

**Signal Processing for mmWave Communication for 5G and Beyond**  
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**Module - 09**  
**Hybrid beamforming concept and Beamforming in MIMO**  
**Lecture - 47**  
**Equalizer Based Detector**

Welcome back welcome to the 5G Communication with millimetre wave and Beamforming. So, today we will be covering the so we have so far discussed the concept of various equalizer parts.

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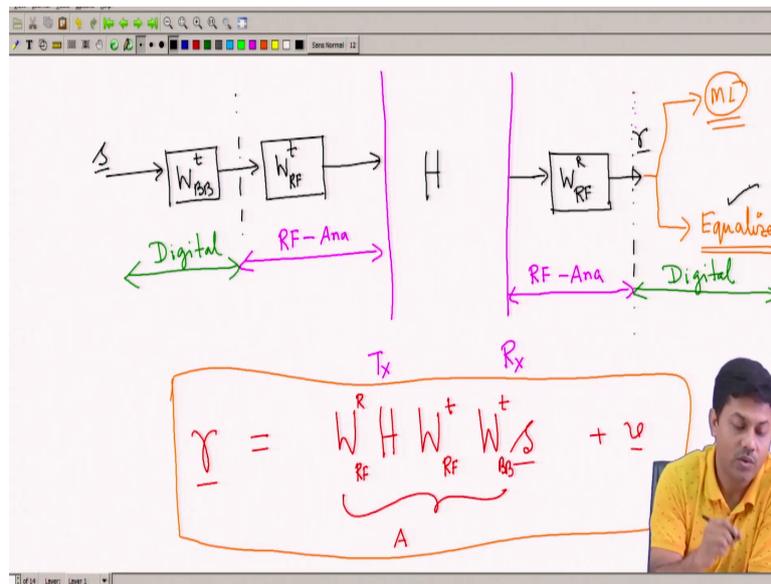
**Concepts Covered**

- Equalizer on low cost.
- Parameter to be configured.
- Data detection .
- Mathematical description of MIMO beamforming.

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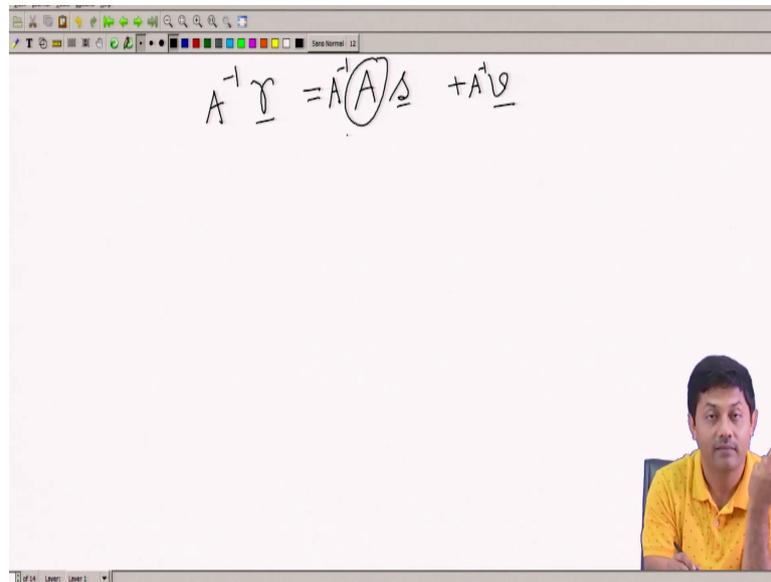
Things that we will be covering are the following concept of received data at the MIMO beamforming part, now we are getting into the detection part.

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So, in the last class if I look at the diagrams if you look at the diagram, so what is there? So, we have just finished the ML part here ok. Now, we will be going to the equalizer part. So, this is my data model which I call it A. So, now it will be very straight forward.

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The image shows a video lecture interface. At the top, there is a toolbar with various icons. Below the toolbar, a whiteboard contains the handwritten equation  $A^{-1} \underline{y} = A^{-1} \underline{A} \underline{x} + A^{-1} \underline{v}$ . The matrix  $A$  in the second term is circled. In the bottom right corner, a man in a yellow shirt is visible, gesturing with his hand.

So, my data model is now r ok we have discussed the ML part. Now let us go to the equalizer based, equalizer meaning what? Some linear system some linear equalizer I am talking. So, that mean there will be one more matrix by which I can nullify the effect of A ok. So, what is the best way to nullify the effect of A? You can just invert A multiply with A inverse provided many things are there just do not come into the conclusion I multiply this A inverse here also A inverse and I get back it, but again A may not be invertible first of all.

So, this is not a very elegant way of doing it so I remove that. So, this is an equalizer I am just showing you just to give you feel of what A an equalizer. So, I can create a generalized equalizer structure.

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$$\underline{y} = \underline{A}\underline{s} + \underline{v}$$

$$\underline{\hat{s}} = \underline{W}_{BB}^R \underline{y}$$

1. LS  
 Pseudo-inverse of  $A \rightarrow A^\dagger$   
 $\underline{W}_{BB}^R \rightarrow A^\dagger \rightarrow A^* [AA^*]^{-1}$

2. LMMSE  

$$\underline{W}_{BB}^R \Rightarrow \begin{matrix} R_{sr}^{-1} \\ R_{rr} \end{matrix}$$

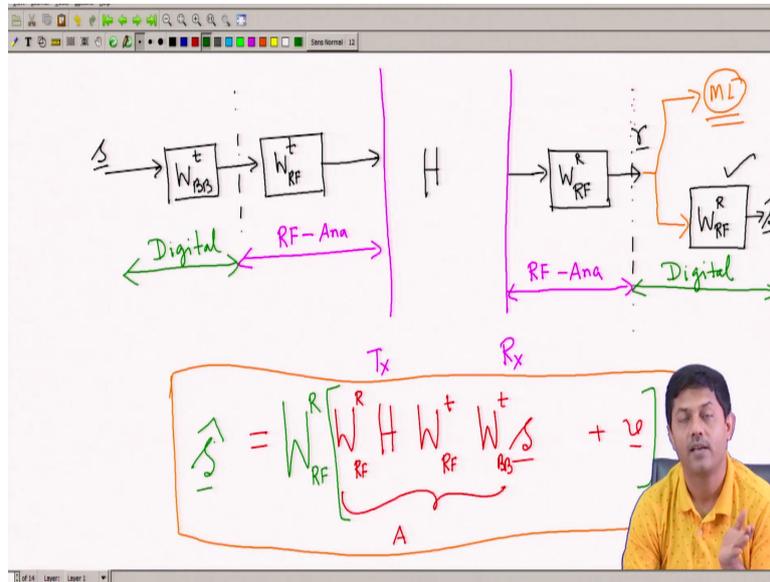
$$\left\{ \begin{array}{l} R_{sr} = E[\underline{s}\underline{r}^*] \\ R_{rr} = E[\underline{r}\underline{r}^*] \end{array} \right.$$

Let us call that equalizer  $W$  baseband, but at the receiver side ok, this is some sort of a matrix. So, what is the purpose of this matrix this matrix  $I$  multiplied by a vector  $r$  then this gentleman will be created. So, this is where the closer estimation of  $s$  will come into picture. So, this wont directly give you  $s$ , but at least the closer values of  $s$  then you have to you know you have to just map it to a particular or closer equalizer or closer constellation point that is a separate story.

But we would like to now create a equalizer 1 ok, just like your MIMO system  $y$  equal to  $h x$  plus  $v$ . So, how did you create an equalizer you create just sort of a some create a  $W$  based on some cost function say least square or LMMSE or some other cost function and create a matrix out of it ok. So, which means my which means here when I go back to my data model

here, when it is an equalizer based one more component will be appearing ok. So, what is that component?

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So, it is equalizer that extra component is now I multiplied this one this is what is coming if it is the equalizer that is the only change compared to the earlier diagram ok. So, this is what the extra parts that will be coming. So, what is the change in the data model? Obviously, you will have let me put a different color, so that it will be differentiable that is the only change. Now, will this be r then?

No, it will not be r this should be multiplied by the whole thing and then this should be this will be s cap that is the only model that will be happening. I just change my earlier model just to show you what exactly my. Now, I put it back the figure because I may be referring this figure left and right ok, this is what is going to happen and this has been reflected here this is

the part that extra part that is popping up here ok. So,  $W_{BBR}$  is cap so that is the point which is popping up here ok.

So, now the question would be how do I estimate or how do I create this  $W_{BBR}$ . Now if I know all the four creating  $W_{BBR}$  is a cakewalk, why because I can create different cost function ok. So, if  $A$  is the suppose this model is like this, so how do I get  $W_{BBR}$  if it is just a simple least square, then what will be the case it is nothing but some pseudo inverse ok, it is a pseudo inverse if it is a least curve.

So, what does it mean? It is pseudo inverse of what? Pseudo inverse of  $A$ , denoted as  $A^+$  like that I can say it is not  $t$  is not transport it is like this ok. So, my  $W_{BB}$  in this case would be the pseudo inverse of  $A$ , which will be many way you can define it ok. This is one what if it is LMMSE based linear minimum mean square linear minimum mean square error based equalizer.

So, that will what if that in this case. What is this? If you remember if you open up detection estimation book this will be more of a this is the gate for a linear model ok. What is  $R_{yy}$ ?  $R_{yy}$  is  $E\{y y^H\}$  expectation of your  $s r$ . What is  $R_{yy}$  or  $R_{rr}$ ? Expectation of  $r$  bar  $r$  bar star this is the one ok and how do you get it. So, let me just put some little focus on that, but it will not I will just briefly explain how you exactly get it and from there what is the real things coming to picture here ok.

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L M M S E

$$\min E \|\underline{s} - \hat{s}\|^2$$

$$\Rightarrow \arg \min_{W_{BB}^R} E \|\underline{s} - W_{BB}^R y\|^2$$

$$E (\underline{s} - W_{BB}^R y)^* (\underline{s} - W_{BB}^R y)$$

$$\frac{d}{d W_{BB}^R} \Rightarrow E \quad \Rightarrow 0$$

Solve  $W_{BB}^R$

So, it basically comes if it is an LMMSE if it is really an LMMSE based equalizer. What will happen? Now what is the cost function right the cost function is this  $\bar{s}$  minus  $\hat{s}$  cap minimize it ok, minimize this function over a what? Over this  $\hat{s}$  caps mechanism, so still is not complete so this will be that is the variable of consideration ok minimize this function over all the argument  $W_{BB}^R$  that is the thing.

Now, how do you get that? You expand this function I may not do the whole thing, but I am just giving you a, what is the mod square? It is nothing but this multiplied by that take an expectation ok. If I take the expectation what will happen there will be lot of internal terms will be there I am not again getting into that. So, finally, once whatever comes do a matrix derivation with respect to  $W_{BB}^R$  and do it and then make it 0 solve  $W_{BB}^R$  ok. So, what will come?

So, you can just take few step here it will be expectation of  $s$   $s$  expectation and all things will come into picture. So, let me just do at least for few of them so that it will be easy.

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L M M S E

$$\min E \|\underline{s} - \hat{s}\|^2$$

$$\Rightarrow \arg \min_{W_{BB}^R} E \|\underline{s} - W_{BB}^R y\|^2$$

$$E (\underline{s} - W_{BB}^R y)^* (\underline{s} - W_{BB}^R y)$$


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$$\Rightarrow R_{ss} - 2W_{BB}^R R_{sy} + W_{BB}^{R*} R_{yy} W_{BB}^R$$

Solve  $W_{BB}^R$

$R_{ss} = E(\underline{s} \underline{s}^*)$

So, what it will be? It will be first step  $s$  cross  $s$  expectation that is nothing, but  $R$  of  $s$   $s$  right  $R$  of  $s$   $s$  meaning expectation of  $s$  bar. So, this is your  $R$  of  $s$   $s$  ok, similar way you can just  $W_{BB}^R$  and then lot of stumps will be there  $s$   $y$  and then it will be something like that will be coming into picture. Similar this kind of equation will come from here. Now, once you derivate it once you derivate it, so what will happen and then take a 0. So, this will be something like if I derivate it if you just derivate with respect to  $W_{BB}^R$ . So, it is a matrix derivation, do not get into the scalar derivation.

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$$\Rightarrow W_{BB}^R R_{yy} = R_{sy} \rightarrow \text{After } \frac{d}{dW_{BB}^R} = 0$$

$$W_{BB}^R = R_{sy} R_{yy}^{-1}$$

$$R_{sy} \Rightarrow E\left[\underline{s} (A \underline{s} + \underline{v})^*\right]$$

$$\Rightarrow A^* R_{ss}$$

$$R_{yy} \Rightarrow E[\underline{y} \underline{y}^*] = \left[ A R_{ss} A^* + R_{mv} \right]$$

So, if I derivate it what will happen? So, you can just say derivate and then make it 0 I can straight away I can say it will be R s y it will be R s y W BB; sorry it will be R s y and then this will be just let me just do it this will be the case that will be there. Finally, will be coming like that and then you solve for W BB R which is equal to R of s y into R of y y inverse. So, that is the one which will be coming into picture. So, this comes after doing this and then make it 0, this result will be appearing and from there I can get it.

Now finally, what is that I have to finally get the exact equation of it right. Now what is R s s versus y that is nothing but expectation of your s and your y vector. What is y? A into that is comes into picture right. So, what it would be, you know what it is? This will be A star and R s s that is the only thing will come right. Similarly, what is R y y inverse or R y y? This will be expectation of your y y star right. So, A s minus v, so if I do not do that whole thing do not do that whole math. So, it will be A R s A star plus R of m v that is the only thing will come, now everything is known to you right.

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$$\underline{W_{BB}^R} \Rightarrow A^* R_{ms} [A R_{ss} A^* + R_{vv}]^{-1}$$

$$R_{ms} = \sigma_s^2 [I]$$

$$R_{vv} = \sigma_v^2 [I]$$

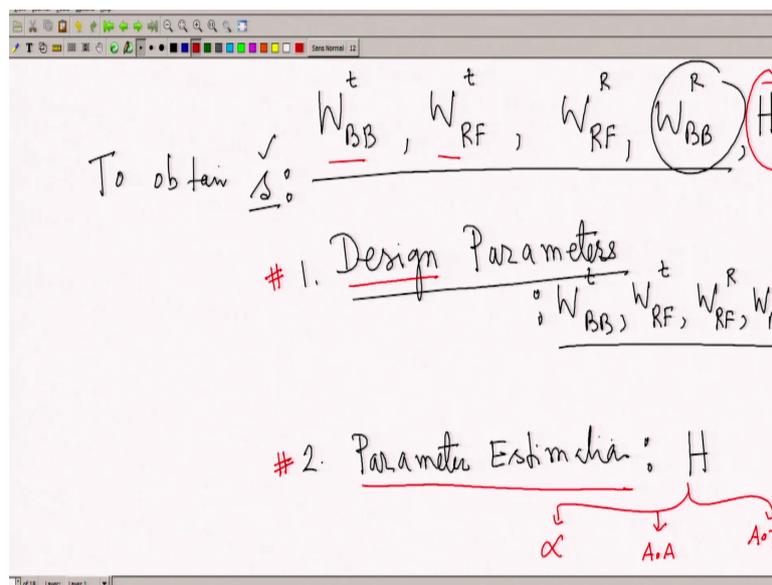
So finally, your equalizer  $W_{BB}^R$  is nothing but that is a closed form expression. So, you know  $A$  you know all the components what is  $R_{ss}$  that is already discussed. What is  $R_{ss}$ ? That is the covariance matrix of my data. So, now if all my data vectors are independent and identically distributed, because that is the data I am sending it. So now, what is the simplest form it can take? It is it takes  $R$  kind of a diagonal form. So, it will be  $\sigma_s^2$  and then there is a diagonal matrix  $I$  identity matrix right.

Because if  $s$  is coming from same you know same constellation points not the same constellation point, but the same constellation set having the power spectral density of  $\sigma_s^2$  that is my  $R_{ss}$ . Now, what about the  $R_{vv}$ ? Now that is also noise covariance matrix, now if it is an independent and identically distributed noise that is will also be  $\sigma_v^2$  into  $I$ . So, these two are known because this you can estimate, it this is in your control

because that is the power spectral density of  $y$  transmitted signal,  $A$  is known what else everything is known.

So, I can get back my  $W_{BB}^R$  ok. So, that is the closed form expression if I take an LMMSE. If I take some other equation if I take some other cost function that you have to estimate you have to see what it comes around, when we cannot say but it really comes around. But there is a catcher what I have done is not a very optimal way of obtaining all my parameter. So, which means that finally, how many parameters I need to find out in my whole chain of transmitter and receiver.

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I need to find out or I need to at least obtain  $W_{BB}$  at the transmitter side, then  $W_{RF}$  at the transmitter side, then  $W_{RF}$  at the receiver side and  $W_{BB}$  at the receiver side; this 4 matrices

I need to find out to obtain  $s$  that is my transmitted vector right. So, to obtain  $s$  I need to find this or I need to estimate all these 4 matrices ok.

Now, I have two I have actually multiple approaches that I have explained as far as the WBBR is concerned, I have what I have not thrown any light that how do you first of all how do you even get these a one more point  $H$  is also there. Of course, you have to have a channel right, unless there is a channel estimation if you do not know the channel nothing can be.

So, all 5 such you know matrix I need to find out to really come up with the  $s$ . But look at this 5 out of these 5 matrix who are my design parameters and which comes as a system parameter, meaning where you do not have a control rather it come because it is there in the system. So, the design parameters are 4 that mean, that is the one which you design it who are the design parameters this, from this philosophy several things will evolve now. So, out of these 5 matrices which I have shown to get back my  $s$  how many of them are design parameter.

What is meant by design parameter? Design parameters are those which you design ok. Now you can see that out of all this except this  $H$  all force are design parameters you cannot design channel that is given that is basically a physics ok. So, that depends on the environment, but rest of the force it is in your control you design ok. So, which means that now I split the problem in two ways.

Now, design parameters are the one which you design the channel is the one which you obtain which you get by default, you do not have any control only thing you can do is that estimating the  $H$ . So now, I am slowly moving to another interesting point is that how do you obtain all these 4. So, I split this problem in two part ok, the first part is I split it in my design part design parameter, second part of the problem is parameter estimation part because my next modules will be dominating these components.

Now, here I am only estimating  $H$  part. So, when I say parameter estimation I am essentially talking of my channel estimation part. When I do a design parameter design designing the you know system parameters there are basically these 4 parameters this 4 matrixes ok. Now, I take

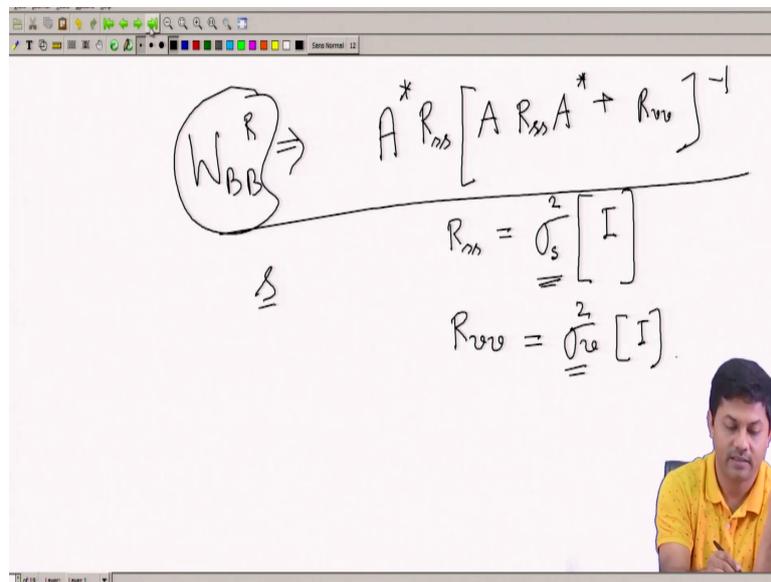
a 2 step approach, the first step approach is that given channel how do I do. So now, you can see that I mean you cannot do all these things together right, I mean you cannot optimize with respect to H; that mean, you cannot design H.

So that mean, you cannot create an optimization framework or any design framework where you are designing h rather you are estimating H. So, so clearly these two parameter two terms are very important design and there is a parameter estimation ok. So, when I go for the first one I assume that my second is actually done, that mean if I know my channel I can design the rest of the 4, that is the philosophy I am trying to here see say here ok that mean given H I can design all this force that will be easy for me.

Now, what does the H contain if you remember what does the H contain, channel contain? This channel was having what is that channel path gain alpha then you have the angle of arrival, then you have angle of departure. So, which means that if I know angle of arrival if I know angle of departure and if I know the individual path gains, I am done with my channel estimation because that is the model we have created long back for a millimetre wave. What is the approximated channel model that comes into effect ok that is the first part of the story.

So, the as a step wise who is the first who is the second, you can realize that this should be my first one I have to first estimate my angle of arrival angle of departure and alpha. So, that I can now get back my H, once I get back my H can I design my rest of the parameters this 4 parameters ok. So, these are my basic things that we need to understand here. So, I go step by step I take the easier one and then I go back to the old I mean I go to the tougher one. So, easier one is that let us assume channel is known to me ok, so I do not get into the channel unknown problem.

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The whiteboard contains the following content:

- A circled expression  $W_{BB}^R$  with an arrow pointing to the right.
- A horizontal line.
- Below the line, the expression  $\delta$  is written.
- Below the line, the equation  $A^* R_{ms} [A R_{ss} A^* + R_{vv}]^{-1}$  is written.
- Below the line, the equation  $R_{ms} = \sigma_s^2 [I]$  is written.
- Below the line, the equation  $R_{vv} = \sigma_{v0}^2 [I]$  is written.

So, let us assume so my easier one task is that assume H or the channel is known ok assume channel is known. Now moment I give that aspect then I go back to my first problem how do I design my parameters. Now, what I have shown you so far is that I have not put any effort or rather I have not told you what how do I you know how do I estimate the first 3. Now I have shown you a very detailed about the W BB R, but here there is a tab problem here. What I have shown you it is an approach which is slightly a easier approach what it means that, I assume so far that this first 3 are known to me.

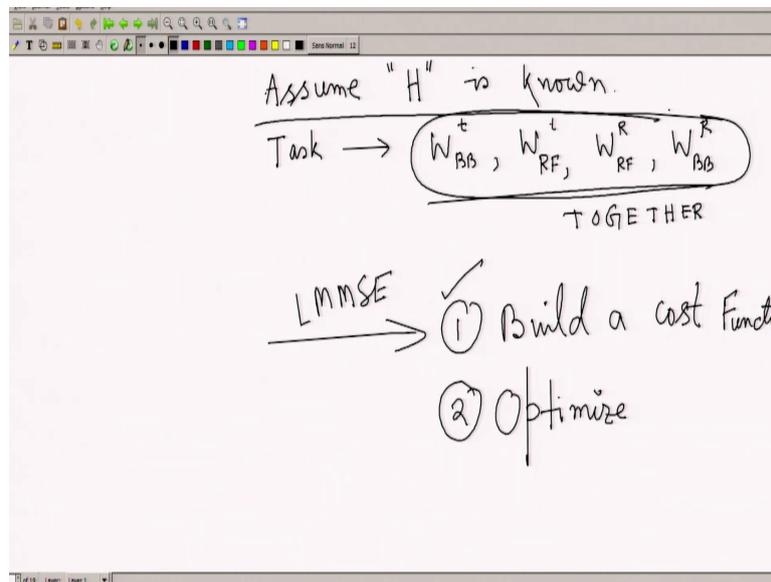
Then I go for W BB R estimation. So, I again I further you know split the problem. So, first problem is that H I assume H is known to me then I go for the design parameter designing the parameter, there also I split the problem two, there I say ok first I know all these three how do

I get the  $WBBR$ . Now, I am so far that is what my explanation right in the all throughout the, because that was the easiest task.

Now I am going for a second approach which is slightly tougher saying that no these three are not known to me and I do not want to visualize this whole three as an  $A$  matrix which I have just drawn. So, can I now jointly estimate this four instead of the three, that is the problem statement that I am trying to make here so instead of  $WBBR$  whichever I have shown here. Now this is more of the fact where  $A$  is known  $A$  meaning these 3 matrices are known, given this  $A$  I know  $WBBR$ .

That mean, three is known I want to go for the fourth one and obviously, your expectation was now how do I know that rest of the three, I mean again first two are known what is the going for the second and so and so forth. But that is more of a you know time taking process that may not be a right optimal way of doing it. Now, the optimal way or the better way of doing is that given  $H$ , how do I estimate all these four matrixes together. So, that is my next level of thinking ok.

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So, which means my task is to design those four parameters together all together ok. So, I do not want to estimate  $W_{BB}^R$  separately given the 3 no all 4 together ok. So, when I go for such kind of things what does it mean? It means that I need some sort of A. So, if you remember the way the whole you know communication system works is that when I go for a parameter design, parameter design always the way you should proceed is that you first I mean build a cost function. Based on the cost function you optimize the cost function.

So, the steps are how do you do that I mean this is the very I mean it is a general model. So, general model is that the process is that first you build a cost function first you build a cost function of your choice, then optimize it then optimize. Optimize the cost function based on the parameters that is the two step that we have built. Now, in the next class we will take

some few examples how do I build the cost function, probably take a very simple example that is your MSE or LMMSE based cost function.

And we will try to do some optimization and try to see how this 4 matrices can be obtained simultaneously ok. So, that would be the next lecture. So, with this I think I have motivated you enough for the complete structure of the things.

Now I am going to more of a joint design joint optimization those kind of thing where these matrixes needs to be evaluated needs to be obtained ok. So, with this I complete this particular class and in the next class we will be try to give some flavour of optimizations and cost function development for this matrix develop ok. So, that is it from my side for today.

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The slide features a dark blue header with the word "Conclusion" in white. Below the header, there is a list of four bullet points, each preceded by a small square icon. The bottom of the slide contains two circular logos on the left (IIT Madras and NPTEL) and the text "IIT Madras" in the center.

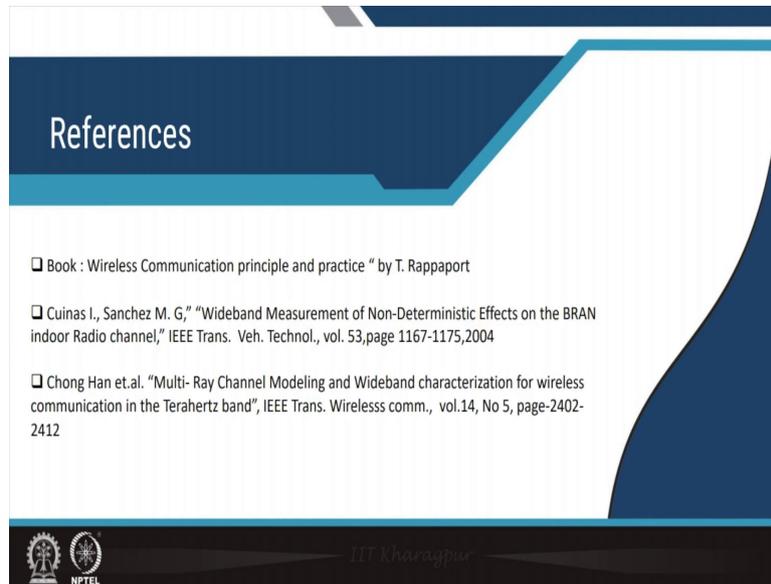
## Conclusion

- We covered more on Equalizer on low cost.
- Parameter to be configured.
- Data detection .
- Mathematical description of MIMO beamforming.

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So, in conclusion we have kind of completed the yeah.

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## References

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- ❑ Cuinas I., Sanchez M. G," "Wideband Measurement of Non-Deterministic Effects on the BRAN indoor Radio channel," IEEE Trans. Veh. Technol., vol. 53,page 1167-1175,2004
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And the reference will be same as what we have.