

Signal Processing for mmWave Communication for 5G and Beyond
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Module - 01
Wireless Channel - A ray tracing model – Part - I
Lecture - 06
General Channel Model

Welcome to Signal Processing course for millimeter Wave and Communication for 5g and Beyond. So, this is modules 1's, Channel Model Part 1 and this is the 6th lecture. We will still continue with the General Channel Model part.

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Today, we will be mostly covering the analog view of the channel and subsequently, we will make inroad to the digital view of the channel because this is where our main interest would be digital part ok.

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The slide contains the following content:

- Equation: $h(\tau) = \sum_{i=1}^N \alpha_i \delta(\tau - \tau_i)$
- Equation: $H(f) = \sum \alpha_i e^{-j2\pi f \tau_i}$
- Equation: $s(f) = \text{Re} \{ s_b(f) e^{j2\pi f t} \}$
- Diagram: A block diagram showing a signal $y(t)$ entering a block labeled 'RF' (Radio Frequency) and 'Analog'. The output is $y_b(t)$. There are also labels 'I' and 'Q' with arrows pointing to the right.
- Diagram: A time-domain plot of $h(\tau)$ showing three impulses at τ_1 , τ_2 , and τ_3 .

So, let us go back to our earlier discussion where we had single channel case, single antenna case ok. Now, if you have single antenna and your view, you are standing at this position ok so, which means you are in RF domain Radio Frequency domain. You have not done demodulation, it is modulation and demodulation, you have just before that stage so, at the RF. So, this is your views in RF view.

So, what we have discussed? We have seen that the channel in tau domain it will be summation of alpha i into Dirac function tau minus tau i if tau i is the individual delay of each and every reflected paths and or the scatterer paths so, that is how the view would be right.

Now, what was the time domain diagram? This was something like that so, that mean at tau 1 there is a path, tau 2 there is a path, tau 3 there is a path and I am plotting the h tau here is what we have seen. Subsequently, what was the frequency? This would be just the Fourier

transformation, it will be summation of $\alpha e^{j 2 \pi f t}$ to the power minus $j 2 \pi f t$ I so, this is how the frequency part would be there. So, anyway so, this is what we have discussed it.

Now, today we will go one step forward. What does it mean? It means that now we are making a demodulation here ok. So, when I say demodulation, there will be two components because there is a I and Q so, this is followed by low pass filter, I am not putting the low pass filter I am just saying that after the demodulation, I will get the in phase and Quadra phase of the same signal. So, this is your analog view. So, this I can say this is my analog view. So, from here, I can say this is my analog, this is my analog view ok.

Now, today we will see what is the channel model when I am standing at the analog. So, that mean now, I am standing here at that point. Now, how would the channel vary? So, it is as if like it is a analog transmission to analog reception in between RF I do not care and so, what will I see the channel as ok?

So, last time we have also shown that if you have any signal; if you have any signals baseband equivalent, baseband is nothing but this analog equivalent basically. So, if you have a signal called $s_b(t)$; $s_b(t)$ because this is what you would transmit from the analog point of view and that is the complex analog signal, and we call it a baseband signal right.

Now, the relationship between the base band and its subsequent RF signal is the following $e^{j 2 \pi f c t}$ this is the relationship right. So, this is your baseband part, this is your RF part.

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$$h(\tau) = \sum_{i=1}^N \alpha_i f(\tau - \tau_i)$$

$$H(f) = \sum \alpha_i e^{-j2\pi f \tau_i}$$

$$s(f) = \text{Re} \left\{ s_b(f) e^{j2\pi f c t} \right\}$$

Now, what happens? I extend the same thing for my received signal. So, what I received as an RF? $y(t)$ can I also write like that? It will also have some baseband equivalent because the $y(t)$ is the RF, this is what it is. So, what was our $y(t)$? $y(t)$ was somewhere here. So, the baseband part will be somewhere here in the analog domain. So, what is the relationship between these two? The relationship between these two would be more of a this one, $y_b(t)$ into $e^{j2\pi f c t}$. So, this is your analog baseband ok. So, I am standing here so, that mean I am seeing $y_b(t)$, I am not seeing $y(t)$ now ok.

At the same time, we also know that your $s(t)$ has a relationship like this ok. So, that is how the relationship goes on here ok. Now, let us see what exactly happens when I convert the whole thing to the $s(t)$ domain. So, now, in a sense, we have also known just from our earlier discussion that this is nothing but $s(t - \tau_i)$, this we have seen it right because if I

transmit $s(t)$, it will be delayed and multiplied by the individual paths gain factor. So, this was in the RF domain, we have seen it.

Now, just replace the $s(t)$ with respect to that this equation. So, this is $s(t - \tau_i)$ you replace it here in this equation. So, let us say call it equation 1 ok replace the value of $s(t)$ inside the 1 so, what you will get? You will get summation of i is equal to 1 to N α_i now here, $s(t - \tau_i)$ I know, what is $s(t - \tau_i)$?

What it would be? It will be the real part that is just replacing so, here replacing everywhere wherever there is a t replace it by you know replace it by $t - \tau_i$. So, it will be this part $t - \tau_i$ ok e to the power $j 2 \pi f c$, I am replacing t by $t - \tau_i$ right so, this would be $t - \tau_i$ simple math ok clear.

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$$\begin{aligned}
 &= \operatorname{Re} \left\{ \sum \alpha_i s_b(t - \tau_i) e^{-j2\pi f_c \tau_i} \times e^{j2\pi f_c t} \right\} \\
 y(t) &= \operatorname{Re} \left\{ \sum \alpha_i s_b(t - \tau_i) e^{j2\pi f_c t} \right\} \\
 \checkmark \quad \underline{y_b(t)} &= \sum \alpha_i s_b(t - \tau_i) \quad \checkmark \quad \left| \alpha_i e^{-j2\pi f_c \tau_i} \right. \\
 \underline{h(\tau)} &= \sum \alpha_i \delta(\tau - \tau_i)
 \end{aligned}$$

So, now, I do some mathematical manipulation, let us see what happens ok. Just I take this. So, there is a real part here and there are certain component in that other part as well ok. So, this part is there. Now, I just take this part out this $2\pi f c$ part out. Now, you can see alpha is also a real part right, alpha is a real quantity so, I can put the alpha inside so, what will happen?

So, summation of so, it is as if like the whole real part I can think of it like that because everything is real alpha i I can just take it inside so, this is $\alpha i s b t$ minus τi I break that other part so, e to the power minus $j 2\pi f c \tau i$ multiplied by e to the power $j 2\pi f c t$, just plain I have broken it, this part I broke it ok.

So, what will happen? This whole thing comes into picture and do some little more mathematics here ok, I group these two, I call it $\alpha i b$, I call it that group it together so, what does it mean? It means I will just let me define it. So, let me group this, we have already shown how to group it.

So, let me define this point $\alpha i b$ basically, it is an alpha and this part together ok, then this is $s b t$ minus τi e to the power $j 2\pi f c t$ look at that where I am saying that $\alpha i b$ is equal to I am defining it $\alpha i e$ to the power minus $j 2\pi f c \tau i$, just a definition; just a definition.

What can you infer from here? This is your $y t$, is it $y t$? What was the signal structure that I have is $y t$ is real part of $y b t$ to the power so, $y b t$ is some base band signal or base band equivalent or analog equivalent of the RF signal $y t$ that is what has happened.

Now, look at this equation and look at this equation, just compare both the thing ok. So, here also there is a real, here also there is a real e to the power $j 2$ so, that means, what is this $y b t$ rest of the thing is $y b t$. So, I can say my $y b t$ is nothing but this component ok. So, this will be there is a summation here of course, $\alpha i b s b t$ minus τ ok.

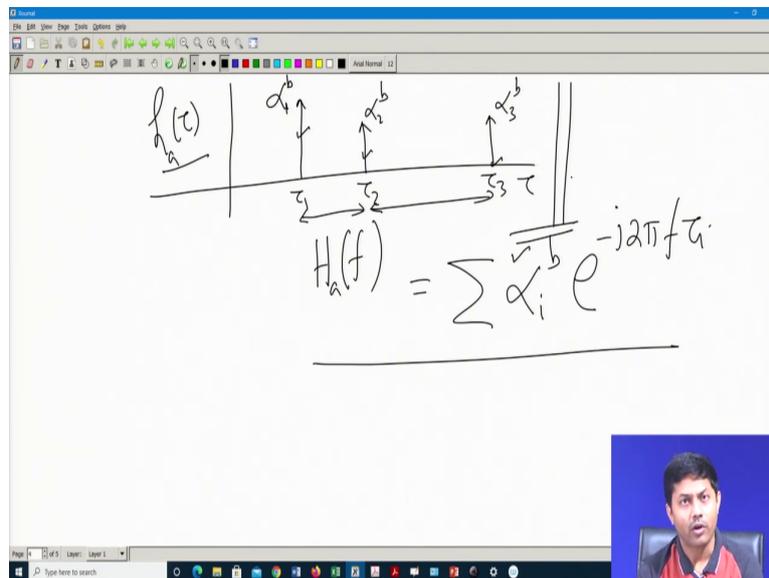
So, what has happened? Look at the RF modeling, RF modeling was what? RF was this model ok and when I take the RF out, the analog baseband comes like that similar, very similar, but only change is that instead of s , now it is s_b , instead of α_i , now it is $\alpha_i b$, $\alpha_i b$ is this one whatever has been written here that is the only difference, but everything structure remains the same ok.

So, what can I conclude? Conclusion is that my channel if I say what is my channel $h(\tau)$ what should be my channel then? If this is the signal model whatever is shown here, channel model will be the same as what I have done in the RF if this is the input-output relationship, what will be the channel?

It will be again the summation of $\alpha_i b$ into Dirac function $\tau - \tau_i$ that is the only difference same thing there is no difference, but there is a catch here, what is the catch? Look at the coefficient now. What was the earlier case, what was the coefficient?

In the earlier case, the coefficients were all real, but the same structure, but now, moment I discard the RF part, my channel structure remains the same, but the coefficient remain complex, coefficient suddenly becomes complex, and they are related by this equation that is the only difference ok. So, that is the view of my analog ok.

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So, which means that if I summarize this analog part, what will I get? I will exactly get wherever my tabs were there, wherever my reflectors were there, but now instead of alpha i or alpha 1, I should say alpha 1 b maybe it is a complex number now this will be alpha 2 b, it is complex number, this will be alpha 3 b complex number, but this tau 1 remains the same that is the only difference got it? So, it just the channel becomes complex, but still in the analog domain that is the only part of the story here ok.

So, this is the only difference when you have an analog, what will be the spectrum then? As it is so, if I have H f which is the spectrum of your analog channel, analog view of the channel, it will be the same alpha i b e to the power minus j 2 pi f theta y, these are the only view that will come into picture ok. So, this is the analog view.

Now, let us get into the digital part now because we have now covered the analog very well, there is no difference between analog view and the RF view except the fact that the coefficient of the summation becomes complex number so, it all depends on my delay so, that is my analog view of the channel so, this is my analog view. So, I will say h tau some analog view so, probably I will put this notation.

Now, once I do that, now what is my next job? My ultimate goal is not that, ultimate goal is to get the digital domain ok because we have to do digitization of our sequences. So, let us now understand the same channel where we have got the RF view, we have got the analog view, what happens when I digitize it? Let us understand that ok, let us understand the digital part.

So, now one part, the position of the you know the reflected paths they do not change between analog view and RF view obviously, right because wherever the reflector path was showing as a delay in the analog, they show exactly at the same point right the τ_1 , τ_2 , τ_3 and so on exactly at the same point, the RF was also there here you notice, the same point wherever it is RF also was the same point.

Now, we will see what happens when I digitize it? Will I get like that or it is totally different ok? One more conclusion, are they uniform I mean is there any relationship between them? No, absolutely no, they are all random numbers so that means, this gap between τ_2 minus τ_1 or τ_3 minus τ_2 .

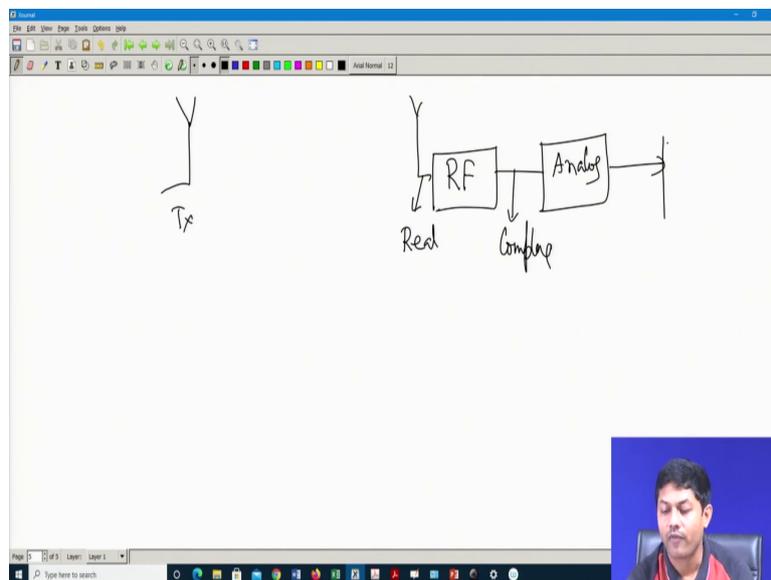
It may take some distribution let me not get into that distribution whether it is a Poisson process or something that you can assume, but what I am trying to say is that that is the random number and that is not equal that means, τ_2 minus τ_1 you cannot assume equal, τ_3 minus τ_2 it is also not an equal.

So, you can say this h a tau it is as if like it is a set given to you, set of data given to you sampled at non-uniform times can I say like that? Suppose this is the tau domain which is the

time domain as if lying I am viewing at tau 1, tau 2, tau 3 which are non-uniformly sampled so, I can think of it that way ok.

So, what does it relate to? So, does it mean that I can think of it some sort of a time series? Probably yes, we will explain that subsequently that whole thing can be modeled as some sort of a time series and this is very important when you go for you know channel model from general channel model to the millimeter wave channel model, we will talk about that later, but now let us get into the digital domain view ok.

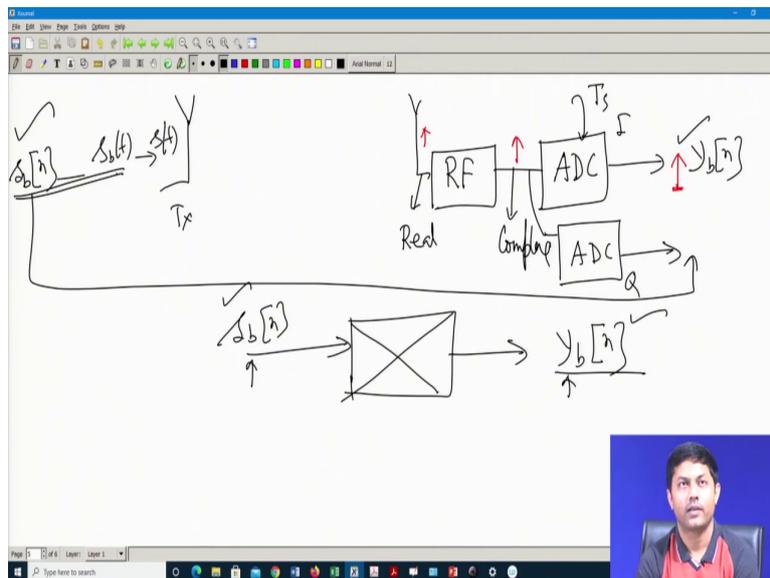
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So, now again I take my same diagram transmitter diagram, this is my receiver diagram ok, this is my RF blocks are there, these are all my analog blocks are here, analog blocks are there and then, what I get is the digital data. Now, obviously, this was real data that means, the I receive a real data.

After the RF cancellation, my data is a continuous time domain, but it is a complex right because the coefficients become $\alpha + j\beta$ that is a complex number so, that is complex analog continuous signal ok and it will do some analog processing like some filter may be there, you know some more block would be there, but at the end of the analog block, what is the last block of analog? So, last block of analog would be the ADC.

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So, if I put the last block of analog, it will be the ADC ok and it gives the data at some sampling rate let us say T_s is my sampling time by which I am sampling the data analog signal, but it is a complex analog signal. So, what does it mean? One ADC will not work you require two ADC ok that is part of the structure. So, you have a two ADC because it is a complex analog signal physically, it will be two separate value will be given so, this is my I, this is my Q, but there are discrete values.

So, now, I am standing here; I am standing here, so far, I was standing here, then I stand here, now I am standing here where I am in the end of digital signal. Now, let us see so, it as if like in the transmitter side also, as if like I have transmitted $s_b n$ that is the discrete view of my baseband signal in between it becomes $s_b t$ after dac in between it becomes s_t after the modulation.

So, now, it is as if like I am viewing the signal from here till here, this is as if like I am transmitting $s_b n$, I am receiving something here so, let us call it $y_b n$, digital view of the baseband. So, it is as if like I am giving $s_b n$ giving to some system that is what our whole channel part and I am receiving $y_b n$; $y_b n$. So, my job is to model this whole black box that is my job ok.

So, now, it is I am sending a digital baseband, I am receiving a digital baseband. So, what is the that in between black box, what is the view of my black box? So, I forget completely about RF, I forget complete about analog, I do not; I do not I mean I do not remember anything in between. So, to me it is a digital data is sent to me it is a digital data I am receiving so, now, what is the in between whole thing ok? Let us get into that.

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$$s_b(t) = \sum_n s_b[n] \text{sinc}(Wt - n)$$

$$y_b(t) = \sum_n y_b[n] \text{sinc}(Wt - n)$$

$$s_b(t) = \sum_i \alpha_i^b s_b(t - \tau_i)$$

Before I get into that, there is one quick comment. Suppose I am given any signal just any signal say $s_b[n]$ discrete values of a signal ok. If I would have sampled it at higher than Nyquist rate or at least at the Nyquist rate, I will get the equivalent time domain signal. So, that means if somebody gives me the discrete value of $s_b[n]$, I can reconstruct $s_b(t)$. If somebody gives me $s_b[n]$, I can reconstruct $s_b(t)$ I mean that sampling theorem assures mass provided I sample at Nyquist rate.

So, what is the relationship between the two if I can reconstruct? What is the relationship between $s_b(t)$ and $s_b[n]$? The relationship is that this will be sinc pulse of $Wt - n$ so, that is the value so, n equal to n you vary it. If you are given that as many as samples you are given, n equal to minus infinite to plus infinite $s_b(t)$.

So, which means that if somebody gives us the discrete values, discrete data $s_b[n]$ which we assume that I would have got it by ah, but I would have got it by Nyquist rate sampling, then I can reconstruct $s_b(t)$ using this formula, this is the standard you know standard reconstruction formulas which is coming from the Nyquist theorem. Now, I will utilize that for my digital view of my channel ok.

So, now, for us, what is the signal, what is the data that I have received? This is $y_b[n]$, this is what I have received. So, what is the relationship between $y_b[n]$ and $y_b(t)$? Similarly, I can draw, $y_b(t)$ I can say what I can say? This is $y_b[n]$ into similar I can do that, I can do that correct.

Now, what is $y_b(t)$? This I can do it. $Y_b(t)$ I can also write it in other way this is one part of the story, this I know, this can be that mean if somebody gives me $y_b[n]$, I can reconstruct like it ok. I am trying to model the digital part so, I am just moving slowly in this particular case so, you have an $y_b(t)$.

So, what is the relationship between $y_b(t)$ and the $s_b(t)$ that is the input signal, what is the relationship between that? So, if you look at this is this was $\alpha_i b_s b_t$ minus τ_i that we have seen it, this was the baseband equivalent of that $s_b(t)$ correct, I can write it.

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$$s_b(t) = \sum_n s_b[n] \text{sinc}(Wt - n) \quad \text{--- (1) ✓}$$

$$y_b(t) = \sum_n y_b[n] \text{sinc}(Wt - n)$$

$$= \sum_i \alpha_i^b s_b(t - \tau_i) \quad \text{--- (2)}$$

Now, $s_b(t)$ continuous signal, what is the relationship between $s_b(t)$ and this one it is discrete version discrete versus its continuous function? This was the relationship so; I should not put anything here so, let us call it equation number 1, let us call it equation number 2 here in this particular context right. So, this is $s_b(t - \tau_i)$ similarly, $s_b(t)$ I got it from the equation number 1 ok.

So, what is my step here? Step is that I got $y_b(t)$ which is equal to α_i , this 1 and then, I suddenly saw hey, $s_b(t)$ has a relationship with $s_b[n]$ because this is where my interest is, I want to; I want to find the relationship between $s_b[n]$ and $y_b[n]$ that is my ultimate goal because this is my data model [FL] data model, this is my data model is $s_b[n]$ is input, $y_b[n]$ is output.

So, I am processing so, that mean I am trying to create some sort of you know equations where only $s_b(n)$ exist and $y_b(n)$ exists nothing else in between right. So, then I can model the channel easily so, that is my goal. So, I have got $y_b(t)$ which is equal to this formula. So, I got it from previous one.

Then, suddenly I saw equation 1 also tells me the relationship between the discrete version of $s_b(n)$ and its $s_b(t)$ which is like this one this relationship this is what I got the relationship right.

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The whiteboard content includes the following elements:

- Equation 1: $y_b(t) = \sum \alpha_i s_b(t - \tau_i)$
- Equation 2: $= \sum \alpha_i \sum s_b(n) \text{sinc}(\omega t - \omega \tau_i - n)$
- Block diagram: A signal $y_b(t)$ enters an ADC block, which outputs $y_b[n]$. The sampling time is indicated as $t = \frac{m}{T_s}$ with a circled T_s .
- Graph: A sine wave is drawn with markers at $-W/2$ and $W/2$.

So, now, let us utilize equation 1 into 2 standard part ok. So, due to space crunch, probably I will go to the next one next file here. So, you have $y_b(t)$ here which is equal to summation of this 1 equation 2 I write it let me write it one more time $s_b(t - \tau_i)$ so, this will be $s_b(t - \tau_i)$

minus tau i ok I put that value. So, this will be alpha i b put that value, put whatever is written here ok, put whatever is written here.

So, what I said? So, wherever I have a t, we just replace t by t, t minus tau i ok. So, this is the equation I got it, replace it. What will I get? I get one more summation here ok, then there is an s b n and sinc right. So, there is s b n and this is sinc, sinc function. Now, instead of W, I have to write W t minus W tau i minus n, this is what I have got it, I replaced it simple replacement, this is what I have got it ok, this is what I have got it ok fine.

Now, so far this is the continuous data. Now, what I do? I do a sampling because that is the ADC does. So, this y b t is the input of what? It is the input to my ADC. So, if I have an ADC here, two ADC will be there because I and Q so, I am not drawing two so, in a pictorially, I am just writing one of the ADC. So, it is as if like I am giving y b t, I want to get back my y b n ok. So, what does it mean? I am sampling it right.

So, let us say I put sample t is equal to m by T s so, that is my sampling rate. So, basically, my sampling rate is T s. What is this T s? It should have some relationship with W as you know. This W is the total band of occupancy that means, the signal is somewhere like this so, this is W by 2, this is minus W by 2, this is my spectrum so, what is my Nyquist state? It will be W.

Now, there is no; there is no need that you have to take only Nyquist state, you can take larger value also so, it is up to you, but let us take a Nyquist, it makes no difference to us.

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$$y_b(t) = \sum \alpha_i^b s_b(t - \tau_i)$$

$$= \sum \alpha_i^b \sum s_b(n) \text{sinc}(Wt - W\tau_i - n)$$

$$y_b(t) \xrightarrow{\text{ADC}} y_b[n] \quad \left[\begin{array}{l} T_s = \frac{m}{T_s} \\ T_s = \frac{1}{W} \end{array} \right]$$

$$y_b(t) \text{ sampled at } T_s, 2T_s, \dots, mT_s$$

Let us say I am sampling at my Nyquist state. So, what does it mean? My T_s I am putting 1 by W_s , that is my sampling rate. At that sampling, this is $y_b(t)$, this will have a continuous wave forms, I am sampling at T_s . So, this is T_s , this is $2T_s$ and so, on and so forth.

So, what is this T_s ? This T_s is this 1 by W , that is the Nyquist state standard. So, that mean each and every small t will be replaced by integer value multiplied by the W right that standard so, m by T_s I replace it so, what does it mean? So, I replace it here.

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$$y_b\left(\frac{m}{T_s}\right) = \sum_i \alpha_i^b \sum_n s_b(n) \text{sinc}(m-n-\tau_i w)$$

$$= \sum_n s_b[m-l] \sum_{i=1}^N \alpha_i^b \text{sinc}(l-\tau_i w)$$

$m-n=l$

So, now, what will I get? I digitize it. So, that mean y_b , that t I replace it by T by; m by $T s$ because I sampled it right, I sampled like this, I sampled it, this is $m T s$ sampling, I am doing it. So, this is my $y_b t$, now these are my $y_b n$ samples and uniformly samples ok.

So, what will happen y_b and t s? Just see wherever you have a t , you replace it so, you just replace it summation of α_i^b similarly, here $s_b n$ so, it was varying over n and then, you have sinc because it was $w t$ was there so, now, this become m minus n minus τ_i this is what is going to happen, this is what plane going to happen ok.

So, this is over i , this is over s , this is over n , this is over i and do some basic mathematical manipulation here so, I just take this fellow off and I put m minus n is equal to l , I just make some mathematical manipulation. And we will see that this will be $s_b m$ minus l because I

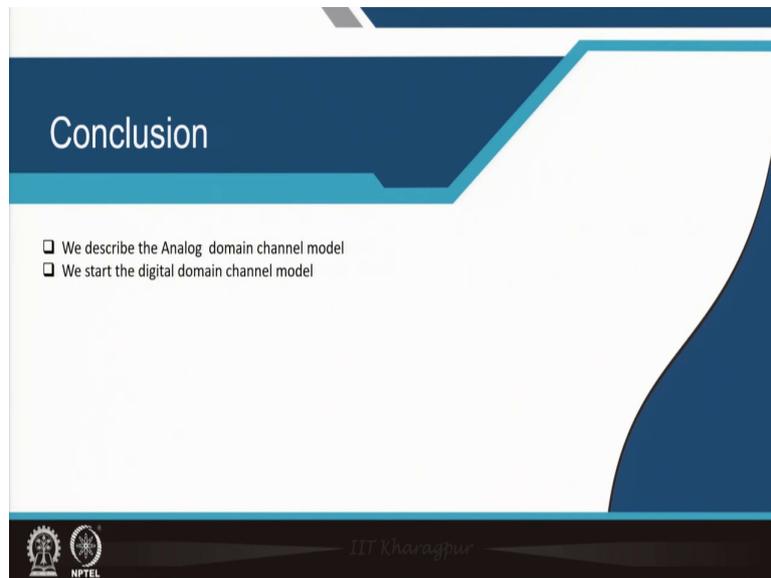
am replacing n by $m - 1$ take this other summation inside i is equal to 1 to N , this will be over you know m so, it will be over 1 .

So, now, this will be $\alpha_i b_{i-1} - \tau_i$, there is a W here, I was missing that W , this will be the case, the standard case what I will be happening ok, this is what I am getting. So, now, what is the complete thing? Now, I separate this fellow with a different color, then we will see what happens when I separate that part it is a summation so, it basically a double summation ok.

Now, let us see what happens when I separate that? I am just whole thing I am assuming some you know one component. Let us understand in the next class, what exactly that part and that defines my channel ok. So, in this class, we stop it, we just introduce the digital part and in the next subsequent class, we will now start modeling this red part because I am now getting a relationship right, this is what your $y_b[n]$, this is what your $s_b[n]$ and this is exactly something we will see that what exactly this part is.

As if like I am getting a relationship between this one $y_b[n]$, I am getting some sort of a relationship and this black box has something to do with this one, we will define that in the next class what exactly that component is that will be the channel as simple as that, we will see that ok.

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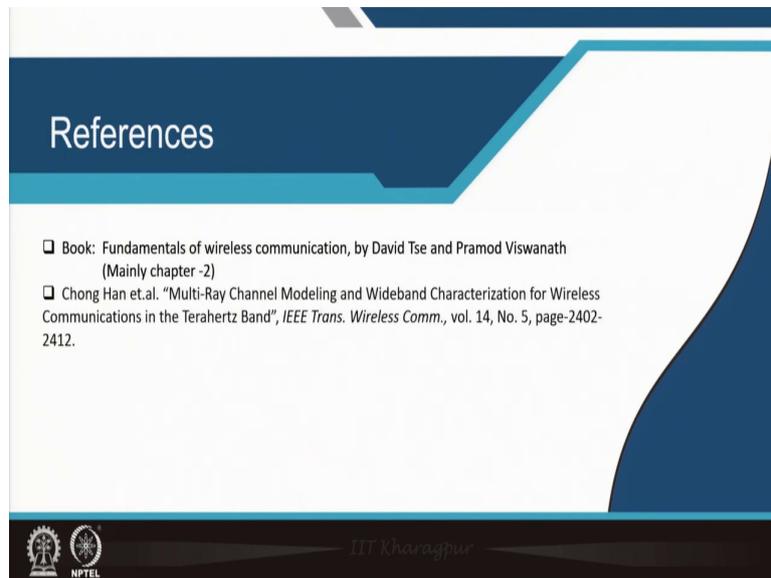
Conclusion

- We describe the Analog domain channel model
- We start the digital domain channel model

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So, now, we will stop it for this class today and we will try to see what we have covered. So, we have described the analog domain channel model and then, we start the digital domain channel model ok. In this next class, we will now get into the details of the digital domain of the channel model.

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References

- ❑ Book: Fundamentals of wireless communication, by David Tse and Pramod Viswanath (Mainly chapter -2)
- ❑ Chong Han et.al. "Multi-Ray Channel Modeling and Wideband Characterization for Wireless Communications in the Terahertz Band", *IEEE Trans. Wireless Comm.*, vol. 14, No. 5, page-2402-2412.

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And the references I have not changed the references so far see the same book and the same paper which I have earlier also said, and it is mainly the chapter 2 of that book.

Thank you.