Fundamentals of Electrical Engineering
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Lecture - 27
Capacitors & Inductors (Contd.)

So come back to the next one hope we are understanding the things right and thinking all these points I am solving many problems for you, it will help you a lot right to understand that all the all these things whatever is there.

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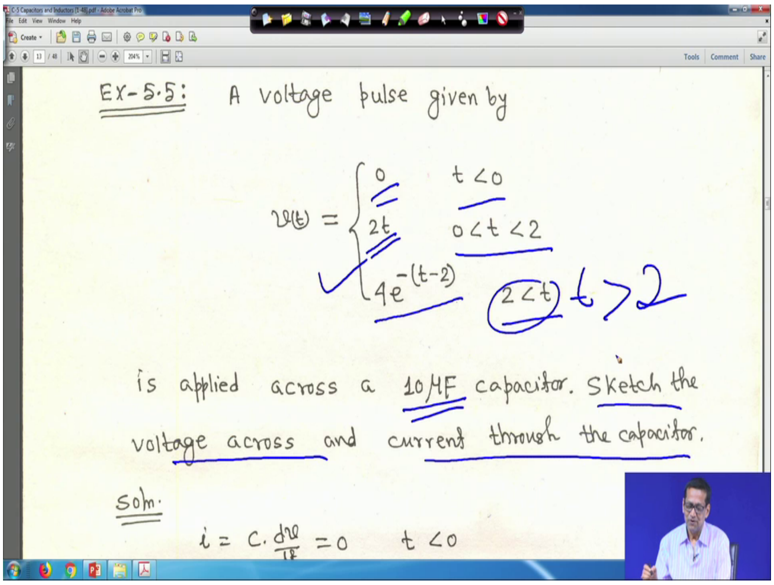

And next we will take a voltage pulse given by this right. So, it is given that v t is equal to 0 for t less than 0; that means, pulse history right. And when v t is equal to 2 t that will linear relationship with time in between 0 and 2 second right and your another thing is given that v t is equal to 4 into e to the power minus t minus 2, when t greater than 2. It is 2 less than t and other this way you can when t greater than 2 right.

So, this way here what you what you call is applied across the 10 microfarad capacitor. I mean this kind of voltage applied to 10 microfarad capacitor for this time duration. Those sketch the voltage across and current through the capacitor; that means, you have to sketch the voltage across the capacitor as soon as current passing through the capacitor, this you have to sketch right.

Now, this voltage is given. So, this voltage you can easily sketch right from you can sketch from 0 to infinity, because t greater than 2; that means, in between 2 t 0 2 here what you call the 2 0 2 to 2 and then this exponential thing for 0 to 2 to infinity right when you put later will see this. So, first let me clear it right. So, we know that is 10 microfarad capacitor right C value is given 10 microfarad now i is equal to C into dv by dt so for t less than 0. So, it is it is given v t is 0 for t less than 0 that is pulse history. So, i is equal to 0 right.

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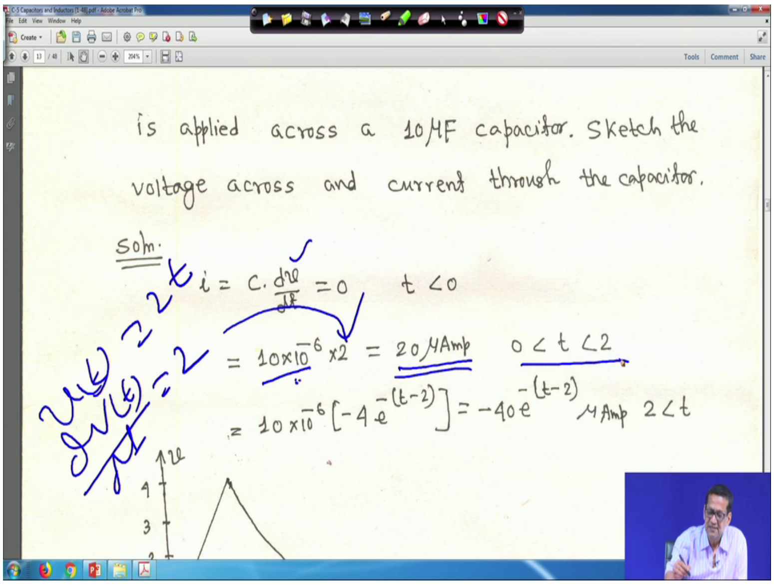

Now, next one is that in between 0 and 2 that is C is 10 into 10 to the power minus 6 microfarad it is given and it is your v t that your v t is equal to 2 t; that means, my dv t by dt is equal to 2 and that 2 is here that 2 is here that C into dv by dt right it is 2.

So; that means, and C is actually 10 micro farads so 10 into 10 to the power minus 6 your farad. If you multiply this will be 20 micro your ampere your what you call the current right in between that interval 0 and 2. And now next one is similarly let me clear it, similarly that dv by dt you take that derivative of this one just 1 minute; that this is v t is given this expression is given 4 e to the power minus t minus 2 you take the derivative of this one right.

So, if you take the derivative of this one, it will become minus 4 into e to power minus t minus 2 right and C is 10 into 10 to the power minus 6. So and is equal to minus 40 to the power minus t minus 2 microampere for t greater than 2 right. So, this way you calculate all vs are given just take the derivative. Now question is that voltage plot this is my in between your what you call v is equal to 2 t just one minute let me move little bit up right. So, if we move little bit up right this v t. So, here this is my v t actually it will be 2, 2 will be somewhere here right.

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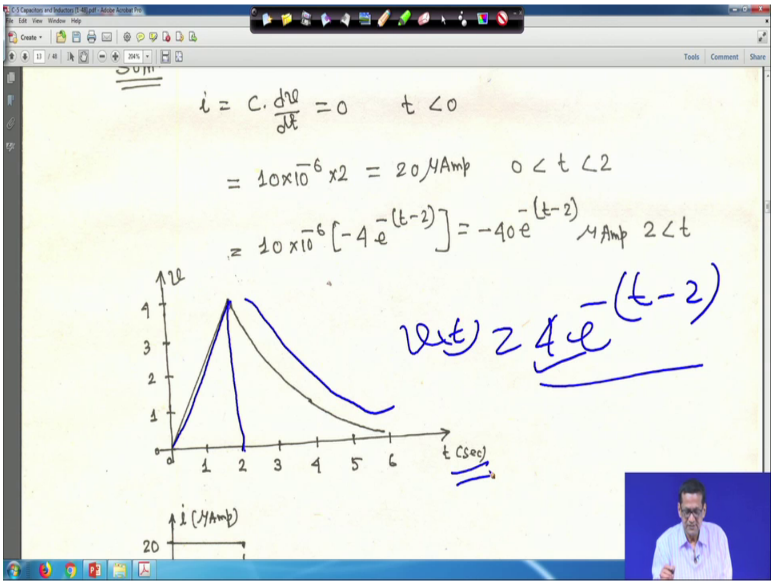

So, it is v t is equal to 2 t and next one your what you call and from 2 this in between 0 and 2 and when t greater than 2 your v t this was given, 4 e to the power minus t minus 2 this was given and this is the plot of your that v t this one that is given in the first problem right. So, this is your plot of v t this is t second right.

So, this is v t and similarly if you plot i t for i for your for t less than 0 it is 0, otherwise for in between 0 to 2 it is 20 micro your ampere and in and if it is greater than t is greater than 2 right.

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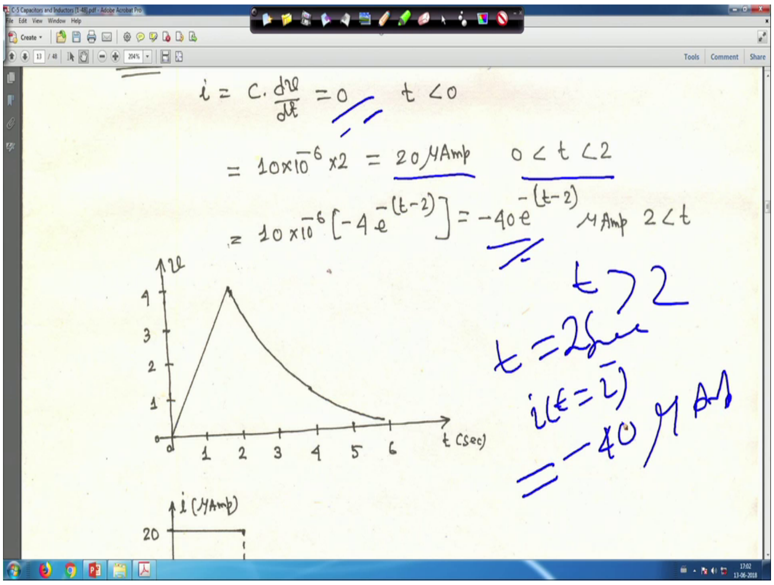

So, it is minus 40 to the power minus t minus 2. Now when t is equal to 2 suppose when t is equal to 2 second right then this your i t is t is equal 2 is equal to it will be minus 40, because e to the power 0 it will become when you put 2, it will be minus 40 microampere at t is equal to 2 right.

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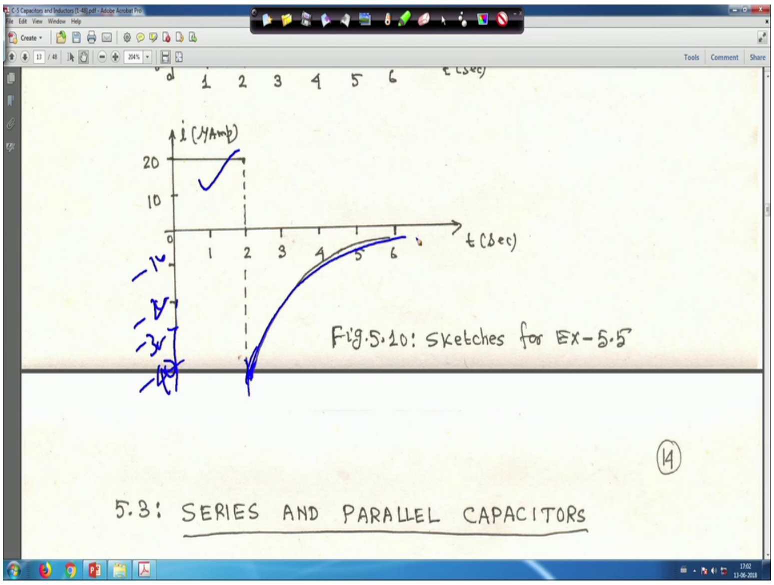

So; that means, first one if you see that that 20 microampere, it is basically your constant. So, in between 0 to 2 right let me move little bit up. So, this is your 0 to 2 that is your 20 microampere. Now plot is not completed this is your minus 10, minus 20, then minus 30 then it will be somewhere minus 40 right and it will start from minus 40 somewhere and then it will go like this right.

So, this is your stage of what you call that current. So, that is what we are and your what you call this equation from this equation, from this equation, from this equation this is the plot right. So, this will hope you have understood this right.

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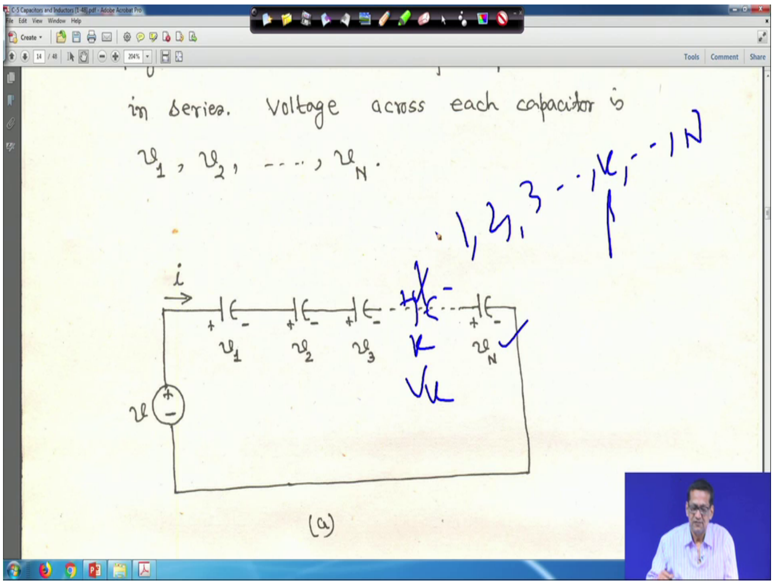

Next one is series and parallel capacitors. So, figure 11 shows n number of capacitors are connected in series. Suppose you have this figure you have n number of capacitors and across each capacitor is voltage is v 1 v 2 v 3 up to v n in between some kth capacitor that is also there right. I mean some if you make v 1 v 2 v 3 like this somewhere some plus some dot dot dot dot dot some kth capacitor is also there and voltage bearing is v k right. So and n is the last one right.

I mean it is 1 2 your then 3 up to kth capacitor then nth. So, somewhere kth capacitor is there right. So, let me clear it. So, all of these capacitors are in series. So, now and total voltage there is v. So, from that one thing is there let me write down for you, for this one thing is there that your v is equal to v 1 plus v 2 plus v k plus v n right this is your what you call that voltage v this is the supply voltage v right.

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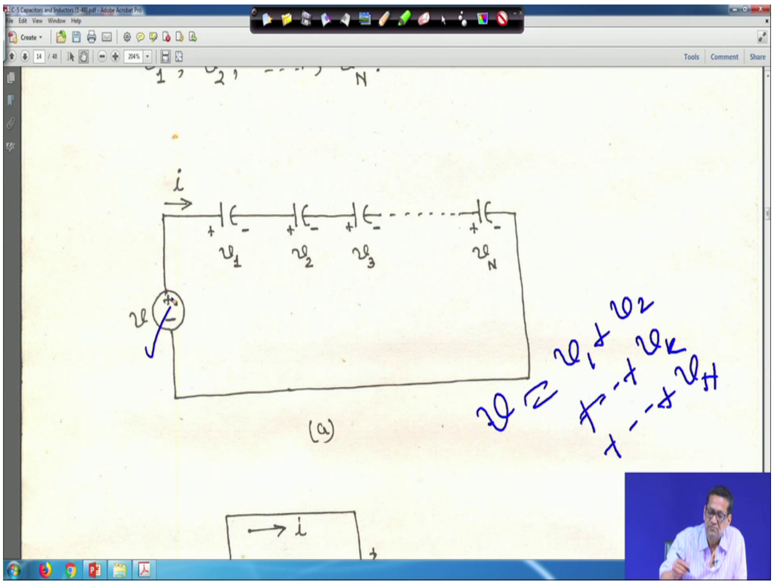

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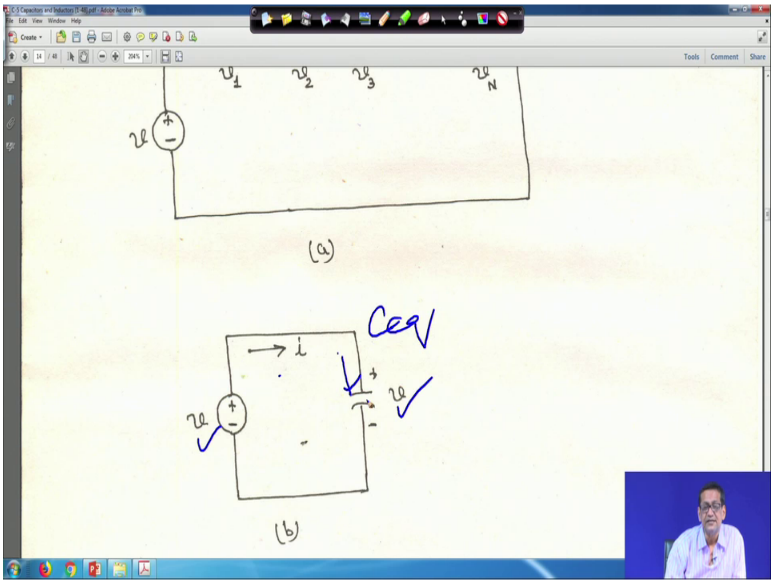

Now, let me clear it. So now if you want to make here one here what you call that all this is equivalent circuit so whatever voltage is applied at this capacitor will be say it is C eq, and all and all these voltage all these voltage is v. So, nearer so naturally only current flowing is i naturally voltage across this also will be v right and this capacitor is C eq equivalent circuit. So, let me clear it.

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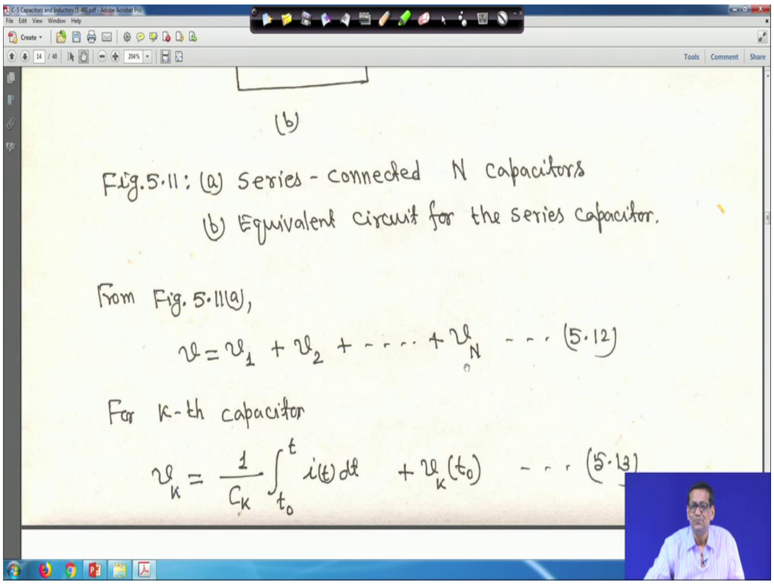

So, this is series connector equivalent circuit of the series capacitor, this equivalent circuit and this is C eq right. So, v is equal to I told you v 1 plus v 2 up to v n

Now, for kth capacitor we know the voltage v is equal to 1 upon c t 0 to t i t dt right we know that so earlier we have studied this. So, this one instead of v we are writing kth capacitor v k, and value of kth capacitor is C k 1 upon C k time is t 0 to t it dt plus v k t 0 right. I told you in the pasts at some times minus infinity some time that volt here what you call voltage across the capacitor in this voltage across the capacitor was 0. So, that is why v k minus infinity was 0 only v k t 0 is kept here right.

So, this is equation 13. So, this is for v k; that means, v 1 is equal to 1 upon C 1 t 0 to t i it what you call dt plus v 1 t 0. Similarly v 2 is equal to 1 upon C 2 then t 0 is equal to your t, i t 0 to t i t dt, then v 2 t 0 like this because all the capacitors are in series.

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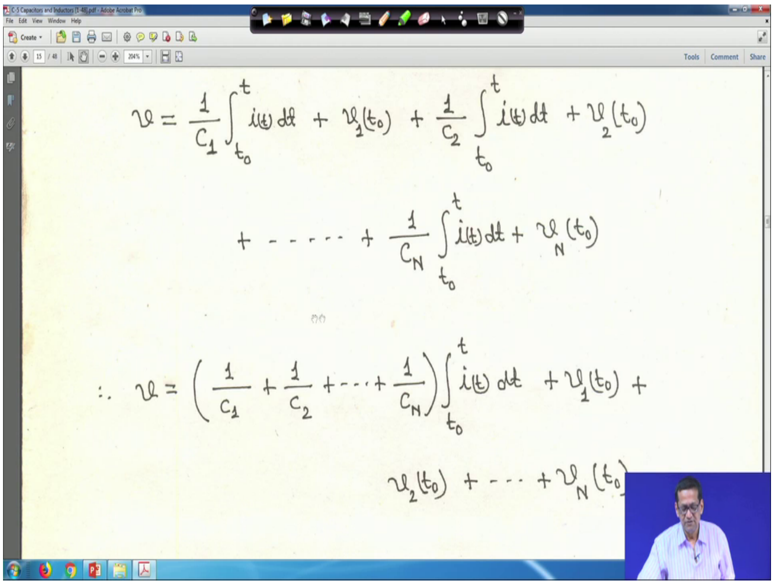

So, let me clear it therefore, we can write v is equal to 1 upon C1 t 0 to t i t dt plus v 1 t 0 this is for capacitor 1.

Similarly, for capacitor 2 it is 1 upon C 2 t 0 to t i t dt plus v 2 t 0. So, similarly for nth capacitor up to n 1 upon C N t 0 t i t dt plus t n t 0 right. Now your now you next a term you can write you take 1 upon C 1 plus all collect all the term t 0 to i t dt right therefore, 1 upon C 1 plus 1 upon C 2 up to 1 upon C N into bracket close into t 0 to t i t dt plus v 1 t 0 plus v 2 t 0 up to v N t 0.

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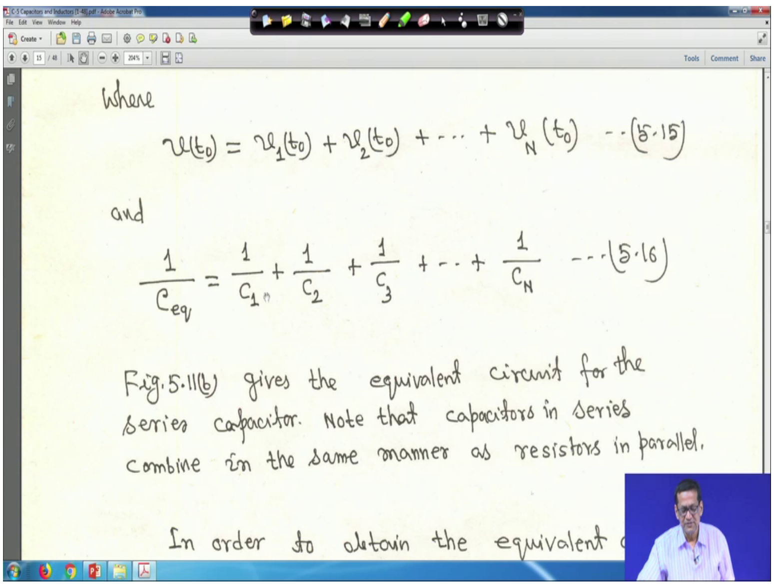

You can write or you can write 1 upon C eq; that means, C eq equivalent t 0 to t i t dt plus v t 0 this is equation 14. Now v t 0 is equal to v 1 t 0 plus v 2 t 0 plus v N t 0 right whatever is there because circuit is like this, this is the circuit all initial at t is equal to t 0 it is v 1 t 0 v 2 t 0 v 3 t 0 and that means, v t 0 here will be v 1 t 0 v 2 t 0 sum upon right.

So, here it is your that is your v t 0 is equal to all these thing therefore, if you compare this equation 14 with equation your with this equation right so; that means, 1 upon C eq is equal to 1 upon C 1 plus 1 upon C 2 up to 1 upon C N that is all summation of reciprocal, it is something like this when we are obtaining your resistance in parallel. So, same thing, but capacitor when capacitor will be in series that calculation will the way you made parallel resistor your combination when you are trying to find out equivalence resistance for parallel resistor, for series connection of capacitor it is just your what you call just opposite to that yeah opposite to the resistors of parallel that here what you call the you obtain mathematically.

So, it gives the equivalence circuit for the series capacitor, note that capacitors in series combine in the same manner of resistance in parallel. The way we find out equivalent resistance in parallel right for capacitor we can come back compute same way when they are in series; that means, here you have capacitor here what you call here you have capacitor C 1 C 2 C 3 then C N and this capacitor is C equivalent right.

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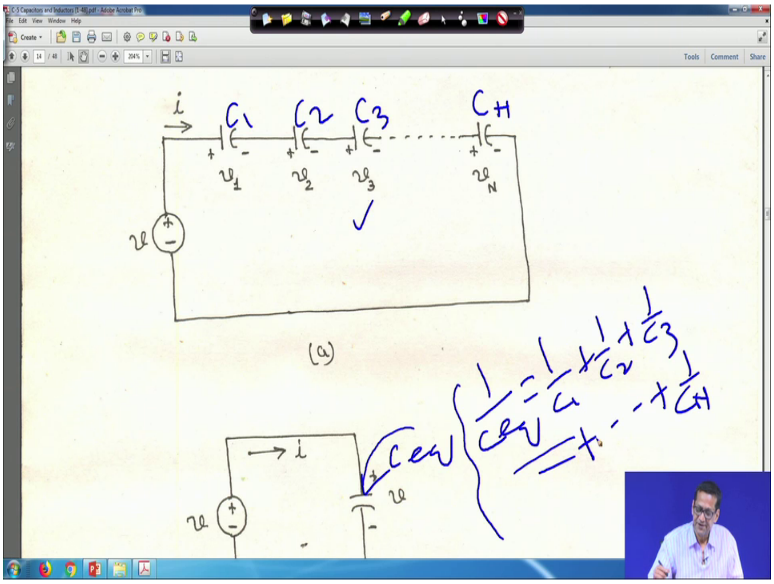

Therefore 1 upon C eq when capacitors are in series, it is 1 upon C 1 plus 1 upon C 2 plus 1 upon C 3 plus 1 upon C N right. I mean it is it is same calculation the way you calculated register in parallel, capacitor also you have to calculate same way when they are in series right.

So, this is and this capacitor is your C eq and 1 upon C eq is equal to this one right. So, let me clear it. So, that is that we have made right. So, from that we can find out if you know C 1 C 2 C 3 and all you can calculate C eq right.

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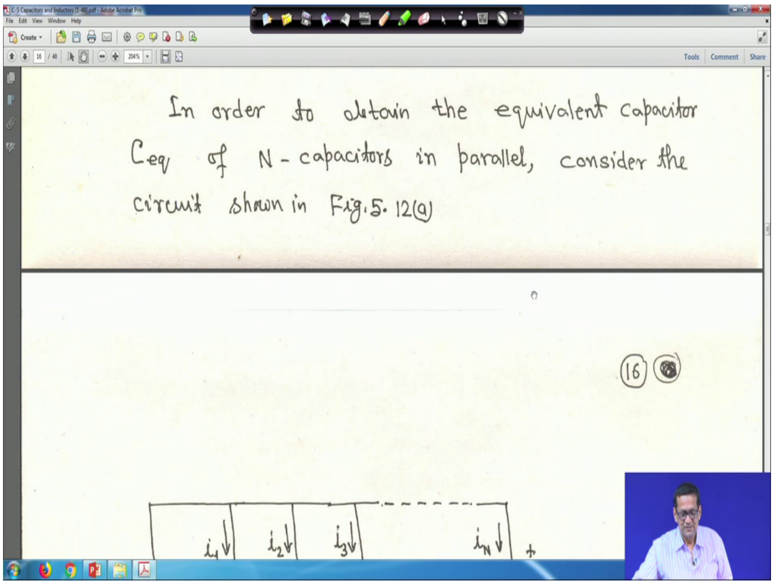

So, this is what you get for series connection.

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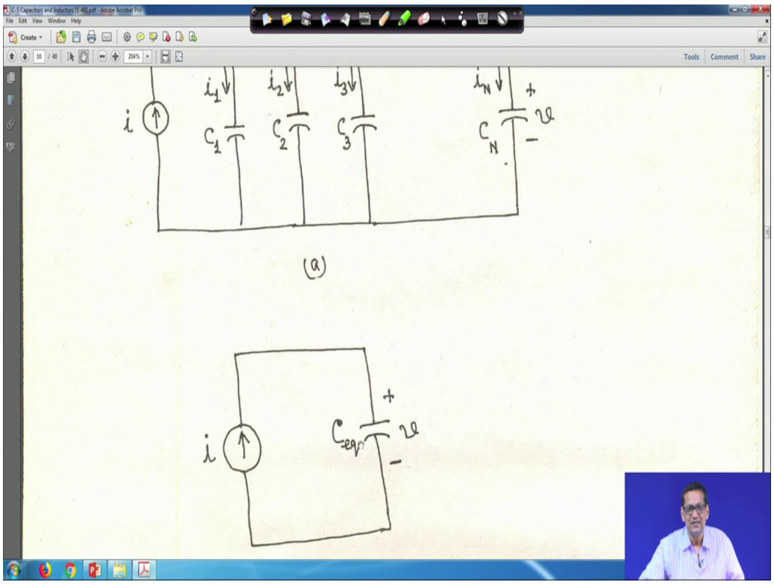

Now, if capacitors are connected in parallel right it will be your what you call when they are connected in parallel it will be like same manner when you calculate resistors in series right. In this case current source is taken and just for analysis right and C 1 C 2 up to C N capacitors are in parallel right and voltage v. So, all they are all are in parallel. So, at volt C 1 C 2 C 3 what you call this voltage is given v because all are in parallel right.

So; that means, voltage remain same and this is C eq equivalent for parallel connection. So, if you go for mathematics if you that i is equal to this current i, i is equal to i 1 plus i 2 up to i N right.

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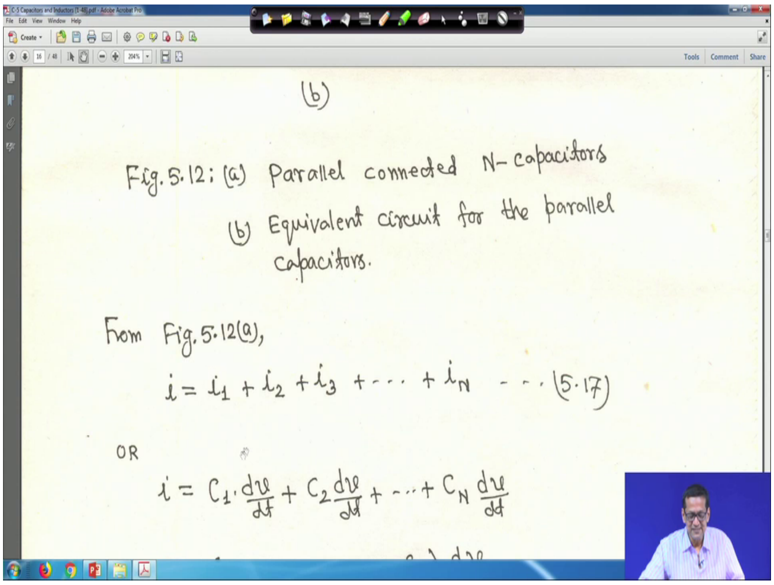

So, this current i is equal to i 1 plus i 2 plus i 3 up to i N and i 1 is equal to C 1 into dv by dt because C 2 because voltage down in parallel. So, voltage v will be same because all the capacitor right. So, that is why it will be C 1 into dv by dt plus C 2 into dv by dt up to C N into dv by dt or you take.

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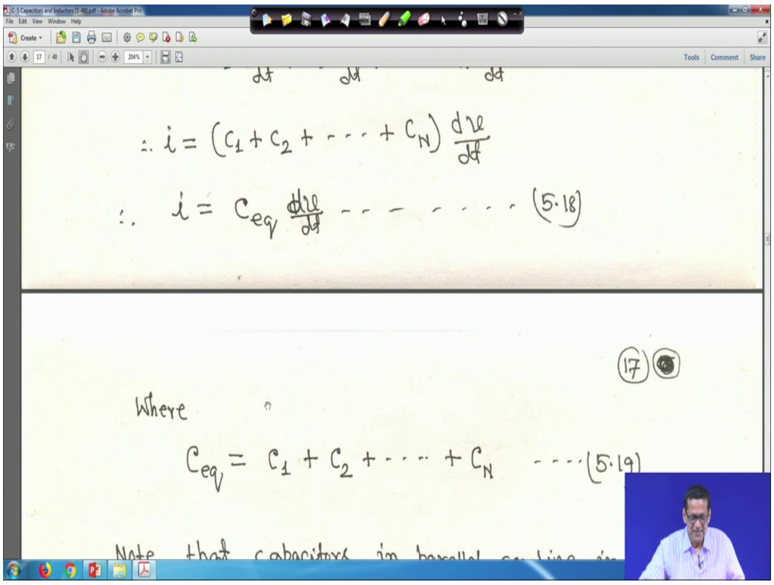

I can write we can write I C 1 plus C 2 up to C N dv by dt or i is equal to C eq dv by dt; that means, C eq is equal to C 1 plus C 2 up to C N sum it up; that means, when resistor are in the series we are summing r 1 plus r 2 up to r n say when the capacitors are in parallel at that time their value will be that thing will be summed up right. Just your what you call it is the way the in a manner when you add the resistance in series in that case of the capacitors are in parallel you have to just add them up right.

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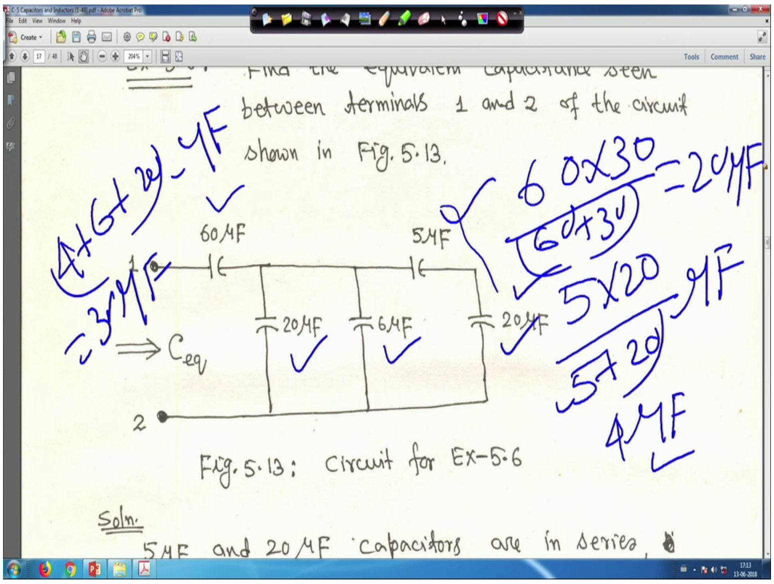

So, note that capacitors in parallel combining the same manner as resistor in series. So, at and this is the here what you call this is the equivalent circuit and C eq is equal to C 1 plus C 2 plus C 3 up to plus C N right. So, if you take this example find the equivalent capacitance between your we have seen between terminals 1 and 2 of the circuit you have to find out the equivalent capacitor load. Here if you look in the circuit that 5 microfarad and 20 microfarad are in series; that means, you have to combine it like the way you do it when resistors are in parallel.

So, it will be equivalent will be 5 into 20 right divided by 5 plus 20 because these 2 are in series microfarad right. So, it will be basically your it is your 25 and 100. So, it will become 4 microfarad right. So, equivalent will be 4 microfarad therefore, what will happen that their equivalent is 4 microfarad those; that means, this 4 microfarad then 6 microfarad and 20 microfarad are in parallel, parallel means just a combination just addition. So this equivalent of this 4 micro farad so its parallel means it is a combination. So, it is 4 plus 6 plus 20 microfarad that is your 30 microfarad right and just if you add it, it will be like this right.

Therefore and with that this 60 and 30 again are in here what you call your what you call are in series. So, at that time you have to go for like a parallel combination right. So, this will be 30 micro farads and again we will see 60 and 30 if you have make it add it up because they are in parallel so they are in series. So, at that time it will be 60 into 30 by 60 plus 30 that is 90; that means, it will be your 20 microfarad equivalent will be 20 microfarad that will be C eq, that will be the answer.

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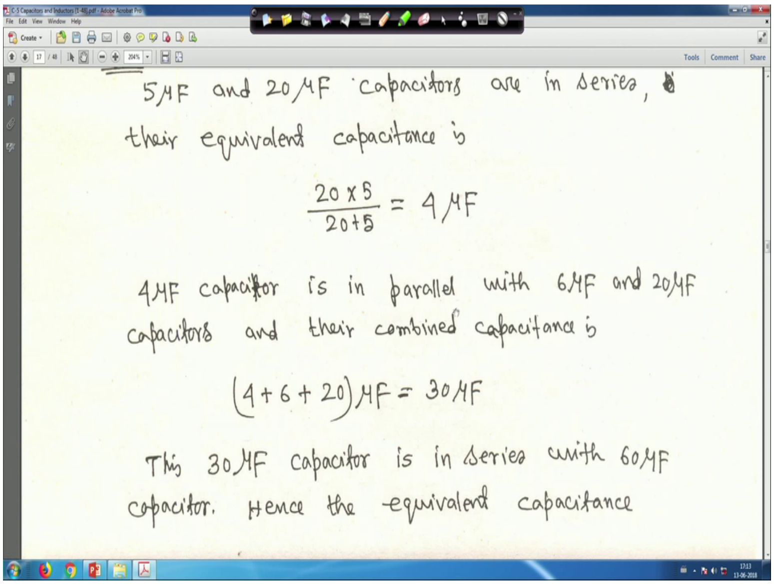

So, solution is there in there later right. So, if you look into this 20 and 5 are in parallel series. So, it will be 4 microfarad. Now 4 microfarad is in parallel with 6 microfarad and 20 microfarad capacitor all are written here and their combine will be 30 microfarad you add them up, now 30 microfarad capacitor in series with 60 microfarad capacitor.

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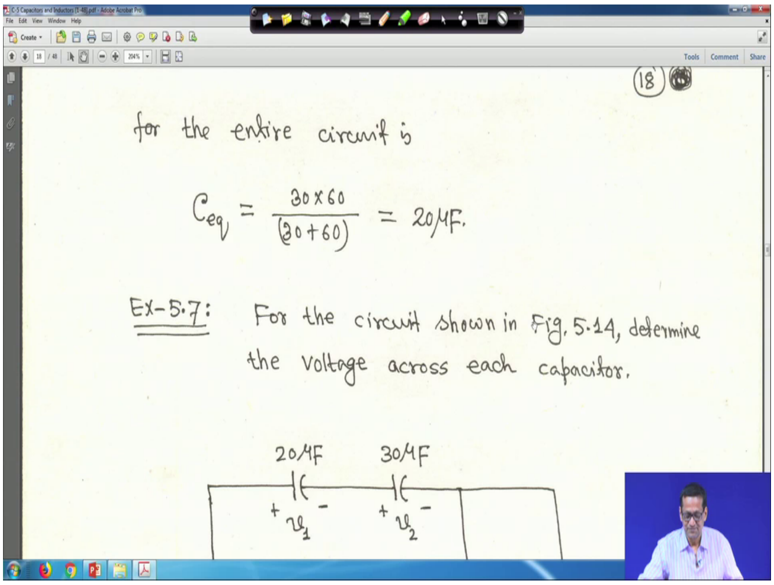

Therefore you have to again series means it is like a your find equivalence in parallel, for same way for series one you have to make it for capacitor. So 30 into 60 by 30 plus 60 so 20 microfarad right.

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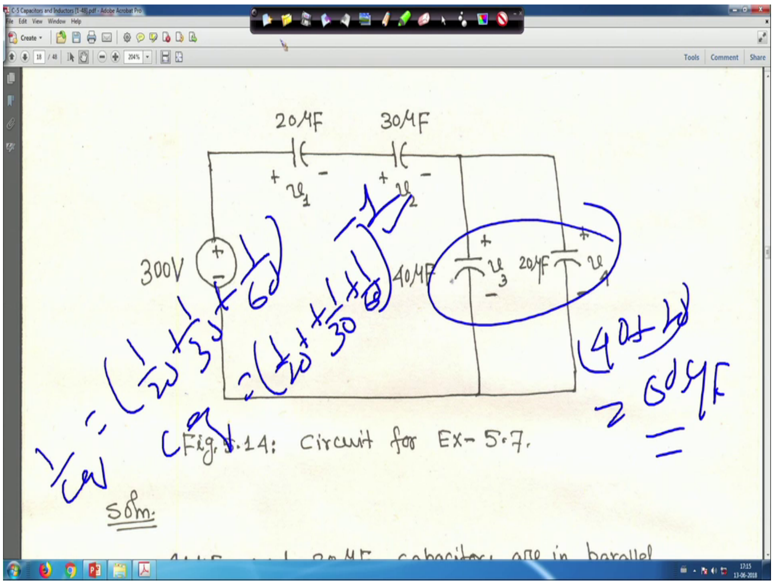

Next one is another example. So, this is the answer another example is for the circuit shown in figure 14 determine the voltage across each capacitor. So, this is a circuit is given 300 volt this is v 1 v 2 this voltage is v 3 and v 4 all the capacitor value are given 20 microfarad, 30 microfarad, 40 microfarad and again here it is 20 microfarad right these loop. This 20 and 40 they are in parallel right. So, when how will that 40 and 20 microfarad are in parallel. So, what you do you add them up 40 plus 20, so 60 microfarad right.

Then 60 30 and 20 are in series first these are parallel and you add them up 40 plus 20 60 microfarad and then 60 30 and 20 they are in series; that means, you have to find out the equivalent one.

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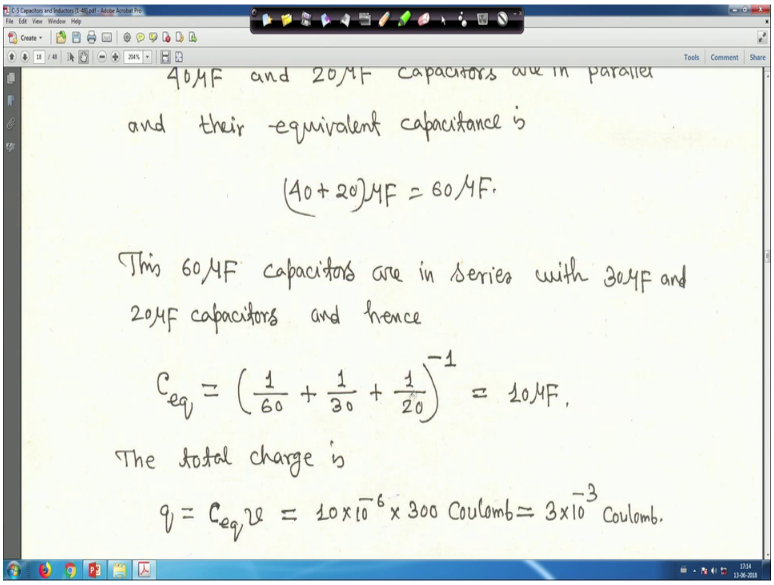

That C eq will be 1 upon C 1 plus 1 upon C 2 plus 1 upon C 3 equivalent that one to the power minus 1 that will give you 10 micro farad, because here this 2 are in parallel right. So, their equivalent is 40 plus 20 right is equal to 60 microfarad. Now 20 30 and 60 you here what you call they are in series; that means, 1 upon C eq say is equal to 1 upon 20 plus 1 upon 30 plus your 1 upon 60; that means, C eq is equal to 1 upon 20 hope it is readable for you I am drawing on the circuit, 1 upon 60 to the power minus 1 just reciprocal of that that is why minus 1 is given right.

So, let me clear it. So, that is why you get your what you call 10 microfarad. Now total charge is q is equal to C eq into v right because this is C eq and total charge q will be C eq into v, v is given 300 volt and it is 10 microfarad. So, 10 into 10 to the power minus 6 farad into 300 coulomb so 3 into 10 2 the power minus 3 coulomb, that is q right total charge.

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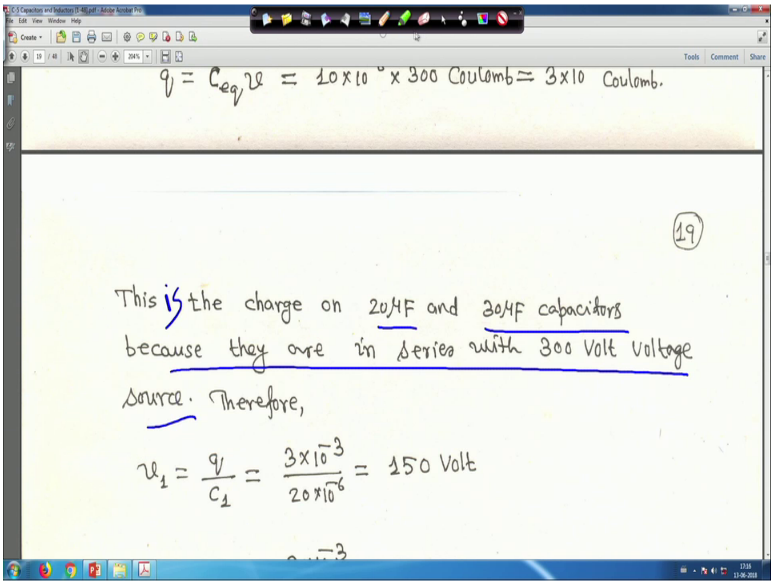

Now this actually it will be is this is the charge on 20 microfarad and 30 microfarad capacitor, because they are in series with 300 voltage source right.

So, this total charge q just let me clear it right. So, 20 minus microfarad and 30 microfarad capacitors are in series, if you look into that 20 and 30 microfarad capacitors are in series. Actually whatever q we have found out that is basically the from equivalent capacitor that is the total charge right, but as 20 and 30 microfarad capacitors you are in series right.

Therefore this is that your what you call this is the charge on 20 and your this is actually there will be an is right this is the charge and 20 microfarad and 30 microfarad capacitor, because they are in series with the 300 volt source therefore, we know q is equal to c v from the formula. So, v 1 will be q upon C 1. So, q 1 q you have got this one. So, this is 150 volt and v 2 will be q upon C 2. So, it will be 100 volt right.

So, as you have got v 1 and v 2 that is 150 and 100 volt respectively, we have to we can easily find out v 3 and v 4 right using you can apply KVL also. So, note that applying KVL. So, now, if you apply KVL in this equation, if you apply KVL in this equation sorry in this circuit if you apply KVL Like this and moving clockwise.

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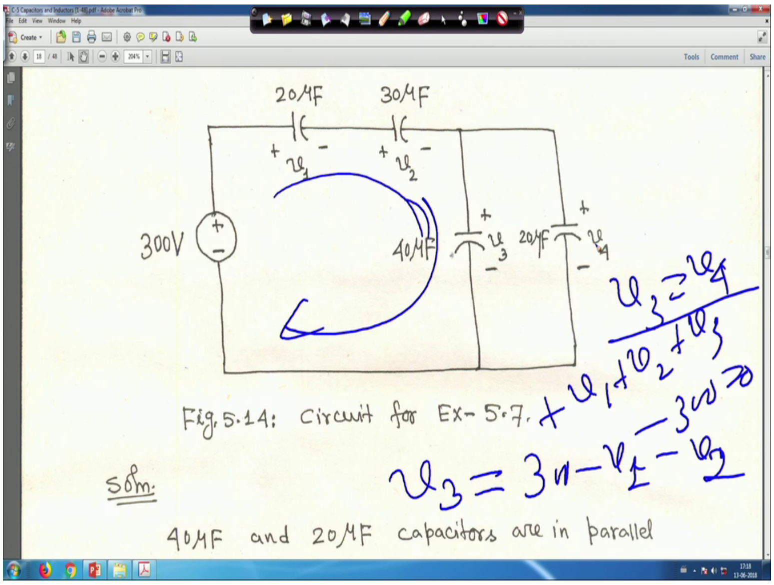

So, it will be plus v 1 plus v 2 plus v 3 minus 300 is equal to 0; that means, my v 3 is equal to your 300 minus v 1 minus v 2 right it is v 1 minus v 2 and this 2 capacitors are in parallel; that means, v 3 is equal to v 4 right because this v these 2 are in parallel

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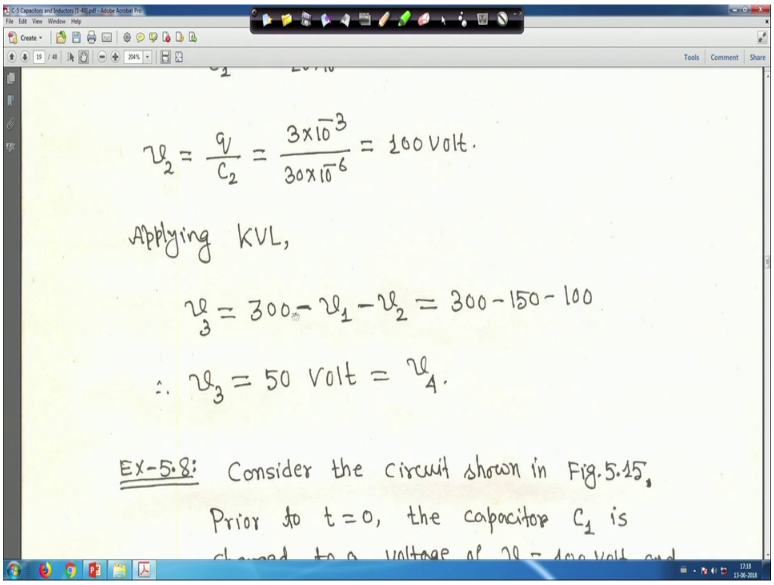

So, go to that that calculation so it will be 300 minus. So, 50 here what you call it is 50 volt v 3 is equal to an v 3 C 3 and c 4 are in parallel so same voltage. So, v 3 is equal to v 4 is equal to 50 volt right.

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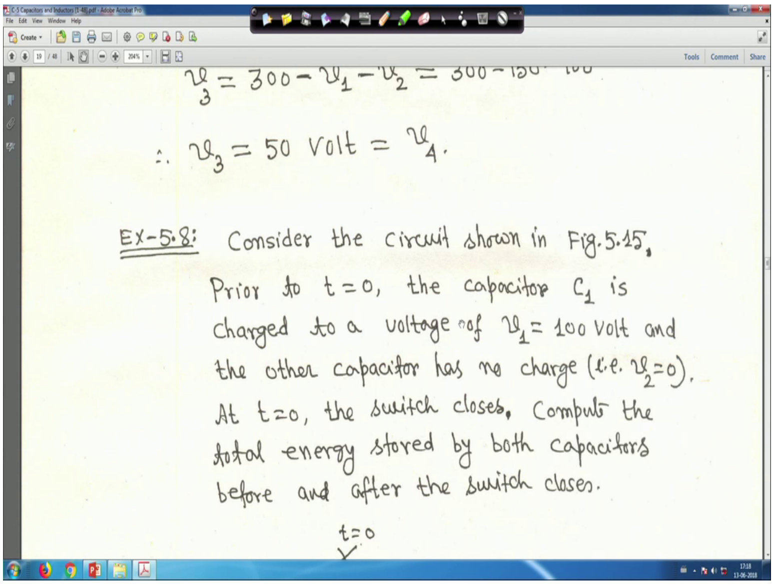

So, next is consider the circuit shown in figure 15 prior to t 0 t is equal to 0, the capacitor C 1 is charged to a voltage v 1 is equal to 100 volt and the other capacitor has no charge that is v 2 is equal to 0 right that is uncharged. At t is equal to 0 the switch closes compute the total energy stored by both capacitor before and after the switch closes right.

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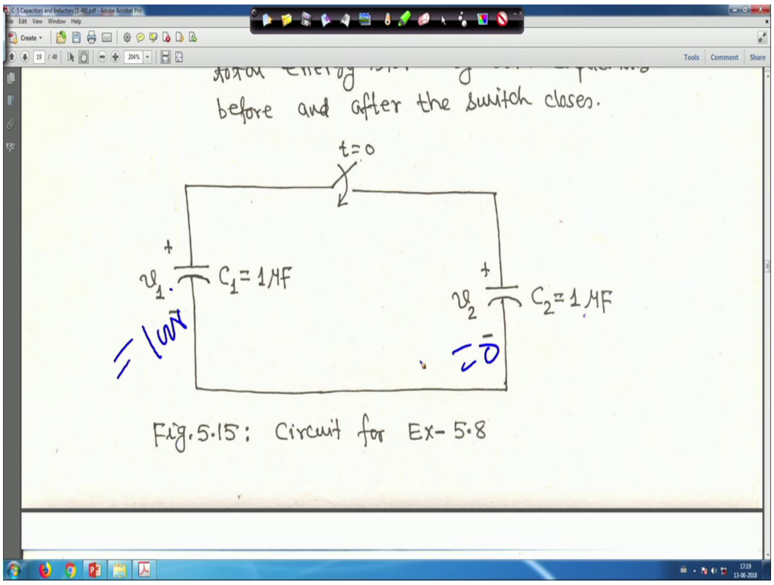

So, this is the diagram it says that before closing the switch this voltage v 1 here what you call 100 volt, and v 2 was 0 volt right capacitor this one is uncharged. And you have to calculate before closing the switch first is the total energy and after closing the switch what is the totalize; here will see some funny thing right. So, question is that before you this you here what you call this 1 microfarad both are 1 microfarad each, but here v 1 is voltage is given before switch is closed it is 100 volt, but here v 2 is equal to 0. So, for this one it will be half C 1 v 1 square right that is half into 10 to the power minus 6 into 100 square because v 1 is 100 given and here it is 100 volt is given that v 1 and here it is 0 right so before this your hold on.

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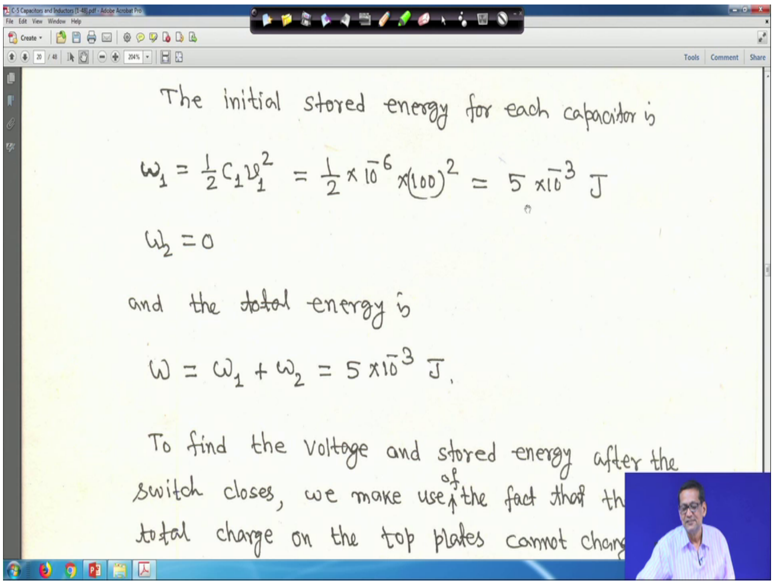

So before this your initial store energy and before closing the switch half C 1 v 1 square. So, it is coming 5 into 10 to the power minus 3 Joule and second case voltage v 2 was 0. So, half C 2 v 2 square 0 so directly we are writing it is 0 therefore, total energy w is equal to w 1 plus w 2 that is 5 into 10 to the power minus 3 joule, this is the value.

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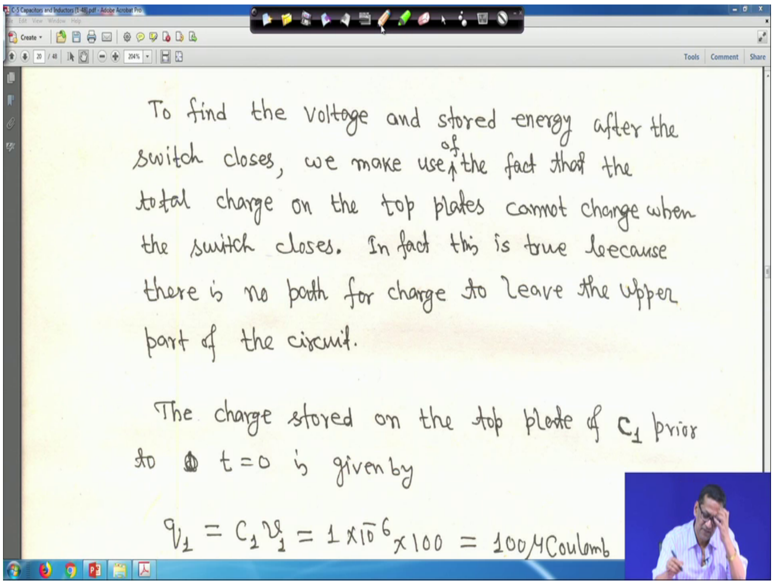

Now, here the thing now to find the voltage, now to find the voltage and stored energy after the switch closes right we make use of the fact that the total charge on the top plate cannot change when the switch closes right we have to make use of the fact right. In fact this is true because there is no path for charge to leave the upper part of the circuit right.

So, idea is something like this. So, we make use of the fact that the total charge on the top plates cannot your what you call change when the switch closes. So in fact, this is the true because there is no path for charge to leave the upper part of the circuit. Now the charge stored on the top plate C 1 prior to t 0 is given by q 1 is equal to C 1 into v 1 that is C 1 is 1 micro farad. So, 1 into 10 to the power minus 6 into 100 that is 100 micro coulomb that is the charge stored on the top plates C 1 right.

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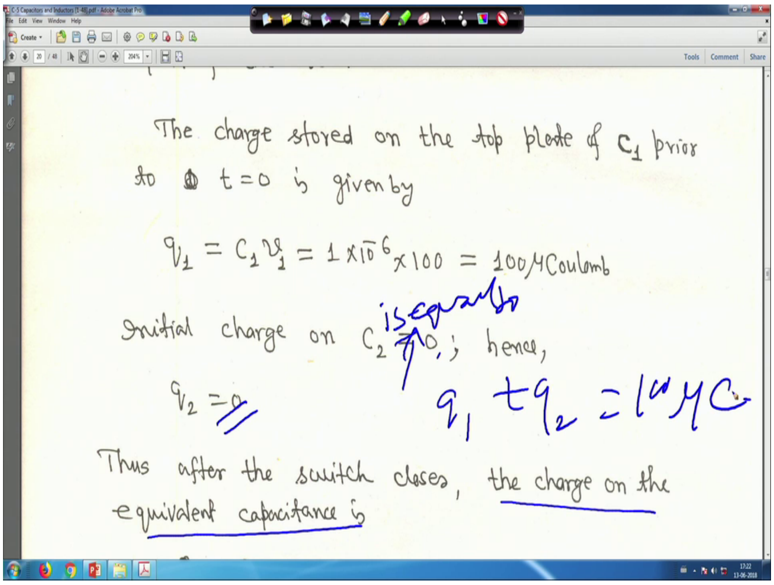

So, let me let me clear it and similarly for q 2 the charge on C 2 is equal to 0 initial charge on C 2 is equal to actually 0 right. So, I should not write C 2 is equal to 0 initial charge on C 2 right better I write is equal to your 0 right. So, otherwise it will carry different meaning right. So, hence q 2 is equal to 0. Thus after the switch closes the charge on the equivalent capacitance is, when switch is closed the charge on the equivalent capacitance is it will be q 1 it will be q 1 plus q 2 that is also 100 micro coulomb right.

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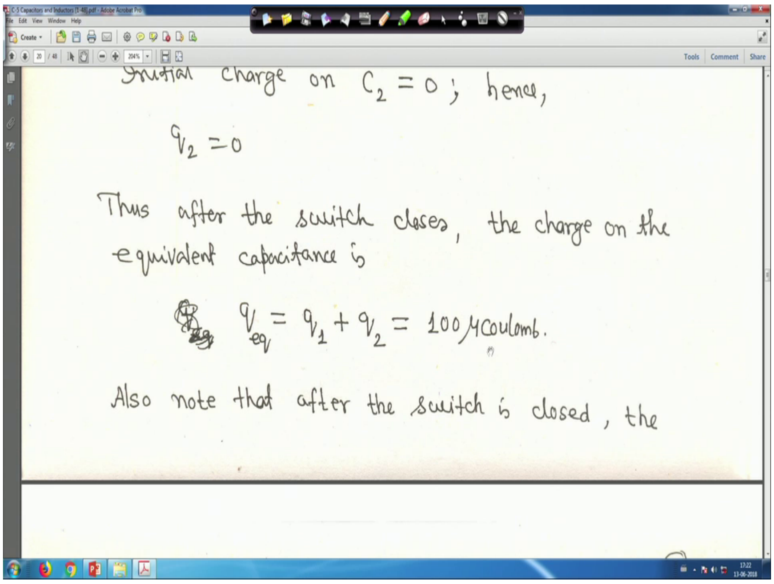

So, that means, it is 100 micro coulomb. Now also note after the switch is closed the capacitors are in parallel, we have an equivalent capacitance; that means, when look if we go back to the circuit that when switch is closed when switch is closed say it is.

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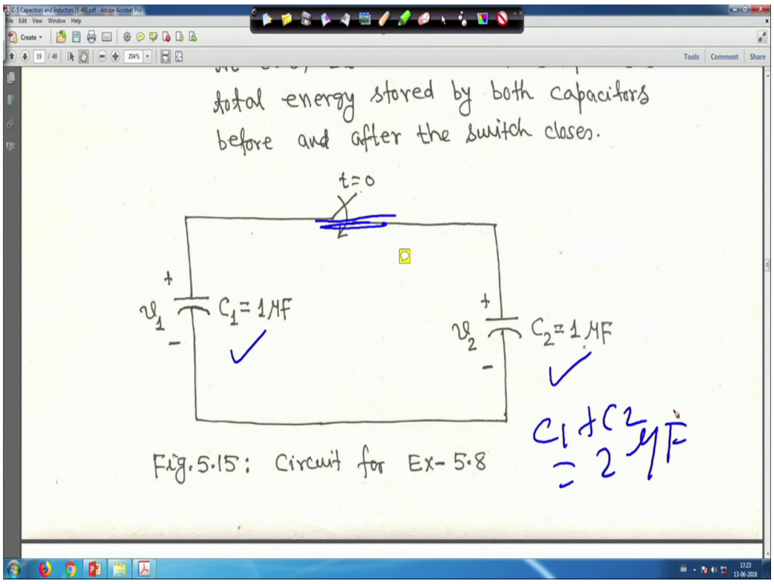

When switch is closed this C 1 and C 2 are in parallel; that means, an both are equal right; that means, it will C 1 plus C 2 that is it will become 2 microfarad because both are in parallel when switch is closed right.

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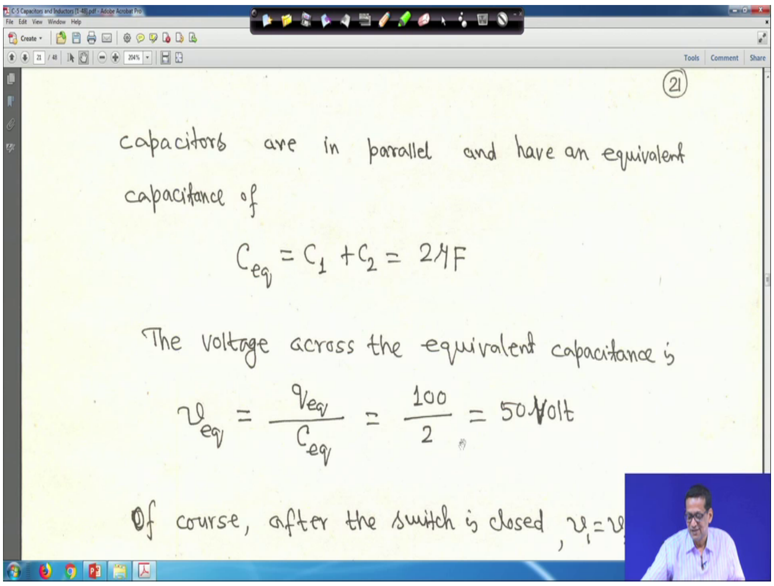

So, in this case sorry let me clear it. So, here so this capacitors are in parallel we have an equivalent capacitance is 2 microfarad. Now the voltage across the equivalence capacitance is now v eq will be q eq upon C eq. So, v eq and it 100 micro coulomb and C eq is equal to 2 your what you call microfarad. So, it is basically 100 by 2 because if the micro coulomb and it is microfarad. So, micro 10 to the power minus 6 cancel right. So, it will be 50 volt right

Now, question is of course, after the switch is closed v 1 is equal to v 2 is equal to v eq because they are in parallel. So, both voltage will be equal now we compute the stored energy with the switch closed.

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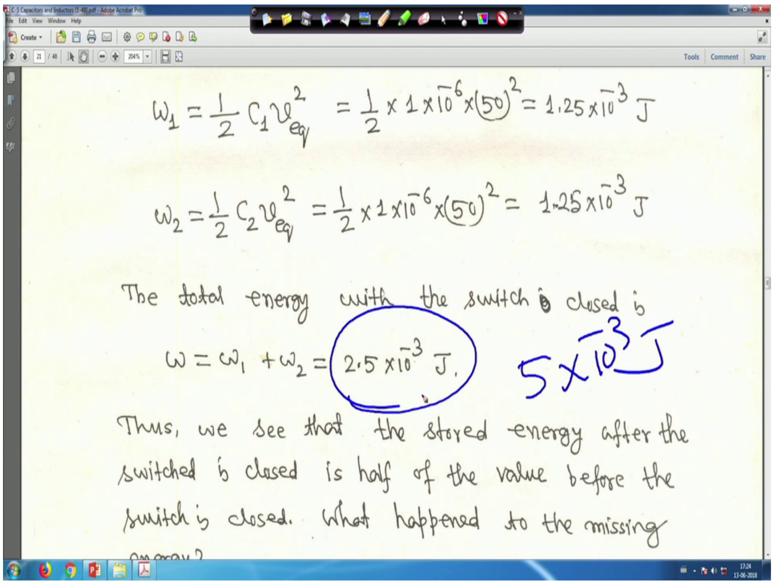

Now, as switch closed w 1 will be half C 1 v eq square, it will be half into 10 to the power minus 6, 1 into 10 to the power minus 6 into 50 square so 1.25 into 10 to the power minus 3 that is Joule right.

Similarly, w 2 is equal to half C 2 v eq square half into one 1 into 10 to the power minus 6 into 50 square that is 1.25 into 10 to power minus Joule so minus 3 Joule. Now total energy if you sum it up w 1 plus w 2 it was 2.5 into 10 to the power minus 3 Joule; that means, before switch closed it was 5 into 10 to the power minus 3 Joule, that is when switch was before closing the switch. But after closing the switch it is coming 2.5 into 10 to the power minus 3 Joules; that means, the energy balance is not there; that means, some 50 percent of the energy is missing then where that energy has gone right; that is that is why this problem I have taken that where that energy has gone.

So, to do this to you know to justify this right. So, we see that the stored energy after the switch is closed is half of the value before switch is closed right. So, what happens to the missing energy, because 50 percent is missing.

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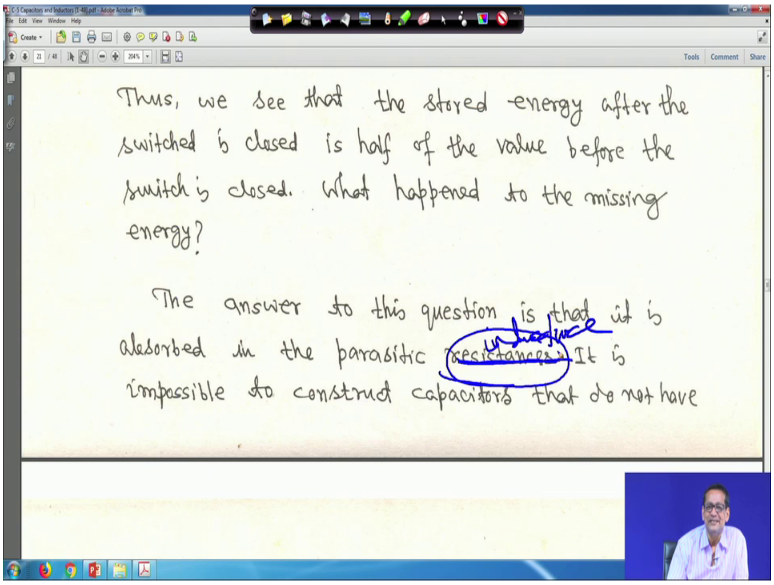

Answer to this question is that it is absorbed in the your what you call, I will it is absorbed in the not parallel I will not put parasitic resistance it is absorbed in the your what you call parasitic inductance right. It is parasitic inductance it is impossible to construct right. So, this one is striked out right

So, it is impossible to construct capacitor that do not have your what you call that parasitic resistance are inductance how?

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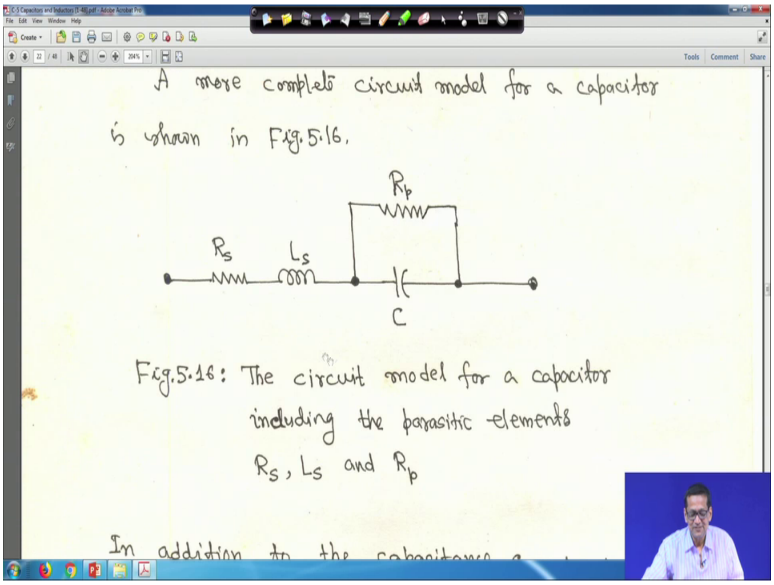

So, some parasitic effects is there for example, in more complete circuit model of a capacitor is shown like this, this is my C, but with that R s L s and R p is there right, because conducting foils right you have some your what you call some resistance.

So, if you take the resistance that will be R s right for conducting plate of the capacitor where we are getting it, and as the current flowing through the capacitor there must be some magnetic field, because of that some parasitic inductance also will be there. All though there very value is very small and in between that that no nothing no material is a pure insulator right some conduction will be there. So, you are because dielectric we are taking in between the parallel plates, but some conduction will be there because of that some R p will be there right that R p resistance we take right this R p.

So, that is why this is actually as a whole the here what you call that for a parallel plate capacitor, but we neglect R p we neglect L s and we neglect R s also. Otherwise that 50 percent energy missing if you had considered that you should have found actually it is here in the parasitic inductances right, that because it is missing means; that means, some where it has gone; that means, some where it is stored. So, it will be in the parasitic inductances that that is why this example is taken right.

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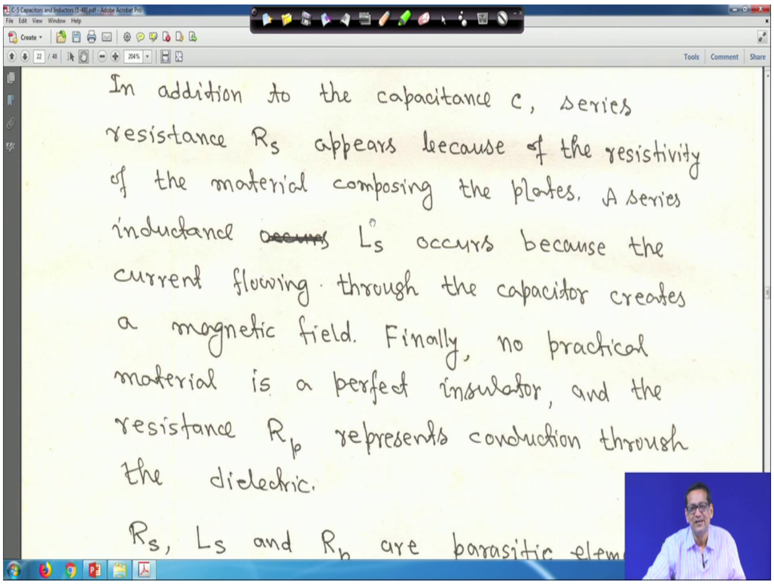

This is actually complete diagram of a parallel capacitor, but R s, L s and R p will be neglected. Now all these right of g r whatever I told right all these right of g r. So, whatever I said it is ok, but you just go through it right. So, I am moving it little bit slowly.

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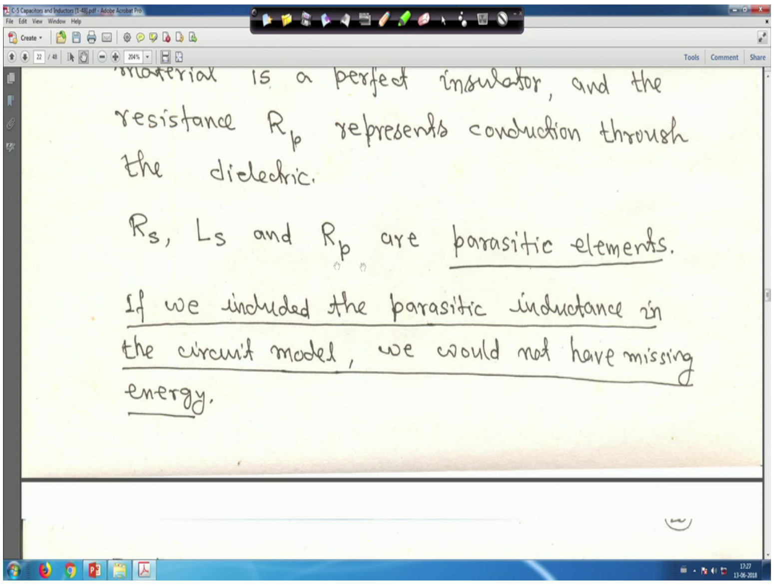

So, R s, L s, R p are the parasitic element, if you include the parasitic inductance in the circuit model we would have not have missing that 50 percent energy, but we have not this example intentionally I have taken, to give you a favor that what you call energy has gone right.

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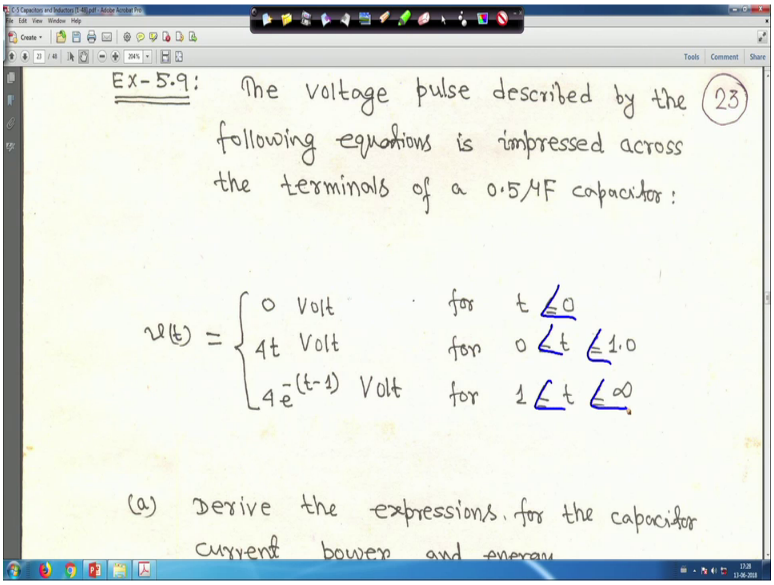

So, another one we will take this thing this; the voltage pulse described by the following equation impressed across the terminals of 0.5 microfarad capacitor right. So, v t is 0 volt, for t actually here it will be just one minute it will be t less than it will be t less than right this we make right. So, 0 volt for t less than 0 and 40 volt for in between 0 and 1 second and 4 e to the power minus t to minus 1 volt in between 20 your what you call your between 1 and infinity t greater than 1 right.

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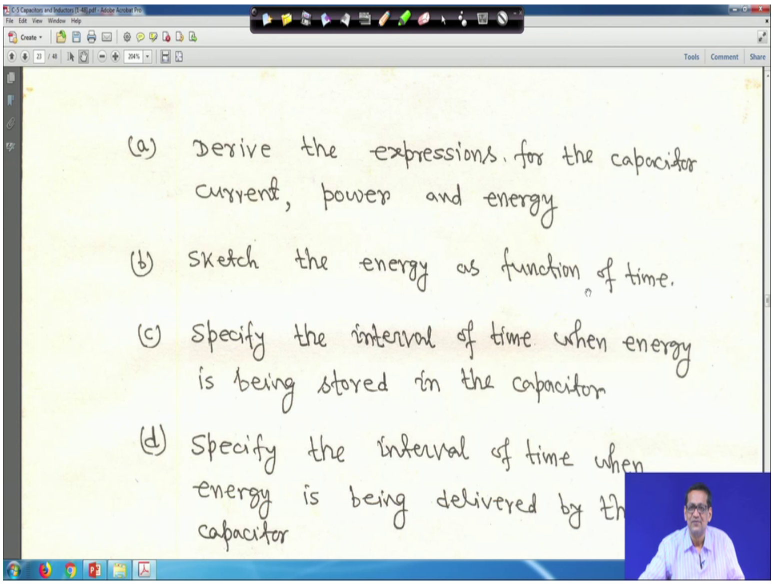

So, what we have to do is that, we have to find out all these derive the expression for the capacitor current power and energy, this is now a, sketch b sketch the energy as function of time, c specify the inter interval of time when the energy is being stored in the capacitor right. I mean you have to when energy stored what you call you have to specify the time or specify the interval of time energy is being delivered by the capacitor that also we will see that based on certain convention we find easily.

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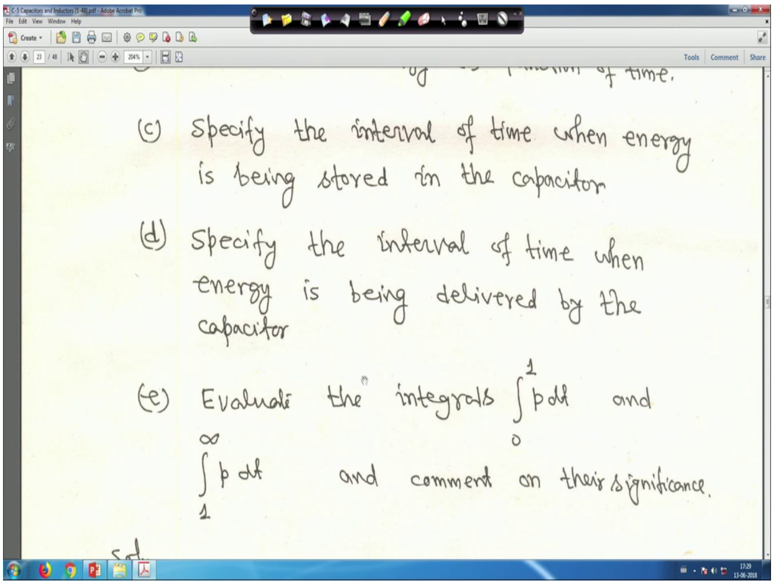

Evaluate the integrals 0 to 1 p dt and 1 to infinity p dt and comment on the on their significance so.

Thank you very much we will be back again.