

Fundamentals of Electrical Engineering
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Lecture – 40
Single Phase AC Circuits (Contd.)

So, we are back again. So, I hope you have understood this.

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AS PHASORS :

$\Sigma x = \frac{1}{\sqrt{2}}(5 + 10 \cos 60^\circ) = \frac{1}{\sqrt{2}}10$
 $\Sigma y = \frac{1}{\sqrt{2}}(0 + 10 \sin 60^\circ) = \frac{1}{\sqrt{2}}8.66$
 $SUM = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \frac{1}{\sqrt{2}}\sqrt{10^2 + (8.66)^2} = 13.23 \cdot \frac{1}{\sqrt{2}}$
 $\alpha = \tan^{-1} \frac{\Sigma y}{\Sigma x} = \tan^{-1} \frac{8.66}{10} = 40.9^\circ$

SO $i = 13.23 \sin(\omega t + 40.9^\circ)$

WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS :

$\vec{I}_1 = \frac{5}{\sqrt{2}}(1 + j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$
 $\vec{I}_2 = 10 \angle 60^\circ$

So, whenever you will draw phasor diagram, it should be in rms value. Keep it in your mind. Do not take your peak value right. Do not take, it is we and the way I told leading lagging lagging little bit understanding (Refer Time: 00:35) AC circuit. Anyway, you understood this way everything will find too easy right. So, so next is this is your an exercise is given to you find out $i_1 - i_2$ by both methods. 2 methods I have shown and answer is also given here. This is your exercise, you will do it right.

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WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:
 $\vec{I}_1 = \frac{5}{\sqrt{2}}(1 + j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$ ✓ $i_2 = 10 \sin(\omega t + 60^\circ)$
 $\vec{I}_2 = \frac{10}{\sqrt{2}}(\cos 60^\circ + j \sin 60^\circ) = \frac{10}{\sqrt{2}}(\frac{1}{2} + j \frac{\sqrt{3}}{2}) = \frac{10}{\sqrt{2}} \angle 60^\circ$

MULTIPLICATION OF $\vec{I}_1 \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ \cdot \frac{10}{\sqrt{2}} \angle 60^\circ = 25 \angle 60^\circ$
 QUOTIENT OF $\vec{I}_1 / \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ / \frac{10}{\sqrt{2}} \angle 60^\circ = 0.5 \angle -60^\circ$

Ex. $i_3(t) \rightarrow$ $i_1(t)$
 $i_2(t)$ GIVEN: $i_1(t) = 71 \cos \omega t$
 $i_2(t) = 100 \sin(\omega t - \frac{\pi}{4})$
 FIND $i_3(t)$.
 SOLVE IT BY PHASOR DIAGRAM Ans: $50.7 \sin \omega t$

So now, expressing the current in rectangular or polar form right. Now, whatever these thing; so for example, for example, suppose I_1 is 5 by root 2 angle 0 and I_2 is equal to your what you call I_2 is equal to it was taken know that your 10 sin omega t plus 60 degree right. This we will taken; so angle 60 degree and this is a peak value. So, when you do represent when you are representing like phasor right, it will be 10 by root 2. It is this is the rms value and angle 60 degree right.

So, if you multiply this I_1 and I_2 , it is just for this thing. So, it will be 5 into 10 50 by 2; so 25 and 0 degree plus 60 degree. So, ultimately it will given 60 degree and if you divide I_1 by I_2 , it will be 5 by 2 angle 0 degree and 10 by 2 angle 60 degree. It will be 0.5 angle minus 60 degree. It is something like this suppose numerator if you have angle theta 1 and denominator.

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WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:
 $\vec{I}_1 = \frac{5}{\sqrt{2}}(1+j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$ $\frac{\theta_1}{\theta_2} = \frac{\theta_1 - \theta_2}{\theta_2}$
 $\vec{I}_2 = \frac{10}{\sqrt{2}}(\cos 60^\circ + j \sin 60^\circ) = \frac{10}{\sqrt{2}}\left(\frac{1}{2} + j \frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{2}} \angle 60^\circ$

MULTIPLICATION OF $\vec{I}_1 \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ \cdot \frac{10}{\sqrt{2}} \angle 60^\circ = 25 \angle 60^\circ$ ✓
 QUOTIENT OF $\vec{I}_1 / \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ / \frac{10}{\sqrt{2}} \angle 60^\circ = 0.5 \angle -60^\circ$

Ex. GIVEN: $i_1(t) = 71 \cos \omega t$
 $i_2(t) = 100 \sin(\omega t - \frac{\pi}{4})$
 FIND $i_3(t)$.
 SOLVE IT BY PHASOR DIAGRAM Ans: $50.7 \sin \omega t$

If you have angle theta 2, then this one will be angle theta 1 minus theta 2 right. So, this angle will be subtracted from this one. If you write angle in the numerator and your what you call in the numerator right.

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WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:
 $\vec{I}_1 = \frac{5}{\sqrt{2}}(1+j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$ $\frac{2 \theta_1}{5 \theta_2} = 0.4 \frac{\theta_1 - \theta_2}{\theta_2}$
 $\vec{I}_2 = \frac{10}{\sqrt{2}}(\cos 60^\circ + j \sin 60^\circ) = \frac{10}{\sqrt{2}}\left(\frac{1}{2} + j \frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{2}} \angle 60^\circ$

MULTIPLICATION OF $\vec{I}_1 \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ \cdot \frac{10}{\sqrt{2}} \angle 60^\circ = 25 \angle 60^\circ$ ✓
 QUOTIENT OF $\vec{I}_1 / \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ / \frac{10}{\sqrt{2}} \angle 60^\circ = 0.5 \angle -60^\circ$

Ex. GIVEN: $i_1(t) = 71 \cos \omega t$
 $i_2(t) = 100 \sin(\omega t - \frac{\pi}{4})$
 FIND $i_3(t)$.
 SOLVE IT BY PHASOR DIAGRAM Ans: $50.7 \sin \omega t$

Now, suppose if it is given for example, suppose 2 angle theta 1 divided by 5 angle theta 2. So, 2 by 5 is 0.4 angle theta 1 minus theta 2 right. This way you can write.

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WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:
 $\vec{I}_1 = \frac{5}{\sqrt{2}}(1 + j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$ $\frac{20}{5\sqrt{2}} = \frac{2}{\sqrt{2}}$
 $\vec{I}_2 = \frac{10}{\sqrt{2}}(\cos 60^\circ + j \sin 60^\circ) = \frac{10}{\sqrt{2}}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{2}} \angle 60^\circ$

MULTIPLICATION OF $\vec{I}_1 \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ \cdot \frac{10}{\sqrt{2}} \angle 60^\circ = 25 \angle 60^\circ$
 QUOTIENT OF $\vec{I}_1 / \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ / \frac{10}{\sqrt{2}} \angle 60^\circ = 0.5 \angle -60^\circ$

EX. $i_3(t) \rightarrow$ $i_1(t)$ $i_2(t)$ GIVEN: $i_1(t) = 71 \cos \omega t$
 $i_2(t) = 100 \sin(\omega t - \frac{\pi}{4})$
 FIND $i_3(t)$.
 SOLVE IT BY PHASOR DIAGRAM Ans: $50\sqrt{2} \sin \omega t$

Now, again let me clear it. Suppose 2 by angle theta 1 by 5 by angle theta 2. Suppose, this numerator angle theta 1 minus 3, but if it is denominator if you want, then it will be 2 by 5. Then, if you write this way, if you require it will be theta 2 minus theta 1, right.

So, the way you want. So, in this case, you are this case you are it is 5 by root 2 by 10 by root 2. It will be 0.5.

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WORK OUT THE SUBTRACTION OF ($i_1 - i_2$) BY BOTH METHODS
 Ans: $8.66 \sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:
 $\vec{I}_1 = \frac{5}{\sqrt{2}}(1 + j0) = \frac{5}{\sqrt{2}} \angle 0^\circ$ $\frac{\angle 0^\circ}{\angle 60^\circ} = \angle 0^\circ - 60^\circ = \angle -60^\circ$
 $\vec{I}_2 = \frac{10}{\sqrt{2}}(\cos 60^\circ + j \sin 60^\circ) = \frac{10}{\sqrt{2}}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{2}} \angle 60^\circ$

MULTIPLICATION OF $\vec{I}_1 \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ \cdot \frac{10}{\sqrt{2}} \angle 60^\circ = 25 \angle 60^\circ$
 QUOTIENT OF $\vec{I}_1 / \vec{I}_2 = \frac{5}{\sqrt{2}} \angle 0^\circ / \frac{10}{\sqrt{2}} \angle 60^\circ = 0.5 \angle -60^\circ$

EX. $i_3(t) \rightarrow$ $i_1(t)$ $i_2(t)$ GIVEN: $i_1(t) = 71 \cos \omega t$
 $i_2(t) = 100 \sin(\omega t - \frac{\pi}{4})$
 FIND $i_3(t)$.
 SOLVE IT BY PHASOR DIAGRAM Ans: $50\sqrt{2} \sin \omega t$

And it was actually angle 0 degree by angle 60-degree numerator 0-degree denominator 60 degree that was actually coming 0 degree minus 60 degree is equal to angle minus 60

degree. That is why it is minus 60 degree right. So, hope this is from your this is from your higher secondary mathematics in complex numbers chapter right class 11.

So, let me clear it. So, one numerical is given that we will solve it. This is one your numerical is given i 1 i 2 and i 3. You have to find out here you will put what you call KCL i 3 is equal to i 1 plus i 2 right. And accordingly you will do it. So, whatever things are given, it is given i 1 t is given this one i 2 t is given. This one you have to find out what you call i 3. The answer you have to find out find i 3 here it is given find i 3 t. So, answer is given 50 root 2 sin omega t. This we will do another exercise right.

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ELEMENTARY CIRCUITS:

i) PURELY RESISTIVE CIRCUIT

THE INSTANTANEOUS CURRENT THROUGH THE CIRCUIT

$$i_R = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

I_m IS THE MAXIMUM VALUE OF THE CURRENT GIVEN BY $I_m = \frac{V_m}{R}$.

RMS VALUE OF THE CURRENT

$$I_R = \frac{\text{RMS VALUE OF VOLTAGE}}{\text{RESISTANCE}} = \frac{V}{R}$$

$$= \frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$$

IN PHASOR NOTATIONS

Now, next is the steady state response of R L C circuit to sinusoidal input. So, we will considered 1 by 1: first, elementary circuit, a purely resistive circuit. Suppose, in this case, in this case circuit is purely polarity is instantaneous. This plus minus polarity is instantaneous right. And in this case, your voltage source V is equal to V m sin omega t. So, in that what you call you we have to find out I R and power loss etcetera right.

So, in that case, generally we write say i R is equal to V upon R. Because, this is the voltage V is equal to V m sin omega t and this is the R it is a resistive circuit. So, it is V is equal to V m sin omega t. So, it is V m upon R sin omega t and this is your is equal to you can write I m sin omega t and I m is equal to actually V m of upon R. This is the maximum value of the current. We are not making here rms value and other thing right now.

Now, rms value of the current is that I_R is equal to rms value of voltage by the resistance right. So, V_m is the maximum value. So, its rms value is equal to V_m by root 2. So, rms value of the current is your V_m upon root 2 divided by R , that is equal to actually I_m by root 2. Because, I_m is equal to V_m by R . So, here if you put I_m is equal to V_m by R it is I_m by root 2 right. This is what you call that current. Now, question is that that this is your V is equal to $V_m \sin \omega t$ V is equal to $V_m \sin \omega t$ right. Let me let me clear it again. Then it will be easier.

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ELEMENTARY CIRCUITS: $V = V_m \sin(\omega t)$

i) PURELY RESISTIVE CIRCUIT
 THE INSTANTANEOUS CURRENT THROUGH THE CIRCUIT

$i_R = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$

I_m IS THE MAXIMUM VALUE OF THE CURRENT GIVEN BY $I_m = \frac{V_m}{R}$

RMS VALUE OF THE CURRENT
 $I_R = \frac{\text{RMS VALUE OF VOLTAGE}}{\text{RESISTANCE}} = \frac{V}{R}$

$= \frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$

IN PHASOR NOTATIONS

Suppose, V is equal to $V_m \sin \omega t$, right; so, in this case, this is low angle associated that it is ωt plus 0 degree right. If you if you your what you call if you put it in a phasor form right. So, you take rms value. Therefore, we can write say V is V arrow you are putting is equal to V_m by root 2 angle 0 degree V_m by root 2 is rms value of the voltage right. Similarly, current also we have got I_R is equal to $I_m \sin \omega t$; that means, my is this current is I_R . This is moving like this current is I_R .

So, my I_R is equal to somewhere I_m writing here I_R is equal to my I_m by root 2 angle 0 degree. Because, here also no angle associated with that both V and I are angle 0 degree; that means, they are in the same phase. They are in the same phase, that is where it has been done. Here, they are in the same phase both I_R and V both are in the same phase right.

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I_m IS THE MAXIMUM VALUE OF THE CURRENT GIVEN BY $I_m = \frac{V_m}{R}$.
 RMS VALUE OF THE CURRENT $I_R = \frac{\text{RMS VALUE OF VOLTAGE}}{\text{RESISTANCE}} = \frac{V}{R}$
 $= \frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$

IN PHASOR NOTATIONS
 $\vec{V} = V \angle 0^\circ = V(1+j0) = V+j0$
 $\vec{I}_R = I_R \angle 0^\circ = I_R(1+j0) = I_R+j0$

CURRENT WILL BE IN PHASE WITH VOLTAGE. THE VOLTAGE AND CURRENT IN AC CIRCUIT IS RELATED BY IMPEDANCE FUNCTION. THE IMPEDANCE FUNCTION MUST TELL TWO IMPORTANT FACTS:
 a) THE RATIO OF V_m TO I_m (OR V TO I) AND

So, both angle 0. So, both are in the same phase. Now, that means, V is equal to we can write V angle 0. So, V is the magnitude, sorry V is the magnitude of the voltage. So, V an angle 0 means cos 0 plus j sin 0 cos 0 is 1 and sin 0 0. That is why, you can write V plus j 0. Similarly, for current I R angle 0; so cos 0 plus j sin 0 degree. So, that is becoming I R plus j 0 1 plus j 0 right. So, current will be in phase with the voltage. This is the current will be in phase in the voltage right. So, power will come will come next to power right. So, let me clear it.

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$= \frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$

IN PHASOR NOTATIONS
 $\vec{V} = V \angle 0^\circ = V(1+j0) = V+j0$
 $\vec{I}_R = I_R \angle 0^\circ = I_R(1+j0) = I_R+j0$

CURRENT WILL BE IN PHASE WITH VOLTAGE. THE VOLTAGE AND CURRENT IN AC CIRCUIT IS RELATED BY IMPEDANCE FUNCTION. THE IMPEDANCE FUNCTION MUST TELL TWO IMPORTANT FACTS:
 a) THE RATIO OF V_m TO I_m (OR V TO I) AND
 b) THE PHASE ANGLE BETWEEN THE WAVES OF VOLTAGE AND CURRENT (PHASORS \vec{V} AND \vec{I}).
 A SPECIAL TYPE OF NOTATION IS REQUIRED TO SIGNIFY THE TWO PROPERTIES OF IMPEDANCE FUNCTION e.g. $Z \angle \theta$.
 Z SIGNIFIES THE MAGNITUDE OF IMPEDANCE AND ANGLE θ SIGNIFIES THE RELATIONSHIP OF VOLTAGE & CURRENT.

So, in this case, current will be in phase with voltage. So, and your what you call then AC circuit is related by the impedance function. Now, question is that impedance function must tell 2 important fact; the ratio of V_m to I_m or V to I . That means, if you in AC circuit, you basically, we will term with impedance. So, impedance will be Z right, impedance will be Z right.

So, we will put Z bar a complex number is equal to you can write either V_m by I_m this one or is equal to you can write V_m by root 2 divided by I_m by root 2. That is, rms value rms value of the voltage rms value of the current or peak value of the voltage peak value of the current right. So, this is actually complex your what you call your complex number.

So, if Z impedance is not a phasor quantity, I will come later right. So, it will be either it will be rms value by rms value of voltage by rms value of current this magnitude. This is magnitude only. It is angle this is a magnitude only; so better not to bar now. So, it is magnitude only. Angle we will see later. So, it is it is actually either peak value of the voltage by peak value of the current the magnitude of the impedance or rms value of the voltage or rms value of the current that is what has been main said here right.

So, so it can be in the case of what you call resistive circuit, in the case of resistive circuit, this one actually only R is there. It is not impedance; actually is a complex quantity.

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THE INSTANTANEOUS CURRENT THROUGH THE CIRCUIT

$$i_R = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

I_m IS THE MAXIMUM VALUE OF THE CURRENT GIVEN BY $I_m = \frac{V_m}{R}$.

RMS VALUE OF THE CURRENT = $\frac{I_m}{\sqrt{2}}$

RMS VALUE OF VOLTAGE = $\frac{V}{R}$

$\frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$

IN PHASOR NOTATIONS

$$\vec{V} = V \angle 0^\circ = V(1+j0) = V+j0$$

$$\vec{I}_R = I_R \angle 0^\circ = I_R(1+j0) = I_R+j0$$

CURRENT WILL BE IN PHASE WITH VOLTAGE.

$Z = R + j0 = R$

But, in the case of R this is this is only R right. So, Z is equal to we can write R plus j 0 right. So, X is not there. Reactance part we will see later. So, X is not there.

So, Z is equal to here you can make Z is equal to R plus j 0 is equal to we can write Z angle 0. Because, what you call this is your 0 and this is R. So, it is complex number right. It is a complex number. So generally, you know that if a plus j b is the complex number right, then this angle theta is equal to tan inverse your b by a right this you know.

So, in that case, in that case, if it is made R plus your 0 j 0; that means, a is equal to R right and b is equal to 0. So, it will be 0 by R. So, theta is equal to 0 that is why angle is 0 right. So, that is why Z is equal to this Z is the magnitude and Z is equal to basically R only resistance.

So, this is your and this we represent no arrow. When we will go through it by, you know writing error, I will I try to rectify everywhere. Do not put arrow, that this is not a phasor quantity. It just to represent complex number you put Z bar. It is not a phasor quantity. If voltage and current both are phasor quantity, but Z is not a impedance, it is not a phasor quantity right.

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A SPECIAL TYPE OF NOTATION IS REQUIRED TO SIGNIFY THE TWO PROPERTIES OF IMPEDANCE FUNCTION eg. $Z \angle \text{ANGLE}$.
 Z SIGNIFIES THE MAGNITUDE OF IMPEDANCE AND ANGLE GIVES THE PHASE RELATION OF VOLTAGE & CURRENT.

THE IMPEDANCE FOR A PURE RESISTANCE IS $Z_R = R \angle 0^\circ$.

POWER: INSTANTANEOUS POWER $p = v i = V_m I_m \sin^2 \omega t$
 Since $\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$, $p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$

SO AVERAGE POWER $P = \frac{1}{T} \int_0^T p dt$

$$P = \frac{1}{T} \int_0^T \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t \right) dt$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V \cdot I$$

ENERGY PRODUCED IS CONVERTED INTO HEAT AND DISSIPATED.

So, next is this is the notation this is the notation right. Now, next that is here written example Z angle. So, everything is written the. Now, next is the power, the impedance

for a pure resistive circuit. Here, it will be bar only, no arrow right. Many places it is there. This is my writing error. So, please I am rectifying it.

So, this is actually R angle 0 . Whatever I told, you right Z is equal to R . R ; angle 0 . Now, instantaneous power because, AC circuit it is time bearing. So, instantaneous power p is equal to v into i right. So, v is equal to $V_m \sin \omega t$ and I is equal to $I_m \sin \omega t$. So, if you multiply this, it will be $V_m I_m \sin^2 \omega t$ right. This is thing or p is equal to that $\cos^2 \theta$ is equal to $1 - \sin^2 \theta$. Therefore, $\sin^2 \theta$ is equal to $1 - \cos^2 \theta$ by 2 .

So, here it is $1 - \cos^2 \omega t$ by 2 . So, if you do. So, it will become $V_m I_m$ by 2 minus $V_m I_m$ by $2 \cos^2 \omega t$. This you know. So, once if you done then the, so the average power you take p is equal to $\frac{1}{T} \int_0^T p dt$ right, the average power.

So, if you take average power, then this one you are this is actually this 2 if you look at the plot, this is a constant term $V_m I_m$ by 2 right. So, this is my $V_m I_m$ by 2 line. This is my $V_m I_m$ by 2 line. It is shown and another term is double frequency minus $V_m I_m$ by $2 \cos^2 \omega t$ right. So, this one is my minus $V_m I_m$ by $2 \cos^2 \omega t$. Now, when you plot together, if you plot together, this thick line, this thick line, this one is your p right, this is the p plot. So, when you try to your integrate over the total time interval the average power right. So, p is equal to your $\frac{1}{T} \int_0^T$ your $V_m I_m$ by 2 minus $V_m I_m$ by $2 \cos^2 \omega t dt$.

If you just integrate this, you will get it is V into I . V is the rms value and I is the rms value of the voltage and I is the rms value of the current right. If you integrate it using that relationship right, ω is equal to 2π by T . This relationship you will get V into I and it is V_m by $\sqrt{2}$ that is V and I is equal to I_m by $\sqrt{2}$; that means, my V is equal to V_m by $\sqrt{2}$ V is the rms value and I is equal to my I_m by $\sqrt{2}$ that is the rms value.

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AND CURRENT

A SPECIAL TYPE OF NOTATION IS REQUIRED TO SIGNIFY THE TWO PROPERTIES OF IMPEDANCE FUNCTION eg. $Z \angle \text{ANGLE}$. Z SIGNIFIES THE MAGNITUDE OF IMPEDANCE AND ANGLE GIVES THE PHASE RELATION OF VOLTAGE & CURRENT.

THE IMPEDANCE FOR A PURE RESISTANCE IS $Z_R = R \angle 0^\circ$.

POWER: INSTANTANEOUS POWER $p = v i = V_m I_m \sin^2 \omega t$

Since $\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$, $p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$

SO AVERAGE POWER $P = \frac{1}{T} \int_0^T p dt$

$P = \frac{1}{T} \int_0^T \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t \right) dt$

$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V \cdot I$

ENERGY PRODUCED IS CONVERTED INTO HEAT AND DISSIPATED.

So, basically, it is a v into i because pure resistive circuit. So, this is the power v into i and energy produced is converted into heat and dissipated right. Whatever energy you will produce, that will be dissipated in the resistor because, is a pure resistive circuit right. So, this is your v into i. So, for resistive circuit with AC supply V is equal to $V_m \sin \omega t$ power is equal to $V \cdot I$ is equal to p is equal to $v \cdot i$. But, in this case, V should be rms value of the voltage and I should be rms value of the current right.

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ii) PURELY INDUCTIVE CIRCUIT

Since $v = L \frac{di_L}{dt} = V_m \sin \omega t$

$di_L = \frac{V_m}{L} \sin \omega t dt$

$i_L = -\frac{V_m}{\omega L} \cos \omega t = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$

(constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

$i_L = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$

$= I_m \sin(\omega t - 90^\circ)$

$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = I_L \angle -90^\circ$

$\vec{V} = V \angle 0^\circ$

$\vec{I}_L = I_L \angle -90^\circ$

Next is that, next is a purely inductive circuit little bit little bit understanding. Here, we have to I mean, what I have done actually, this first taken one single element at a time such that, we will try to see our concept how is it right. So, next is the purely inductive circuit, purely inductive circuit.

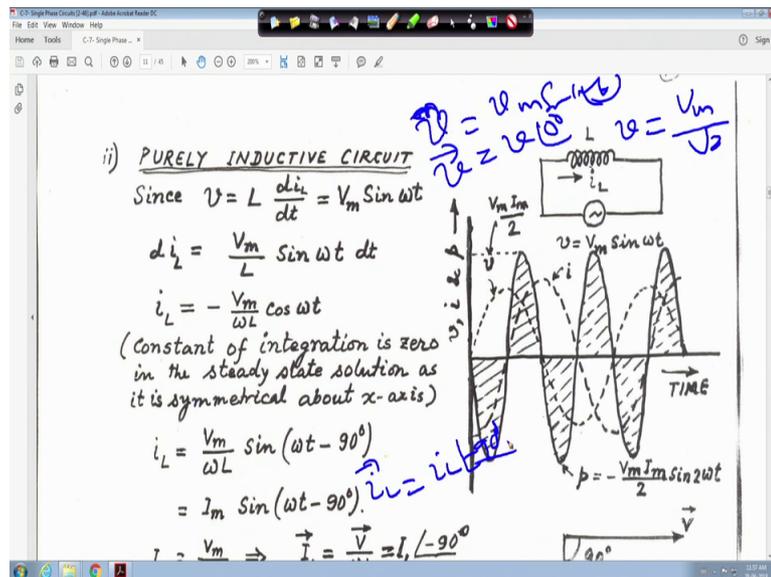
So, in this case, instantaneous polarity will be there. I have missed it. It is plus minus say, it is inductive circuit, it is L no other element is there and current flowing through this is i L and instant it and polarity instantaneous polarity right. So, in that case, we know that V is equal to $L \frac{di}{dt}$ right is equal and V is equal to it is given V is equal to $V_m \sin \omega t$.

So, $L \frac{di}{dt}$ is equal to $V_m \sin \omega t$ right. Therefore, $\frac{di}{dt}$ is equal $\frac{V_m}{L} \sin \omega t$ dt. If you integrate, if you integrate you will get i L is equal to minus $\frac{V_m}{\omega L} \cos \omega t$ right. So, this constant of integration is 0 in the steady state solution as it is symmetrical about X axis. So, there is no constant of your of integration right.

So, it is simply i L is equal to minus $\frac{V_m}{\omega L} \cos \omega t$ right. So, this i L is equal to minus $\frac{V_m}{\omega L} \cos \omega t$. You can write $\frac{V_m}{\omega L} \sin \omega t - 90^\circ$. How we are writing? So, let me let me clear it; so this one, this $\cos \omega t$. You know that this, this one you can write minus $\frac{V_m}{\omega L} \cos \omega t$, you can write $\frac{V_m}{\omega L} \sin \omega t - 90^\circ$ and this minus is there. When you go this sin term and when minus going inside, it will be $\omega t - 90^\circ$. That is why, i L is equal to $\frac{V_m}{\omega L} \sin \omega t - 90^\circ$ right.

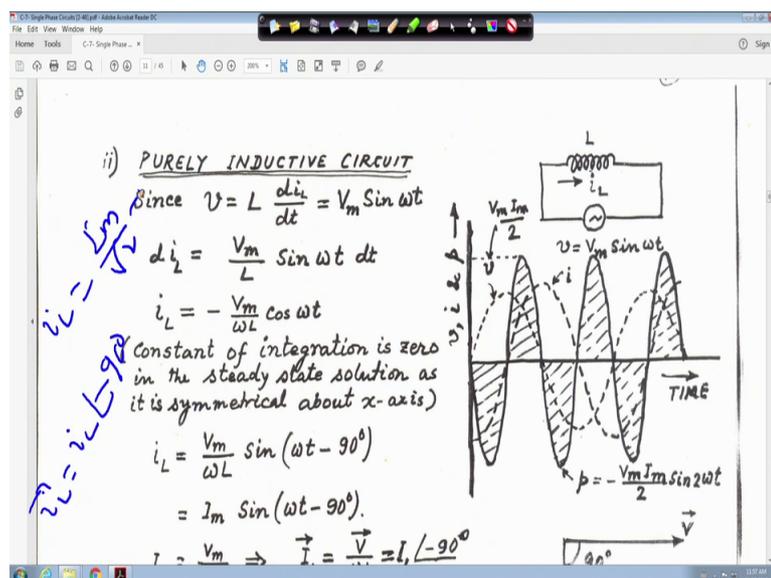
So, this one; that means, this one is equal to it can be written I_m is equal to $\frac{V_m}{\omega L} \sin \omega t - 90^\circ$. That means, this I_m is equal to actually $\frac{V_m}{\omega L}$ I_m is equal to $\frac{V_m}{\omega L}$ right.

(Refer Slide Time: 15:44)



So, let me clear it. So, in this case, in this case, your voltage is V is equal to $V_m \sin \omega t$. So, let me write it here V is equal to $V_m \sin \omega t$ right. No angle associated with that. If you put in this thing, V is equal to $V_m \sin \omega t$ right. V is the rms value, that is $V_m / \sqrt{2}$ and it should be angle 0 degree, say right and V is the rms value. It will be $V_m / \sqrt{2}$ right. Similarly, here current is equal to $I_m \sin \omega t - 90^\circ$.

(Refer Slide Time: 16:28)



So, this i_L ; it can be written it can be written as say i_L . This is phasor quantity say i_L angle minus 90 degree. It is angle minus 90-degree right. Just, just hold on, let me write here. This i_L this i_L is this is phasor quantity. If you write this is the rms value angle minus 90 degree and i_L is equal to it is I_m by root 2 right; that means, it is minus 90 degree.

(Refer Slide Time: 16:49)

$$i_L = -\frac{V_m}{\omega L} \cos \omega t$$
 (Constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

$$= \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

$$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = I_L \angle -90^\circ$$

$$\vec{V} = V + j0 \quad \vec{I}_L = 0 - jI_L \quad \vec{Z}_L = \frac{\vec{V}}{\vec{I}_L} = \omega L \angle 90^\circ$$

THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L = 2\pi f L$.

IN A PURELY INDUCTIVE CIRCUIT CURRENT LAGS BEHIND THE VOLTAGE BY 90° .

That means let me clear it. That means, from here, this diagram, I will come to later; that means, this is my voltage, this is my voltage phasor, that is V angle 0 and current, I told you it will be here. It is written here I_L minus 90 degree.

So, current lags from the voltage by 90 degree from purely inductive circuit and here it should not be arrow it should not be arrow it should be just Z bar the complex number. Now, that means if you look like this right. For example, if you if you look like this v is equal to your V rms value. Say, angle 0 degree and I right, I is equal to I_L angle minus 90 degree right.

So, this is my your this thing, then Z actually, Z actually V arrow divided by your I arrow. That is, whatever it is written here right, whatever is written here, this is we are putting Z_L . So, put Z_L is equal to V and this thing if you do. So, and you know that V is equal to what you call V angle 0 right and I is equal to you know this is rms. It is what you call, it is rms value.

So, I this i_L is equal to your V , your V_m . This is rms value, that is V_m by root 2 and this one also this angle is minus 90 degree. That means, this is I can make just hold on let me clear it. Let me clear. Otherwise becoming clumsy just becoming.

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$$i_L = -\frac{V_m}{\omega L} \cos \omega t$$
 (Constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

$$i_L = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

$$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = I_L \angle -90^\circ$$

$$\vec{V} = V + j0, \vec{I}_L = 0 - jI_L, \vec{Z}_L = \frac{\vec{V}}{\vec{I}_L} = \omega L \angle 90^\circ$$

THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L = 2\pi fL$.

IN A PURELY INDUCTIVE CIRCUIT CURRENT LAGS BEHIND THE VOLTAGE BY 90° .

Diagrams include:

- A graph of current i_L vs TIME showing a sine wave lagging behind a voltage wave.
- A phasor diagram showing \vec{V} on the positive x-axis and \vec{I}_L on the negative y-axis, with a 90° angle between them.
- A complex plane diagram showing $\vec{Z}_L = \omega L \angle 90^\circ$ on the positive imaginary axis.

So, my here I am writing my V arrow divided by I arrow that is my Z_L right is equal to I can write that V_{rms} right. It is V divided by my I_L angle minus 90 degree right. This is my I_L angle minus 90 degree. So, V is equal to V_m by root 2 and I_L is equal to that is rms value I_m by root 2; that means, V_m by your root 2 right. Then, this is root 2 then ωL this actually is equal to ωL angle 90 degree right. So, this is ωL angle 90 degree; that means my Z bar will be ωL angle 90 degree right.

So, same thing same thing here has been made that is this is also put bar. So, V (Refer Time: 19:21) ωL angle 90 degree. Little bit you do, little bit practice is required initially right. So, there should not be any confusion anywhere. Wherever Z bar I have written, it is by mistake. Put simply bar to represent the complex number. Because, Z is not in what you call phasor quantity. So, it is simply complex number right. Real part and imaginary part (Refer Time: 19:35). We see later. So, this is actually ωL 90 degree; that means, for inductive circuit then, let me clear it.

(Refer Slide Time: 19:45)

$$i_L = -\frac{V_m}{\omega L} \cos \omega t$$
 (Constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

$$i_L = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

$$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = \vec{I}_L / -90^\circ$$

$$\vec{V} = V + j0 \quad \vec{I}_L = 0 - jI_L \quad \vec{Z}_L = \frac{\vec{V}}{\vec{I}_L} = \omega L / 90^\circ$$

THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L$

IN A PURELY INDUCTIVE CIRCUIT CURRENT LAGS THE VOLTAGE BY 90° .

So, for inductive circuit, your what you call this is my your Z L; that means, Z L is equal to your L omega. Sometimes, we call X L that is the reactance of the your what you call of the circuit only reactant. So, inductive reactant L omega and omega is equal to 2 pi f. Therefore, my X L is equal to 2 pi f into L right.

Later, when we solve the numerical you will know that. So, this is actually your what you call the magnitude of the above impedance is omega L is called inductive reactance I represented by X L is equal to L omega is equal to 2 pi f into L. Everything is written there here notes actually little bit clumsy because, very closely written here. But, whenever you read it, if you have anything just simply you zoom it. That is all right.

So, next is this that; that means, if you look into this, power will come. Now, if you look into this, that this is my this dash line is the voltage, this is the voltage waveform. Because, V is equal V m sin omega t. So, start from it is starting from 0 and in the case of I, it is lagging by 90 degree. That is why, I is starting from here, I is starting from here right. This is I is lagging by 90 degree.

And this is the expression of I sin omega t minus 90 degree right. When omega t is 0, I m is equal to sin minus 90 degree, so minus I m. So, here it is starting. This is your minus I m. I is starting from here right. So, let me clear it.

(Refer Slide Time: 21:19)

$\vec{V} = V + j0$, $I_L = 0 - jI_L$, $Z_L = \frac{V}{I} = \omega L \angle 90^\circ \downarrow I_L$
 THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L = 2\pi fL$.
 IN A PURELY INDUCTIVE CIRCUIT CURRENT LAGS BEHIND THE VOLTAGE BY 90° .
 POWER & ENERGY:
 INSTANTANEOUS POWER = $p = vi = [V_m \sin \omega t][I_m \sin(\omega t - 90^\circ)]$
 $= V_m I_m (-\sin \omega t \cos \omega t)$
 $= -\frac{V_m I_m}{2} \sin 2\omega t$
 AVERAGE POWER
 $P = \frac{1}{T} \int_0^T p dt = 0$
 THE AMOUNT OF ENERGY DELIVERED TO THE CIRCUIT DURING A QUARTER OF A CYCLE $\frac{T}{4}$
 $W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2} \sin 2\omega t dt$
 NOTE:
 POWER VARIATION IS A DOUBLE FREQUENCY VARIATION (2ω).
 THE AVERAGE POWER ABSORBED IS EQUAL TO ZERO. THE IMPLICATION IS THAT THE INDUCTIVE ELEMENT RECEIVES ENERGY FROM THE SOURCE.

So, next is power. Here power graph is there will come first you come here now. So, that means, in a pure that what I told purely inductive circuit, current lags behind the voltage by 90 degree. This we show got it from phasor diagram the instantaneous power instantaneous power p is equal to $v i$. In a inductive circuit, V is equal to $V_m \sin \omega t$ and you will got I is equal to $I_m \sin \omega t - 90$ degree.

That means, $V_m I_m$ and if you it will be minus $\sin \omega t$ into $\cos \omega t$, because, it is your $\sin \omega t$ minus 90 degree. Take minus \sin out. It will be there. So, it will be $\sin 90$ degree minus ωt . So, that is $\cos \omega t$. That is why, minus $\sin \omega t \cos \omega t$.

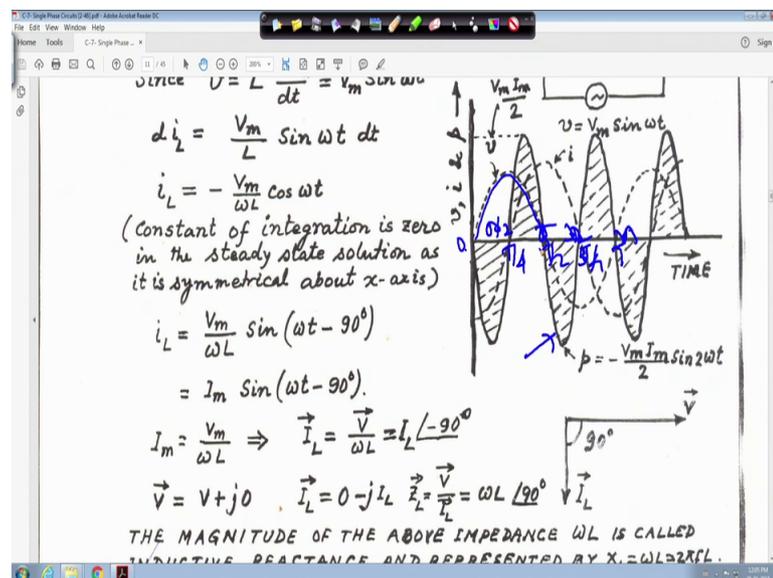
So, this was your multiplying numerator and denominator by 2. So, it will be minus $V_m I_m$ by 2 into $2 \sin \omega t \cos \omega t$ that is, $2 \sin \theta \cos \theta$ that is $\sin 2 \theta$. That is why, it is written minus $V_m I_m$ by 2 $\sin 2 \omega t$. This is p is equal to $v i$. Now, the average power average power 1 to T if you then it will become 0. This you put and integrate it. Use this relationship always $\int \sin x dx = -\frac{1}{x} \cos x$ right. This relationship is always used.

So, if you make average power for inductive circuit $\frac{1}{T} \int_0^T p dt$, it will find 0 because, there is no resistor here. So, there is no question of power in the in purely inductive circuit right. So, let me delete it. So now, in that case if you look that power, this is at this is at double frequency right.

So, if you look into that, that your power variation is double frequency variation of 2 omega. This is 2 omega because, it is 2 omega right. The average power absorbed is equal to 0; that means, the implication is that the inductive element receives energy your from the sources during 1 quarter of a cycle of the applied voltage and returns your exactly the same amount of energy to the driving sources during the next quarter your what you call quarter of a cycle. It is a double frequency.

So, let me clear it. Come to this figure. Now, if you come to this figure, this is your power expression, this is thick line is your power expression right. This thick line is the power expression.

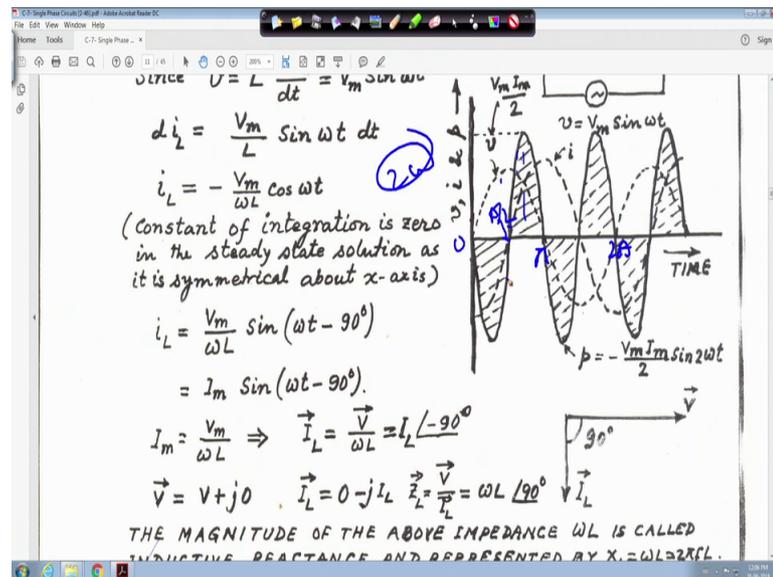
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So, if it is 0 it will be T by 4, it is T by 2, it is 3 T by 4 and it is T. I mean, that means, if it is if you take this one, this is 0, this is what you call pi by 2. This is pi and this is your 3 pi by 2 and this is your 2 pi right.

So, if you look into that, that that this current or voltage. It is actually computing half cycle in your in between 0 and pi right. Let me clear it for.

(Refer Slide Time: 24:16)



Suppose, this is 0, this is pi and this (Refer Time: 24:22) this is 2 pi. So, if you look and look at this point, it is your what you call pi by 2 right. So, whereas current or voltage whatever it is take. For example, voltage it is completing half cycle in pi 0 to pi. Whereas, this power actually it is completing in what you call complete 1 cycle in what you call in pi. Because, frequency is 2 omega right double frequency; that means, in what; that means, this each one is a quarter your what you call your quarter cycle.

So, this each part, it is receiving energy another cycle another quarter. It is what you call sending back. It is your what you call delivering back the sources right. So, 1 1 quarter cycle, it will absorbed another quarter cycle, it will return.

(Refer Slide Time: 25:06)

The image shows a whiteboard with handwritten mathematical derivations and notes. The text is as follows:

AVERAGE POWER
$$P = \frac{1}{T} \int_0^T p \, dt = 0$$

THE AMOUNT OF ENERGY DELIVERED TO THE CIRCUIT DURING A QUARTER OF A CYCLE
$$W_L = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t \, dt$$

$$= \frac{V_m I_m}{2 \left(\frac{4\pi}{T}\right)} \left[\cos \frac{4\pi}{T} t \right]_{T/4}^{T/2}$$

$$= \frac{V_m I_m}{2\omega} \quad (\because \omega = \frac{2\pi}{T})$$

$$= \frac{(\omega L I_m) I_m}{2\omega} = \frac{1}{2} L I_m^2$$

NOTE:
POWER VARIATION IS A DOUBLE FREQUENCY VARIATION (2ω). THE AVERAGE POWER ABSORBED IS EQUAL TO ZERO. THE IMPLICATION IS THAT THE INDUCTIVE ELEMENT RECEIVES ENERGY FROM THE SOURCE DURING ONE QUARTER OF A CYCLE OF THE APPLIED VOLTAGE AND RETURNS EXACTLY THE SAME AMOUNT ENERGY TO THE DRIVING SOURCE DURING THE NEXT ONE-QUARTER OF A CYCLE.

So, let me clear it. So, that is what I have written here. What it has been written here that, the average power absorbed is equal to 0 that is here it is given. You please do this integration. It is simple thing. The implication is that the inductive element receives energy from the source during 1 quarter of a cycle. I told you that because, in 0 to pi, it is completing 1 cycle. So, that means, in 2 in 2 pi, it is completing 2 cycles because it is a double frequency.

So, that is why, it receives energy from the source during 1 quarter of a cycle of the applied voltage and returns exactly the same amount of energy to driving source during the next 1 quarter of the cycle right. And the amount and the amount of energy delivered to this thing circuit during this.

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$$di_L = \frac{V_m}{L} \sin \omega t dt$$

$$i_L = -\frac{V_m}{\omega L} \cos \omega t \quad T = 2\pi$$
 (Constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

$$i_L = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

$$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = I_L \angle -90^\circ$$

$$\vec{V} = V + j0 \quad \vec{I}_L = 0 - jI_L \quad \vec{Z}_L = \frac{\vec{V}}{\vec{I}_L} = \omega L \angle 90^\circ$$
 THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L = 2\pi fL$.

So, we will take it integration will take $T/4$ to $T/2$ why, if you come to this if you come to this, this is if we take this is 0, then this point is $T/4$ and this point is $T/2$ and this is $3T/4$ and this is T . T is equal to if you take this way, T is equal to 2π right. So, this is; that means, we have to integrate from $T/4$ to $T/2$. Therefore, right.

(Refer Slide Time: 26:21)

THE AMOUNT OF ENERGY DELIVERED TO THE CIRCUIT DURING A QUARTER OF A CYCLE

$$W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2} \sin 2\omega t dt$$

$$= \frac{V_m I_m}{2 \left(\frac{4\pi}{T}\right)} \left[\cos \frac{4\pi}{T} t \right]_{T/4}^{T/2}$$

$$= \frac{V_m I_m}{2\omega} \quad (\because \omega = \frac{2\pi}{T})$$

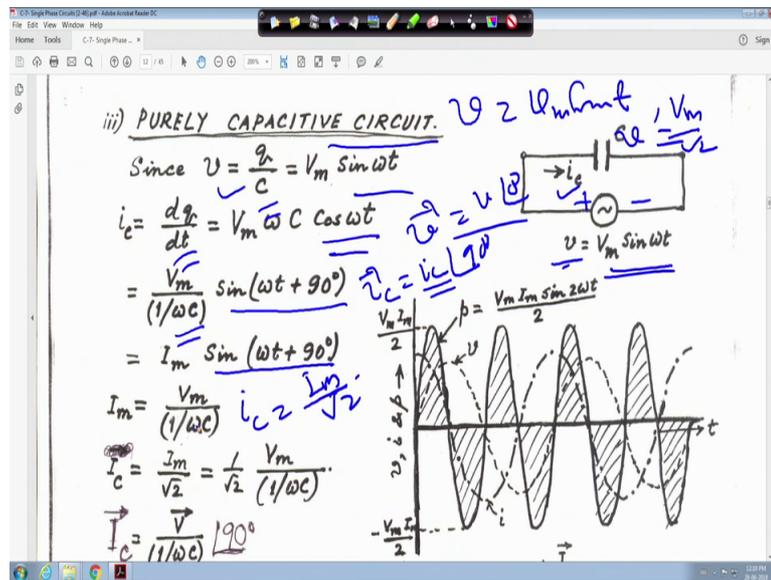
$$= \frac{(\omega L I_m) I_m}{2\omega} = \frac{1}{2} L I_m^2$$
 POWER VARIATION IS A DOUBLE FREQUENCY VARIATION (2ω). THE AVERAGE POWER ABSORBED IS EQUAL TO ZERO. THE IMPLICATION IS THAT THE INDUCTIVE ELEMENT RECEIVES ENERGY FROM THE SOURCE DURING ONE QUARTER OF A CYCLE OF THE APPLIED VOLTAGE AND RETURNS EXACTLY THE SAME AMOUNT ENERGY TO THE DRIVING SOURCE DURING THE NEXT ONE-QUARTER OF A CYCLE.

Therefore, if you do, so, if we do, so, the $T/4$ to $T/2$ and it is minus minus $V_m I_m$ by 2 sin 2 omega t dt, if you integrate and simplify use this relationship omega is equal to 2π by 2, it will be simply half $L I_m^2$. That means, half L right into this I_m^2

is equal to you can write L into I m by root 2 square; that means, L into I rms square right.

So, whatever we know that in an AC circuit, we know that energy stored is half L I m square. So, if somebody says half L I square means, it is R m peak value, not the rms value. If you take the rms value, it will be L into I rms square right. That is why, it is half L I m square right.

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Similarly, similarly, purely capacitive circuit right, purely capacitive. Same philosophy will be applied. Same philosophy you follow the way inductor just it will be opposite right. So, in this case, in the case of purely capacitive circuit; we know q is equal to C V from DC circuit analysis we have seen. So, V is equal to q by C is equal to V is equal to V m sin omega t. Same thing we have taken and instantaneous polarity is there plus minus right and that means, if you take; that means, i C is equal to dq by dt. You know that.

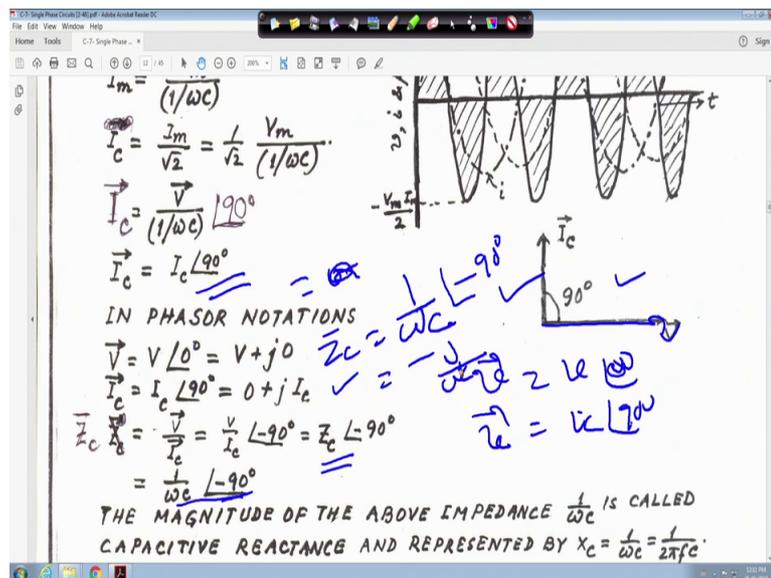
Therefore, this one should be V m omega C cos omega t right. This is what you call, if you to take the derivative of this one that i is equal to dq by dt. So, it will be V m your omega C cos omega t.

So, you can write sin 90 degree plus theta is equal to cos theta. Therefore, you can write this 1 V m upon I C is equal to V m upon 1 upon omega C sin omega t plus 90 degree

right. That means, my V is equal to my V is equal to $V_m \sin \omega t$ right; that means, my rms value is V_m by root 2 that is my V is equal to V_m by root 2 right. That means, my phasor notation. It will be $V \angle 0^\circ$. That means, this one you can write. Similarly, for current, we got $I_m \sin \omega t$.

So, this one in the current notation say I_c , we are writing this current is I_c , we are writing I_c . So, i_c is equal to we can write $i_c \angle 90^\circ$ because it is coming $\omega t + 90^\circ$. Therefore, that I_c actually is equal what you call this I_c is equal to somewhere here I am writing that I_c is equal to I_m by root 2, because, this is rms value. Whenever you write phase value that it is rms value. So, I_c is equal to I_m by root 2 and this is I_m is given V_m upon ωC right.

(Refer Slide Time: 29:22)



So, similar way if you proceed right. You will get that what you call you will get your Z . Similarly, the way the way I have mention. So, in this case, if you draw V . Actually, I told you V is equal to $V \angle 0^\circ$ and I_c is equal to your $I_c \angle 90^\circ$. So, in this case, current is plus 90 degree and this is 0 degree. So, this is my reference phasor V and this is I . So, I is leading V for purely capacitive circuit by 90 degree or other way V lags from I_c by 90 degree right.

So, this is my phasor diagram right and similar way, if you move similar way, if you move, you will see that your Z_c actually is equal to ωC by 9 angle minus 90 degree in this case right. The way I have done for inductive circuit, same way you do it.

You will get the; that means, my this is Z_C is equal to the magnitude that capacitive reactance is equal to $1/\omega C$ and its angle is your what you call minus 90 degree.

So, put bar; that means, it is actually 0, it is actually forget about 0. It is actually minus j upon ωC . Say, $\cos 90$ degree minus $j \sin 90$ degree.

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$$I_m = \frac{V_m}{(1/\omega C)}$$

$$I_c = \frac{I_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{V_m}{(1/\omega C)}$$

$$I_c = \frac{V}{(1/\omega C)} \angle 90^\circ$$

$$\vec{I}_c = I_c \angle 90^\circ$$

IN PHASOR NOTATIONS

$$\vec{V} = V \angle 0^\circ = V + j0$$

$$\vec{I}_c = I_c \angle 90^\circ = 0 + j I_c$$

$$\vec{Z}_c = \frac{\vec{V}}{\vec{I}_c} = \frac{V}{I_c} \angle -90^\circ = Z_c \angle -90^\circ$$

$$= \frac{1}{\omega C} \angle -90^\circ$$

THE MAGNITUDE OF THE ABOVE IMPEDANCE $\frac{1}{\omega C}$ IS CALLED CAPACITIVE REACTANCE AND REPRESENTED BY $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$.

$$\angle 90^\circ = \cos 90^\circ - j \sin 90^\circ$$

$$\frac{1}{\omega C} \angle 90^\circ = \frac{-j}{\omega C}$$

Let me clear it. Angle minus 90 degree is equal to $\cos 90$ degree minus $j \sin 90$ degree is equal to minus j and multiplied by this ωC . So, it will be j by ωC . So, you are $1/\omega C$ angle minus 90 degree is equal to minus j by ωC right. This is j minus j by ωC . So, this is actually the magnitude. Actually ωC is called the capacitive reactance right.

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CAPACITIVE REACTANCE AND REPRESENTED BY $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
 IN A PURELY CAPACITIVE CIRCUIT THE CURRENT LEADS THE VOLTAGE BY 90° .
POWER & ENERGY:
INSTANTANEOUS POWER

$$p = v i = [V_m \sin \omega t] [I_m \sin(\omega t + 90^\circ)]$$

$$= V_m I_m \sin \omega t \cos \omega t = \frac{1}{2} V_m I_m \sin 2\omega t$$
 POWER VARIATION IS AGAIN A DOUBLE FREQUENCY VARIATION (2ω) AND HENCE AVERAGE POWER ABSORBED IS EQUAL TO ZERO.

$$P = \frac{1}{T} \int_0^T p dt = 0$$

HERE, ALSO, THE CAPACITOR RECEIVES ENERGY FROM THE SOURCE DURING THE FIRST QUARTER OF A CYCLE AND RETURNS THE SAME AMOUNT DURING THE SECOND QUARTER CYCLE, etc.

$$W_c = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t dt$$

$$= \frac{1}{2} \frac{V_m I_m}{\omega} = \frac{1}{2} C V_m^2$$

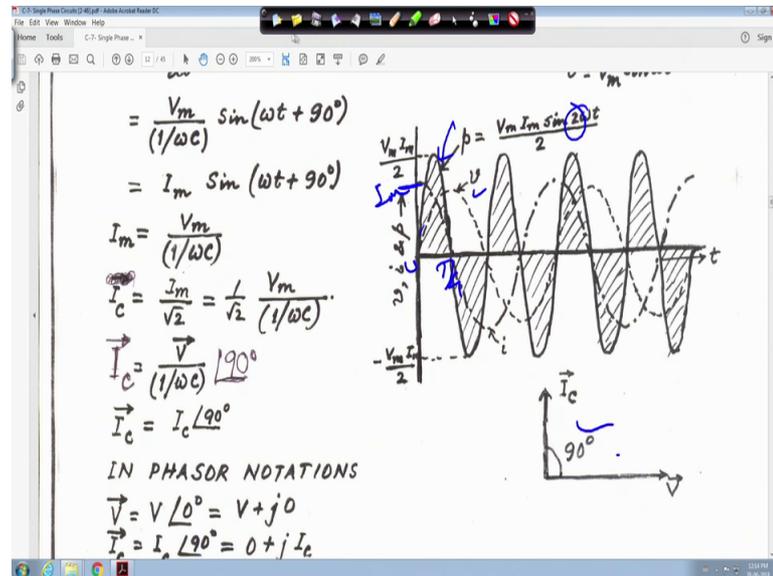
So, this ωC is actually called that you are what you call that that capacitive reactance right and represented as X is equal to 1 upon ωC and ω is equal to $2\pi f$. So, 1 upon $2\pi f C$.

So, in purely capacitive circuit, the current leads the voltage by 90 degree. I showed you and power and energy right. So, just let me clear it. So, the power and energy the in a purely capacitive, your what you call that in your instantaneous power same as before p is equal to $v i$. If you make $V_m \sin \omega t$ into $I_m \sin \omega t + 90$ degree and if you just simplify this one, this one you make it, it will be $V_m I_m \sin \omega t$. Because, $\sin 90$ degree plus $\omega t \cos \omega t$ multiply numerator and denominator by 2 . So, it will be $2 \sin \theta \cos \theta$. So, 2 is $\sin \omega t \cos \omega t$. So, $\sin 2\omega t$, right.

And if you take the power P is equal to $\frac{1}{T} \int_0^T p dt$ average power, if you take, it will become 0 right. So, power variation is again double frequency because, this is actually double frequency 2ω right and is equal to 0 because there is no resistor there. So, where power will be, there will be no power. So, average power will be 0 , but if you take the here also, the capacitor receives energy from the source during the first quarter of the cycle and returns to the same amount during the second quarter of cycle.

For example, for example, you come to this what you call this phasor this diagram. This is power plus this is voltage current here in the in the case of current is leading. That is why, when this is this is your I_m this is your maximum on if the current.

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And current is leading by 90 because reaching is faster than the voltage. So, current is leading 90 degree. This is I_c is equal to what you call peak value I_m , that is V_m upon $1/\omega C$ and power is that double frequency. So, if it is 0, it is T by 4 then T by 2 and so on right.

So, same philosophy; so current in what you call in 2π , it will make one cycle. But in 2π , power will make 2 cycles. It will complete 2 cycles right. So, in this case if you do this 0 to T by 4 that only quarter cycle 0 to 2 by your what you call $4 V_m I_m$ by 2 $\sin 2\omega t$ dt. If you integrate this, it will become half $C V_m$ square. You know the store energy in capacitor as we have studied in DC half $C V$ square.

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THE VOLTAGE BY 90°.

POWER & ENERGY:

INSTANTANEOUS POWER

$$p = v i = [V_m \sin \omega t][I_m \sin(\omega t + 90^\circ)]$$

$$= V_m I_m \sin \omega t \cos \omega t = \frac{1}{2} V_m I_m \sin 2\omega t.$$

POWER VARIATION IS AGAIN A DOUBLE FREQUENCY VARIATION (2ω) AND HENCE AVERAGE POWER ABSORBED IS EQUAL TO ZERO.

$$P = \frac{1}{T} \int_0^T p \, dt = 0$$

HERE, ALSO, THE CAPACITOR RECEIVES ENERGY FROM THE SOURCE DURING THE FIRST QUARTER OF A CYCLE AND RETURNS THE SAME AMOUNT DURING THE SECOND QUARTER CYCLE, etc.

$$W_e = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t \, dt$$

$$= \frac{1}{2} \frac{V_m I_m}{\omega} = \frac{1}{2} C V_m^2$$

$\frac{1}{2} C V_m^2 = C \cdot \frac{V_m^2}{2}$

It is becoming half $C V_m^2$; that means, half $C V_m^2$ square right; that means, this one can be written $C V_m^2$ by root 2 square.

That means, is equal to $C V_{rms}^2$ square right; that means, in AC circuit, whenever you talk about for your $C V_m^2$, $C V_m^2$ square means your V is the peak value right or you have to make C into V_{rms}^2 square. That is, the energy stored in AC circuit and the capacitive circuit right. So, this is a purely capacitive circuit. So, what we have learnt that for purely resistive circuit power absorbed is, we have seen that is v into i right. In the in the case of a purely inductive circuit, power absorb is 0 average power absorb is 0.

Similarly, for capacitive circuit right, but energy in the pure inductive circuit; it will be half $L I_m^2$ square and for the capacitive 1. It will be half $C V_m^2$ square. This integration is very simple. You can do yourself.

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$$P = \frac{1}{T} \int_0^T p \, dt = 0$$

$\omega = \frac{2\pi}{T}$

HERE, ALSO, THE CAPACITOR RECEIVES ENERGY FROM THE SOURCE DURING THE FIRST QUARTER OF A CYCLE AND RETURNS THE SAME AMOUNT DURING THE SECOND QUARTER CYCLE, etc.

$$W_e = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t \, dt$$
$$= \frac{1}{2} \frac{V_m I_m}{\omega} = \frac{1}{2} C V_m^2.$$

But, all the time you need to use this relationship sorry omega is equal to 2 pi by T. This there is a relationship you will use right, with this.

Thank you very much. We will be back again.