

Electrical Machines - I
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Lecture - 25
Regulation: Its Expression (Contd.)

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Regulation :- $R \approx \frac{I_2' r_{e1} \cos \theta + I_2' x_{e1} \sin \theta}{|V_1|}$ for lag. p.f. Lec-25

Make Algebraic equ. $I_2' \approx I_1$ (neglect no load current I_0) ($I_1 = I_0 + I_2'$)

$r_{e1} = r_1 + a^2 r_2$ $x_{e1} = x_1 + a^2 x_2$

$\cos \theta = \text{p.f. of the load}$ $R = \frac{I_2' r_{e1} \cos \theta \pm I_2' x_{e1} \sin \theta}{|V_1|}$

\rightarrow for lag p.f.
 $-$ for leading p.f.

$R = \frac{I_2' r_{e1} \cos \theta - I_2' x_{e1} \sin \theta}{|V_1|}$ for leading p.f.

Welcome to lecture 25 on Electrical Machines - I, we are discussing regulation and in our last class we found out an approximate, but very useful expression for regulation.

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δ small for a well designed the $OC \approx OM$

$R = \frac{OC - OA}{OC} \approx \frac{AM}{OC}$

$OC - OA \approx OM - OA = AM$

now $AM = AN + NM$

$= I_2' r_{e1} \cos \theta + B \phi$

$R \approx \frac{(I_2' r_{e1} \cos \theta + I_2' x_{e1} \sin \theta)}{|V_1|} \times 100$

And it was like this that finally, this is the thing that regulation will be $I_2 r_e \cos \theta + I_2 x_e \sin \theta$ by V_1 . Mind you, this is an algebraic equation ok. So, regulation of a transformer is equal to I_2 dashed and if you neglect I_{naught} you can say this is $I_1 r_e \cos \theta + I_2$ dashed $x_e \sin \theta$ divided by magnitude of V_1 supply voltage; very neat formula approximately equal to and I told you that to determine determination of regulation by actual loading the transformer is almost an impossibility because of the fact that if the rating of the transformer is very large kVA, large voltage ratings.

To get that value of load in the laboratory is out of question. Therefore, these expressions comes very handy and it is often used very widely used formula. So, this is the approximate, what are each term? I_2 dashed which is approximately equal to I_1 because you can neglect no load current, neglect no load current I_{naught} . Recall that I_1 is equal to phasor sum of I_{naught} plus I_2 dashed. So, this if you neglect this is I_1 , r_e is equivalent resistance of the coils referred to primary side. So, this expression I have found out with referred to primary side r_2 , x_e is equal to leakage reactants of primary side plus a square into x_2 .

So, this is the expression [FL] and what is $\cos \theta$? $\cos \theta$ is equal to power factor of the load. Mind you, I have found out this expression for a lagging power factor load. Similarly, I could find out the expression of the power factor for a leading power factor load.

In that case, what it would; what would happen? That is I will draw V_2 dashed and load power factor is suppose; leading suppose it is like this I_2 dashed then V_2 dashed plus I_2 dashed r_e will be like this which is small, I am I am drawing in a larger scale plus this is I_2 dashed r_e and this is $j I_2$ dashed x_e and this would have been your V_1 , is not; and what I am telling this angle is small because of what not like this it will come, it will be very small that is V_2 dashed then this, then this.

This is V_1 this is the actual thing it will be, but in a larger scale so that I understand every component and δ is small same argument and once again you can approximate it, you can derive this expression and what is θ ? θ is this angle load power factor angle of V_2 dashed and if you do the same exercise regulation expression will then be equal to I_2 dashed which I am not doing, r_e this I leave it as an exercise, minus I_2 dashed $x_e \sin \theta$ divided by V_1 .

Mind you, these expressions are algebraic expressions; note this is algebraic expression no phasor, algebraic equation. Similar is this one algebraic equation here no phasor, I_2 dashed is only number what is the magnitude, r_{e1} is number; no $j \times e_2$ etcetera. Therefore, it will be like this.

So, for leading power factor, this is the expression, this for lagging power factor and this is for leading power factor and combining these two you can say that regulation is it would same thing I am writing $r_{e1} \cos \theta$ plus minus I_2 dashed $\times e_1 \sin \theta$ and this is worth remembering and easy to remember and magnitude of V_1 . Plus sign for lagging power factor load and minus sign for leading power factor load.

Ok, what this regulation actually means? V_1 is constant primary voltage it means changing secondary terminal voltage, change in secondary terminal voltage expressed as a percentage of rated voltage. Numerator gives you change in magnitude of voltage divided by rated voltage, that is the regulation is and all these expression after I get this, I will be able to transform it with respect to the secondary side as well if I please. For example, I will say that I mean these I am just pointing out it will only take some time, but you must be understanding this one.

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$$\%R = \frac{I_2' r_{e1} \cos \theta \pm I_2' x_{e1} \sin \theta}{|V_1|} \times 100 \quad I_2' = \frac{I_2}{a}$$

$$= \frac{\frac{I_2}{a} (r_{e1} \cos \theta \pm x_{e1} \sin \theta)}{|V_1|}$$

$$= \frac{I_2 r_{e2} \cos \theta \pm I_2 x_{e2} \sin \theta}{\underbrace{|V_1|/a}} = \frac{I_2 r_{e2} \cos \theta \pm I_2 x_{e2} \sin \theta}{|V_{20}|}$$

This is equal to refer to primary side, I have defined these $r_{e1} \cos \theta$ plus minus I_2 dashed $\times e_1 \sin \theta$ divided by magnitude of V_1 . Everything could be written in terms

of secondary side ok, how it can be written? Because of the fact, I_2 dashed is nothing but I_2 into a, I_2 dashed is nothing but a into I_2 . No.

Student: (Refer Time: 08:57).

I_2 dashed.

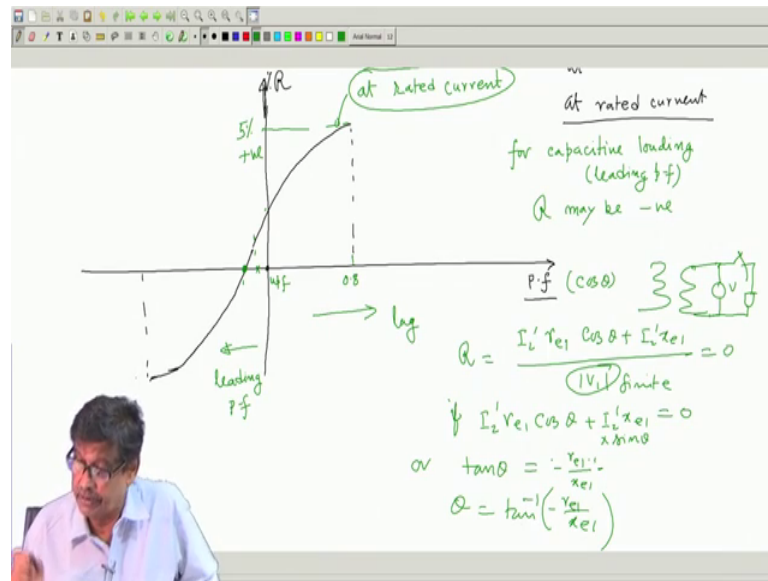
Student: (Refer Time: 09:03).

Is equal to I_2 by a, is not? So, I_2 dashed a I_2 , I_2 by a sorry. So, it will be I_2 by let us see it hopefully things will come correctly I_2 by a I can write, $r e 1$ is equal to I can write in terms of $r e 2$ like this. Power factor does not change $\cos \theta$ plus minus. Similarly, this is I_2 by a magnitude of I_2 by into a square into $x e 2$ into $\sin \theta$ divided by magnitude of $V 1$. Now these a goes one a.

So, and this common a small a, I will bring it down. So, I can write it as $I_2 r e 2 \cos \theta$ plus minus $I_2 x e 2 \sin \theta$ divided by $V 1$ by a, is not? And what is $V 1$ by a, is the a no load voltage $V 2 0$. So, this is same as $I_2 r e 2 \cos \theta$ plus minus $I_2 x e 2 \sin \theta$ same expression but the only thing you must be careful this is $V 2 0$.

So, either of them you use, but as you know you stick to one, you always transfer it to the primary side and get these values and these are algebraic equation. So, the numerator actually is the difference of the magnitude of no load to full load voltage. [FL] Before that, I will just try to sketch some curve, what will be the order of value? This into 100 will give you percentage regulation ok; this is per unit regulation, percentage regulation. [FL] How this curve will look like, if I sketch it?

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So, you sketch this curve in this way at least one curve I will sketch. So, this is the value of regulation, I will sketch percentage regulation. The regulation curve and this is suppose you sketch power factor and that rated current at sorry at rated current. Rated current means if secondary current is rated primary too will have rated current; you do not have to bother. So, load current is rated means primary side current is also rated.

So, the a typical regulation curve will look like this one, this is a I will just sketch it. Something like this and this point is suppose unity power factor cos theta against cos theta I am sketching. This side is leading power factor; no, lagging power factor and this side is leading power factor and this regulation curve.

I have plotted for example at rated current I must specify that at rated current and as I was telling you at rated current, power factor 0.8 lagging suppose 0.8 lagging, regulation will be small or high like efficiency is 99 percent, 98 percent that is common but regulation should be low it may be about say 5 percent, got the point? At unity power factor also there is regulation is positive, what is regulation? Regulation is no load voltage minus the voltage with load connected divided by the no load voltage.

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$$\begin{aligned} \%R &= \frac{I_2' r_{e1} \cos \theta \pm I_2' x_{e1} \sin \theta}{|V_1|} \times 100 & I_2' &= \frac{I_2}{a} \\ &= \frac{\frac{I_2}{a} (r_{e1} \cos \theta \pm x_{e1} \sin \theta)}{|V_1|} \\ &= \frac{I_2 r_{e2} \cos \theta \pm I_2 x_{e2} \sin \theta}{|V_1/a|} = \frac{I_2 r_{e2} \cos \theta \pm I_2 x_{e1} \sin \theta}{|V_{20}|} \end{aligned}$$

Therefore, if it is positive means the numerator is positive. No matter either you use this expression or that one and the same thing you will get same result. Therefore, as you can see if you want to calculate regulation at unity power factor; that means, theta is 0 then it will be simply $I_2 r_{e1} \cos \theta$ into 1 divided by V_1 .

Put the corresponding power factor, how I plotted that. Fixed up theta at rated current I know the rated current, knowing the rating of the transformer $r_{e1} \times e_1$ I know from OCSC test, V_1 is rated voltage I know. So, I can calculate and this is how I will get this curve. Only one thing you note that, regulation this side is positive and this is negative regulation.

So, it looks like for capacitive loading; for some capacitive loading that is leading power factor, regulation maybe negative indicating that with respect to the voltage when no load is connected when you connect a leading power factor load terminal voltage will rise. Unlike a battery, battery with an internal resistance R you connect a load always the voltage will fall, positive regulation means voltage falls.

So, for leading power factor load, if you draw the phasor diagram correctly you there will be some power factors for which the length of this phasor, I drew some phasor diagram here the length of this will be more than this, got the point? Therefore, regulation may be negative sometimes and voltage may rise and if you see this

expression and this curve, there may be some power factor you will be able to say, you will be able to predict if the transformer is.

So, this curve must be attached with this tag that is at what current you are doing at full load current and you are varying power factor. So, for rated current there exist a point, when the regulation may become 0 indicating that no load condition record these voltage, this your voltmeter. No load condition, record this voltage. Load it at rated current and at this power factor which is leading, you will find there is no change in voltmeter reading it means that.

So, regulation may be come 0 and I can predict what power factor at what power factor regulation will become negative because of the fact regulation is with respect to the primary side. If it becomes 0, this is finite it cannot be 0, finite and rated value applied voltage.

So, this will become 0 if $I_2^2 r_{e1} \cos \theta + I_2^2 x_{e1}$ is equal to 0 then only it will be 0.

Student: $\sin \theta$.

$\sin \theta$, into $\sin \theta$; this will be 0 or you can easily see $\tan \theta \sin \theta$ by $\cos \theta$ is minus x_{e1} by r_{e1} , is not? Or just opposite it will become that is $\sin \theta$ by $\cos \theta$ is minus.

Student: R by x .

R by x . So, it will be r_{e1} by x_{e1} . So, θ will be negative indicating it is leading power factor ok. So, it is this point. Therefore, is regulation in case of a DC source with the internal resistance are regulation will be always positive voltage will always drop and drop, but here in case of a transformer which is giving you AC supply and you can choose a load power factor of $r_{e1} \tan^{-1}$.

So, load power factor if you choose \tan^{-1} minus r_{e1} by x_{e1} you can say regulation may become 0. Anyway it is only a statement that is what it will happen, but unfortunately I cannot operate a transformer under this condition because load is always lagging type.

Anyway, this a point what should be what noting that is all and regulation maybe negative sometime. So, one should not get surprised I have connected a load and voltage has risen, maybe you have then connected a capacitive load whose leading power factor is greater than 1 not for all capacitive load it will be negative or 0 because if the capacitive load of this power factor, regulation will become positive.

So, one can get a number of curves at different current levels. For example, that lower current level the curve will shift below which I am not drawing. It will become regulation will become less. So, it will be above.

Student: (Refer Time: 22:24).

It will, that is what I am telling this is this one. Suppose on the same curve I want to show regulation at reduced current then curve will be like this.

Student: (Refer Time: 22:37) amplitude (Refer Time: 22:39).

Amplitude will get reduced, you have understood; you can scale this thing. So, it will change in its amplitude, but it will follow the same curve. Anyway this you can easily verify therefore, the regulation of a transformer we have understood.

The difficulty of regulation concept, it is otherwise very simple, it directly loaded, but directly loading a big transformer in the lab is out of question. Therefore, these simple expressions will come very handy because it is an algebraic expression, easy to remember. Tell me; what is the current transformer is delivering at what power factor I will be able to tell you what will be the change in voltage.

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$$R = \frac{I_2' r_{e1} \cos \theta + I_2' x_{e1} \sin \theta}{|V_1|} = 2.5\%$$

at rated current and at 0.8 pf lag

$$R = \frac{I_2' r_{e1}}{|V_1|} \cos \theta + \frac{I_2' x_{e1}}{|V_1|} \sin \theta$$

p.u. resistance

p.u. reactance

$$R = \cos \theta + \frac{I_2' r_{e1}}{|V_1|} \cos \theta + \frac{I_2' x_{e1}}{|V_1|} \sin \theta$$

Algebraic Equat.

10KVA, 200V/100V, 50Hz, 50A, 100A

$\frac{I_2' r_{e1}}{200} = \frac{50 \times r_{e1}}{200}$

$\frac{I_2' x_{e1}}{500} = \frac{50 \times x_{e1}}{500}$

resistance voltage drop in the primary referred to the primary side

leakage reactance voltage drop in the primary referred to the primary side

r_{e1} , x_{e1} , 50A

$V_1 = 200V$

Some more words about this regulation; for example, regulation is $I_2' r_{e1} \cos \theta + I_2' x_{e1} \sin \theta$ or I_1 whatever you write $I_2' r_{e1} \cos \theta + I_2' x_{e1} \sin \theta$ by V_1 , is not? This is regulation. Now, in this expression this I_2' as I told you if you neglect no load current this is as good as I_1 .

So, you can calculate regulation given a particular current and particular power factor of the load. Now this one therefore, one must say along with this number whatever you get say 2.5 percent I get, I must write at rated current and at 0.7 power factor lagging. I must specify these two things along with this number, you cannot simply say regulation is 5 percent at what load current and at what load power factor angle.

So, this is at 0.7 power factor lag; you must specify this then only it becomes complicated, complete. I, this expression can be written as only one point I will tell, r_{e1} by $V_1 \cos \theta + I_2' x_{e1} \sin \theta$, what will be the regulation of an ideal transformer? $r_{e1} = 0$, $x_{e1} = 0$, it will be 0. For ideal transformer regulation is 0 because there is no internal drops which will be taking place. So, it can be written like this.

Generally, as I told you regulation is calculated at rated current and at 0.8 power factor ok. So, I know this number is very important, calculate regulation at rated current and zero point power factor 0.8 power factor lag. Now, look at this term; this equation once

again I am repeating, it is algebraic equation; no phasor involved. No terms is a phasor here magnitudes [FL].

Suppose, let us take that transformer 1 kVA for easy calculation or say 10 kVA, 200 volt 100 volt 50 Hertz transformer. What is the rated current of the HV side? 50 ampere, what is the rated current of the LV side? 100 ampere. What this number $I_2 \text{ dashed } r_{e1}$ will mean? It will mean, this $I_2 \text{ dashed } r_{e1}$ is 50 into r_{e1} it will mean that. Suppose this is my primary side 1 and this is side 2.

So, $I_2 \text{ dashed } r_{e1}$ by V_1 is this 200 and this will be equal to 50 into r_{e2} by 200. That is the equivalent circuit if you look at, here is your V_1 200 volt and this is your $r_{e1} \times I_1$ and rated current is 50 ampere, is not? Load is connected here whatever it is.

Therefore, what this $I_2 \text{ dashed } r_{e1}$ mean? It means this voltage drop at that rated current 50 into r_{e1} I am so sorry this is r_{e1} this will be r_{e1} . So, with respect to the primary side or HV side in this case, this number tells you voltage drop in the internal resistance of the transformer in volts. This is the; if I write in language, I will write voltage drop in resistive voltage drop equivalent resistive voltage drop in the transformer.

Similarly, $I_2 \text{ dashed } x_{e1}$ by 500 is the leakage reactance voltage drop in transformer, refer to your primary side if you want to write that also you better write, refer to primary side and what is the denominator? Denominator is rated voltage of the primary side.

So, this number this ratio is actually how much of this total rated voltage is consumed by the resistance of the transformer; if it comes out to be say this number after calculating if it comes out to be 5 percent, I will say oh, in the resistance 5 percent of the this 200 volt out of these at rated current some voltage will be dropped across here, some voltage will be dropped across here.

That magnitude of that voltage divided by the rated voltage is nothing but how much of this total voltage rated voltage is consumed by the resistance, how much of this total rated voltage is consumed by this reactance. And this one in books, they will write this is called the per unit resistance of the transformer and they will write it like this, what is epsilon r? Epsilon r is this one per unit resistance.

So, what is per unit resistance of a transformer? You simply calculate transformer is operating how much of the rated voltage, is dropped in the resistance this number you would like to have more or less definitely less. Out of this total rated voltage lesser and lesser get consumed by the transformer is better. So, that regulation will improve.

Similarly, this number this is the epsilon x, this is called epsilon r, epsilon x is the per unit reactance per unit reactance of a transformer. In numbers if I say suppose for this transformer if I say I will just tell this and go.

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Handwritten calculations on a whiteboard:

$$10 \text{ kVA}, \quad 200 \text{ V} / 100 \text{ V}$$

$$50 \text{ A} \quad 100 \text{ A}$$

$$r_{e1} = 2\%$$

$$r_{e1} \text{ p.u.} = .02$$

$$\frac{50 \times r_{e1}}{200} = .02$$

$$\frac{100 \times r_{e2}}{100} = .02$$

Suppose I say for last statement suppose I say that 10 kVA transformer and 200 volt by 100 volt 50 Hertz etcetera and rated current I know, it is 50 ampere this side rated current is 100 ampere.

If I say instead of giving you the value of r_{e1} , I say r_{e1} is equal to 2 percent I say that, what that does that mean? It means per unit resistance percentage. So, r_{e1} per unit is 0.02, is not? And this I am telling what it is? It is nothing but that is r_{e1} value is not explicitly given in Ohm suppose, it is given in terms of per unit values then r_{e1} per unit is 0.02. Means what is the absolute values of r_{e1} I want to calculate then I will say oh, r_{e1} is rated current of this site is 50, r_{e1} this into r_{e1} actual ohmic value divided by 200 is your 0.02 and from which r_{e1} can be calculated, are you getting? Similarly, with x_{e1} .

So, in some problems instead of giving you absolute values of r_e and x_e per unit values are given, but physically try to understand per unit values means that at rated current what will be the voltage drop in r_{e1} or r_{e2} . Similarly, this is to 100 into r_{e2} divided by the rated voltage here; that will be also 0.02 because of that ratio business.

Therefore, given knowing the rated currents and the equivalent per unit values. So, per unit values will never be specified in terms of primary or secondary because from whichever side you calculate that value will remain same. Please think about this and read it, solve problems. We will continue with this next time.

Thank you.