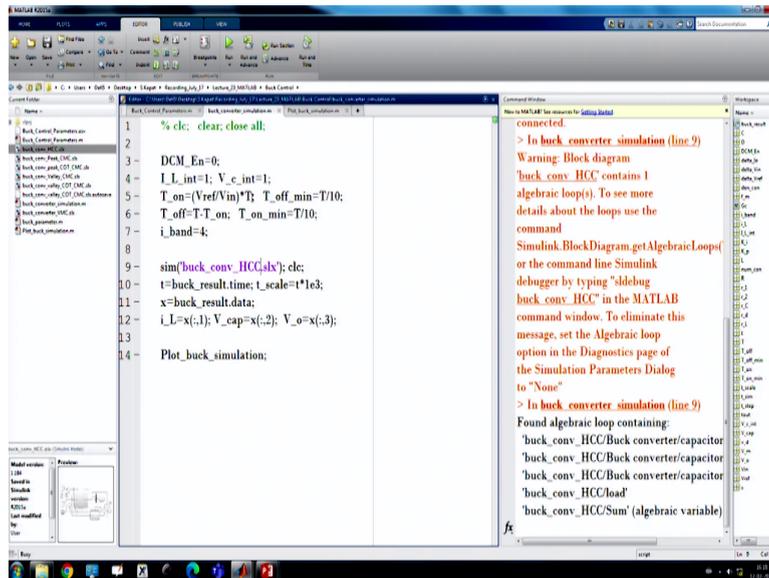
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So that means, if we want to obtain the small-signal model for the same constant on time control and the peak current mode control, they should be different because for the same controller, the response is different. So, the constant on time seems to be overdamp response. There is no overshoot here ok.

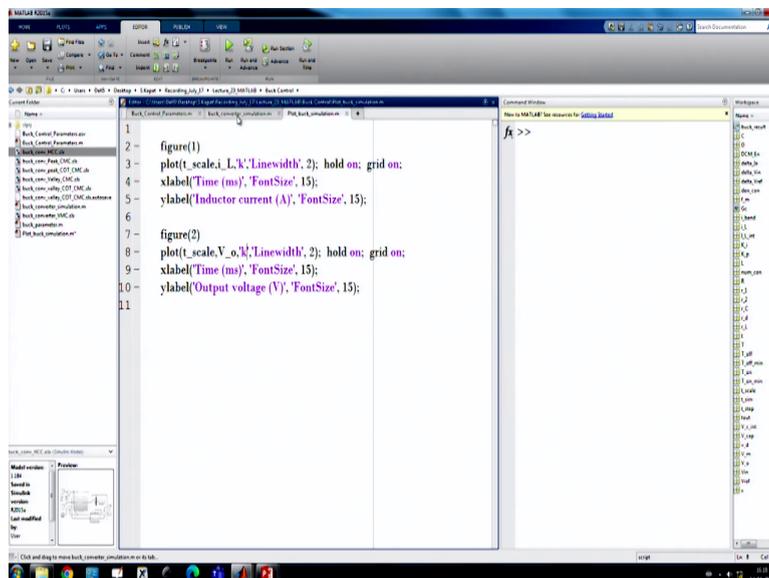
But whereas, the peak current mode control is giving a high overshoot because it is much higher, it is roughly around 10 percent 10 to 15 percent overshoot is we can get it from here ok. We also want to compare the hysteresis control ok. So, now we are going for hysteresis control.

(Refer Slide Time: 39:09)



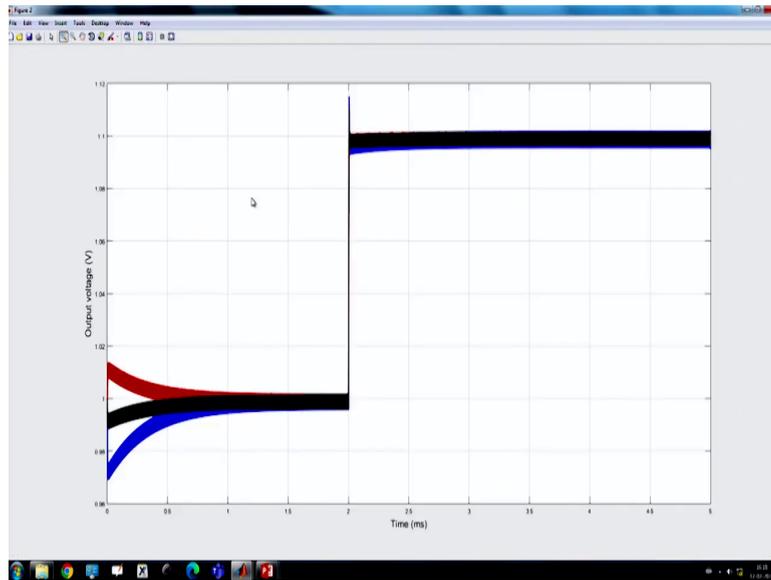
So, let us compare the hysteresis control and we will use a different color.

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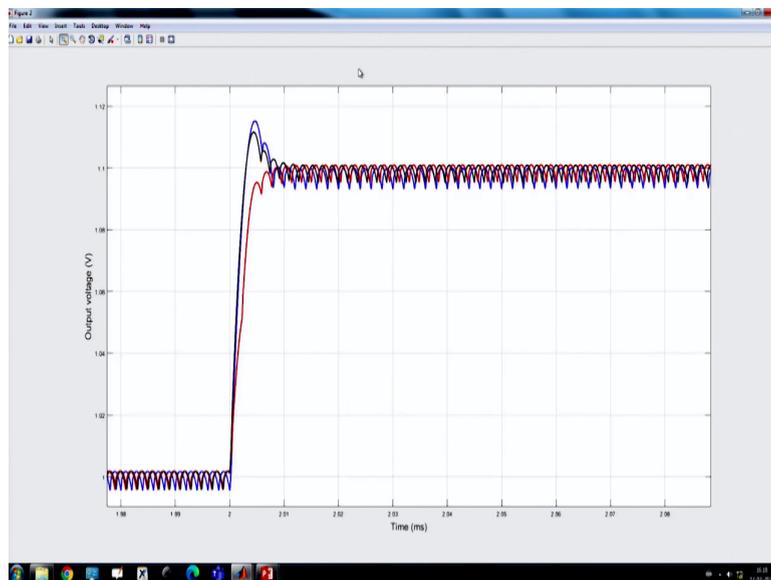
So, for hysteresis we will use let us say black color and let us run it.

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So, of a same configuration we are now running hysteresis control all this are same.

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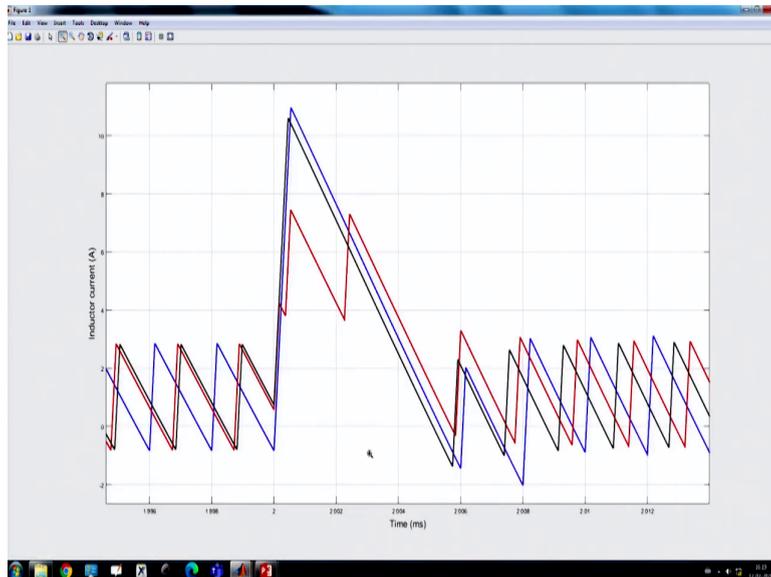


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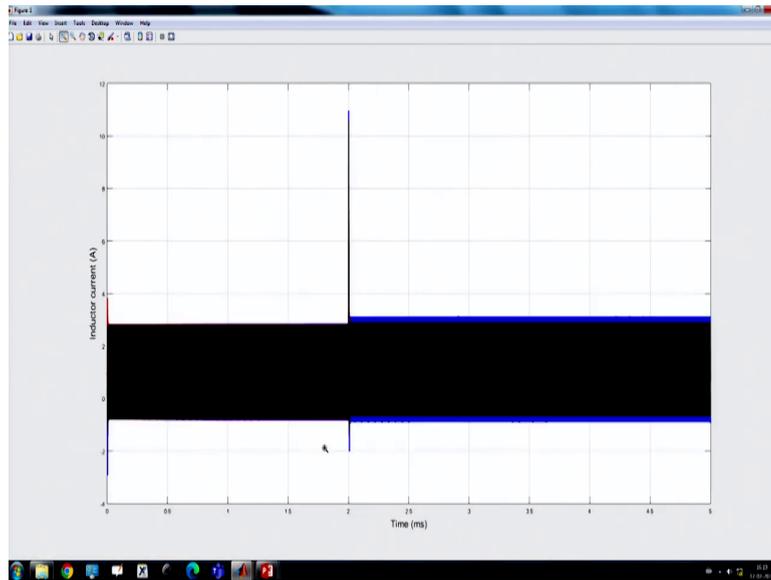


Now in hysteresis control you see this is something similar to our peak current mode control ok.

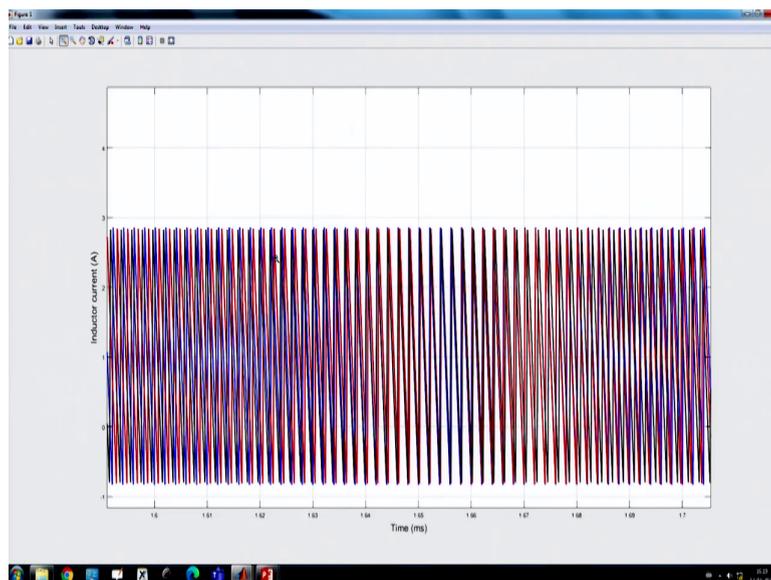
(Refer Slide Time: 39:32)



(Refer Slide Time: 39:33)

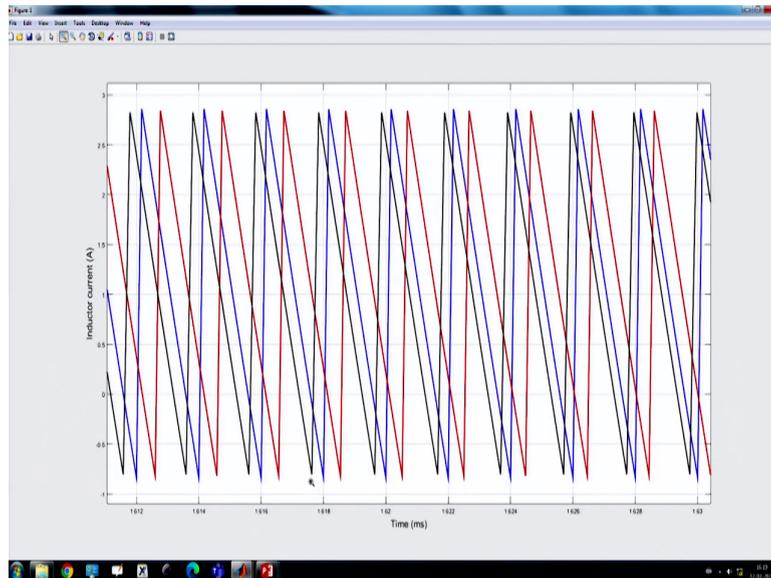


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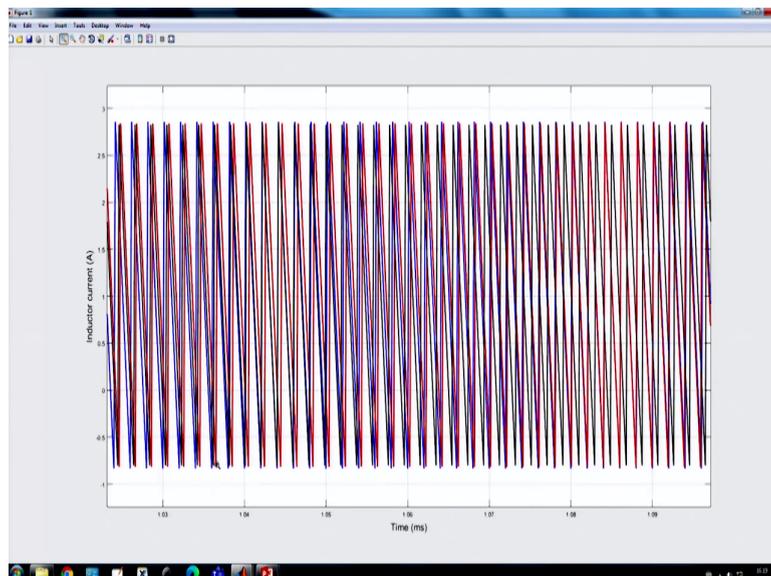
And if you go to this block, you will find hysteresis control current is within a band.

(Refer Slide Time: 39:37)

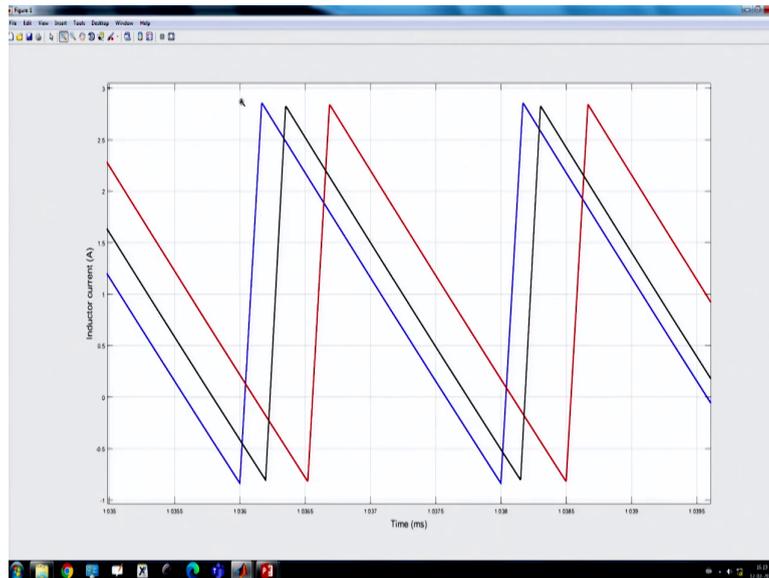


But though we have set 4 ampere band, but it is not actually 4 ampere it is less than that ok. Because if you go back to the actual MATLAB file, we set the current hysteresis band to 4 ampere and I have discussed. If you close the outer loop, the actual current ripple will not be equal to the hysteresis ripple.

(Refer Slide Time: 39:56)



(Refer Slide Time: 39:58)



And we will you know whenever we will discuss at the end I will refer some paper and show it is actually when you fix only current reference they are true otherwise the actual current ripple will be less. Because of black color you will see the current ripple is I mean it is smaller than 4 because minus 1 to 3 is the 4 ampere band, but it is smaller than this ok.

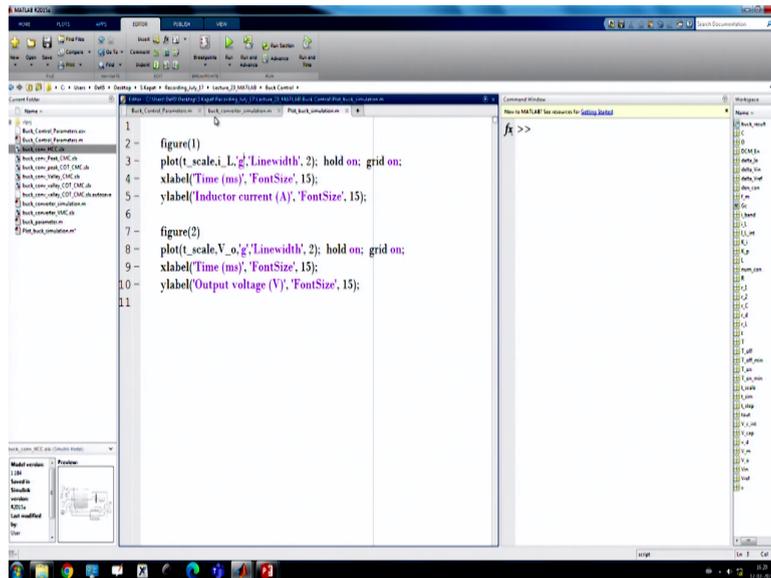
(Refer Slide Time: 40:24)

```
1 % cke; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=1;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_peak_COT_CMC.slx'); cdc;
10 t=buck_result.time; t_scale=1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Warning: Discontinuities detected within algebraic loop(s), may have trouble solving > In buck_converter_simulation (line 9)

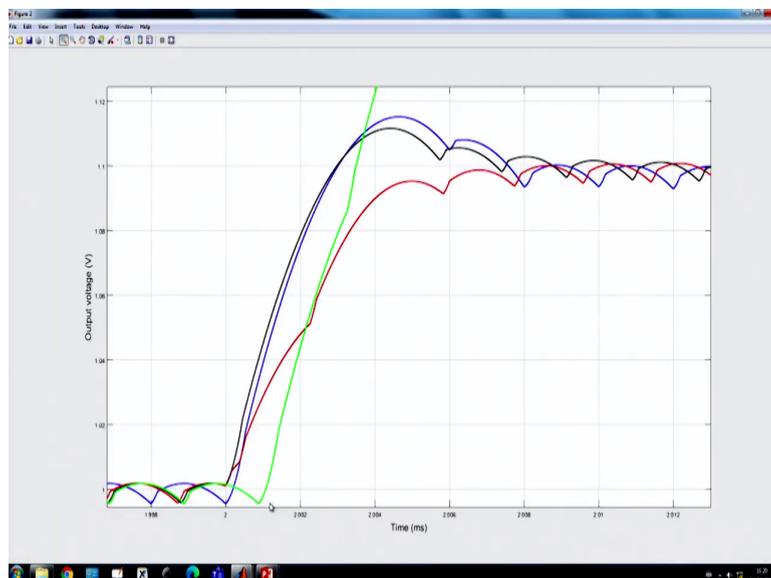
So, you have compared the last one which is left is that we need to check now peak one, so peak one.

(Refer Slide Time: 40:29)

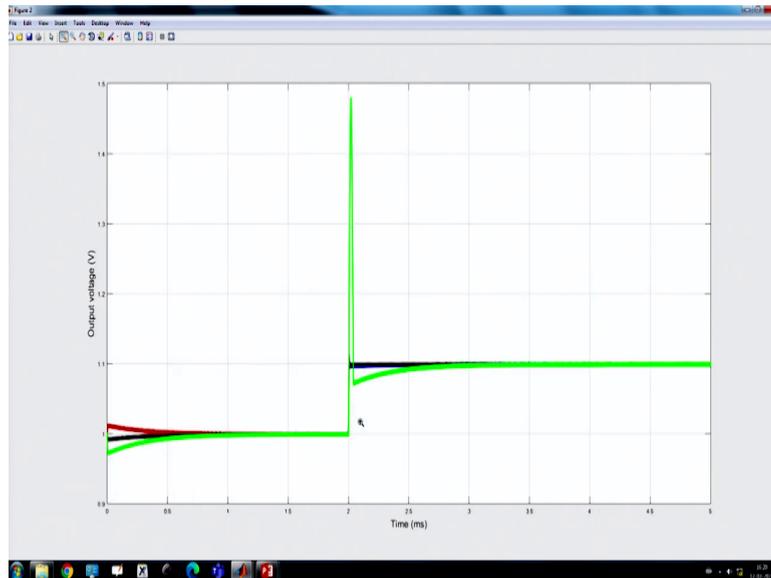


Because we need to evaluate there you know compare, how can you compare their performance? So, the fourth one is this the peak; that means, the constant off time control ok. But valley current we cannot use because the low duty ratio it will be unstable ok.

(Refer Slide Time: 40:46)

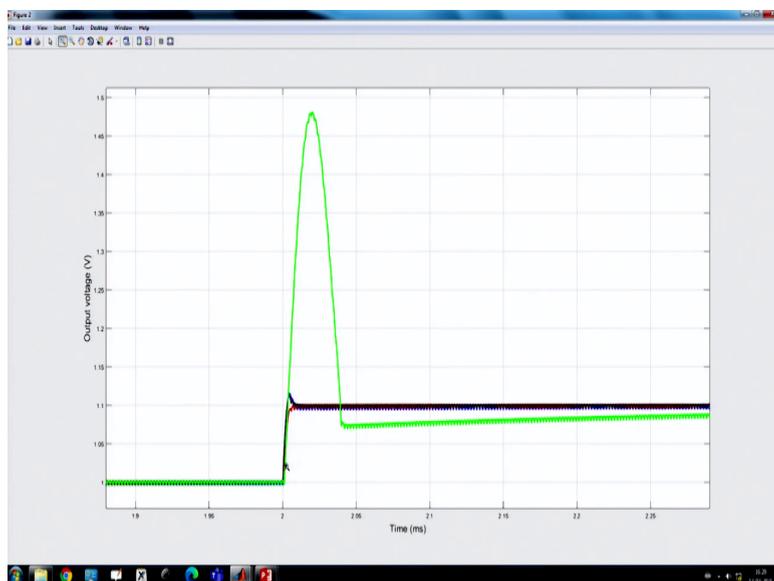


(Refer Slide Time: 40:48)

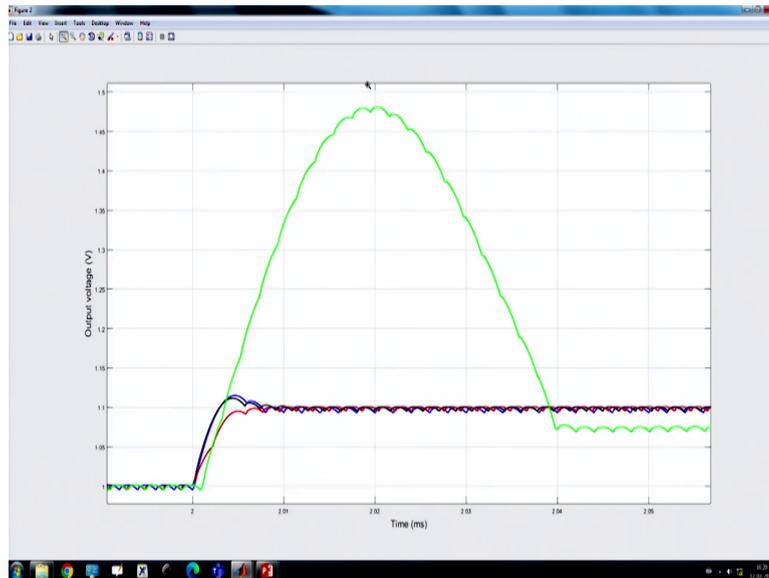


And you will see in this case, it has an unacceptably high overshoot using constant off time control.

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(Refer Slide Time: 40:54)



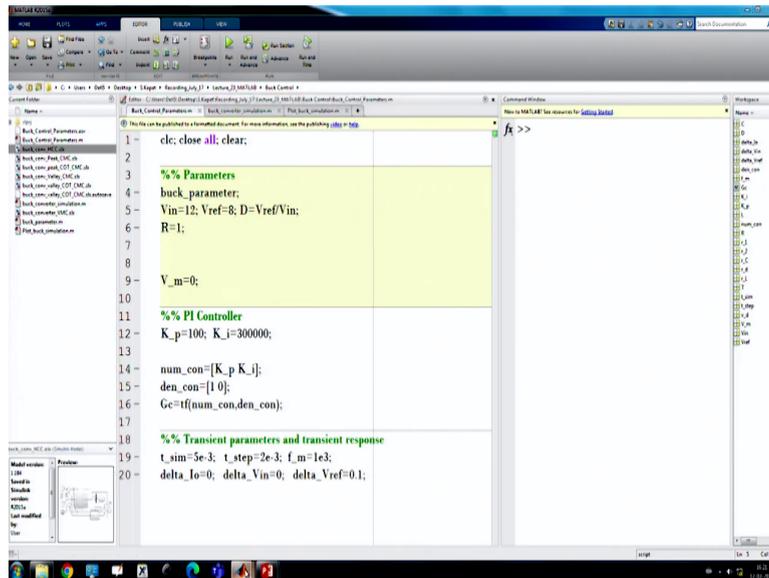
Now, coming back to; that means, we can check the audio susceptibility. So, if you want to check the audio. So, we have checked this point the low duty ratio operation transient response we have checked. Now we are going to check for high duty ratio operation. Let us go for a high duty ratio operation. So, in high duty ratio operations, our objective is operate around 8 volt input.

(Refer Slide Time: 41:18)

```
1 % clear; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=0;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_peak_COT_CMC.slx'); clc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Here, 12 volt is the output and 8 volt is the input ok.

(Refer Slide Time: 41:27)

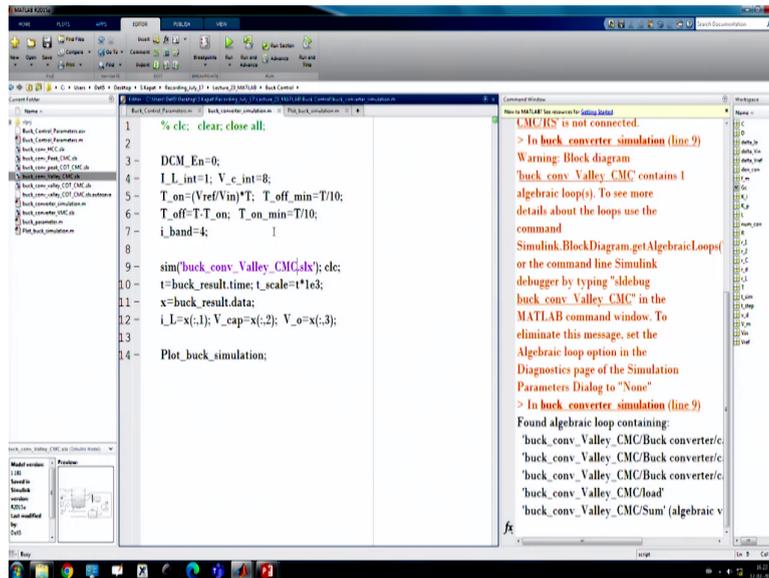


```
1 = clc; close all; clear;
2
3 %% Parameters
4 buck_parameter;
5 Vin=12; Vref=8; D=Vref/Vin;
6 R=1;
7
8
9 V_m=0;
10
11 %% PI Controller
12 K_p=100; K_i=300000;
13
14 num_con=[K_p K_i];
15 den_con=[1 0];
16 Ge=tf(num_con,den_con);
17
18 %% Transient parameters and transient response
19 t_sim=5e-3; t_step=2e-3; f_m=1e3;
20 delta_Io=0; delta_Vin=0; delta_Vref=0.1;
```

So, 8 volt we are setting reference voltage to be 8 volt same controller we want to compare 8 volt ok. Now, as we discussed that we should not use peak current mode control here; because peak current mode control our duty ratio is it is 8 volt by 12 volt. That means, if I go back, I am talking about this scenario where initially here we took 1 by 12; that means, that was our equivalent duty ratio, which is equal to 0.083. It is a low duty ratio operation.

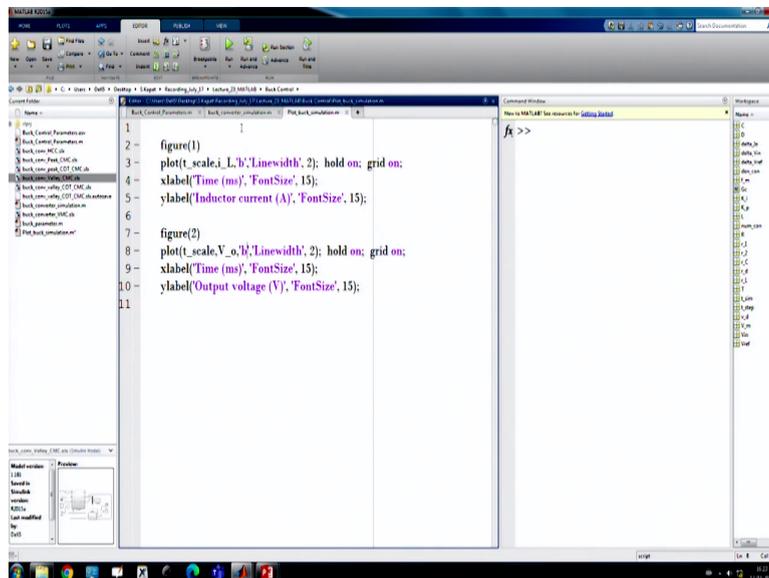
Now, we are talking about 8 volt to 12 volt ok and which if you write in terms of equivalent duty ratio it will be how much? So, point you know 75, 0.75 duty ratio; that means, it is 4 by; that means it is 2 by 3. So, it is 2 by 3, 2 by 3 not 0.75, I am sorry it should be 0.67. It is; that means, it is equal to 2 by 3. It is nothing, but 0.67; that means, it is higher than 0.5. Now let us go to the MATLAB. So, here we should first use valley current mode control. So, this is our valley current mode control.

(Refer Slide Time: 42:56)



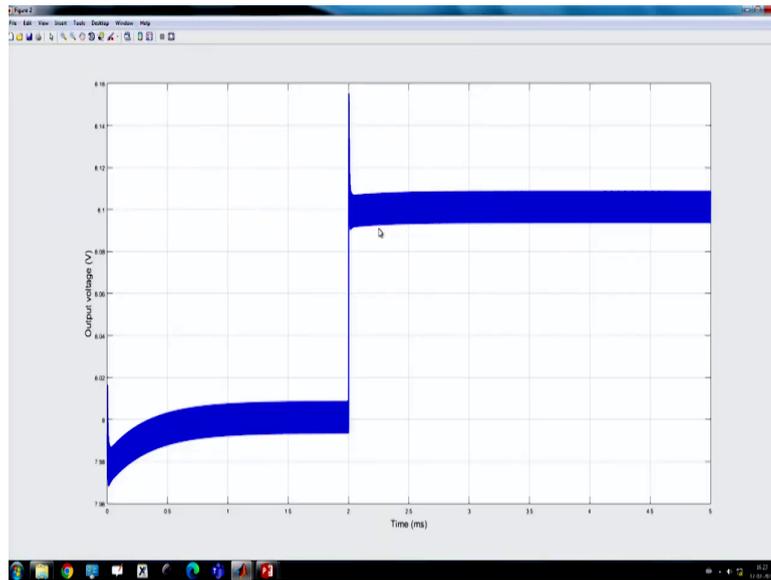
Here we are talking about valley current mode control ok and we want to first simulate using blue color, so blue color.

(Refer Slide Time: 43:02)



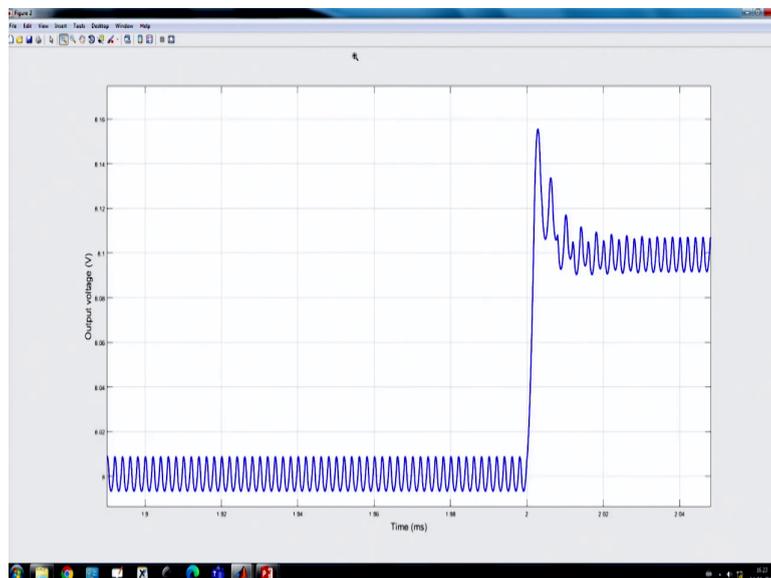
So, here we are making a reference transient response.

(Refer Slide Time: 43:10)

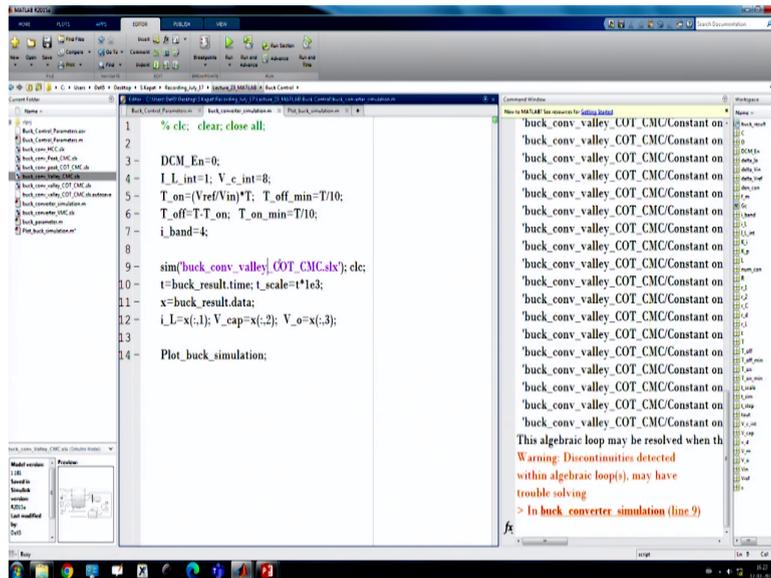


And you see, it is stable ok under valley current mode control and this is a response time ok.

(Refer Slide Time: 43:16)



(Refer Slide Time: 43:20)

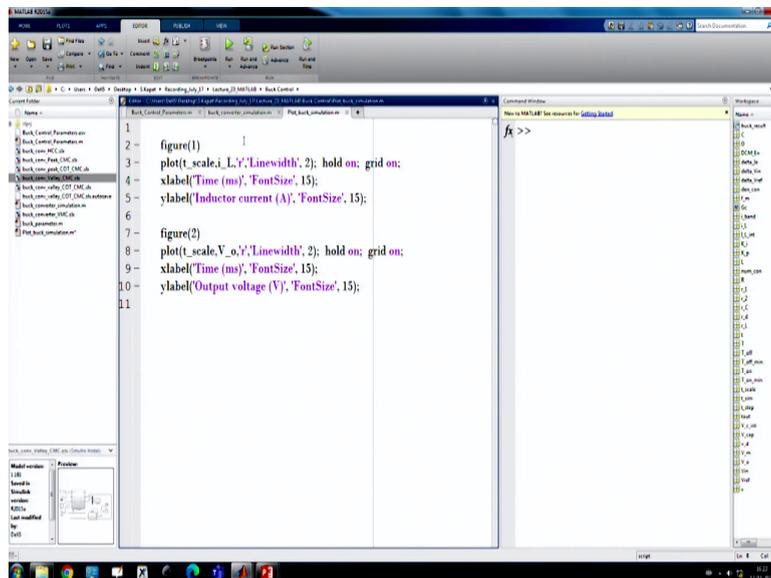


```
1 %clc; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_valley_COT_CMC.slx'); clc;
10 t=buck_result.time; t_scale=t*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Warning: Discontinuities detected within algebraic loop(s), may have trouble solving
> In buck_converter_simulation (line 9)

Now we are going for the same using valley constant on time; that means, they are analogous, but modulator structure is different.

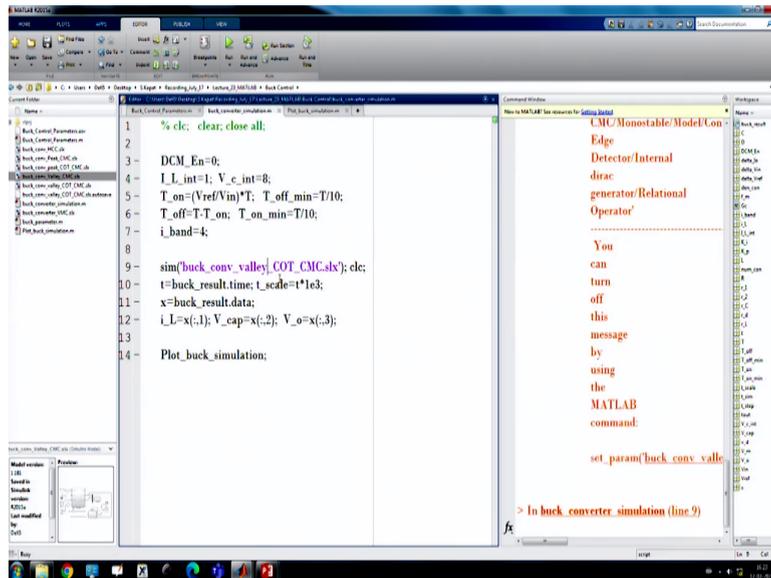
(Refer Slide Time: 43:29)



```
1
2 figure(1)
3 plot(t_scale, I_L, 'r', 'LineWidth', 2); hold on; grid on;
4 xlabel('Time (ms)', 'FontSize', 15);
5 ylabel('Inductor current (A)', 'FontSize', 15);
6
7 figure(2)
8 plot(t_scale, V_o, 'r', 'LineWidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11
```

So, now we are using I am using red color ok.

(Refer Slide Time: 43:34)



```
1 %clear; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_valley_COT_CMC.slx'); clc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

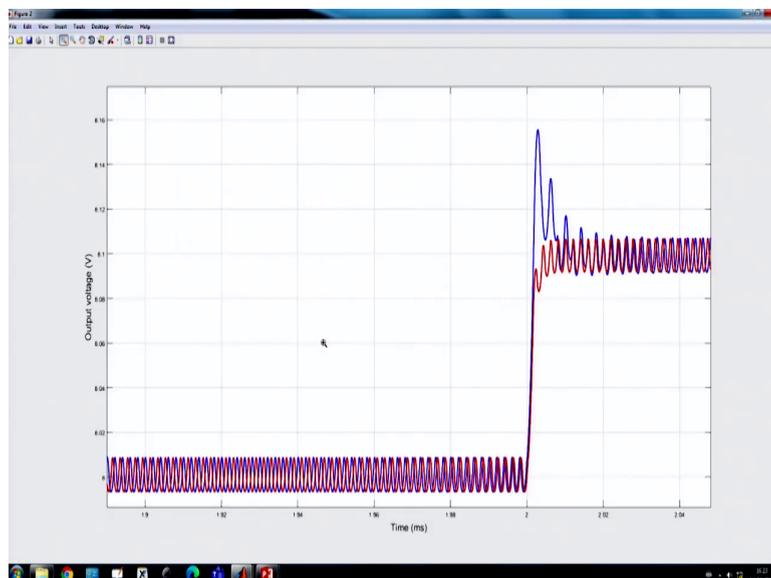
CMC Monostable Model/Con-
Edge
Detector/Internal
dirac
generator/Relational
Operator'

You
can
turn
off
this
message
by
using
the
MATLAB
command:

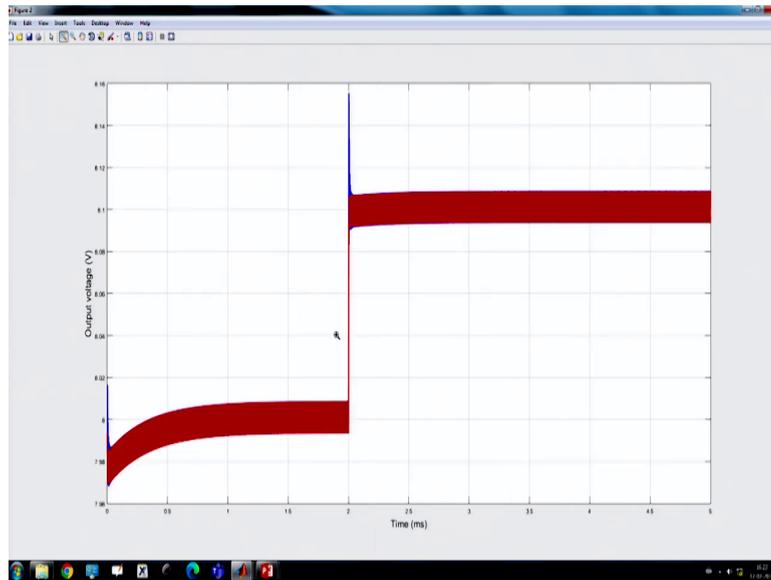
set_param('buck_conv_valley',
> In buck_converter_simulation (line 9)

So, this is constant on time controller. This is analogous to valley current mode control and I want to compare this with our fixed frequency valley current mode control and see what is the how is the performance.

(Refer Slide Time: 43:46)

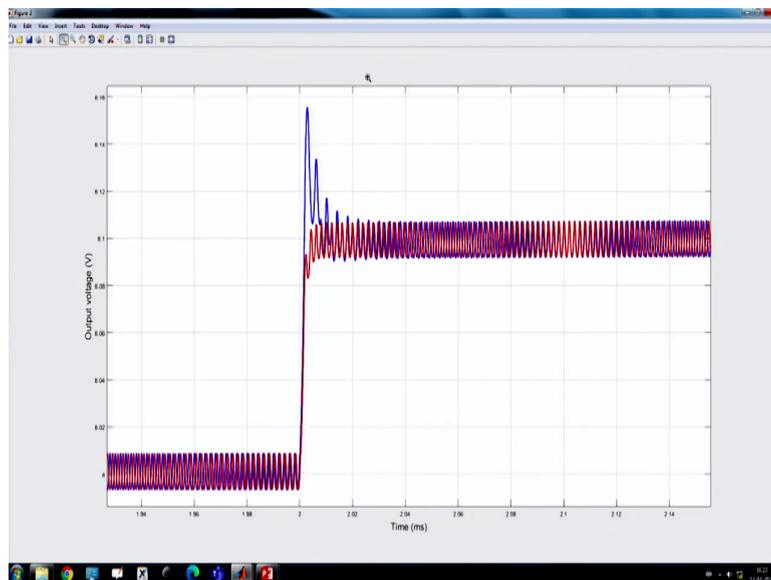


(Refer Slide Time: 43:48)

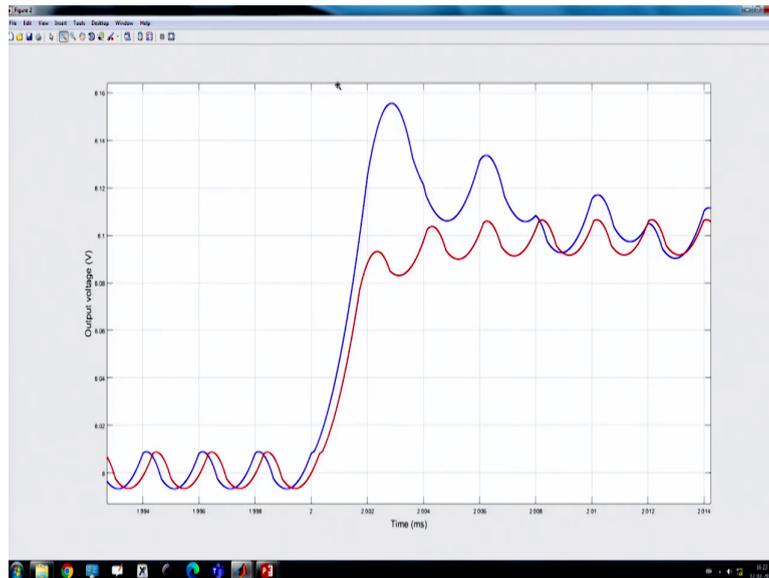


And we have already plotted.

(Refer Slide Time: 43:50)

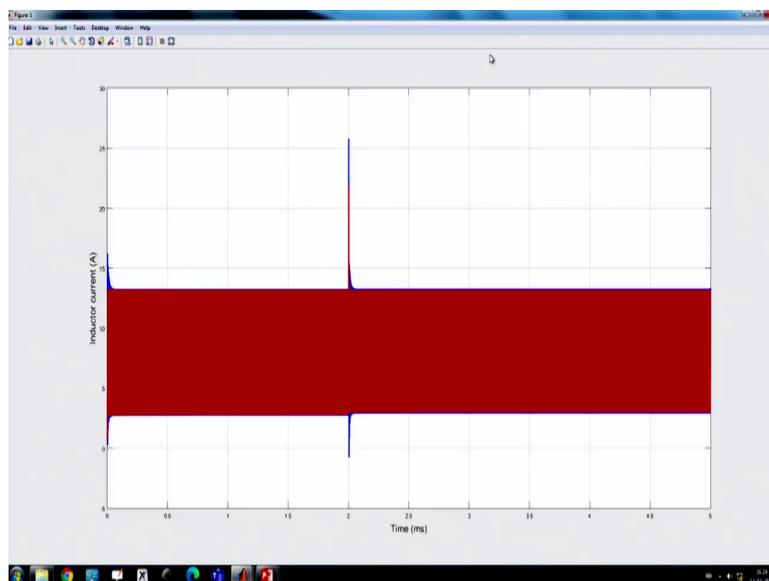


(Refer Slide Time: 43:53)



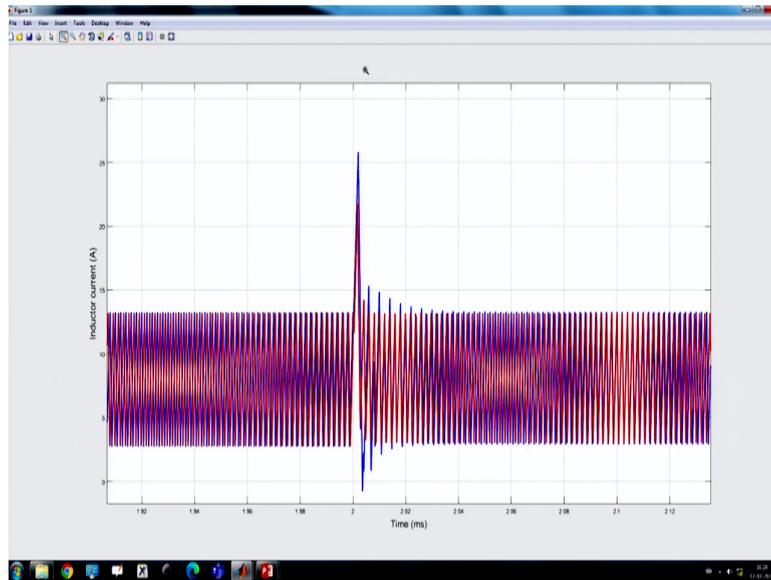
So, this looks like valley current mode control that the constant on time is superior in terms of response, you know.

(Refer Slide Time: 43:57)

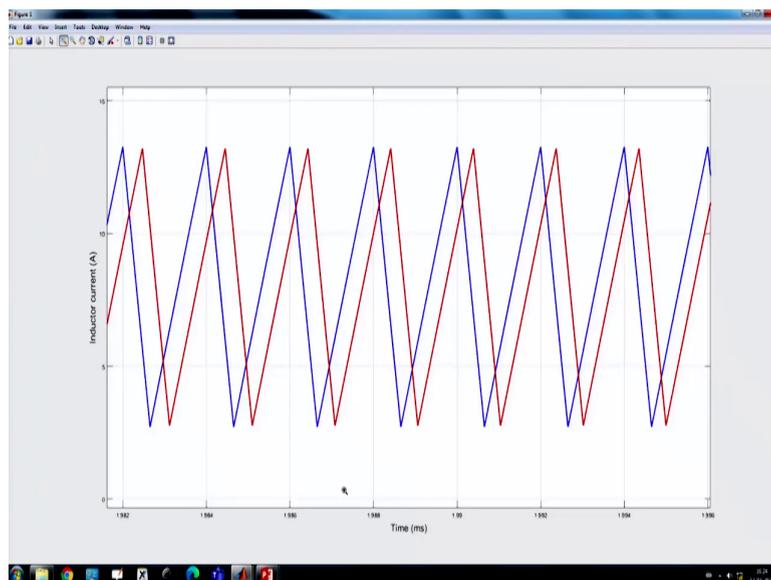


And if you go to the inductor current waveform, so, this is the inductor current waveform ok.

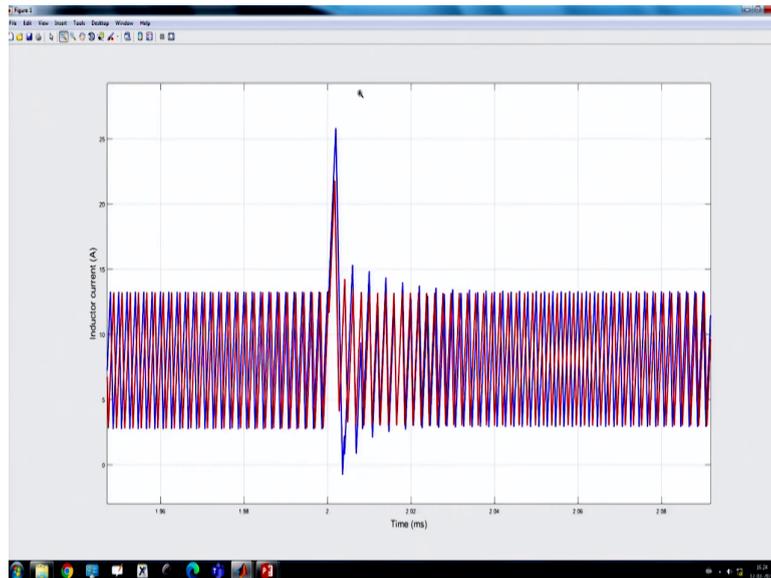
(Refer Slide Time: 44:02)



(Refer Slide Time: 44:04)

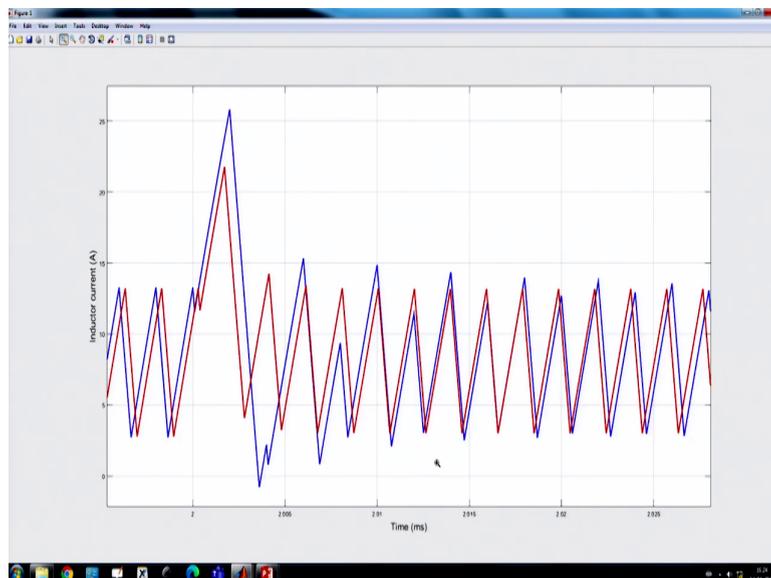


(Refer Slide Time: 44:07)



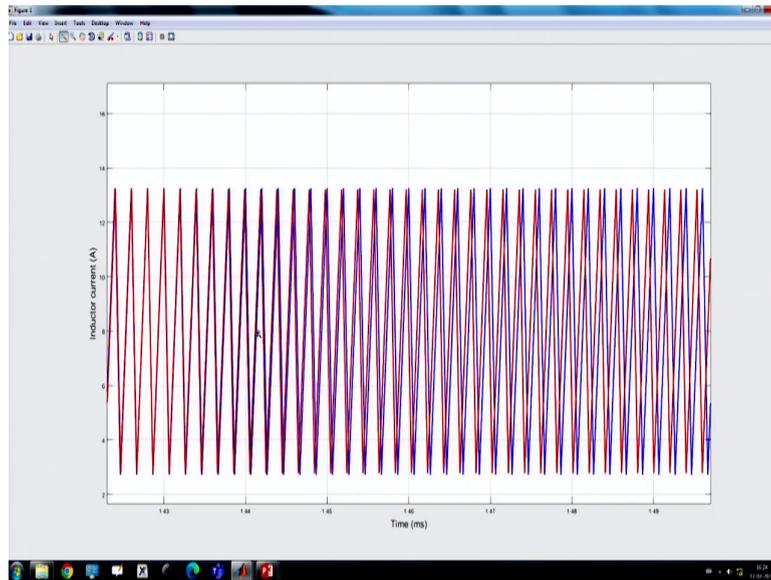
.So, they are designed to achieve close to the same duty ratio.

(Refer Slide Time: 44:10)



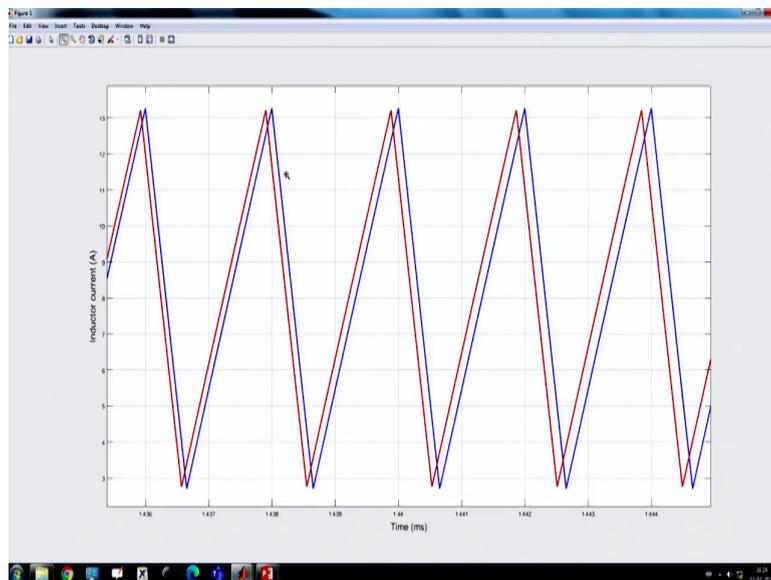
So, this is a response time ok.

(Refer Slide Time: 44:14)



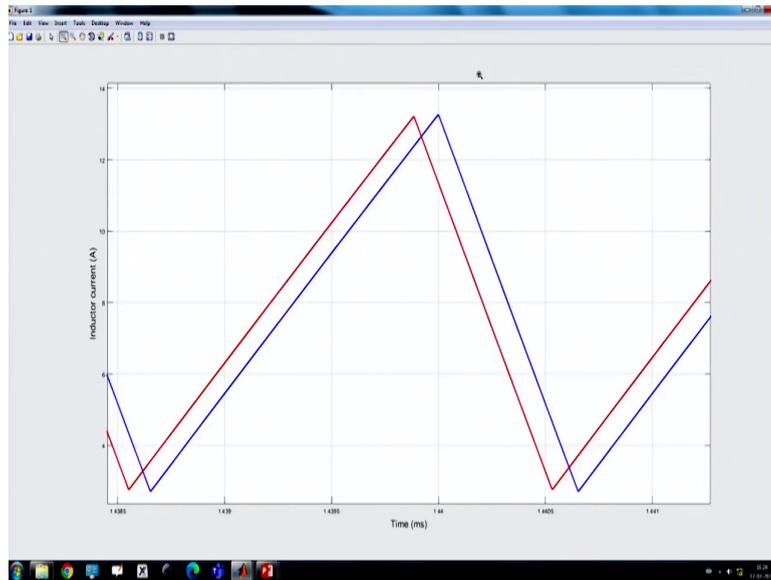
So, this is a very high duty ratio operation.

(Refer Slide Time: 44:16)



If you see the duty ratio they are very high duty ratio operation.

(Refer Slide Time: 44:18)



Because, duty ratio is around 0.67 ok.

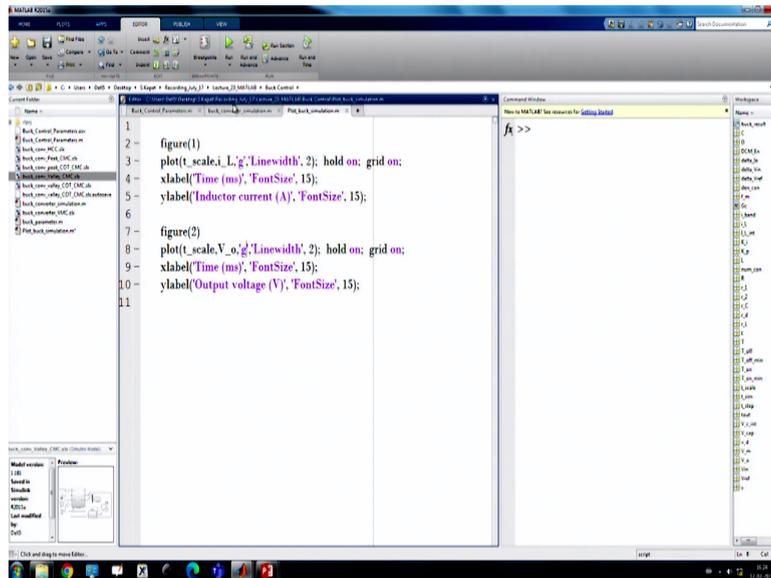
(Refer Slide Time: 44:28)

```
1 %clc; clear; close all;
2
3 DCM_Ea=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=4;
8
9 sim('buck_conv_peak_COT_CMC.slx'); clc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Warning: Discontinuities detected within algebraic loop(s), may have trouble solving > In buck_converter_simulation (line 9)

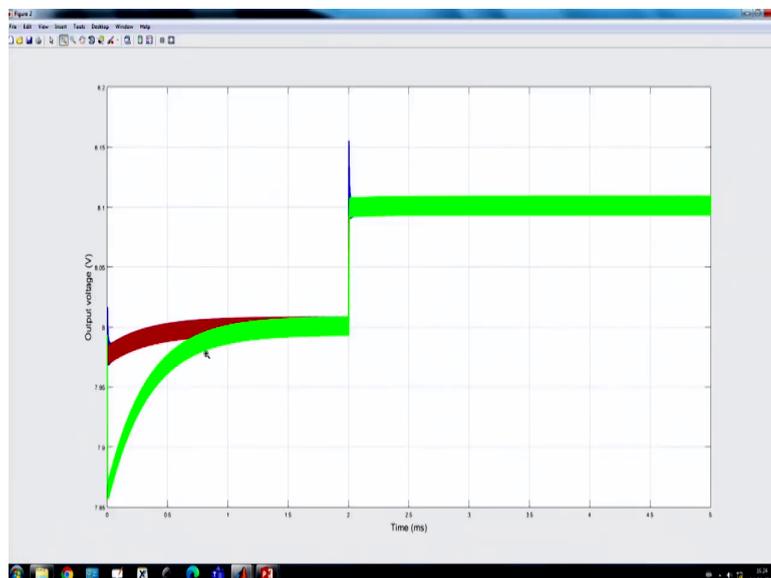
Next we want to compare with peak one, for this case peak one.

(Refer Slide Time: 44:33)

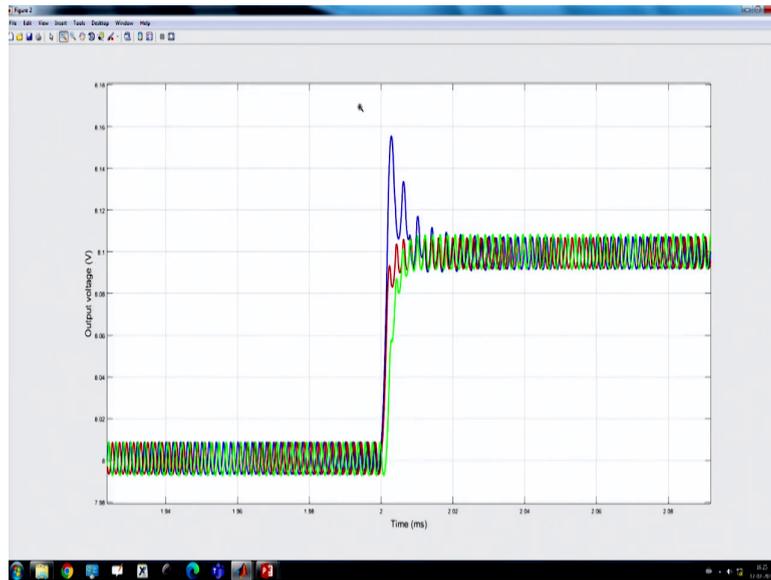


And we want to use green color here; this is a peak. That means constant off time control. But we cannot use a fixed frequency peak current mode control because it will be simply unstable ok. And then last one we will see after this will be hysteresis control.

(Refer Slide Time: 44:51)

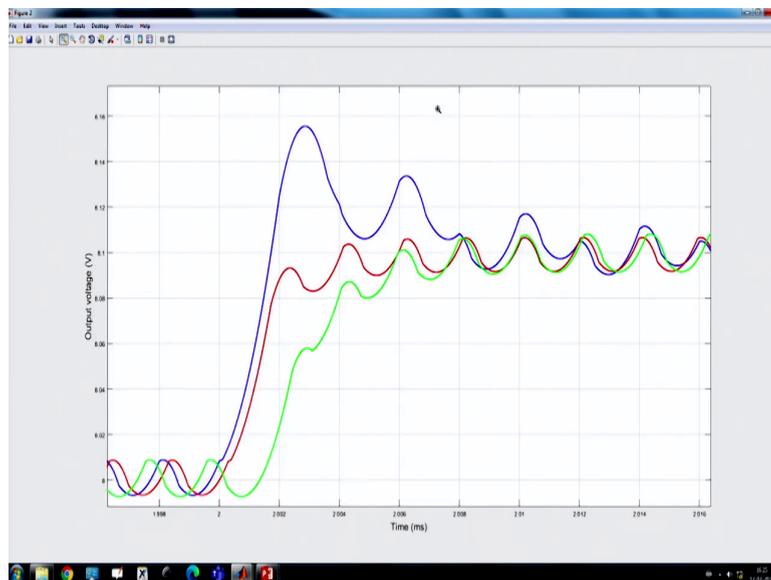


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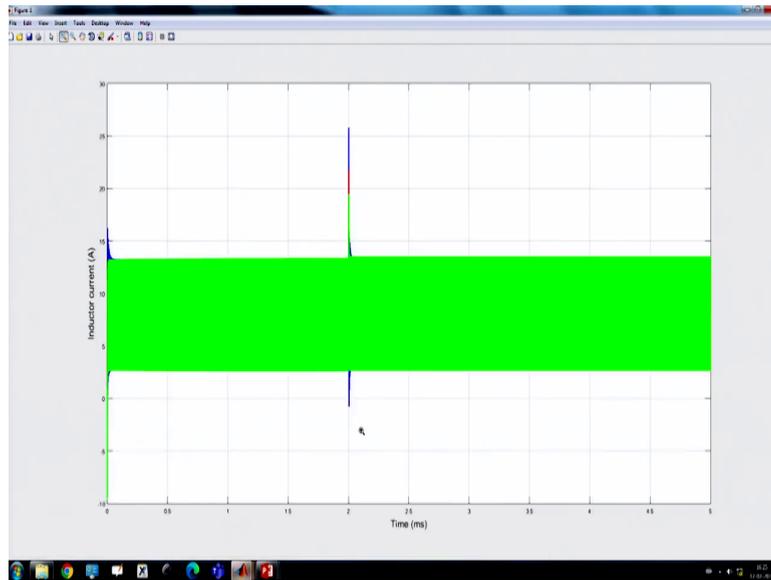
So, in this case, this one is also fine.

(Refer Slide Time: 44:56)

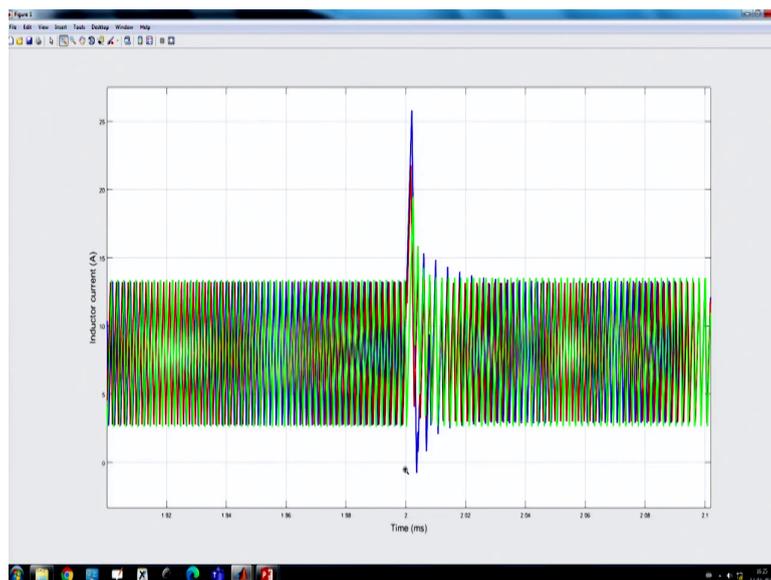


It looks also fine under this condition.

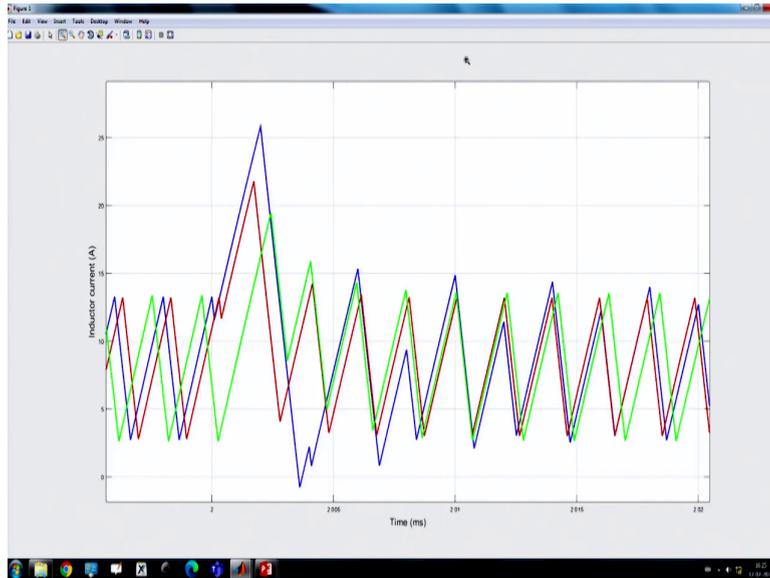
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(Refer Slide Time: 45:02)



(Refer Slide Time: 45:04)



So, if you go to this modulator and if you see the waveform, so these are the comparative response. And if you go to the voltage waveform, so will come at a time; that means, you can do all this audio susceptibility, everything that you can verify that we have discussed ok. Now, the last part we want to check the current loop stability ok. So, let us go to the current loop stability and we want to compare that means, if the current loop you know if you are close to 6.

(Refer Slide Time: 45:30)

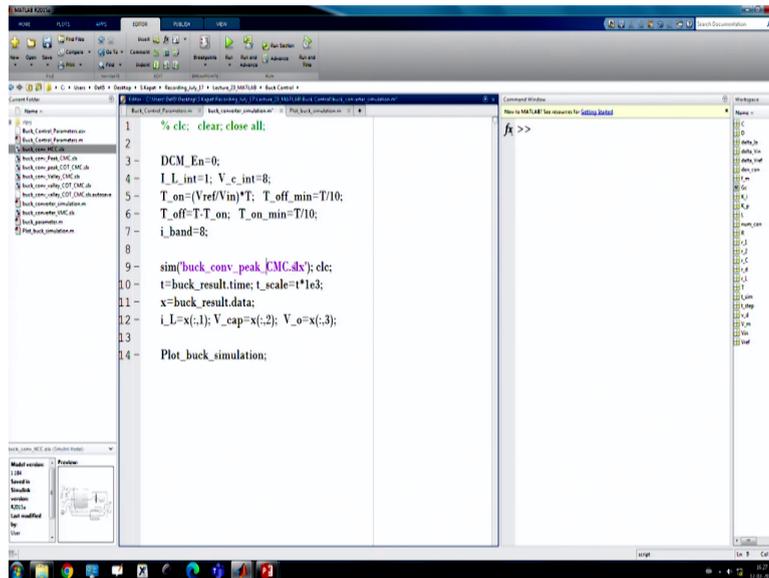
```

1 = clc; close all; clear;
2
3 %% Parameters
4 buck_parameter;
5 Vin=12; Vref=6; D=Vref/Vin;
6 R=1;
7
8
9 V_m=0;
10
11 %% PI Controller
12 K_p=100; K_i=300000;
13
14 num_con=[K_p K_i];
15 den_con=[1 0];
16 Ge=tf(num_con,den_con);
17
18 %% Transient parameters and transient response
19 t_sim=5e-3; t_step=2e-3; f_m=1e3;
20 delta_Io=0; delta_Vin=0; delta_Vref=0.1;

```

Let us say we are close to 6.

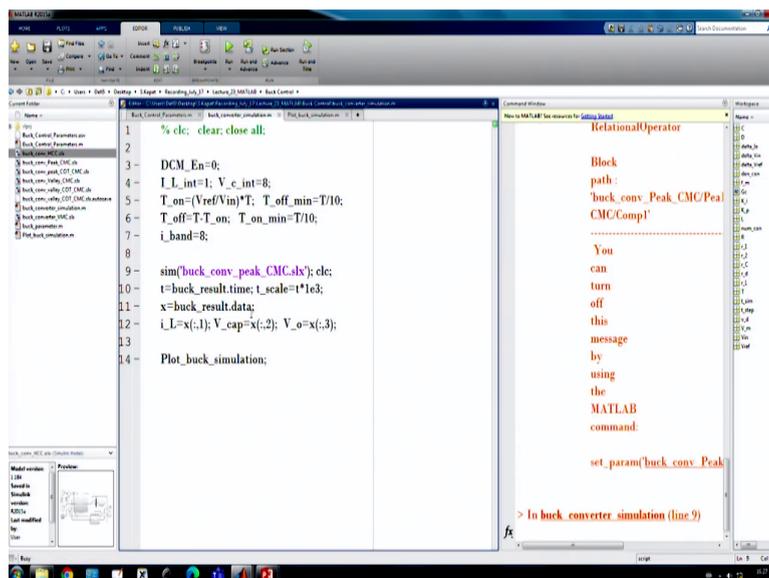
(Refer Slide Time: 45:35)



```
1 % cfc; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=8;
8
9 sim('buck_conv_peak_CMC.slx'); cfc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

And we are setting a band of 8 ampere band close to 6. So, first we want to see what will be the stability under peak current mode control, valley current mode control and we are not applying any transient. Because we do not want to see any transient here, we just want to run it for ok. So, let us see we want to see there is no ramp compensation ramp is 0. So, if we see peak current mode control, peak current mode control, first let us run.

(Refer Slide Time: 46:07)



```
1 % cfc; clear; close all;
2
3 DCM_En=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=8;
8
9 sim('buck_conv_peak_CMC.slx'); cfc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

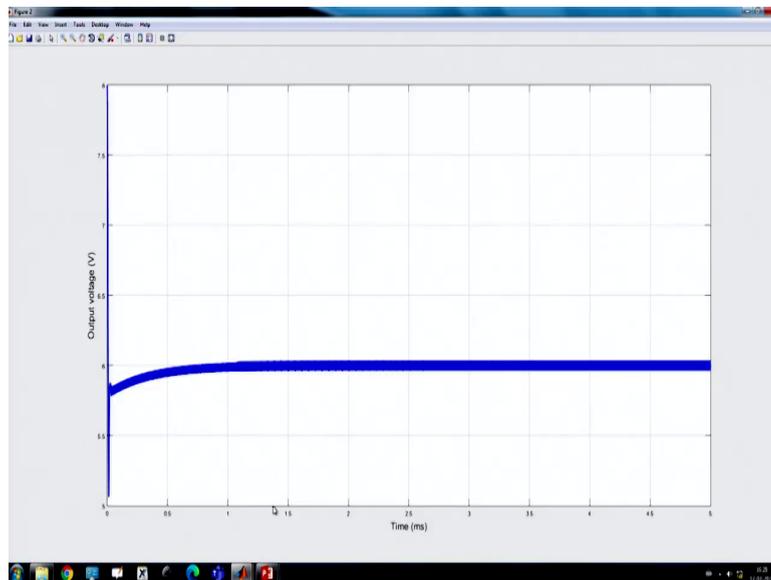
Warning: In buck_converter_simulation (line 9)
RelationalOperator
Block
path:
'buck_conv_Peak_CMC/Pea
CMCCompl'

You
can
turn
off
this
message
by
using
the
MATLAB
command:

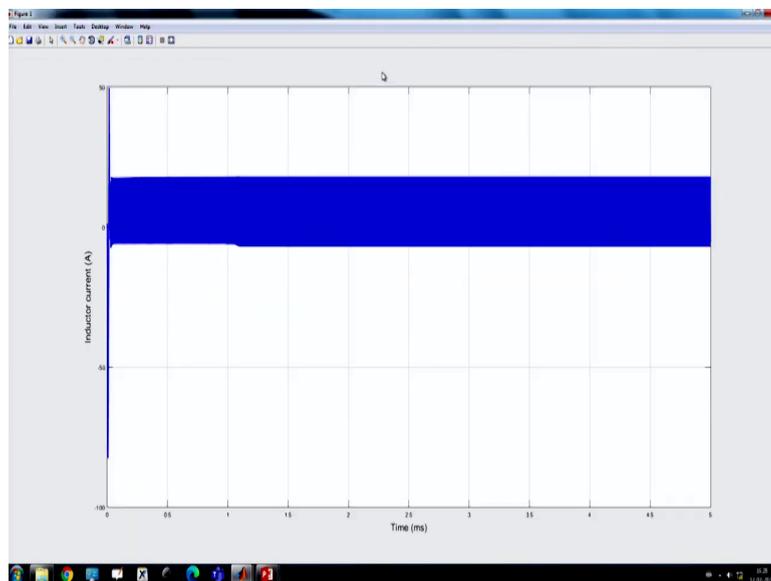
set_param('buck_conv_Peak
> In buck_converter_simulation (line 9)

And we want to see what happens in the peak current mode control. So, this is like our input output voltage, like a duty ratio is close to 50 percent. It is just in the, we are setting the critical duty ratio boundary 0.5 ok yeah.

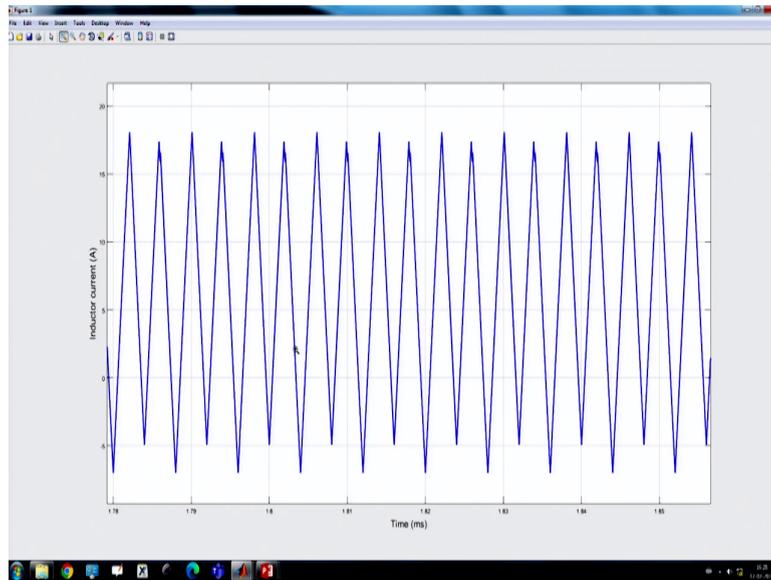
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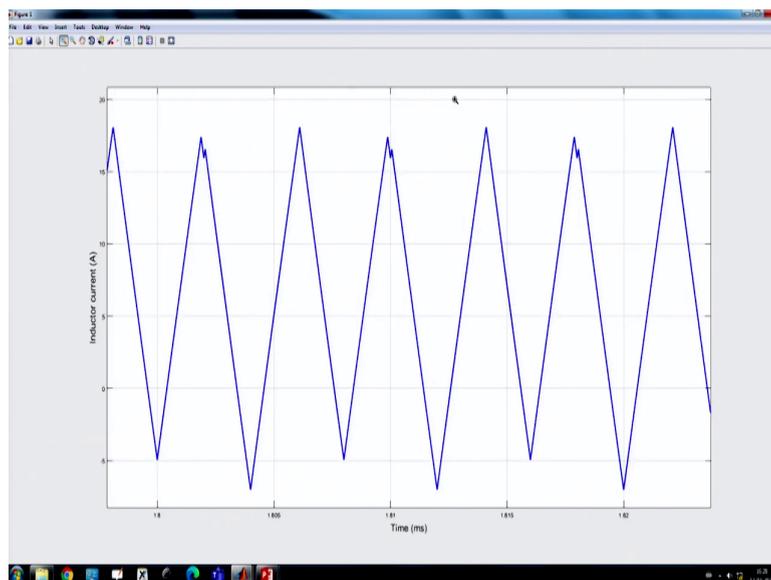
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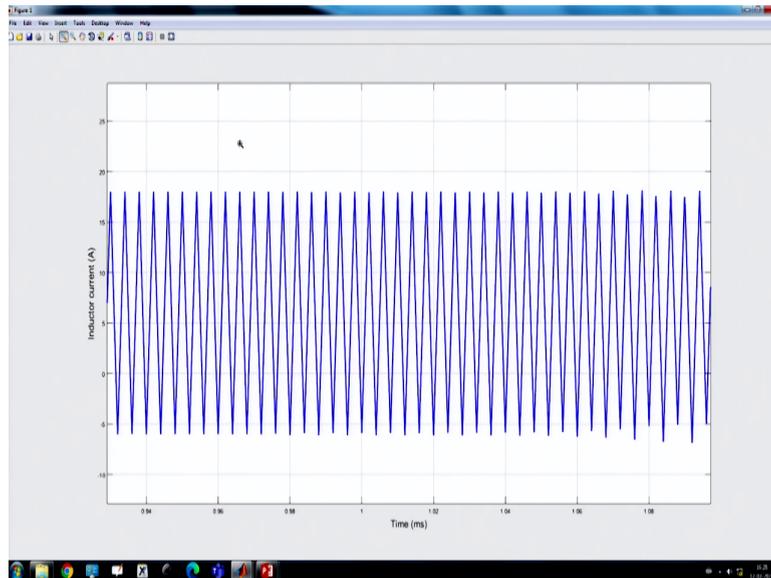


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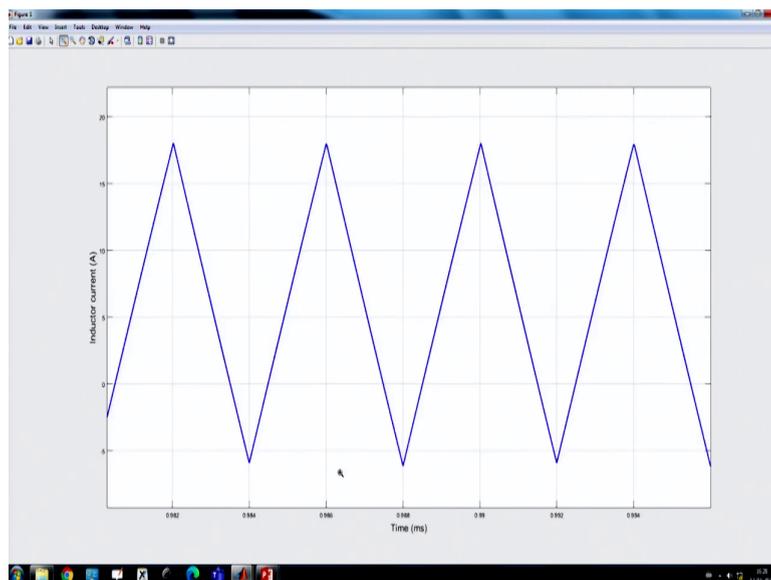


So, if you see the inductor current waveform, you will see this subharmonic oscillation.

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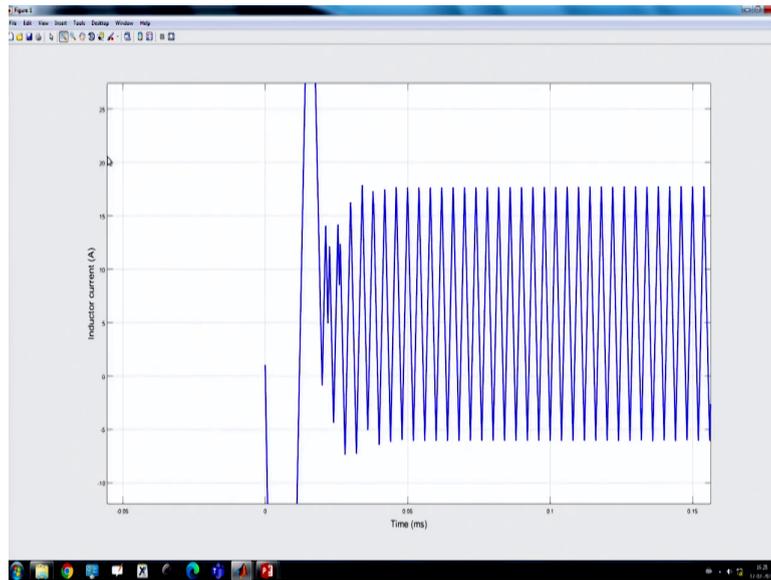


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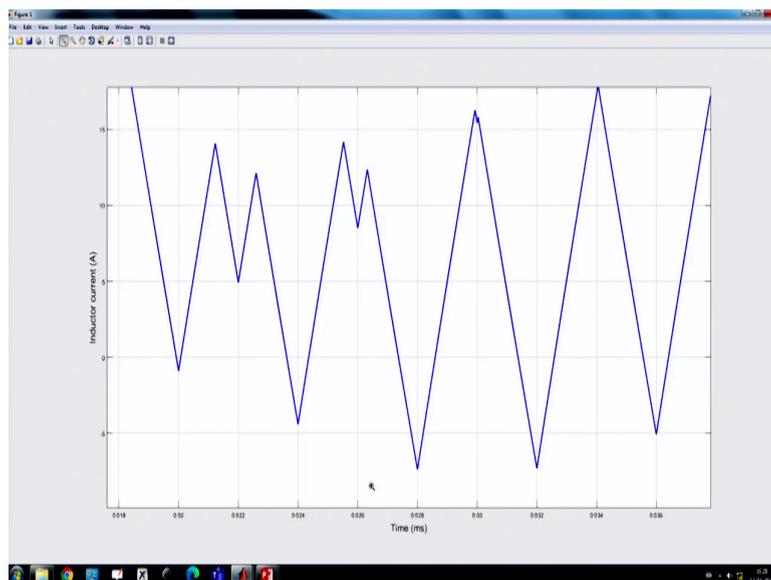


Because, if you go here, our switching period is set to a point. You know it should be 2 microsecond, but it is showing 4 micro second that means, it is totally unstable.

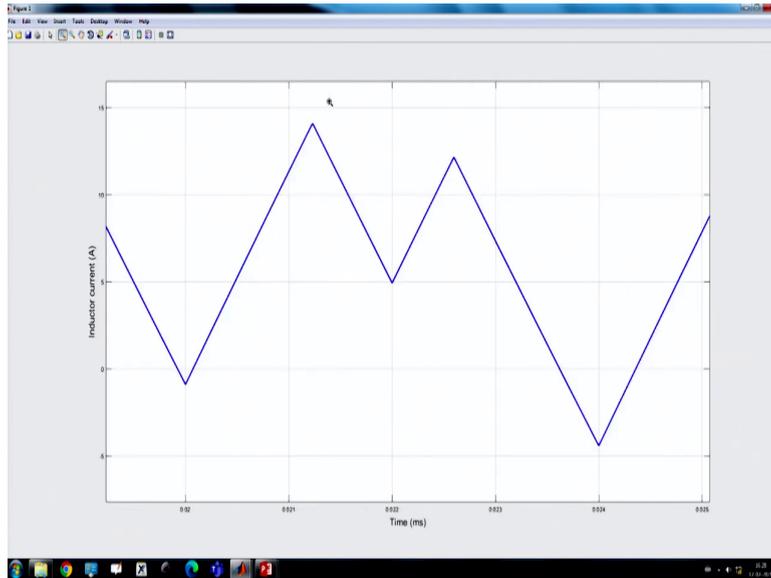
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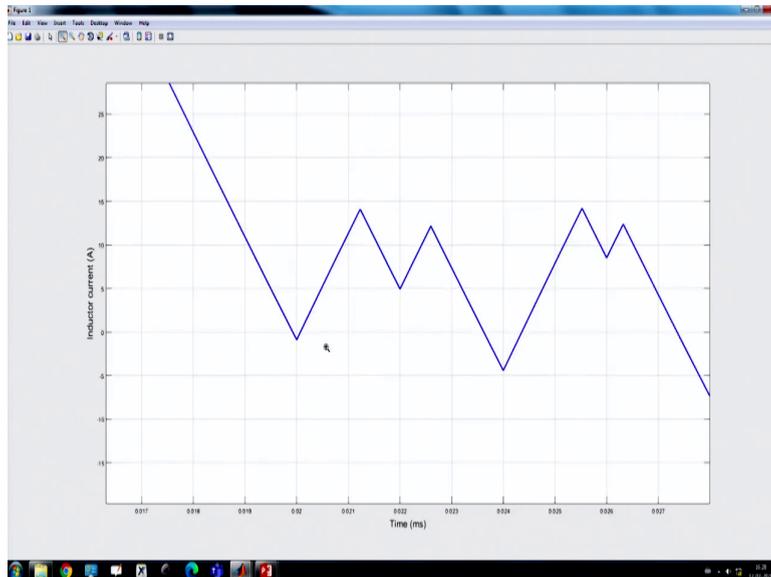


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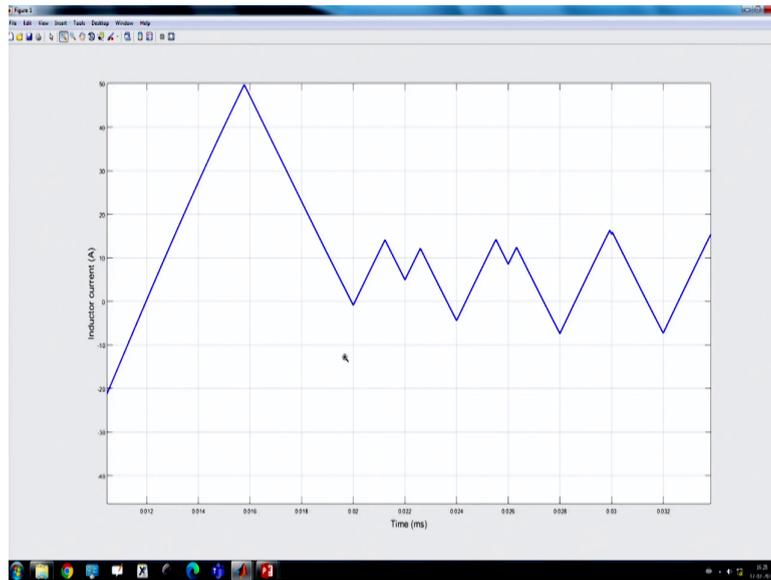


You see, it starts from here and if you take a few cycles at the beginning, this is like from this to this is our 2 micro second, so it is totally unstable.

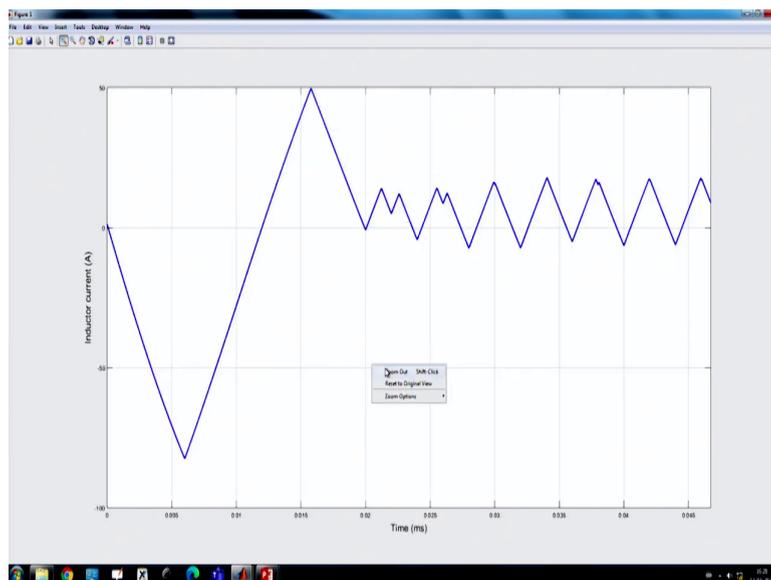
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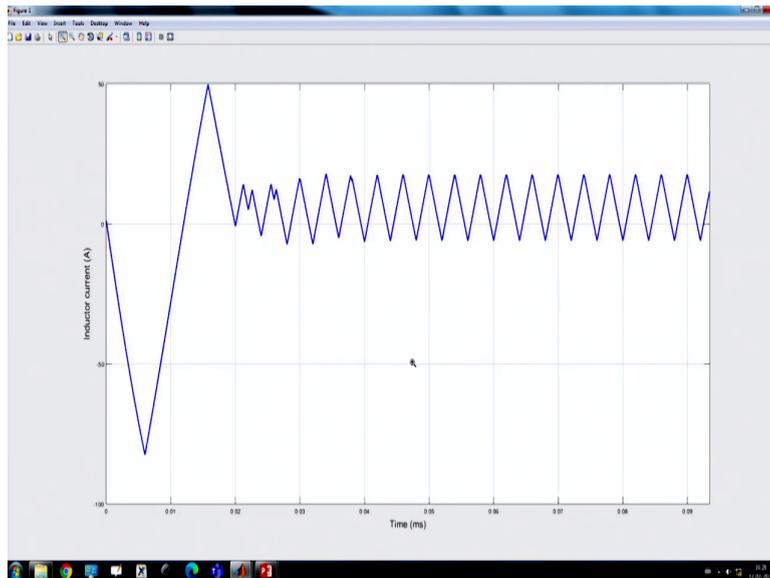
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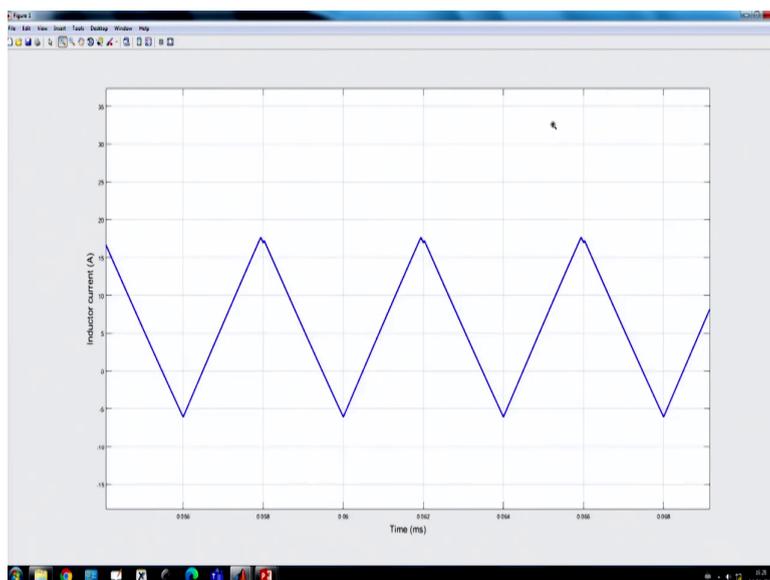
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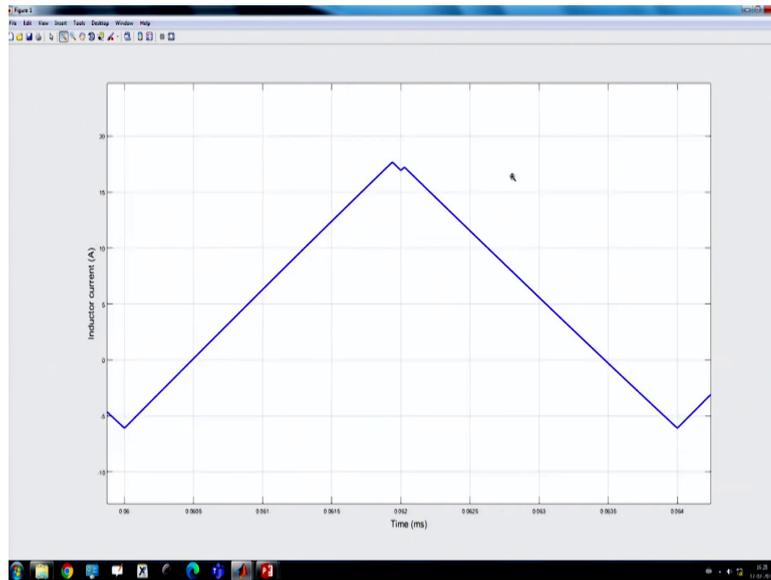


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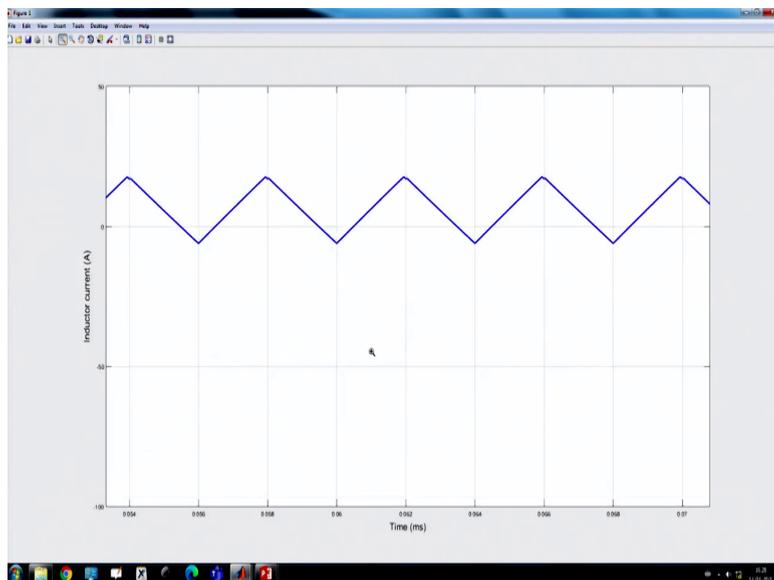
And slowly this current loop become unstable, and it looks like your switching frequency becomes half.

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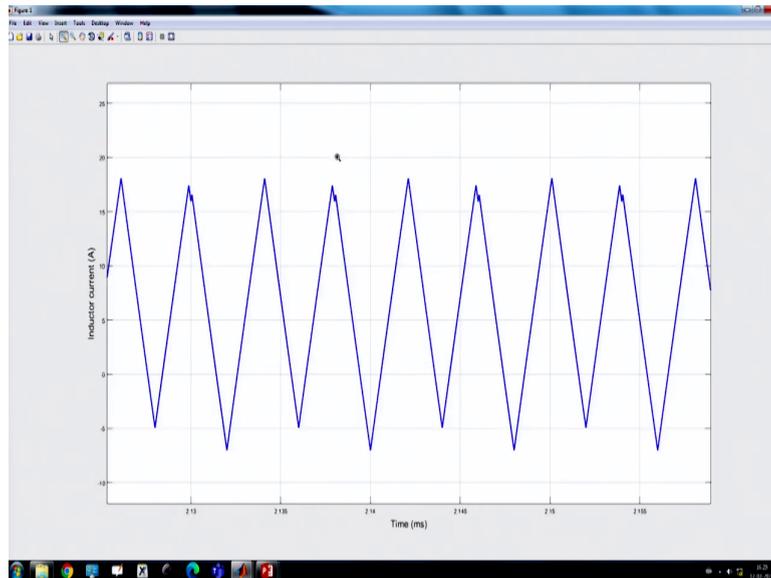


But yes, you can see if you very closely look at this there is a subharmonic instability.

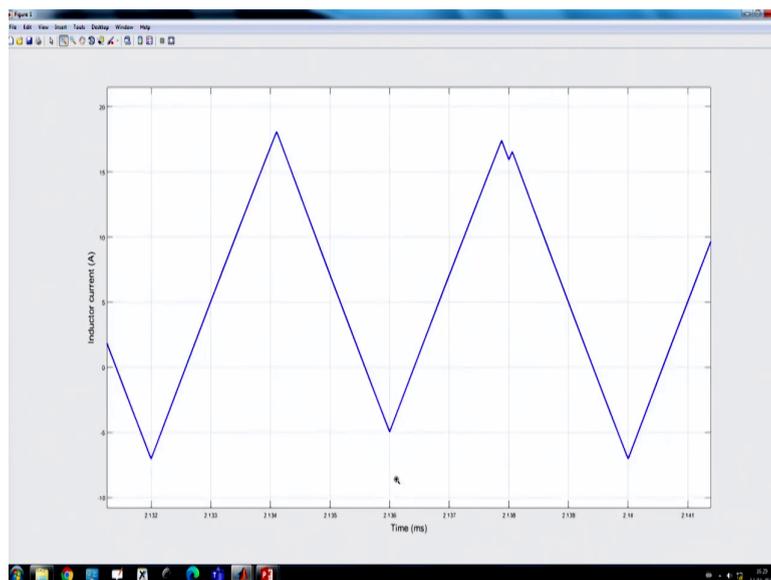
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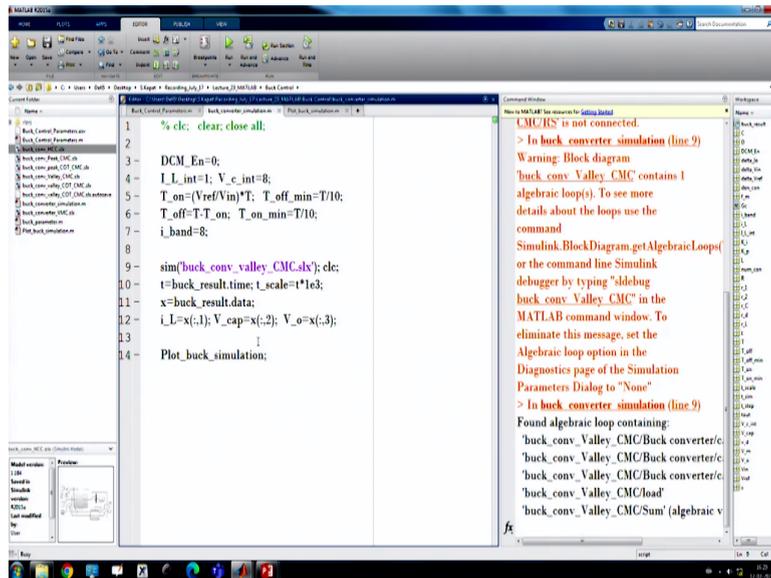


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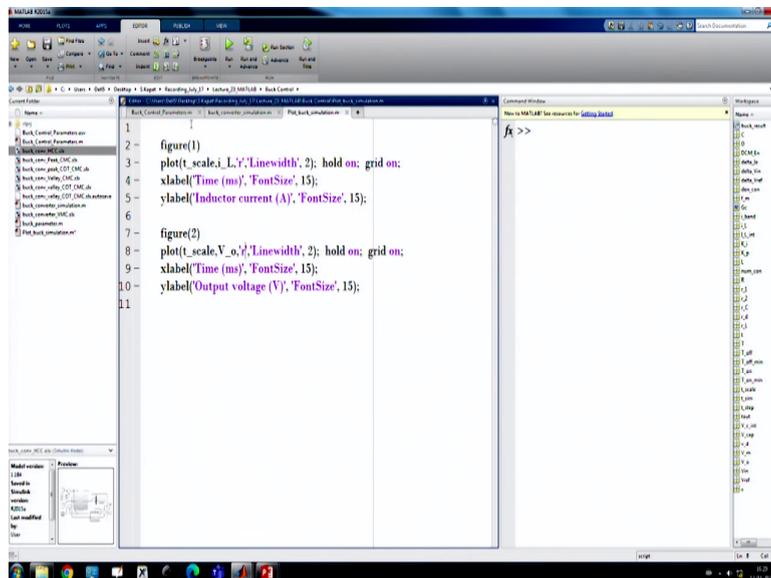
So, this is my 2 microseconds and if you wait for long time, then it will look like your you know if this is a totally unstable behaviour.

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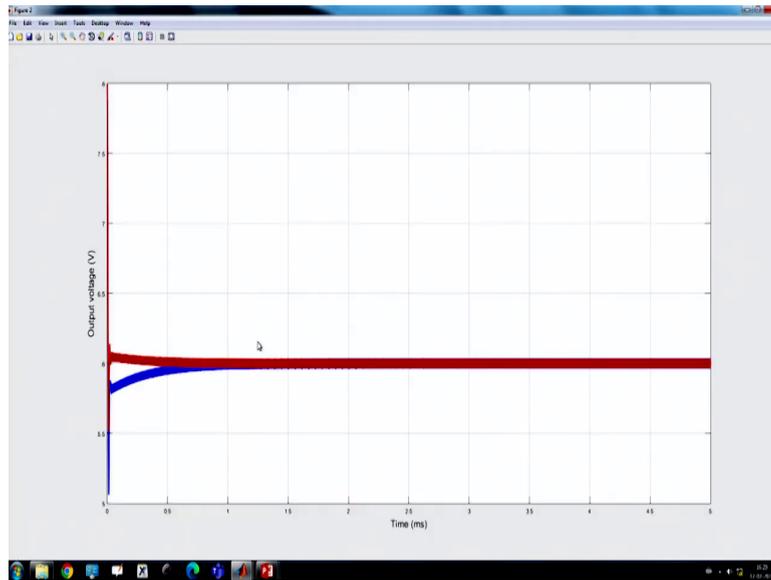
Now we want to run the same scenario with valley current mode control valley current mode control and we want to use red waveform.

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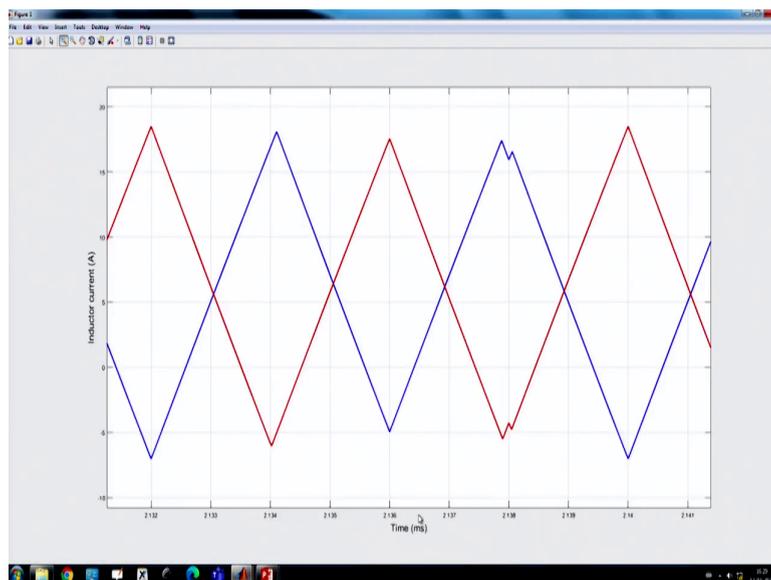


So, this is a borderline case where, but even with the borderline. There is too much instability.

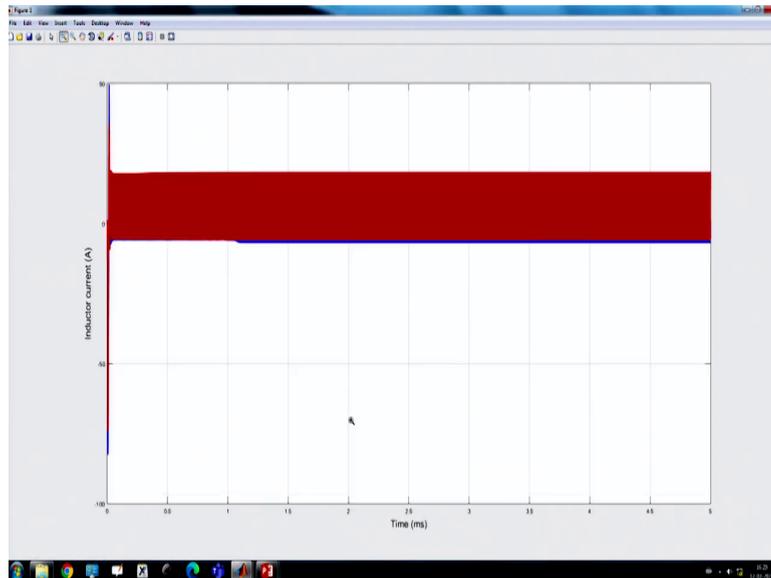
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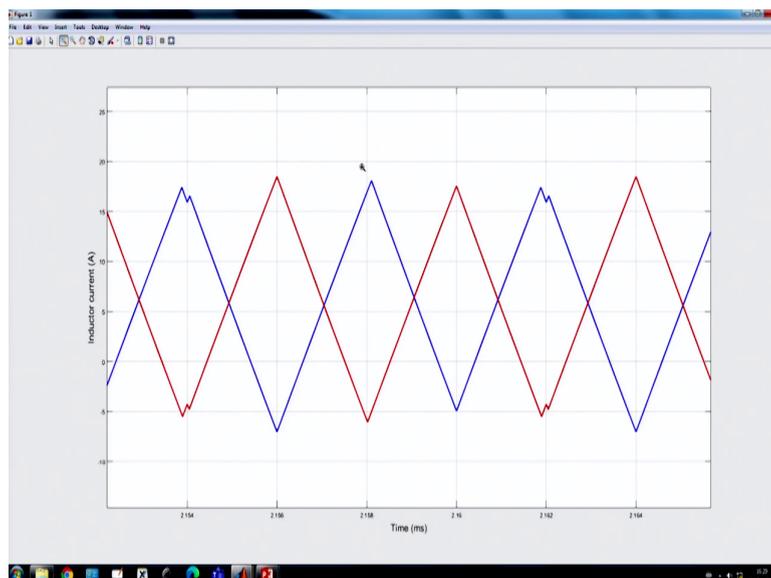
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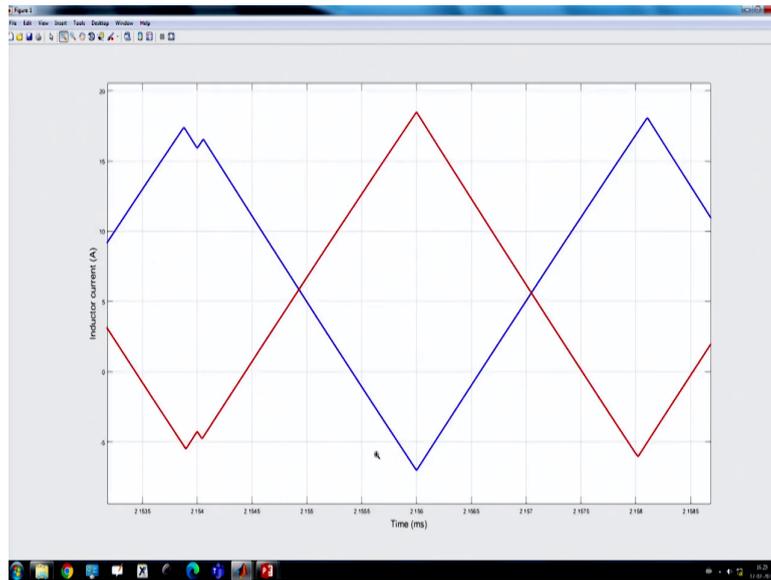


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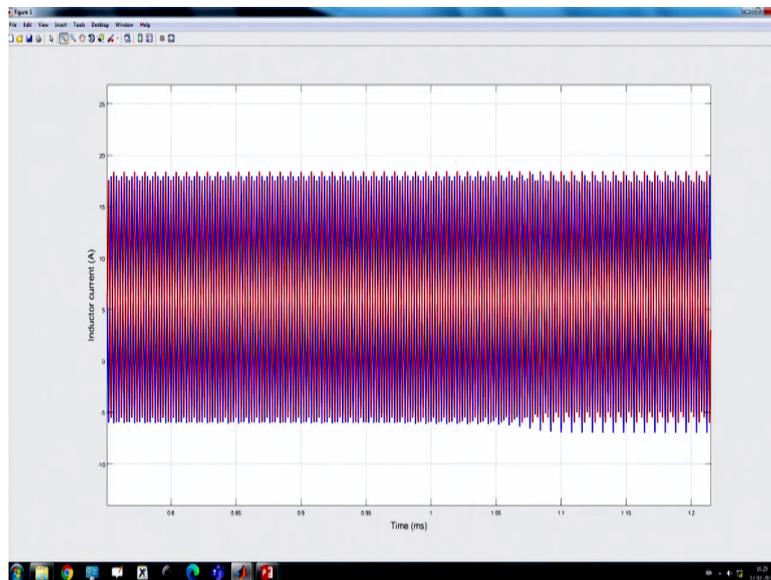
So, if you go to the current waveform here, the similar kind of stability problem will also find for valley current mode as well.

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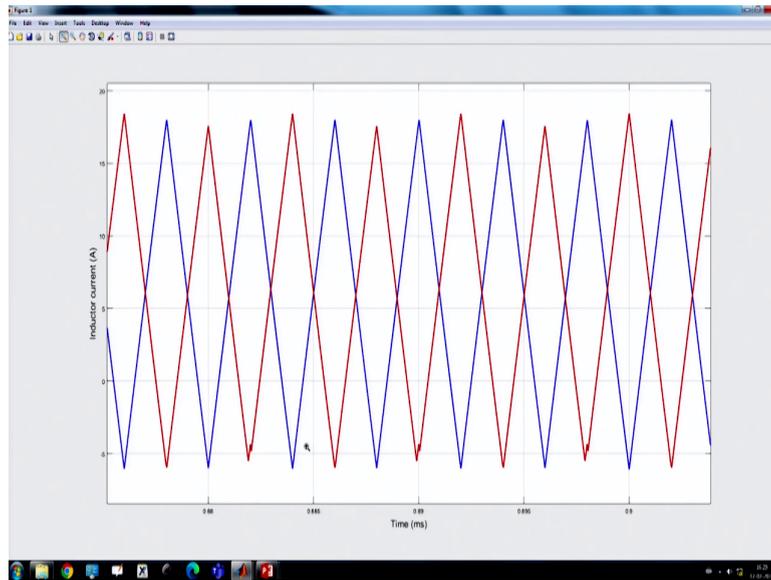


So, it is also not stable; it is also unstable because the waveform looks like it is you know they it is ok.

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So, both of them are unstable.

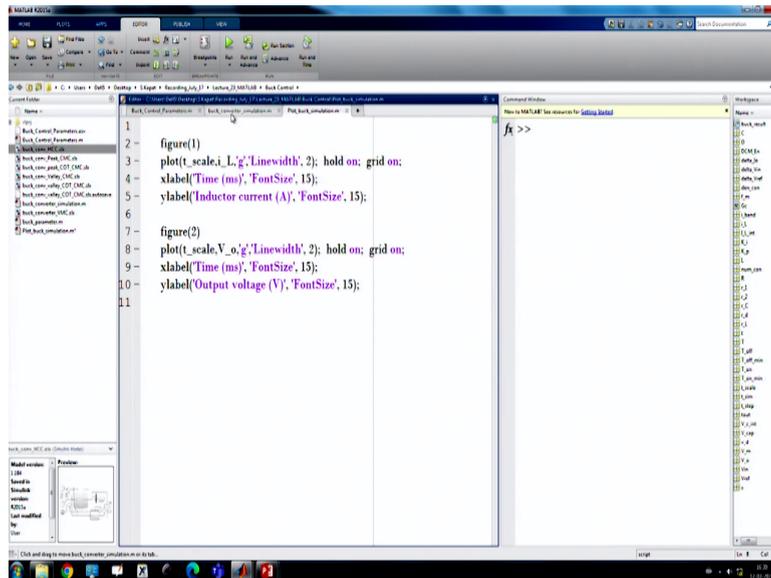
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```
1 % cdc; clear; close all;
2
3 - DCM_En=0;
4 I_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=8;
8
9 sim('buck_conv_valley_COT_CMC.slx'); cdc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

Warning: Discontinuities detected within algebraic loop(s), may have trouble solving > In buck_converter_simulation (line 9)

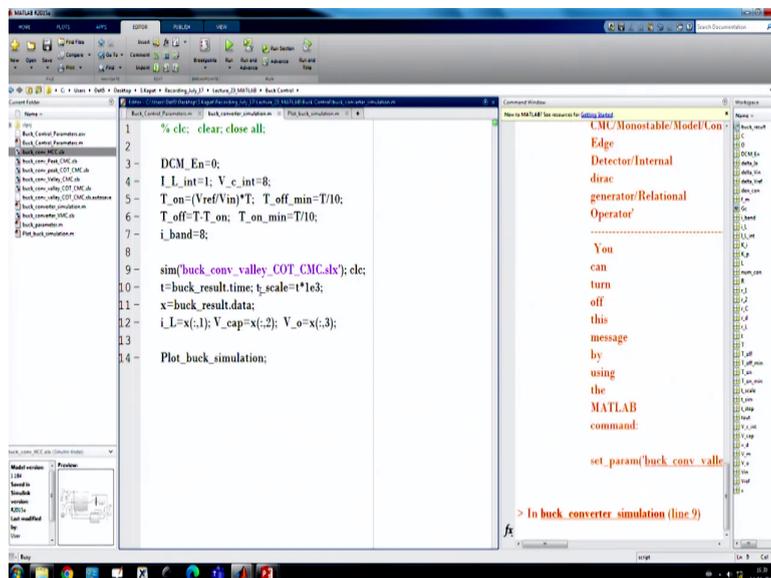
And if we want to operate now constant on time; that means, you know valley COT that we are using green color.

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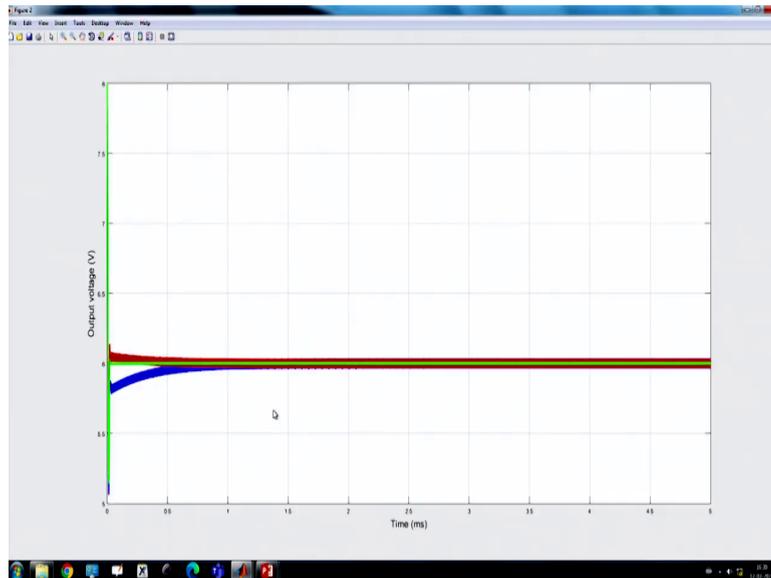
Let us say valley, this is now using constant on time and if you run it.

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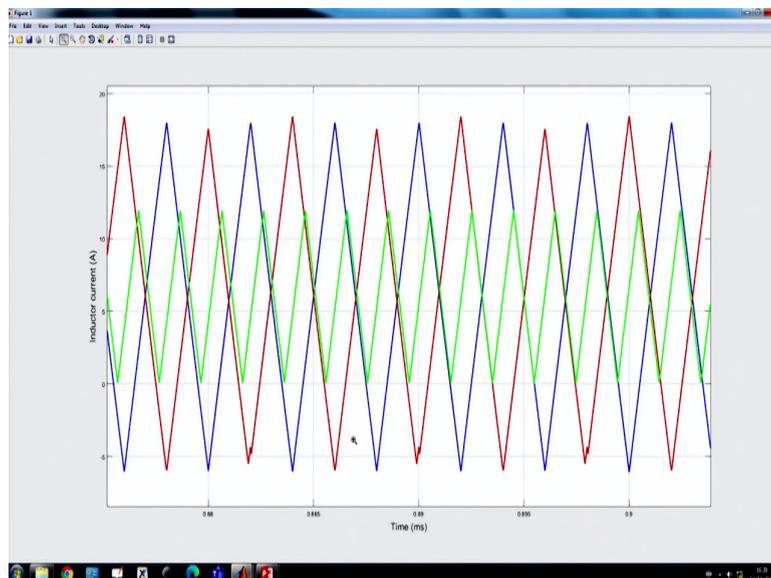
So, under the same condition we are now running constant on time using green color ok. And this green color will show the actual you know the stable current waveform and that we want to see.

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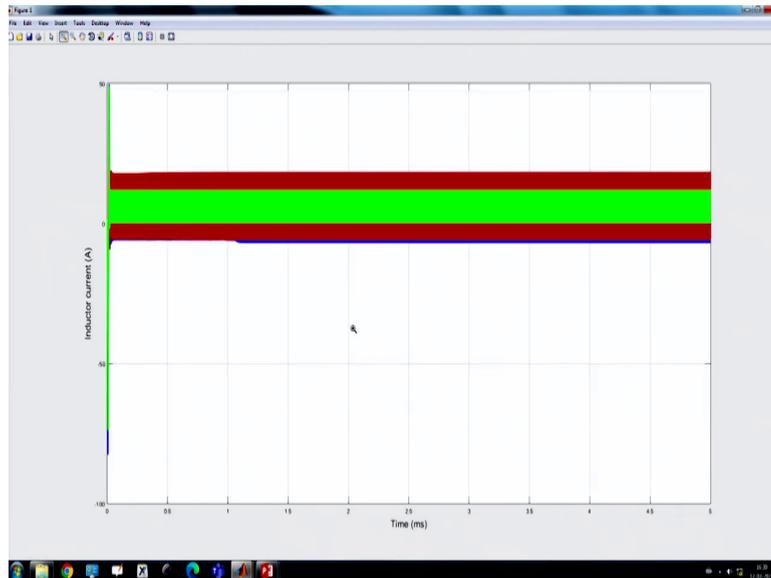


Because, in both peak and valley we got totally unstable behaviour.

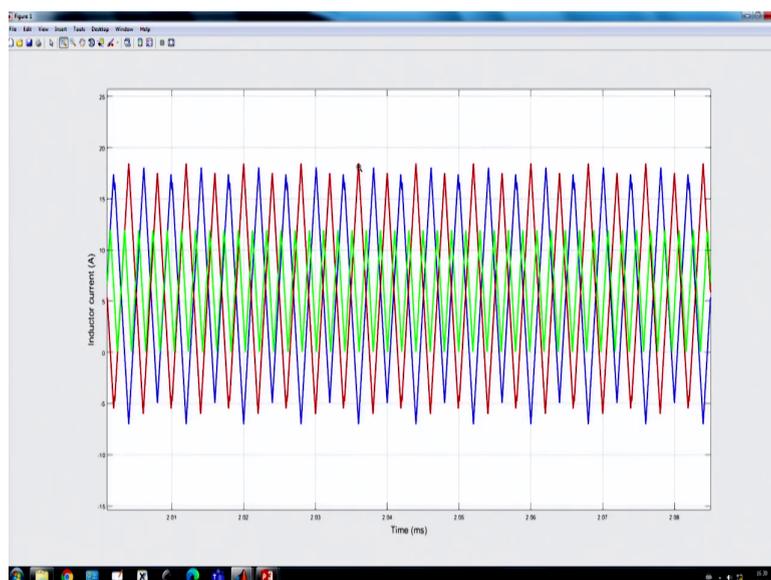
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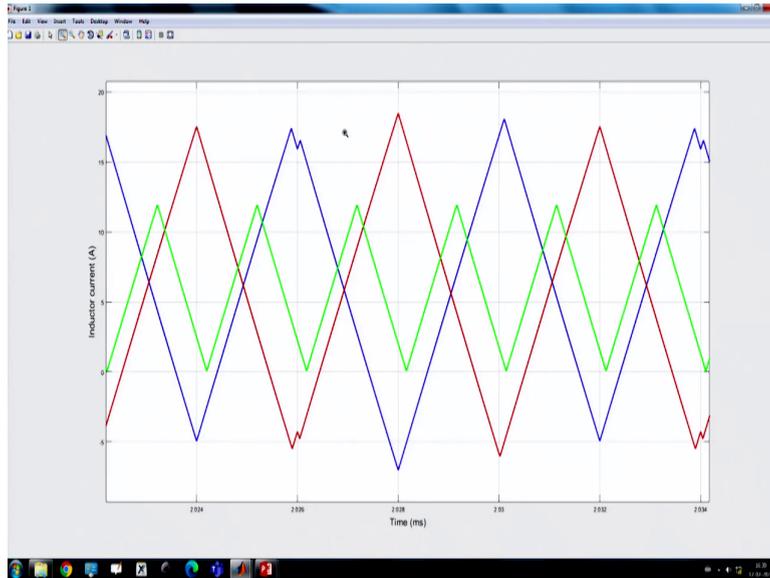
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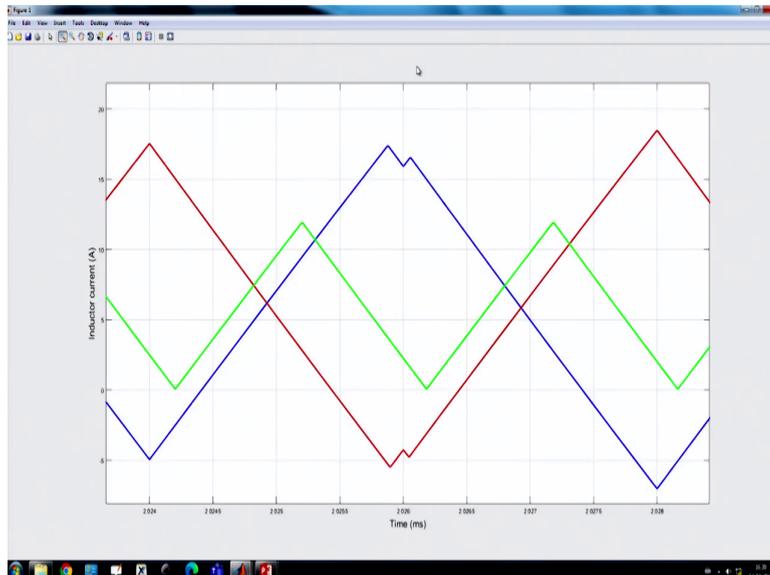


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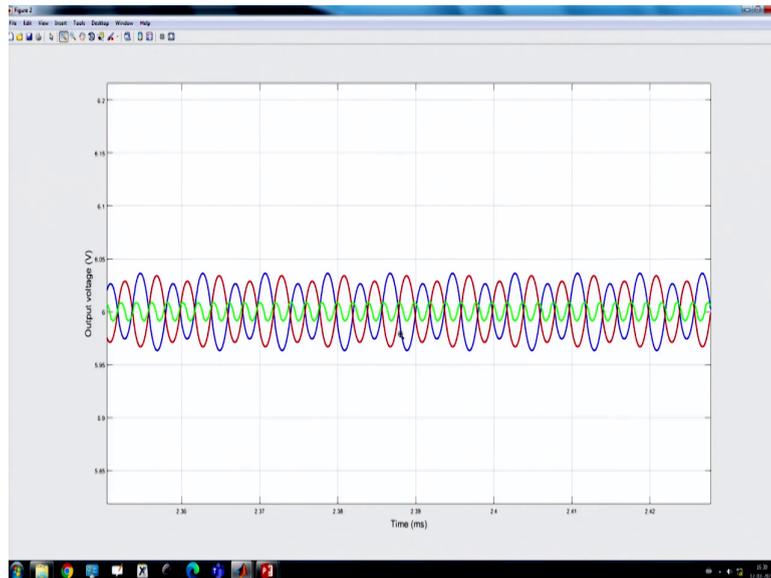
Now if you go to the green color, you know this is actual switching frequency because it is perfectly stable ok.

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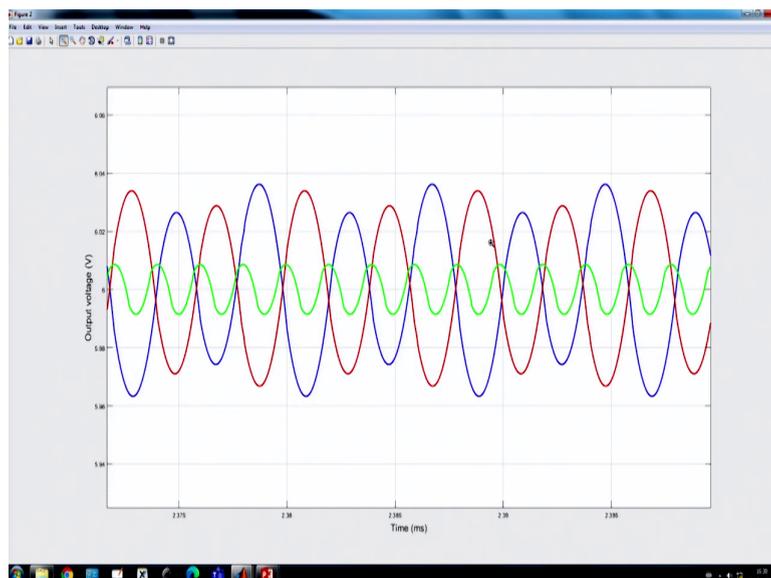


But here, the current ripple has significantly increased because of the unstable current loop and this is also evident from our voltage waveform.

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It is also evident from voltage waveform ok. So, this is too much you know about instability. So, constant on time and if you go to constant off time, you can see a peak.

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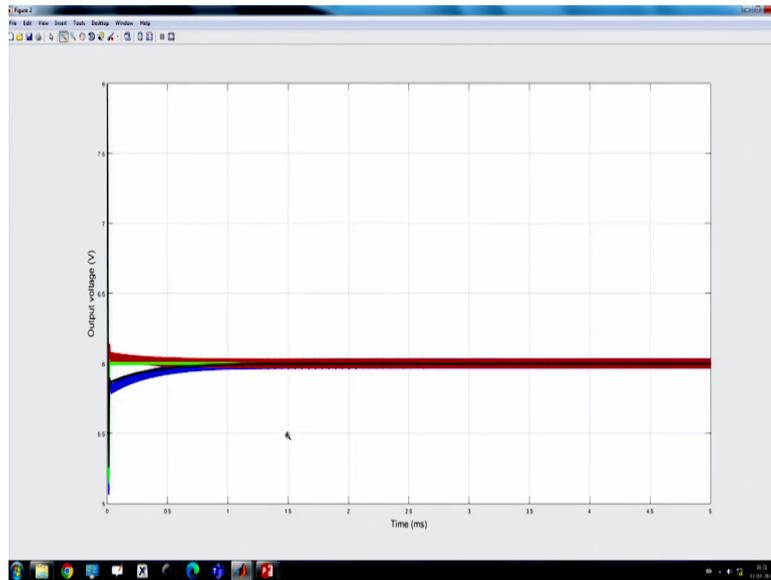
```
1 % clc; clear; close all;
2
3 DCM_En=0;
4 L_L_int=1; V_c_int=8;
5 T_on=(Vref/Vin)*T; T_off_min=T/10;
6 T_off=T-T_on; T_on_min=T/10;
7 i_band=8;
8
9 sim('buck_conv_peak_COT_CMC.slx'); clc;
10 t=buck_result.time; t_scale=1*1e3;
11 x=buck_result.data;
12 i_L=x(:,1); V_cap=x(:,2); V_o=x(:,3);
13
14 Plot_buck_simulation;
```

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```
1 figure(1)
2 plot(t_scale*i_L,'k', 'LineWidth', 2); hold on; grid on;
3 xlabel('Time (ms)', 'FontSize', 15);
4 ylabel('Inductor current (A)', 'FontSize', 15);
5
6
7 figure(2)
8 plot(t_scale*V_o,'k', 'LineWidth', 2); hold on; grid on;
9 xlabel('Time (ms)', 'FontSize', 15);
10 ylabel('Output voltage (V)', 'FontSize', 15);
11
```

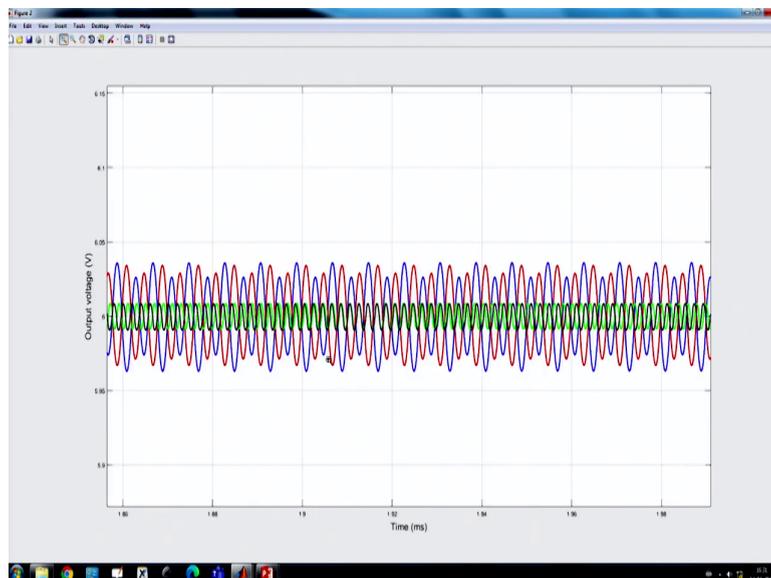
So, peak and you can if I use a black color, this is a last one to compare and hysteresis is, of course, stable; there is no question of instability. Then the last one will show that constant off time is also stable, perfectly stable. There is absolutely no problem ok yeah.

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So, this is showing the constant off time control ok.

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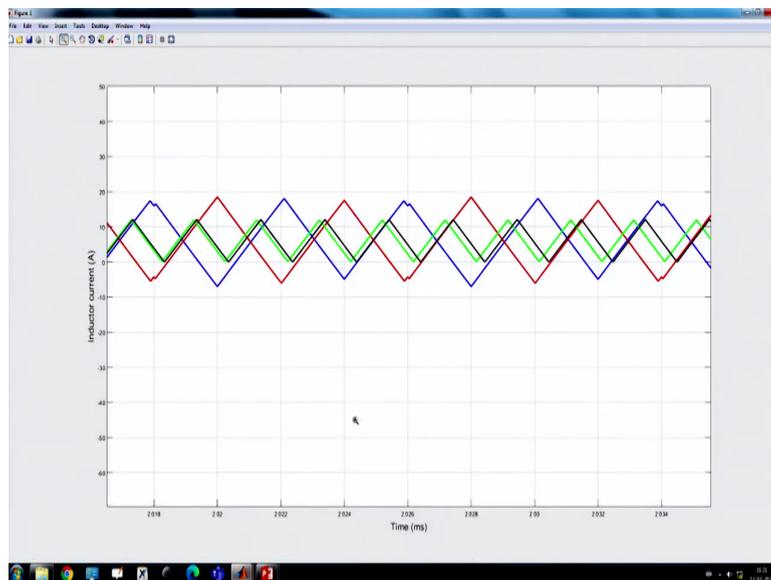


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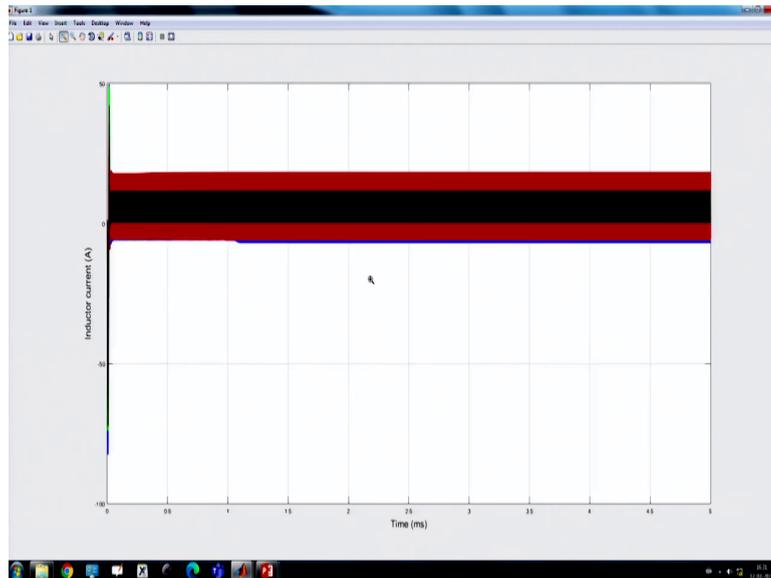


So, constant off time control they are perfectly stable.

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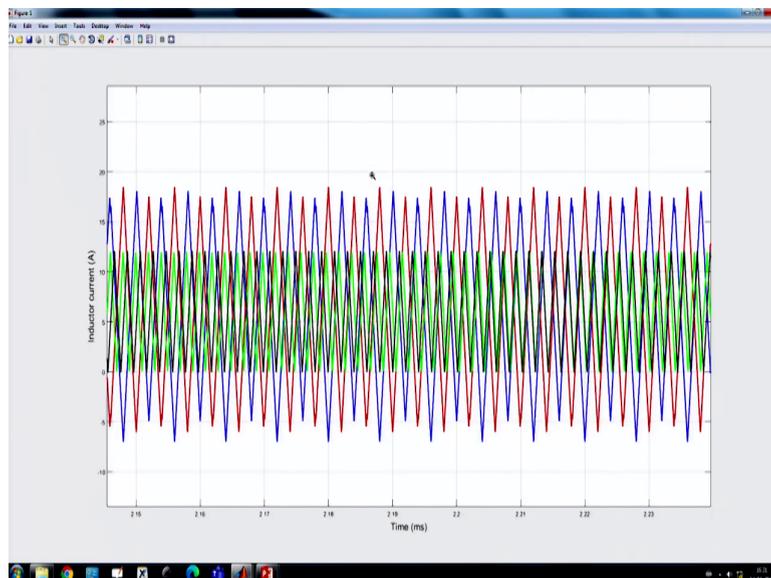


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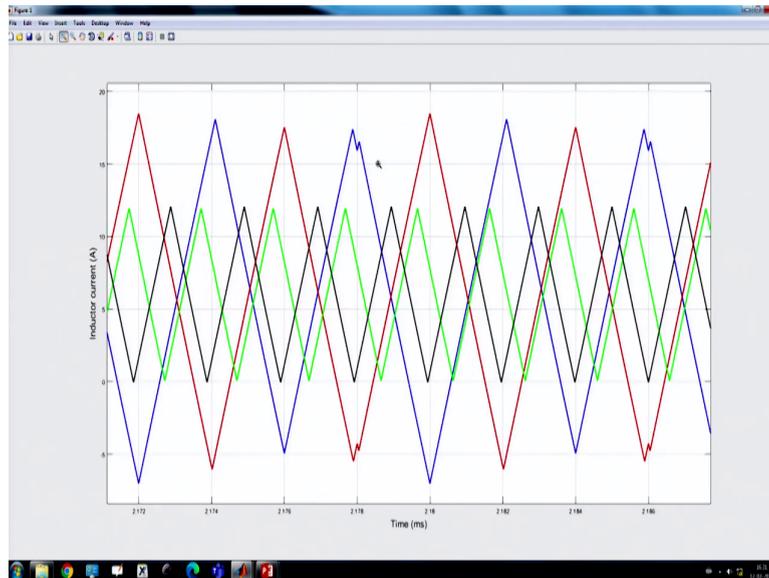


And if you go to the current waveform, it is stable.

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But, the problem with the peak and valley, so you need to have put ramp compensation for stability and that will slow down the transient response, ok. And that we will discuss with the design system not now. So, we have you know justified and you can check the audio susceptibility of the current mode control as a very excellent line regulation. So, there is absolutely no problem it will reject any supply disturbance.

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Worst Case Inductor Current Ripple – Buck Converter in CCM

Modulation Technique	Current Ripple (Δi_L)	Worst case scenario
Pulse width modulation	$\frac{T}{L} \times V_o \left(1 - \frac{V_o}{V_{in}} \right)$	Highest input voltage
Constant on-time modulation	$\frac{T_{on}}{L} \times (V_{in} - V_o)$	Highest input voltage
Constant off-time modulation	$\frac{T_{off}}{L} \times V_o$	Insensitive to operating conditions

Handwritten notes on the right side of the slide include: $i_{avg} = i_{LED}$, $\Delta i_L = \frac{\Delta i_L}{2}$, i_{LED} , and "Battery". A small video inset shows a man speaking.

So, now, if we want to consider worst-case scenario the pulse width modulation, you know if you take the ripple current. That means, the ripple current for highest input voltage we know the worst case for a buck converter and this is the expression that we have discussed.

If we use constant on time control the worst case again with highest input voltage, but the expression of current ripple is different. But under constant off time control, it is insensitive to input voltage operation. That is why constant off time control is used for you know even for LED driving or also sometime battery charging. Because your ripple current, if you fix more or less the output voltage, that means, your Δi_L is independent of input voltage.

So, if there is an input voltage, if the output voltage is more or less fixed suppose if you are talking about a driving an LED, then if you know this value then you can create a band. So; that means, you can set the if this is our reference. You know suppose I need to track some LED current. So, then my reference current will be simply i_{LED} plus Δi_L by 2 since Δi_L by 2 is known for this particular case. So, you can implement very fast average current control here ok.

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Worst Case RMS Inductor Current – Buck Converter in CCM

Modulation Technique	RMS Current ($i_{L,rms}$)	Worst case scenario
Pulse width modulation	$\sqrt{I_o^2 + \frac{1}{12} \left[\frac{TV_o}{L} \left(1 - \frac{V_o}{V_{in}} \right) \right]^2}$	Highest input voltage and highest load current
Constant on-time modulation	$\sqrt{I_o^2 + \frac{1}{12} \left[\frac{T_{on}}{L} (V_{in} - V_o) \right]^2}$	Highest input voltage and highest load current
Constant off-time modulation	$\sqrt{I_o^2 + \frac{1}{12} \left(\frac{V_o T_{off}}{L} \right)^2}$	Highest load current

Handwritten note: $I_{rms}^2 = I_o^2 + \frac{\Delta i_L^2}{12}$



So, worst case RMS current this is consistent with the inductor current, because RMS current expression is nothing, but we know that what is the RMS current expression. It is nothing, but I average for any waveform into ΔI_L square by 12 that we have discussed. So, if the ripple current increases, if the Δi_L is constant, then RMS current will also increase.

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Switching Frequency – Buck Converter in CCM

Modulation Technique	Switching frequency (f_{sw})	Worst case scenario
Pulse width modulation	$f_{sw} = f_{ext}$	Insensitive to system and operating conditions
Constant on-time modulation	$f_{sw} = \frac{1}{T_{on}} \times \left(\frac{V_o}{V_{in}} \right)$	Highest switching frequency at lowest input voltage
Constant off-time modulation	$f_{sw} = \frac{1}{T_{off}} \times \left(1 - \frac{V_o}{V_{in}} \right)$	Highest switching frequency at highest input voltage

Constant on-time low duty



Switching frequency, that is a one of the major drawback in variable frequency control. So, in the fixed frequency it is constant, but in constant on time you see it will be maximum highest switching frequency will happen at the lowest input voltage, ok. That means, for low duty ratio operation, the switching frequency will be low, so you need to adjust ok. But for high and low input voltage condition or the high duty ratio the switching frequency can be very high and then we have to regulate.

For constant off time, if you see the expression, the highest switching frequency happens at highest input voltage. That is why we have to carefully select the off time. And that is why the constant on time is a very preferred choice for you know for low duty ratio operation. So, for low duty ratio operation for VRM because your on time itself is very small, you can just fix it.

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Worst Case Inductor Current Ripple – Boost Converter in CCM

Modulation Technique	Current Ripple (ΔI_L)	Worst case scenario
Pulse width modulation	$\frac{T}{L} \times \left[\frac{V_{in}}{V_o} (V_o - V_{in}) \right]$	Input voltage equals to half of the output voltage
Constant on-time modulation	$\frac{T_{on}}{L} \times V_{in}$	Highest input voltage
Constant off-time modulation	$\frac{T_{off}}{L} \times (V_o - V_{in})$	Lowest input voltage



So, worst case ripple for boost converter pulse width modulation, the current ripple of the boost converter is this. And this is worst when the input voltage is equal to the half of the output voltage; that means, at 0.5 duty ratio. For constant on time modulation, the ripple current is maximum, when the input voltage is highest ok.

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Switching Frequency – Boost Converter in CCM

Modulation Technique	Switching frequency (f_{sw})	Worst case scenario
Pulse width modulation	$f_{sw} = f_{ext}$	Insensitive to system and operating conditions
Constant on-time modulation	$f_{sw} = \frac{1}{T_{on}} \times \left(1 - \frac{V_{in}}{V_o} \right)$	Highest switching frequency at lowest input voltage
Constant off-time modulation	$f_{sw} = \frac{1}{T_{off}} \times \frac{V_{in}}{V_o}$	Highest switching frequency at highest input voltage -

Handwritten notes:
 Monoshot timer in digital domain
 All digital PLL
 TDC
 D high
 D low



So, switching frequency for the boost converter also can be discussed. Under fixed frequency control, this is constant. Under constant on time control, the switching frequency is maximum

when the input voltage is lowest and under constant off time control, it is maximum at the highest input voltage. That means, this is when the D is high and when the D is low, ok.

So, then these are the challenges in variable frequency and we need to regulate the switching frequency by means of. So, it is now possible because if you implement this monoshot timer, if you implement the monoshot timer in digital domain in digital domain then you can use an all digital PLL ok.

Because in the monoshot timer, if it is in digital like a counter, then we can use a time to digital converter. That means, just a counter, we can count what is the time period and that can be fed back to a frequency loop by comparing the reference frequency. And then we can adjust slowly adjust either on time or off time, depending upon which regulator you are using. Then you can nearly regulate the switching frequency.

And in this case, you need a slower switching frequency loop frequency regulation loop so that it will not interfere. And in digital implementation these are very easy and that is why the constant on off time control is coming with a digitally assisted. Even though they are implementing it analog in majority of the commercial product, but for the timer adaptation we can use at digital technique. And then you can solve this variable frequency option, but you can get the benefit of the best of their feature.

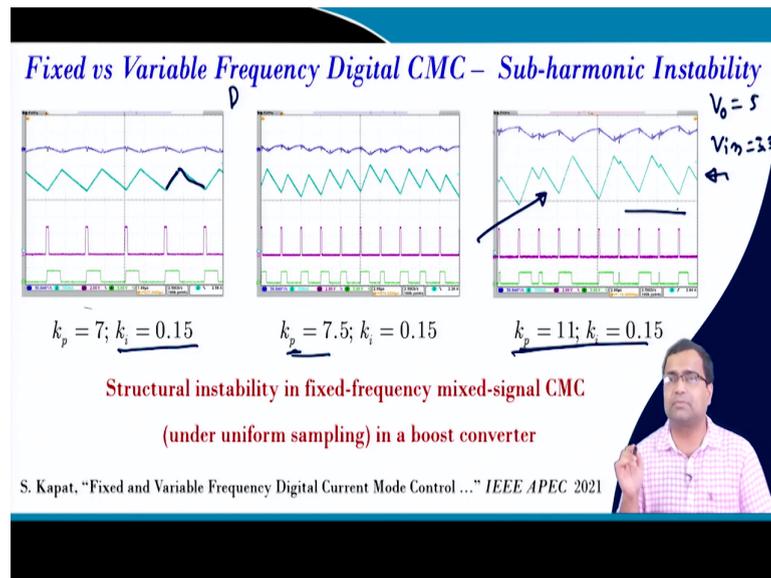
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Fixed vs Variable Frequency Control – Comparison

- Switching frequency →
- Ripple parameters →
- Transient performance →
- Stability status →

S. Kapat, "

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So, variable frequency we have discussed you know fixed frequency and variable frequency control. The one more thing I want to emphasize in fixed frequency verses variable frequency comparison switching frequency, ripple parameter, transient performance and stability status. These are the very important factor. And we have discussed in variable frequency this is one of the drawback, but you can of course, adapt it. The ripple parameters, if we can adapt the switching frequency it will be the same as your fixed frequency.

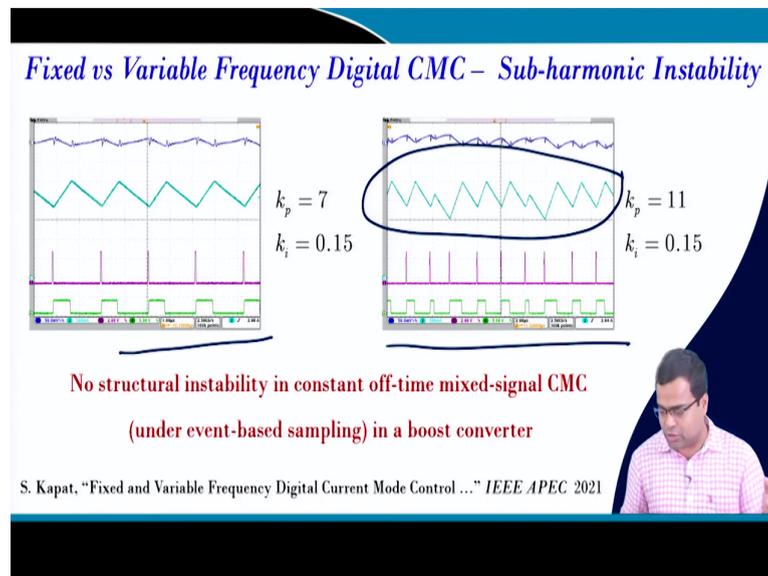
The transient performance can be improved that you have also checked and the stability of the current loop is also checked. But more interestingly in the recent you know in APEC 2021 we have shown that even if you take a peak current mode control in digitally controlled converters. If you simply increase the gain and here the duty ratio is much lower, here your duty ratio is you know here output voltage is 5 volt and input is 3.3 volt.

So, it is much lower duty ratio 0.5 less than and you can see from this on time and off time right. And if we increase, it is a PI controller if we increase slowly the integral gain or proportional gain and this is a discrete time integral gain. Then the current loop will become stable even for low duty ratio operation.

And if you further increase this kind of stability can happen where the duty ratio can saturate. And this is called structurally unstable behaviour, which can lead to a severe problem in terms of ripple parameter, RMS current and it can violate the ripple constant. Because this

will lead to a large voltage ripple also and that is not acceptable and this will increase the conduction loss significantly.

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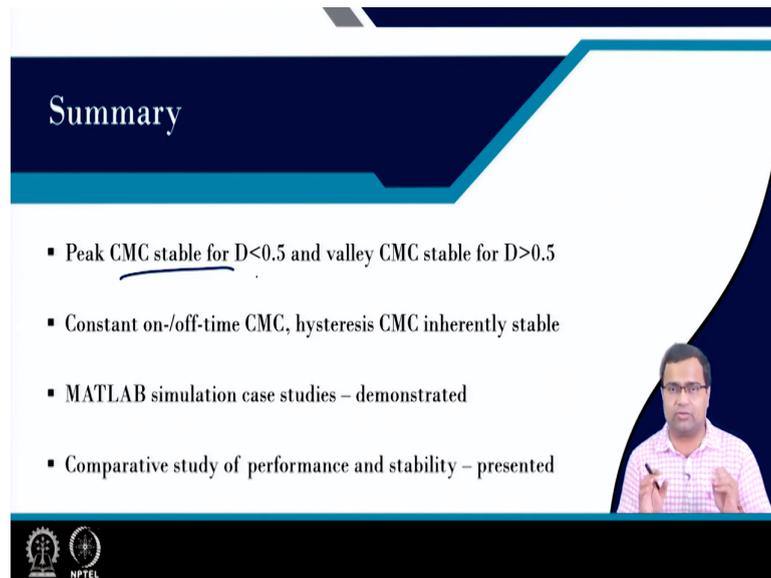


But if you go for variable frequency digital control; that means, if you in variable frequency there is no structural stability, even for the same earlier case if we go to higher proportional gain for the same PI controller under constant off time control which is analogous to peak current mode control there is no structural instability.

So; that means, even though there can be a stability problem in the closed loop because of the sampling delay, but this stability is much within the limit and it is not going to significantly increase the ripple current. So; that means, at the end I will say when you go to digital and if you are talking about fixed frequency and variable frequency, current mode control the variable frequency by inherently provide the inner loop stability there is no problem.

Under close loop also even for low duty ratio, it offers better, superior structural stability and of course, it will offer superior performance. Because if you increase the gain, then you can speed up the response; whereas, in the previous case you cannot because of the stability problem. So, you can actually increase the performance using this variable frequency control.

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Summary

- Peak CMC stable for $D < 0.5$ and valley CMC stable for $D > 0.5$
- Constant on-/off-time CMC, hysteresis CMC inherently stable
- MATLAB simulation case studies – demonstrated
- Comparative study of performance and stability – presented

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So, with this I summarize the peak current mode control stability. We saw it require less than 0.5. In fact, it can be even less. For valley current mode, it should be greater than 0.5 for constant on off time control. In hysteresis control, there is no problem with the current loop.

We have shown some MATLAB case study demonstrated to show their effectiveness. And we have also shown some comparative case study those are also presented in order to show different type of current mode control. So, with this I want to finish it here.

Thank you very much.