Evolution of Air Interface towards 5G
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Lecture - 41
NOMA (Non Orthogonal Multiple Access)

Welcome to the lectures on Evolution of Air Interface towards 5G. So, this is the last lecture for this lecture series and we have talked about the MIMO mechanisms, the different ways to evaluate the performance, we have talked about the practical channel models, and we have mainly talked about the point to point link performance evaluation of most of the systems.

Usually in this system it is also required to evaluate the area spectral efficiency. So, we will briefly talk about the methods used to evaluate area spectral efficiency other than simulation modes, because simulation modes are generally used and that is very very time consuming and we would always propose methods which should be using analytical techniques while being as close as possible to the results obtained by simulations.

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So, we will briefly go back to the classical descriptions which are available. So, when we discussed about fading process, we have talked about the multipath propagation as well as the shadow fading. So, when we talk about the area spectral efficiency area average spectral efficiency, we need to combine both of these together.

We at some point talked about coverage probability or boundary coverage probability in a circular area when there is a base station and users are with a certain cell radius R gamma. So, will recall our discussions over there and we said that the shadowing is log-normally distributed or in dB domain it is distributed in a Gaussian distribution. So, when we have the small scale fading and large scale fading together, you have a composite distribution of the Rayleigh distribution or gamma distribution and shadowing parameter which is log-normally distributed.

So, one can evaluate and show that by two different mechanisms, one can find the composite distribution as the same distribution, it is also known as the shadowed log-normal distribution also called the Suzuki distribution.

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So, this entire thing would result in a composite distribution, which can be approximated as another log-normal random variable.

It is sometimes very useful to model the radio propagation environment as a shadowed Nakagami fading channel. It is mathematically convenient and can closely approximate a Ricean distribution .

The composite distribution of the squared-envelope due to Nakagami fading and log-normal shadowing has the Gamma-log-normal density function

$$p_{\alpha_\xi^2}(x) = \int_0^\infty \left(\frac{m}{w}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left\{-\frac{mx}{w}\right\}$$

$$\times \frac{1}{\sqrt{2\pi}\xi\sigma_\Omega w} \exp\left\{-\frac{(10\log_{10}w - \mu_{\Omega_p\,(dBm)})^2}{2\sigma_\Omega^2}\right\} dw$$

This can be approximated by a log-normal distribution with mean and standard de

$$\mu_{(dBm)} = \xi^{-1}[\psi(m) - \ln(m)] + \mu_{\Omega_p\,(dBm)} \qquad \sigma^2 = \xi^{-2}\zeta(2,m) + \sigma_\Omega^2$$

where $\psi(\cdot)$ is the Euler psi function and $\zeta(\cdot,\cdot)$ is Riemann's zeta functi
When $m = 1$ the approximation is valid for $\sigma_\Omega > 6$ dB
for $m > 2$ the approximation is valid for all ranges of σ_Ω

Now, if this gamma log-normal random variable can be approximated like what you have the log-normal shadowing has the gamma-log-normal density function if this can be approximated as a log-normal distribution then recall what we have just said that we discussed the boundary coverage probability, probability that received signal strength crosses at some distance d is greater than some sensitivity level, we have been able to calculate that because we used the log-normal distribution.

Now when there is multipath and shadowing together, it produces a gamma log-normal distribution and this in literature is known to be it can be approximated as a log-normal. So, now, if you are able to approximate this as log-normal, you can even find the coverage probability and hence the area spectral efficiency using the log-normal random variable which is easy to manage.

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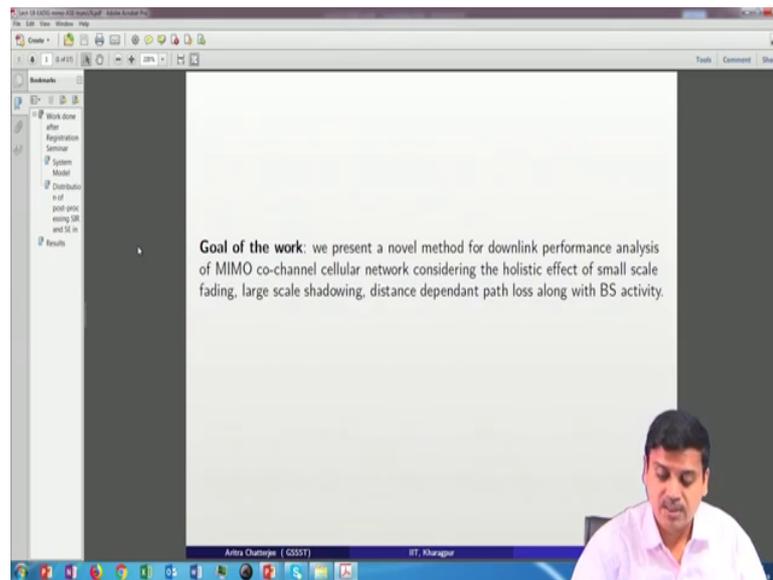
So, that is the summary of one set of things again in this particular work that we are trying to discuss.

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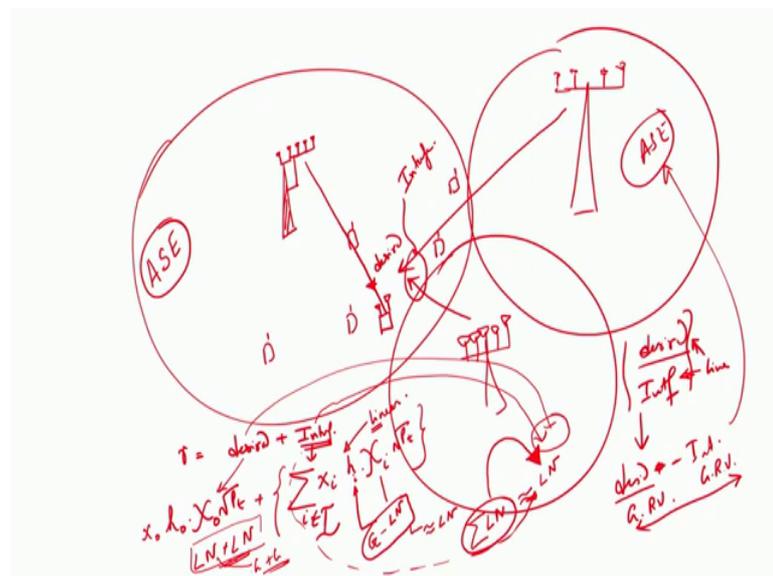
We are talking about a scenario where we have a MIMO communication system as well as it is covenant area; that means, you have a base station and instead of a SISO links we are talking about multiple antenna based link and there are users.

Although we have been describing the scenario, but now we are saying that in this coverage area, what is the area spectral efficiency? And you may recall that this

definition of area spectral efficiency is one of the requirements which has been specified in the fifth generation communication system in the very early lectures that we have been talking about the requirement specifications.

So, usually a huge amount of simulation is done in order to evaluate these performances. What we will show you is some analytical methods which can be used and hence you can very very quickly evaluate the performance with the tools that are available. So, typically under this situation what you have is there would be multiple transmitting antennas as base stations neighbor by nearby.

And this is the typical communication scenario in a fourth generation or fifth generation communication system and a huge amount of system simulation is to be done, because you have to generate all the links calculate the signal power, the interference power, because any signal that is being received by a user let us say who is located here the signal that is being received. So, this is the desired signal and you have interfering signals coming in.

So, the received signal is composed of desired plus interference. Interference is composed of the signal itself times the channel gain along with it sorry I mean we need to correct this expression along with the log-normally distributed shadowing and of course, there is the transmit power.

So, what we see, we write this is a set of interference. This is a Rayleigh distributed random variable in the SNR domain interference domain, this is a gamma distributed random variable, this is a log-normally distributed random variable. So, what we have is a gamma log-normal random variable. So, now we are using the description which we give just a few minutes back this can be approximated as a log-normal random variable and then what we are left with is a sum of log-normal random variables.

So, what we are left with is a sum of log-normal random variables. Again in literature this can be approximated into a log-normal random variable and there is a huge amount of process that goes in towards this log-normal approximation. So, we would be able to translate this detailed interference into one log-normal random variable, the desired signal is in a similar manner desired signal times the channel multiplied by the log-normal random variable and of course, the transmit power.

So, we again have a sum of log-normals, log-normal plus another log-normal, if you are in the dB domain then it is not a problem, we can easily calculate or even in the linear domain it is in the ratio. So, is the dB domain it is a sum of log-normals. So, in the dB domain it is the sum of Gaussian random variable with another Gaussian random variable and then all though this summation is in the linear domain ok, and this summation that we are talking about in the dB domain again it is like that. So, what we have effectively is that we have approximated the interference as log-normal, the desired signal is log-normal again these two together can be converted to a log-normal random variable and hence we can calculate the desired signal to interference.

So, this is in linear domain, this is in linear domain, if you go to dB domain you get a desired minus the interference in the dB domain, in the dB domain this is a Gaussian random variable, this is a Gaussian random variable. So, now, you would be getting a Gaussian random variable again with a modified mean and variance and then one can calculate the area spectral efficiency. All this is said and done this is very true for SISO links; you have to work out the results for the MIMO links. So, we will show you some brief results about the process and also the reference from which we can get the details of this right.

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Signal Model

Table 1 : System Parameters

Number of antennas at BS	N_t
Number of antennas at UE	N_r
Number of multiplexed users in each cell	N_u
Number of parallel streams transmitted from BS	N_s
Number of parallel streams transmitted to a particular user	N_{su}

In the cell of interest (0-th cell), the generic expression for the baseband received signal vector $\mathbf{r}_0 \in \mathbb{C}^{N_r N_u \times 1}$ across all the users belonging to the cell is given as

$$\mathbf{r}_0 = \sqrt{P_T} \mathbf{D}_0 \mathbf{H}_0 \mathbf{W}_0 \mathbf{X}_0 + \sum_{k=1}^{K} \sqrt{P_T} \mathbf{D}_k \mathbf{H}_k \mathbf{W}_k \mathbf{X}_k, \tag{1}$$

For any diversity schemes: $N_{su} = N_s = N_u = 1$.
In case of multi user MIMO schemes, $N_{su} \leq N_r, N_s \leq N_t$ and $N_u > 1$.

So, we have considered a MIMO link and we have been able to write the expression for the received signal as you can see that is given by equation number 1.

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SIR Expression for different MIMO modes

the generalized expression for the post-processed SIR at the typical user for different MIMO modes can be represented as

$$\Upsilon_G = \frac{\Gamma(D_{sh}, D_{sc})\Omega_0 \frac{P_T}{N_s}}{\sum_{k=1}^{K} z_k \Gamma(I_{sh}^k, I_{sc}^k)\Omega_k \frac{P_T}{N_s}}, \tag{2}$$

- D_{sh}, D_{sc}: shape and scale parameters Gamma distribution corresponding to small-scale fading part of desired link.
- I_{sh}^k, I_{sc}^k: shape and scale parameters Gamma distribution corresponding to small-scale fading part of k-th interfering link.
- Ω_0: Lognormally distributed shadow fading along with path loss in desired link.
- Ω_k: Lognormally distributed shadow fading along with path loss in k-th interfering link.
- z_k : Bernoulli random variable capturing the activity of k-th interfering BS.
- P_T: Tx. power from each BS.

And with the details you will find that the received signal to interference plus noise ratio can be written as expression number 2, again it is too much to describe all of these in the short video lecture, but all the reference material available for you to go through them. So, in these, what you will find is that we have a gamma distributed random variable in

the numerator and in the denominator you have a sum of gamma log-normal in the numerator also you have a sum of gamma log-normal.

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With the scale and shaping parameters of the gamma distributed log-normal random variables have been tabulated in this table for different MIMO mechanisms which is the contribution of the work that has been done in our department.

So, now, what you will find is that although we have different MIMO techniques, but they can be characterized as a gamma times another log-normal random variables, it is a gamma log-normal random variable with different scale and shape parameters as described in this. So, now, if we are able to characterize the different SINR for the MIMO links in terms of gamma distribution then we get back to this expression where we have gamma log-normal which can be approximated to another log-normal this we again we have a gamma log-normal which can be approximated to a log gamma log to a log-normal sum of log-normal to be approximated again as another log-normal ratio of log-normals can be easily expressed as a log-normal.

In all this process there has been detailed work which can be found in the reference that will be provided to you and you will be able to calculate the SINRs distribution. Once you know the distribution of SINR you are effectively calculating the SINR at one particular point.
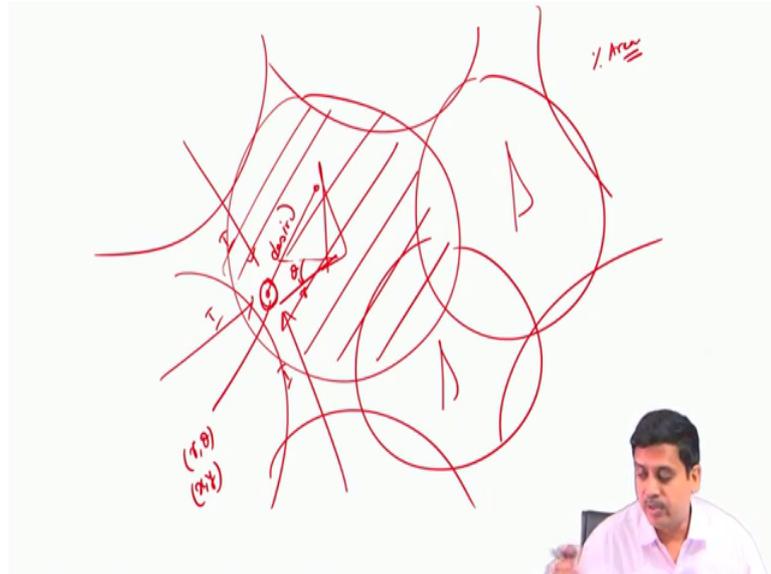
So, one is calculating. So, we have said that there is a circular area; there is another circular area and so on. So, the entire area is filled up with these things and what with that expression what we have been able to calculate is the SINR this is the desired link these are all interfering links at any one particular location. So, this location can be identified by r theta, this is the r and this is theta or you can also identify this is the location x y. And once you have done for the location x y then you can find the coverage area in this region by averaging it over the area. We will use exactly the same mechanism as we had used for percentage area coverage that has been described earlier since we have been able to convert the entire thing to a log-normal random variable.

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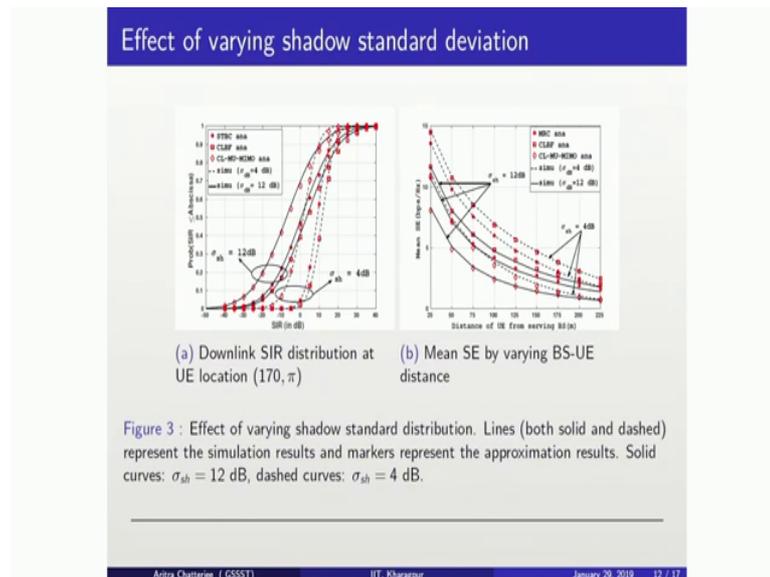
So, the entire work is about converting the MIMO SIR to a log-normal random variable. Is that the spectral efficiency calculations as given on the right hand side curve, matches very closely with the simulation results; that means, you need not go for a heavy amount of simulation if you are using the methods that has been described in this particular work for the different MIMO mechanism; that means, MRC, Closed Loop Beam Forming, Closed Loop MU-MIMO and so on and so forth.

So, this again helps you reduce the huge amount of simulation time for all practical evaluation of area spectral efficiency of MIMO mechanisms whereas, while you are getting the results analytically and quite accurately. So, there is another way of doing it, again for limitations of time we are unable to discuss it, but there are enough details available which one needs to be aware of.

So, summarizing our discussion till this point, what we have done so far is calculated the MIMO gains, we have been able to describe the signal to interference, we have been telling you about the different mechanisms of taking advantage of MIMO through diversity, through spatial multiplexing, through beam forming, through using massive number of antennas, we have also told you about the different advantages and disadvantages of millimeter wave and how it matches with the MIMO with the massive MIMO architecture, we have also describe to you how you can not only in calculate the spectral efficiency, the error probability, you can also calculate the area spectral

efficiency which is of course, a cumbersome stuff task and again for all these different things we have been repeatedly mentioning that instead of going for simulation there are mechanisms by analytical methods which can be used in evaluating the performance of these advance mechanisms in practical scenarios even using analytical methods.

So, with that we would conclude our discussion on MIMO mechanisms of course, there remains a huge amount of things that can be always discussed and it leaves you an opportunity to explore this entire domain. We move to the last part of our lecture, which is about the Non-Orthogonal Multiple Access which is also expect to be one of the major parts of the fifth generation communication system.

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So, the non-orthogonal multiple access is a special mechanism which will briefly describe in this particular next few minutes.
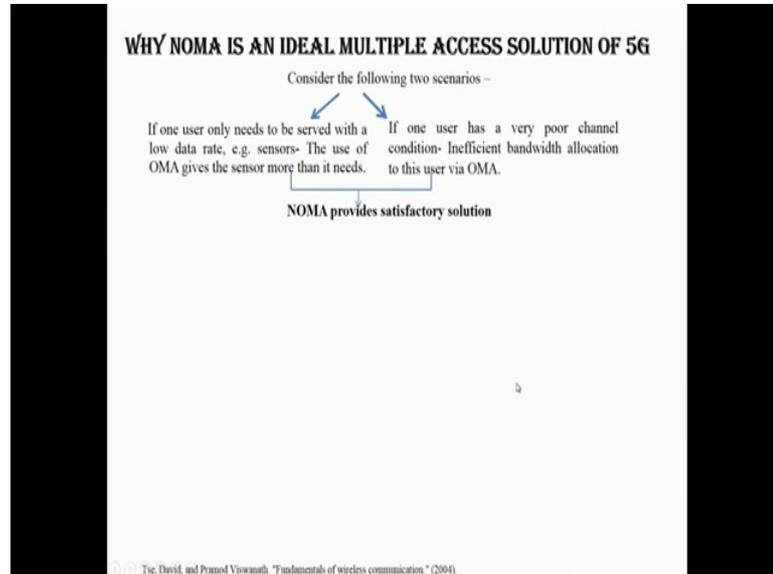
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So, typically in Orthogonal Frequency Division Multiple Access, we have the subcarriers and different users are given different set of subcarriers. There is a standard procedure in LTE and even in the fifth generation.

Whereas in Non-Orthogonal Multiple Access the whole idea is that a particular subcarrier or a group of subcarriers can be given to more than one users. So, we are not talking about MU-MIMO. In MU-MIMO, that is a Multiuser MIMO the same group of subcarriers are given to user 1 and the same group of subcarriers are to be given to user 2, but that requires the use of multiple antennas.

NOMA that is Non-orthogonal Multiple Access is not dependent on the use of multiple antennas, it does not require multiple antenna, it is a fundamental mechanism. Of course, one can go for NOMA with MIMO and there are again lot of works that are available and in the fifth generation, advanced version people are working towards mechanisms by which MIMO can be made enabled with to operate along with NOMA systems. So, we will we have described the MIMO mechanisms earlier will talk about the MIMO mechanism, what the NOMA mechanism here and one can explore the combination of MIMO NOMA, although a large amount of literature is already public now, but still there as there is opportunity towards contributing towards mechanism which can be used by a large number of community in the next version of the fifth generation mobile

communication systems or in the next version of 5G. So, let us look at the fundamental aspect of NOMA without the MIMO scheme ok.

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So, this NOMA mechanism will again define briefly talks about two different signals being sent from the transmitter.

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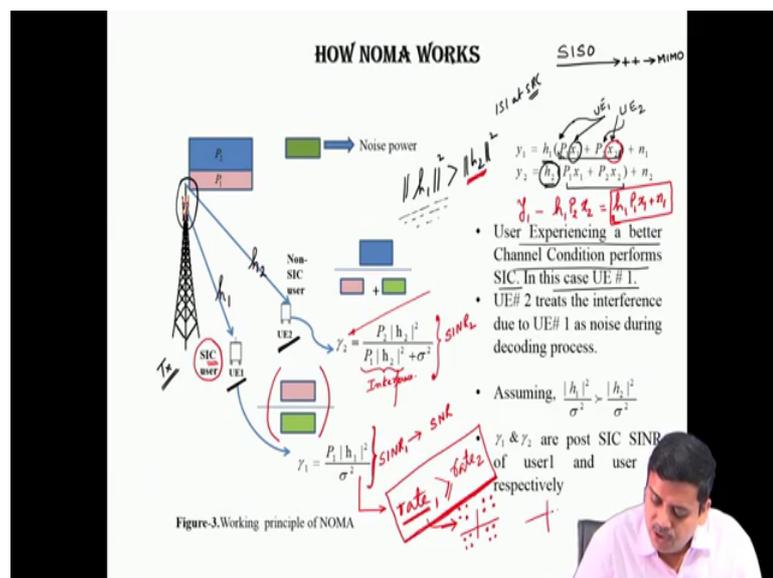
So, this is the transmitter and being sent simultaneously to user 1 as well as to user 2 at the same time without taking advantage of multiple antennas; that means, you can

implement a NOMA even using SISO and of course, you can enhance this thing towards MIMO ok. So, let us look at the SISO system.

So, the signal that is being sent is x 1 is the desired signal for user 1 and x 2 is the desired signal for user 2 ok. And user 1 is given a certain amount of power p 1 and user 2 is given a certain amount of power that is p 2. So, if you look at this composite signal, it is not possible to distinguish between the two signals and there is SIR or ISI at the source you can think of it like this.

The signal received at user 1 if the channel index is h 1, we have only a SISO link then h 1 is multiplied with this and you receive. At the user end 2, you have the channel h 2 and the user 2 receives the same signal at its end, but the coefficient of channel is different right. So, that is the premise based on which we discuss. So, user experiencing a better channel condition performs SIC in this case, it is user 1; that means, we assume that the channel 1s gain is much much larger or it is greater than h 2 we did not put much much larger ok indicating that the signal to interference ratio of the first user is much higher than that of the second user.

So, what it includes is that since the first user encounters a better channel, it can decode x 1 as well as it can decode x 2. If it can decode x 2 and if it knows the power values, then from y 1 it can take out this particular signal to produce h 1, p 1, x 1, plus n 1. If you look at this expression now, this is interference free expression at the receiver and this is primarily happened because the user can perform an interference cancellation at its end. And it has been able to take out the user 2s data from the interference part and it experiences a signal to interference ratio as rather signal to noise ratio. So, for two user case this is signal to noise ratio. Now what you can guess is that the user 2 would not be able to do the same operations because user twos channel condition is weaker than user ones condition because user twos channel condition is weaker it would only be able to decode its own data and it would treat the user ones data as interference right.

So, hence the SIR or SINR experienced at the user 2 is as there and SINR at user 1 is getting converted to SNR in this particular case. When you multiple uses the situation is hierarchical or cascaded effect it is not exactly same, but enhancement of this.

So, what we see that the first user sees signal to noise ratio of its link and an corresponding rate has to be allocated right. User 2s rate has to be allocated as per the

SINR condition in this expression. So, now, this clearly means that rate of user 1 can be typically higher than the rate of user 2. So, again as we have discussed during the MIMO communication, this rate allocation is also an important part of the communication link design.

Without which you are not complete, without which you are only doing a capacity analysis. So, if rate 1 is greater than rate 2 then what you would mean I mean it need not necessarily be, it can be equal to what you would get is that two users one can choose a higher constellation and the other user can choose a lower constellation that is what it would translate to all right.

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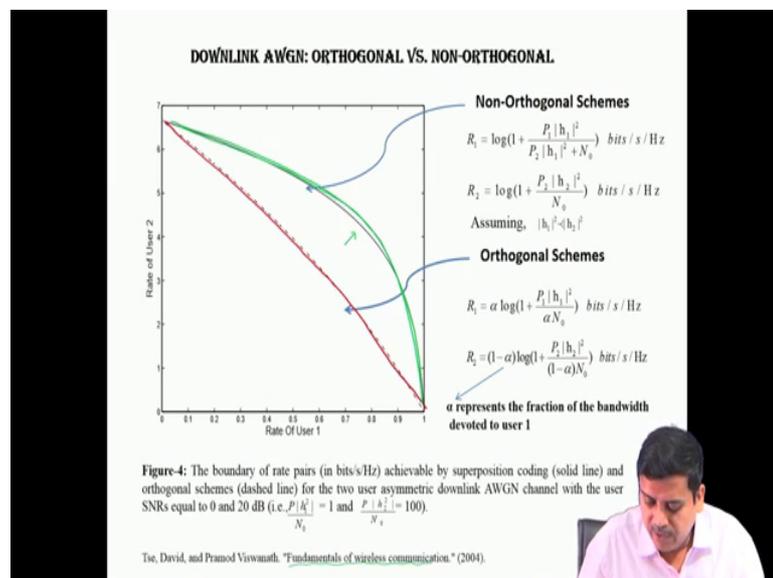
So, now if we get back what we will find is an orthogonal scheme would have a rate region of user 1 and user 2 as given by this curve, where as the rate region for a non orthogonal multiple access scheme can be as per the current curve that I am tracing. So, what we find is that through a detailed analysis one would find out that the rate region; that means, the sum rate of user 1 and user 2 would be higher than that of orthogonal access. So, any increase in the rate is always welcome and hence this particular mechanism one of non orthogonal multiple access is one of the major schemes which 5G community is expecting to get and this can be and the details of these can be found in the book of Fundamentals the wireless communications by David Tse.

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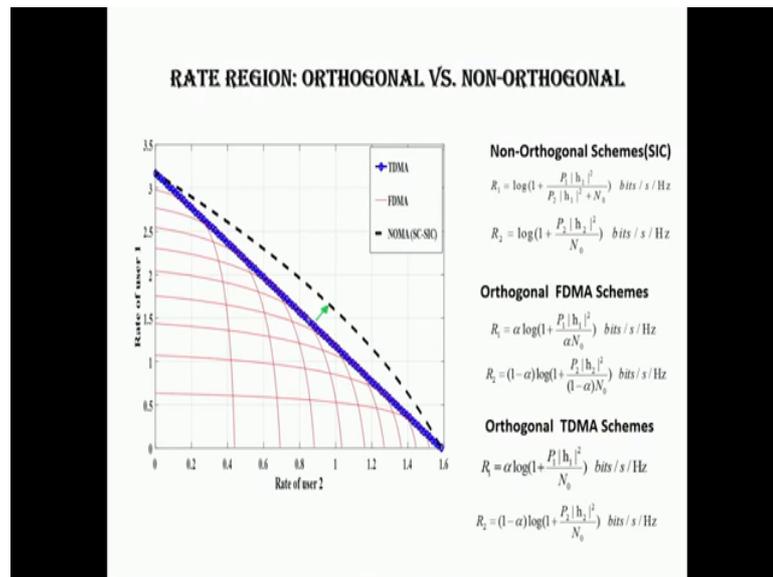
So, one can evaluate the performance of FDM scheme, TDM scheme and NOMA and then one can again get the benefit that one would find in a NOMA system providing a higher throughput in the system.

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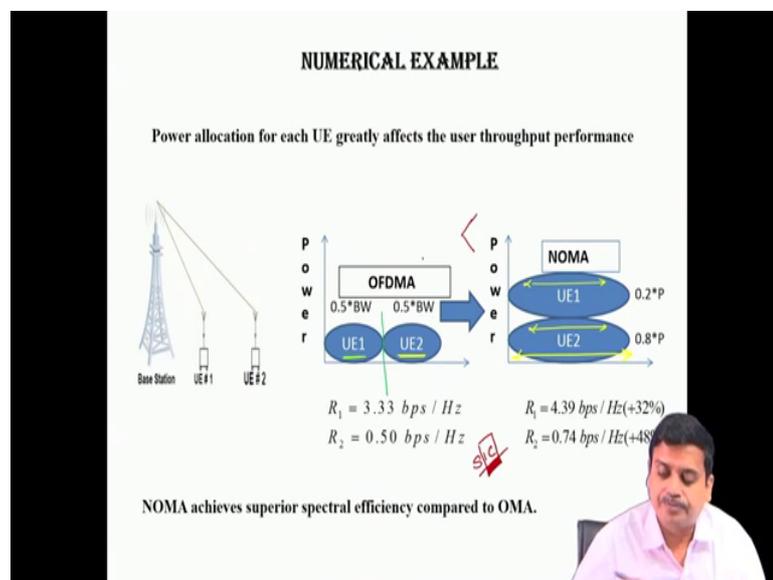
So, what effectively we are seen is that, in OFDMA system the bandwidth is divided right and user 1 is given a set of frequencies, user 2 is given a set of frequencies, whereas in NOMA the entire bandwidth is given to the two users, user 1 uses a different amount of power, user 2 uses a different amount of power and they can be decoded because one

of the user if there are two if it is a two user system can use a successive interference cancellation or rather interference cancellation mechanism how it implements the interface cancellation it is up to the particular receiver algorithm, but since it cancels out the interference, it can actually get a sum throughput which is much higher than a OFDMA system.

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So, in summary we can have multiple users allocated to one band in different bands there can be different number of users. So, what we see is that two users in this band, three users in this band and again this picture represents three users in this band. One can maintain a power constraint that the total power in each of the band is constant which is typically the case in fourth generation system, but this can be changed there is it is possible to go beyond this. This is of course, an implementation constraint where as within the band the power that is allocated to different users is as per the strategy which maximizes the total spectral efficiency.

So; that means, in one band in this case we are saying that user equipment 1, 5 and 8 are allocated in this band user equipment 3 and 6 are allocated here again 2, 5 and 4. So, there is a resource allocation which is again a fundamental aspect of this particular method. So in fact, it talks about allocating resources which is not only power which is the fundamental domain of NOMA, it also talks about a sub carrier or frequency

allocation when you talk of OFDMA may be system which is again the fundamental scheme in fifth generation system.
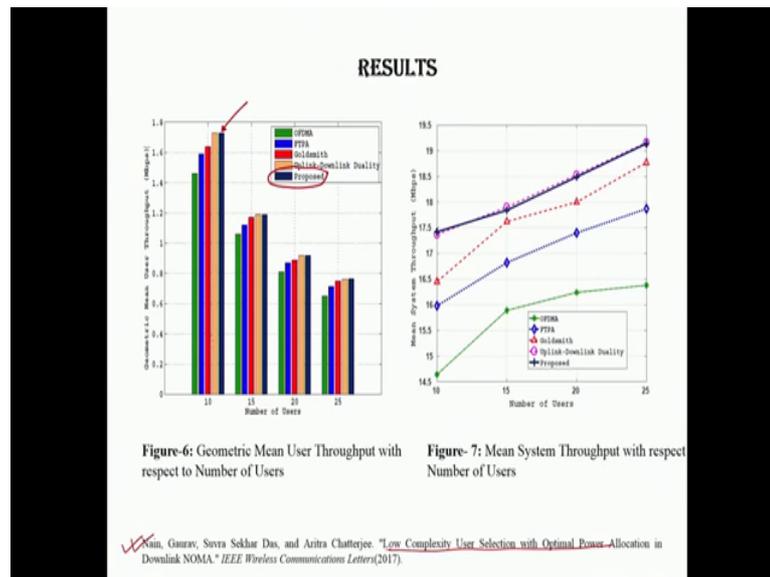
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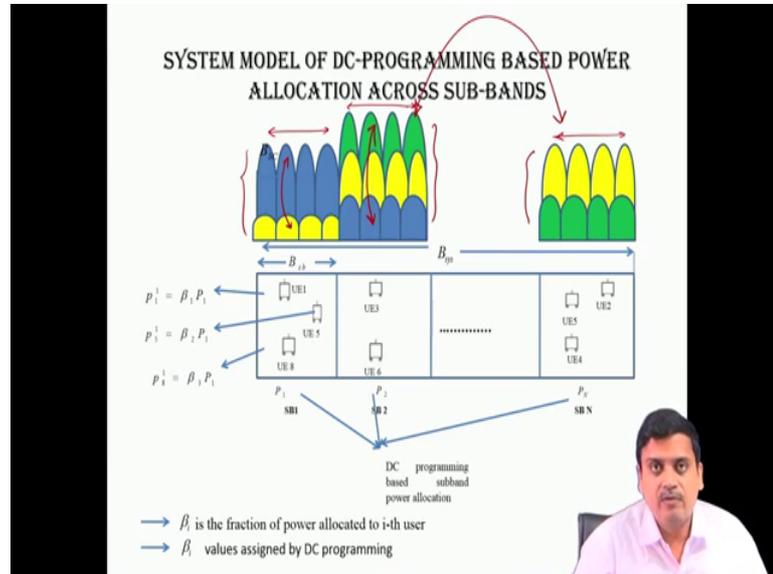
If you go beyond and you can also think of schemes whereby, different amount of powers maybe allocated to different bands to make it most generic and different number of users can be allocated to different bands.

So, this leads to a different problem; that means, you have to allocate users in each band you have to allocate powers to users in each band and you have to also allocate powers across the different bands there is a multiple level of allocation that has to be done.

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Figure-6: Geometric Mean User Throughput with respect to Number of Users

Figure-7: Mean System Throughput with respect Number of Users

Primarily the result that will show is based on our result what we can see is that, this is the one we have talked about some algorithm which is actually available in this particular paper below which says that the low complexity method which is available in the paper which uses a NOMA allocation mechanism is almost same as an uplink downlink allocation mechanism which is very very complicated mechanism and any of these NOMA schemes gives a much superior spectral efficiency compared to a OFDMA system and we have also seen the result as number of users increase. So, these the geometric mean that means the user throughput.

So, as the number users increased per user throughput drops, but some throughput keeps on increasing. So, as we see the increase the number of users, the mean system throughput what we find is pretty high and there is no difference in the uplink downlink duality scheme which is kind of mere optimal whereas, it is much better than the OFDMA based mechanisms.

So, this result is demonstrated based on a certain limiting a certain number of users per band. So, one can keep on doing better works and we have also come up with results, where we show that even better performance can be achieved if you are using this kind of mechanisms, but of course, that is dependent upon the certain number of users in the system.

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Figure-8: Mean System Throughput and Multi User Diversity
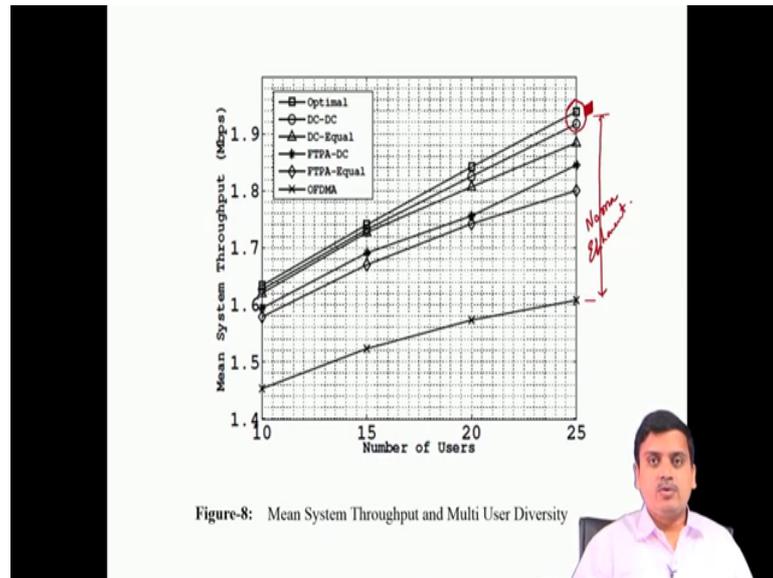
So, in summary what we find is that a mechanisms which use this multiple allocation of power across users user groupings, one can go very close to optimal power allocation, optimal power allocation and optimal user allocation would be exhaustive search of user allocation along with Optimal power allocation mechanism. Where as if we compare the performance with OFDM there is a significant enhancement by virtue of NOMA, which is a major reason why we are interested to use NOMA as an additional mechanism on top of whatever is available for the fifth generation communication system.

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So, of course, NOMA has its various challenges that have been listed in this particular list of work items that can be done for NOMA and what important part we should also mention is MIMO-NOMA, where a lot of active work is going on and one can expect to provide a good contribution in this particular area. So, in summary I mean if we look at all the things that we have been talking about. We have talked about the requirements in the initial few lectures, then we have talked about the various waveforms, we have talked about the frame structure of fifth generation, the particular OFDM numerology, how it started what was the original work.

We have also talked about the various other waveforms which where potential candidate technology for the fifth generation and how this can be enhanced so, that they can provide better performance compared to the scheme that is available till today in the current version of fifth generation that we are going to see.

We have talked about the propagation mechanisms, the different methods that are necessary to evaluate the performance, we have talked about the multiple antenna technologies, which provide us reliability as well as provides a high spectral efficiency, we have also talked about the different access mechanisms namely NOMA which can provide us enhancement in spectral efficiency. So, we have been able to cover at least some portion of the air interface which are expected to be part of the fifth generation as well as the air interface or the technologies which are going to come in the next version of fifth generation or even beyond fifth generation likely to be the sixth generation communication system.

So, we wish you all success in all your future endeavors and hope you would benefit from this course, and do look forward to your feedback by which we can keep on improving the content in future.

Thank you.