Fundamentals of Electrical Engineering

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Lecture - 19
Circuit Theorems (Contd.)

So we will come to that your homogeneity property.

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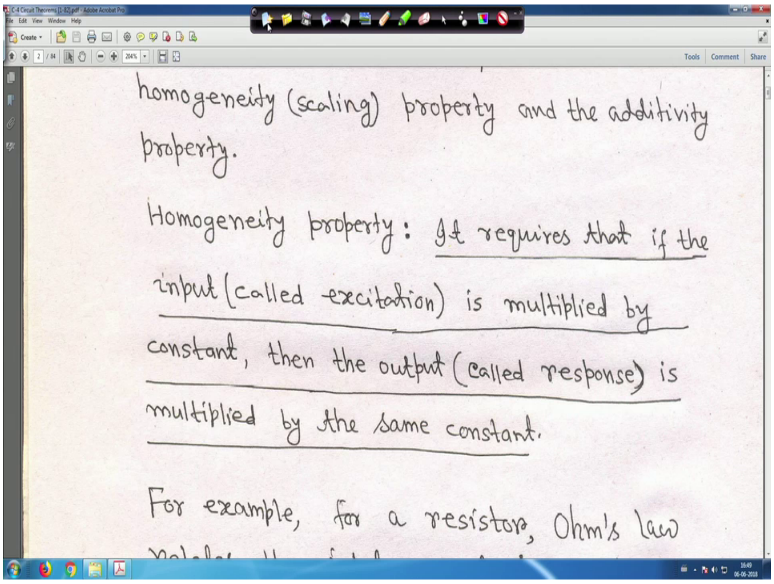

So, basically it requires that if the input that is called the excitation is multiplied by the constant, then the output it is called the response is multiplied by the same constant. That is I have underlined right for example, for a just hold on for example, for a resistor Ohm's law release the input current to the voltage v.

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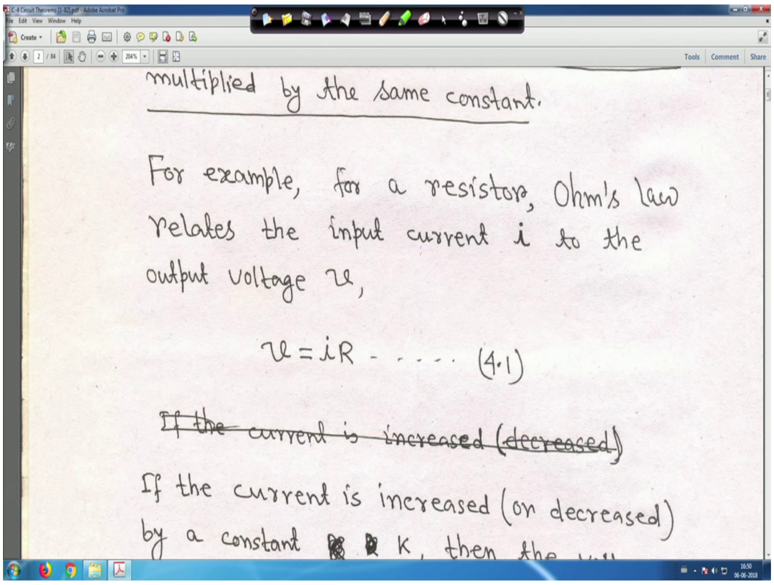

So, v is equal to i R, if the current is increased or if bracket I have written down or decrease by constant k, then voltage increases or decreases correspondingly by k that is ki into R is equal to kv.

So, this is equation 2 right, it chapter 4.2, but I will say 2 3 like this right so, and additivity property.

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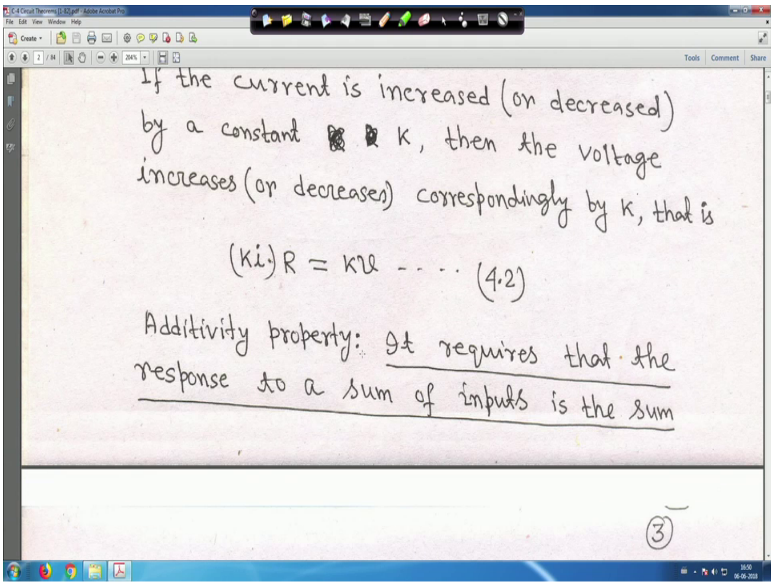

It requires that the response to a sum of the input right is the sum of the responses to each input applied separately. Suppose input you have your input you have what you call more than one input is there and then output you are getting a response, by adding all this input. Now if you consider one at a time at the input and output responses you are getting and sum it up if it is equal to that same as your sum of the inputs where together you are putting at the input and getting the output response. So, additivity property is it requires that the response to a sum of inputs to the sum of the responses to each inputs applied separately.

I mean suppose of the input you have many your what you call many inputs are there. So, consider one at a time and output response you are getting and add this one. Now again at the input all the inputs are there get output response this 2 must be your equal right so, that is call actually additivity property.

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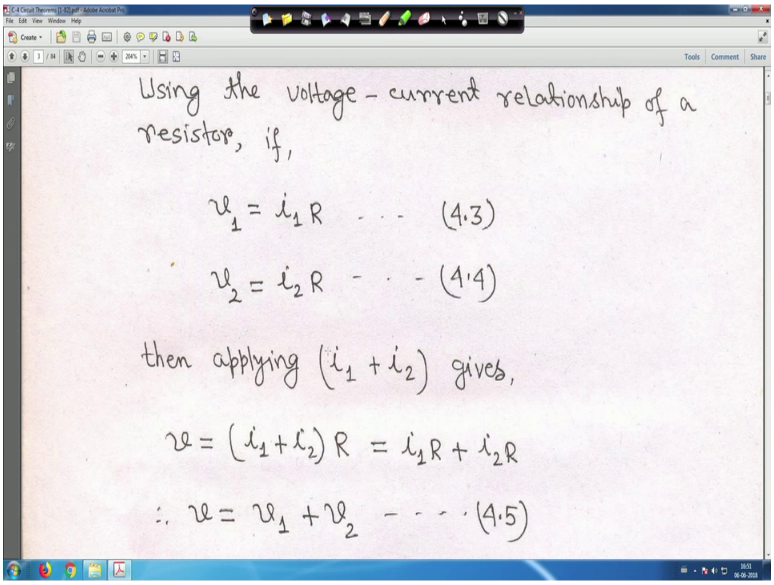

So, using the voltage current relationship of a resistor say if v 1 is equal to i 1 into R right and v 2 is equal to i 2 into R right. By applying i 1 plus i 2 it gives v is equal to i 1 plus i 2 into R that is i 1 r plus i 2 R then v is equal to your v 1 plus v 2 this is additivity property right.

So, that is this condition as to has to be here what you call satisfied. So, therefore, we can say that the resistor is a linear element, because if voltage current relationship satisfied both homogeneity and additivity properties right. Then only you can say that element is linear it has to satisfy both homogeneity and additivity properties right. So, then you can say that it is a linear element; in general a circuit is a linear if it is both homogeneous and additive right.

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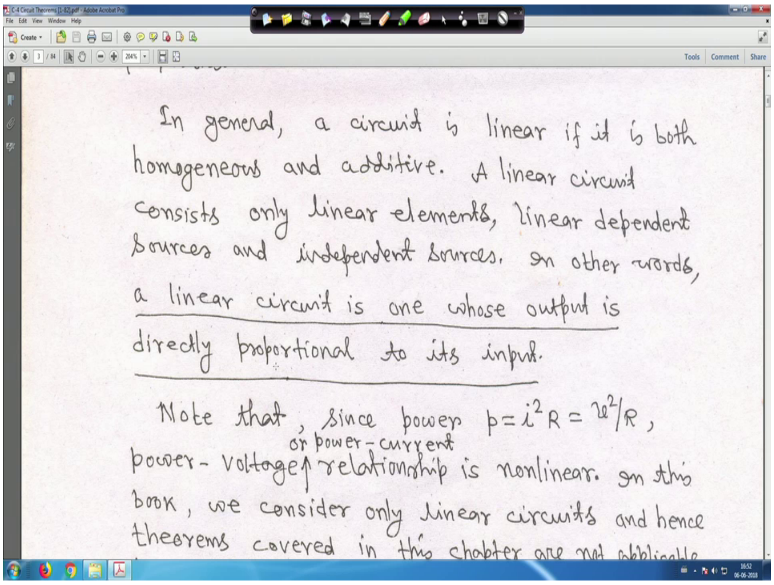

So, a linear circuit consist only linear elements; so, linear dependent sources and independent source. So, if you if I underline it for you so, a linear circuits consists only a linear elements, linear dependent sources and independent sources.

In other words all this are all are underlines a linear circuit is one output is directly proportional to its input right output is directly proportional to its input, then the circuit is called a linear circuit. But note that power p is equal to i square R is equal to v square upon R right it is it is your what you call it is non-linear right. So, power voltage is non-linear. So, it is where I have written somewhere in this book, book is not written actually it will be chapter, it will be chapter the book will never completed actually so, it will be chapter.

So, in this chapter you concentrate only linear circuit and theorems covered in this chapter are not applicable to non-linear circuits so, let me clear it right. So, so in this case not applicable to your power right so, early we go for linear circuit.

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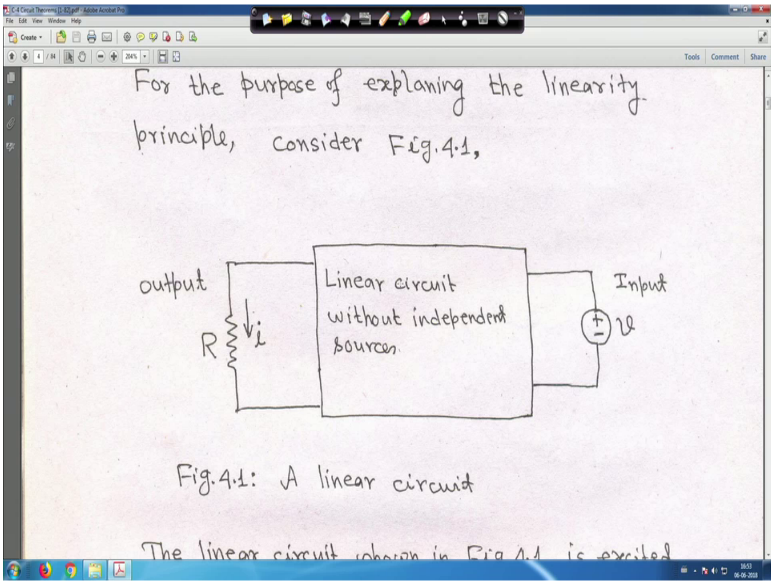

So, for the purpose of the exam explaining, the linearity principle is consider this figure 1 right. So, you have you have one voltage source, you have one voltage source and here one left hand side one resistor load resistor is there R and current is flowing through it and between a linear circuit without independent in between your what you call this portion is a linear circuit, but without any without independent sources and this is voltage, and this is your what you call this is your one load resistance R is their current flowing through it is i right.

Now, question is that suppose if my v is equal to 5 volt, at that time say i is equal to 1 ampere right if I make it v is equal to 10 volt, then i has to be 2 ampere because v has been multiplied by 2 so, i has to be multiplied by 2, if it is if it is happened then a circuit is a linear right. So, that property has as to be your what you call satisfied. So, in this case the linear circuit is excited by voltage source v, I told you here in voltage source v which serves as the input and the circuit is a terminated by load resistance R.

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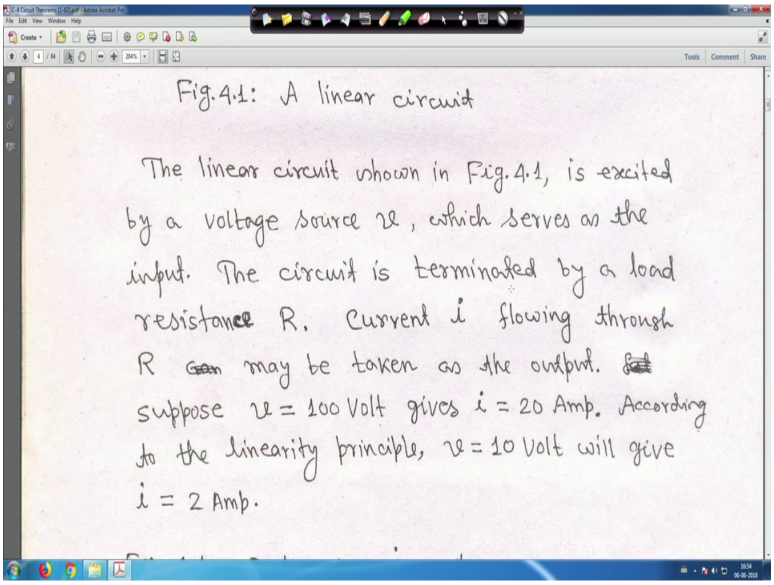

So, I told it that v for example, suppose here I have taken whatever I said same thing suppose if v is 100 volt see at that time current is 20 ampere, but if we make v is equal to 10 volt, the current will be 2 ampere right, because you are multiplying k is equal to say 0.1 right. So, that is that the linear what you call then you can say that it is linearity principle according to that it, it condition has to be satisfied right.

So, next is determine i 0 when v is equal to 3 volts and v is equal to 6 volt of the circuit shown in figure 2 right.

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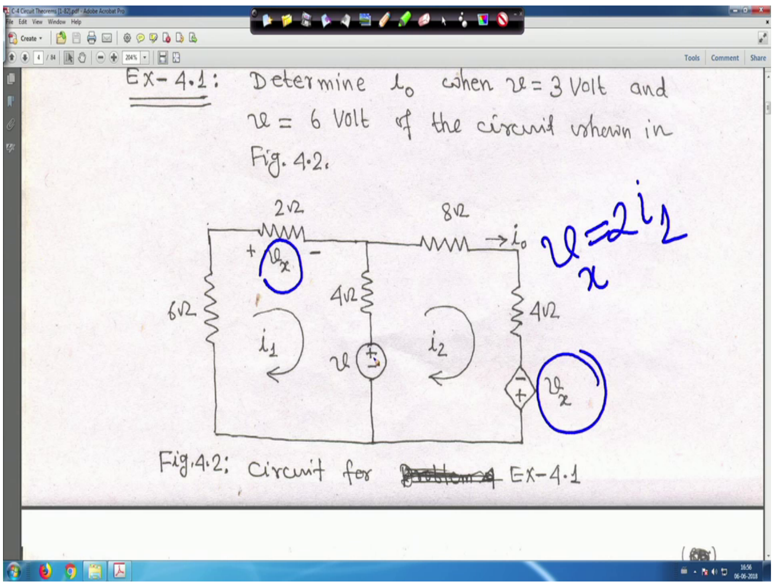

So, in this case you have one dependent voltage source v x look, though out this all this equations KVL, KCL all we have studied now right. So, in this case this voltage is v x and one dependent voltage source is there v x right. So, this v x actually v x is equal to actually 2 into your i 1 this is my v x right. So, in this case what you can what you can do is you apply your and if you see there are 2 mesh in between your what you call one we have of course one voltage source is there it is not a current source. So, we apply KVL and accordingly you solve this one right and prove that whether it a circuit is a linear or not.

So, let me clear it. So, in this case we apply KVL so, this circuit please this mesh 1 and mesh 2 we apply KVL so, not telling it again and again it is understandable to you and this current is i 0. This current is here it is marked that this current is your i 0 here the current is i 0 right so, let me clear it.

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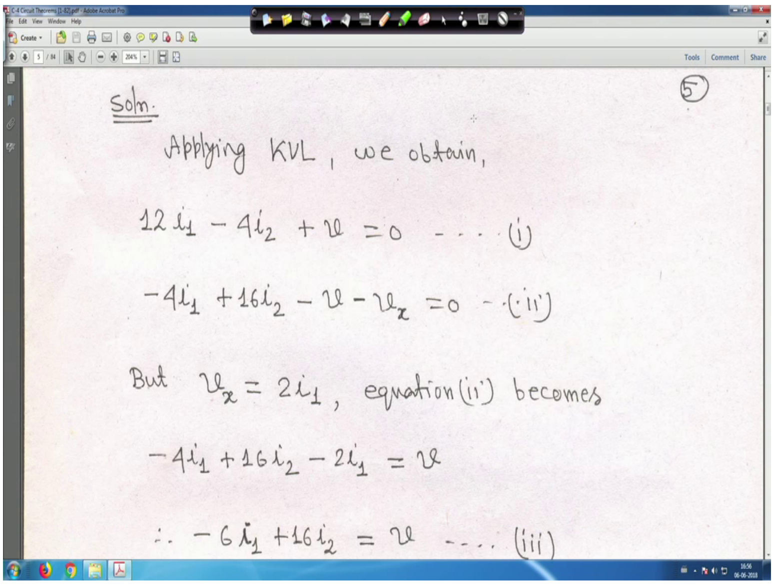

So, KVL if you apply so, two meshes you will get 2 equations right one is 12 i 1 minus 4 i l 2 plus v is equal to a 0 and then will get minus 4 i 1 plus 16 i 2 minus v minus v x is equal to 0, but v x is equal to 2 i 1 right if you look into that, this voltage is v and it is given once you find out i 0 when v is equal to 3 volt and v is equal to 6 volts.

So, after getting this when v is equal to 3 volt you solve the circuit, you will get i 0 is equal to 3 by 28 ampere right.

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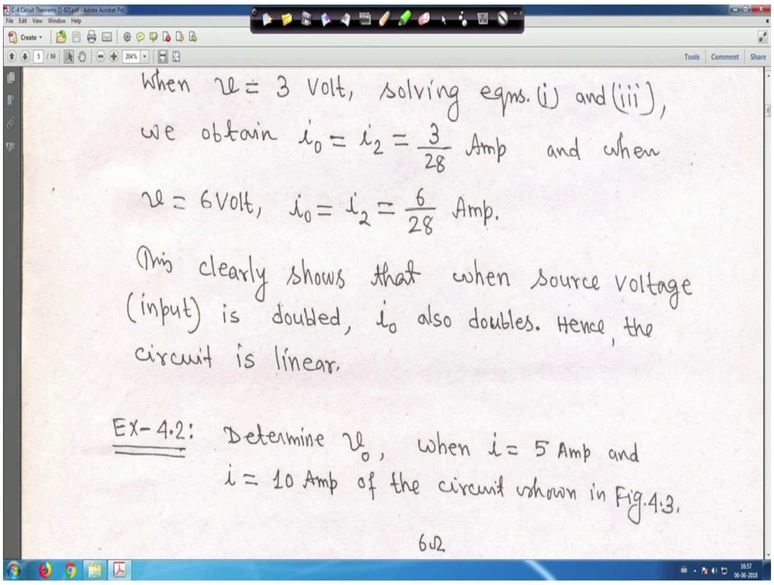

When a v is equal to 6 volt, you see i 0 is equal to 6 by your what you call 28 ampere right so; that means, when it is 3 volt it is 3 by 28 when it is 6 volts voltage is doubled, current also doubled 6 by 28 ampere. So, this clearly shows that when source voltage is doubled i 0 also doubled. So, in the circuit is linear circuit right.

So, similarly another one you take that determine v 0 when i is equal to 5 ampere and i is equal to 10 ampere of the circuit.

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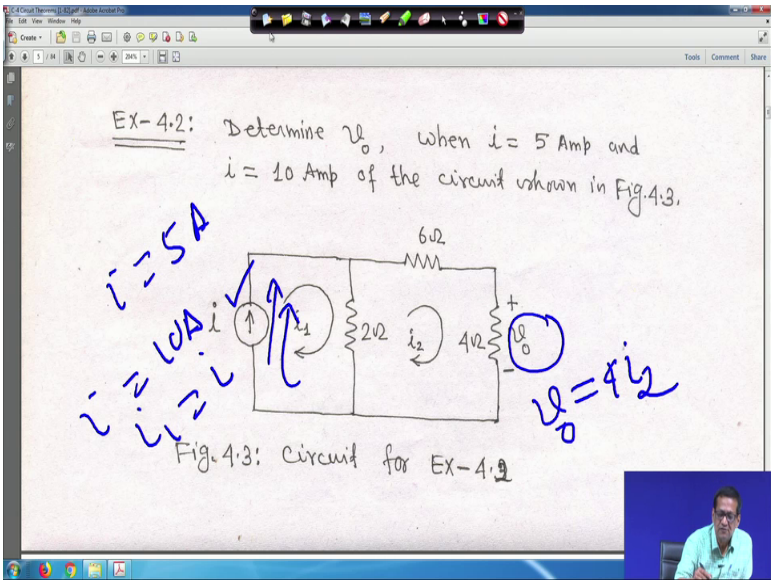

So, i i your what you call this i actually is the current source, a current source is given you find out your what you call v 0, this voltage is v 0. So, v 0 is equal to actually 4 into your i 2 this is v 0 right and in this case you have to find out v 0, when i is equal to 5 ampere. So, this i is 5 ampere another case when i is equal to 10 ampere and if 5 ampere to 10 ampere means the current getting doubled naturally v 0 also you solve it, it has to be doubled right.

So, accordingly you please solve it. So, this current is i i is this direction is this 5 ampere that this is your i 1 same direction. So, basically i 1 is equal to i or i is equal to i 1 right. So, let me clear it. So, if you apply then what you call KVL in your what you call mesh 2 you apply KVL in mesh 2. So, i 1 is equal to I told you and 12 i 2 minus 2 i 1 is equal to 0.

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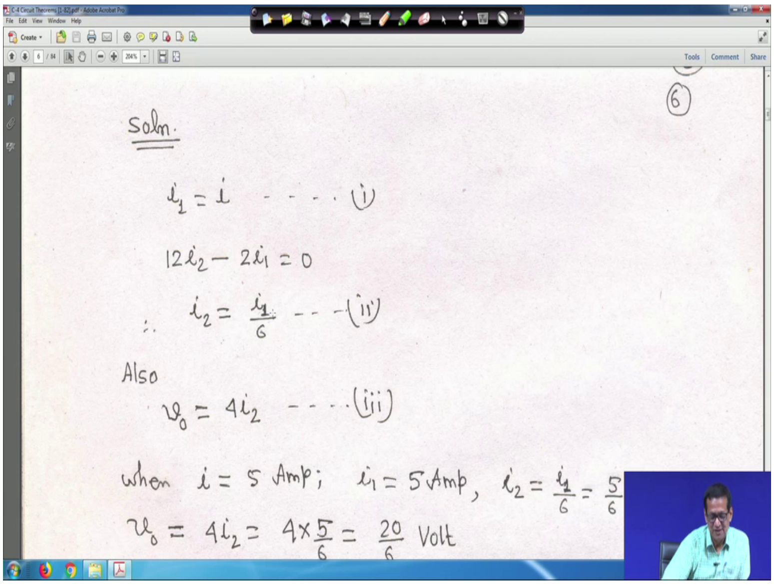

So, all I am not writing here again it is understandable to you right you apply KVL in mesh 2. So, i 2 is equal to i 1 by 6. So, v 0 is equal to 4 i 2. So, i is equal to 5 ampere right.

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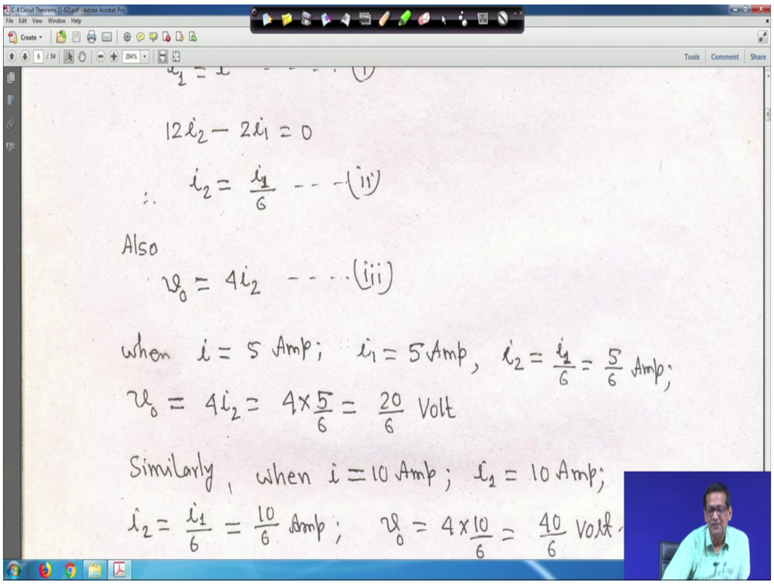

So, that means, i 1 is equal to 5 ampere, I told you i is equal to i 1 therefore, i 2 is equal to i 1 by 6 so, here it is given. So, here it is given that i 2 is equal to your 5 by 6 ampere and v 0 also I told you your 4 by 2 a sorry 4 i 2 and v 0 is equal to i 2 is equal to 20 by 6 volt. Now when i is equal to 10 ampere so i 1 also 10 ampere; so, i 2 is equal to i 1 by 6 or 10 by 6 ampere. So, v 0 is equal to then 4 i 1 so it is 40 by 60 volt.

So, it was 20 by 60 volt when current was 5 ampere, i is equal to 5 ampere, when i was made 10 that is doubled so voltage also had became doubled. So, from that you can tell circuit is a linear right. So, next is superposition principle so, this is something like this.

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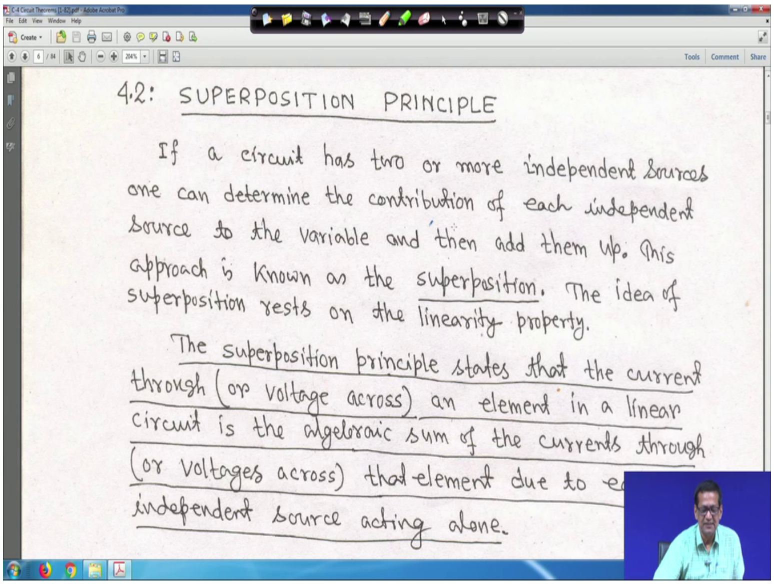

So, if a circuit has 2 or more independent sources one can determine the contribution of each independent source to the variable and then add them up.

Suppose we have 2 or more independent sources, then you consider one at a time other sources should be your turn off right and you add them up right, then these approach is known as a super position. So, idea of superposition rests on the linearity property right. So, the then I have underlined it, the superposition principle states that the current through or voltage across current through an element or voltage across an element, that is why in the bracket, I have write down I current through or voltage across right. An element in a linear circuit is the algebraic sum of the current through or voltage across that element due to each independent source acting alone right.

I mean whenever you applying super position, you consider only one source at a time independent source right only independent source if they dependent source, there will remain intact in the circuit right you cannot turn them off. So, later we will see and that; that means, if consider one at a time, then either current through element or voltage across element independently you make made it and then you then you add it up whatever you will get, you serve the circuit as a whole you will get the same thing right that that is super position. Only thing is that superposition case where this approach is good, but it will compute your, it will consume your more time right.

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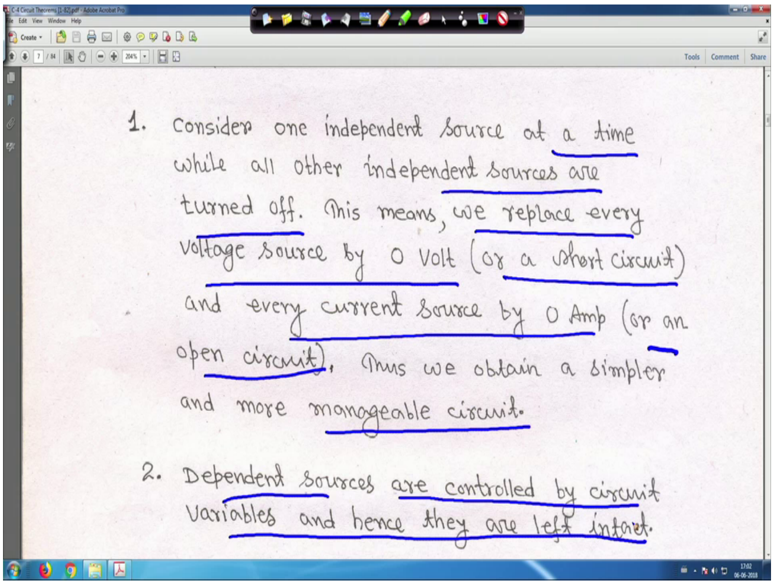

So, that means, your; to apply the superposition 2 things must kept in our mind right. One thing is you consider one independent source that is your you consider one independent source at a time right one only one independent source at a time and all other independent sources are turned off right. This means we replace every voltage source by 0 volt; that means, your short circuit that voltage source should be short circuit, and every current source by 0 ampere that is it is an open circuit right.

We will see later in the taking example few example we will say will take, that we obtained the simpler and more manageable circuit, that if it is a voltage source you short circuit. Take consider one at a time and if it is a current source you open it right such that, because you have to consider only one source at a time right.

Dependent sources and dependent sources are controlled by variables and hence they are left intact so, but if dependent source are there, you just cannot turned it off right because their values are dependent on the what you call current or voltages right. So, that they will give you a intact how will see later right. So, circuit analysis superposition may very lightly involved more work and this is a major disadvantage.

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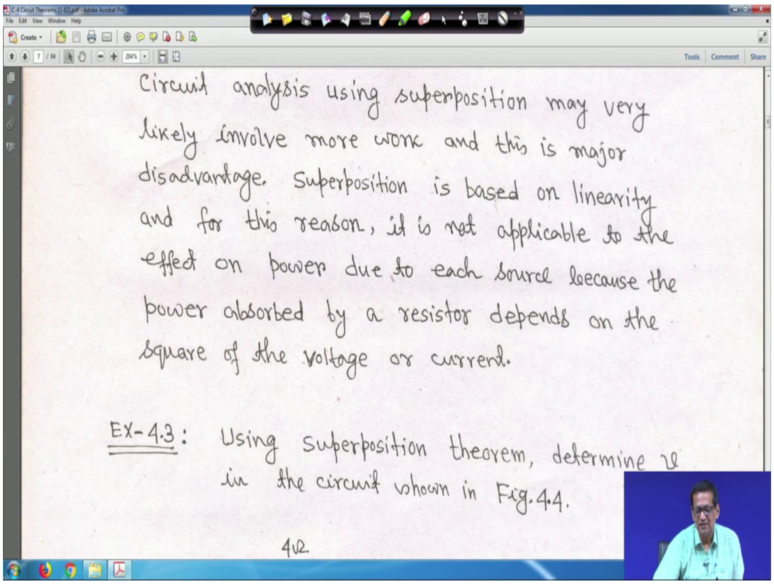

Superposition is best on linearity and the and this reason it is not applicable to the effect on power due to the source, because the power observed by resistor depends on the square of the voltage or the square of the current.

So, it is not applicable for the power, but for the voltage and current it is applicable right. So, using superposition theorem determine v, in the circuit shown in figure this things right.

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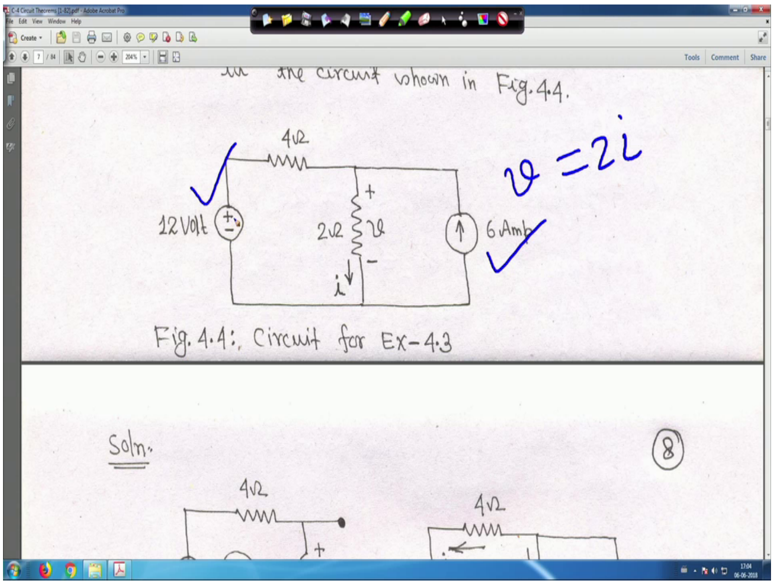

You have to find out what is the what you call; you have to find out that your voltage v in general, v is equal to 2 i that is in general right, because this is current i is flowing using superposition theorem you have to make one voltage source is there, independent voltage source is there 12 volt and one independent current source is there it is 6 ampere right.

So, now how will do it? First thing is by making your applying superposition theorem, what will do is, once you will take only one source at a time look how you can make it. So, in this case when current source is turned off that is your 6 ampere source is turned off then this is the circuit.

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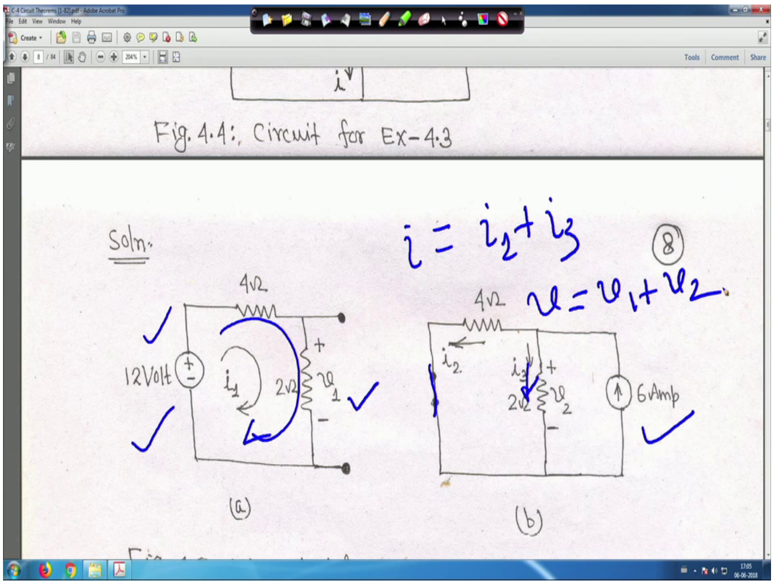

This is the circuit that is your this is the circuit when 6 ampere at that time current is i 1, current is flowing through this i 1 we have taken right and when current source is there, then voltage source this 12 volt voltage source this current source is there now and 12 volt voltage source is short circuit one at a time.

If it is a current source it has to open, if it is a voltage source it has to short right at that time current is taken here is i 3 for this branch it is taken has i 3 right; that means, in that current earlier in the circuit your i actually is equal to your i 1 plus your i 3 right, because this is your what you call this is the voltage v is equal to this voltage v 1 because superposition then is voltage v 2 right.

So, earlier in the circuit actually just let me go to go to the circuit right here this is the current i. So, after breaking after getting this 2 different circuit because of superposition theorem, so now, in this case for 2 ohm resistance, when current source is open it is current i 1 and when your voltage source is shorted, but current source is there i 3. So, basically i is equal to your i 1 plus i 3 and v 1 is equal to your 2 into i 1 and v 2 is equal to 2 into i 3 and v is equal to v 1 plus v 2, because here it is because we are taking one source at a time here it is v 1 here it is v 2 so, basically it will become v 1 plus v 2 right

So, before moving this; so in this case it is very easy to find out what is the i 1 current. So, if you apply KVL here it will be 4 plus 2, it is the simple circuit right nothing is here.

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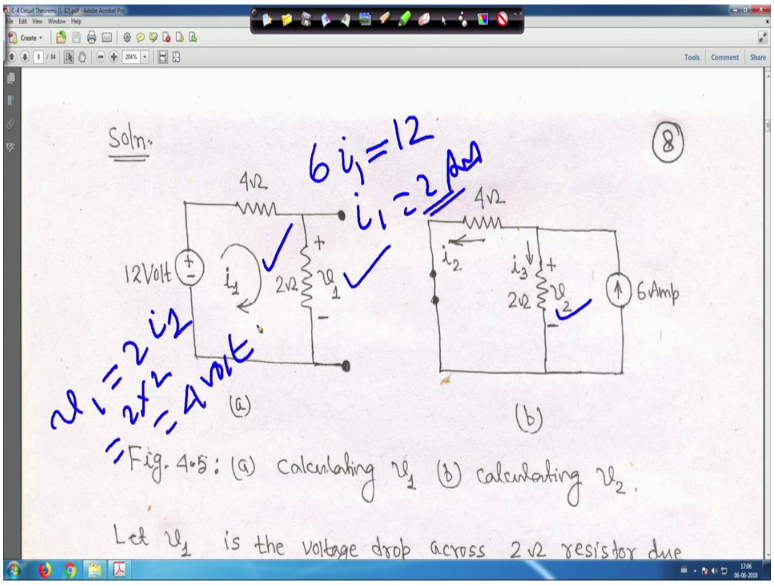

So, basically 6 i 1 is equal to 12 therefore, i 1 is equal to 2 ampere or this one right so; that means v 1, v 1 is equal to 2 i 1 right. So, i 1 is equal to 2 ampere so; that means, v 1 is equal to 2 into 2 is equal to 4 volt right this is your v 1 right.

Similarly, you just let us clean it; similarly if you solve this circuit it is a basically we have at the beginning we have studied know current division method right. So, this is basically a parallel circuit, this 4 ohm and this 2 ohm there are in parallel right and 6 ampere current is entering here this 4 ohm and 2 ohm are in parallel, then what will be then if current division method what will be i 3?

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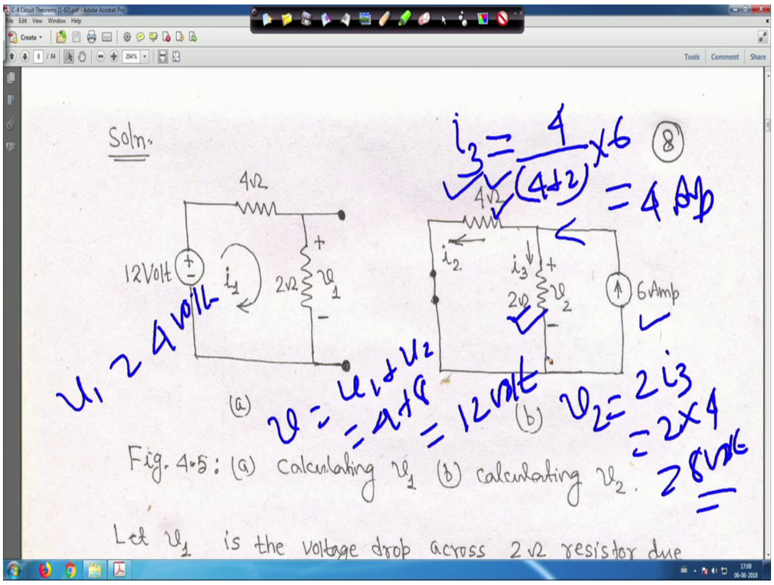

Directly I can write i 3 is equal to right. So, it is through this you have to find out i 3. So, this stance is 4. So, 4 by 4 plus 2, this is 4 and this is 2 into your 6 so, that is actually 4 ampere. So, i 3 is equal to 4 ampere directly from the current division method you can find way out right.

And if your i 3 is equal to 4 ampere then v 2 is equal to your 2 into i 3 that is 2 into 4 that is your 8 volt right. So, here we got v 1 is equal to 4 volt right and here we got v 2 is equal to 8 volt then what is v then, v will be is equal to v 1 1 plus v 2 that is 4 plus 8 that is your 12 volt right. So, that is this is this is v as per superposition theorem, but you have to verify by solving let me clear it, but you have to verify by solving this circuit. As a whole you solve this circuit, and find out that your what you call that this v is equal to 12 volt or not this is also solve for.

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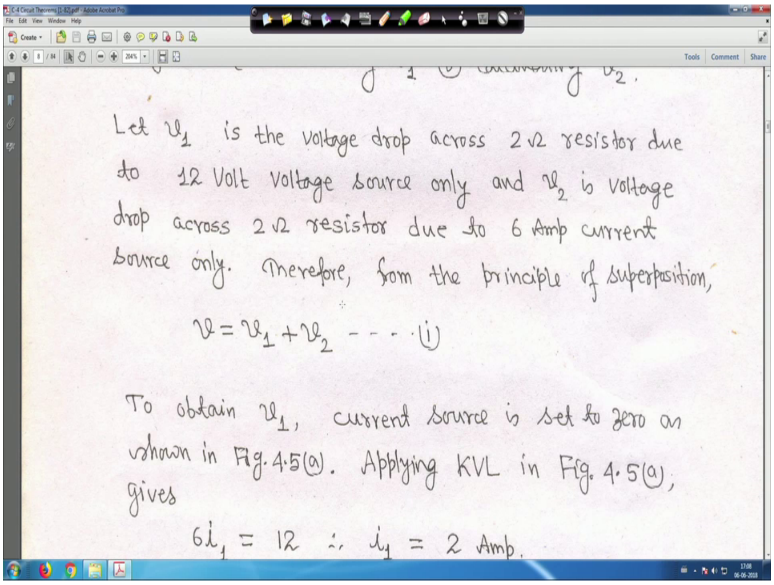

So, whatever is whatever I said everything is everything is done for you.

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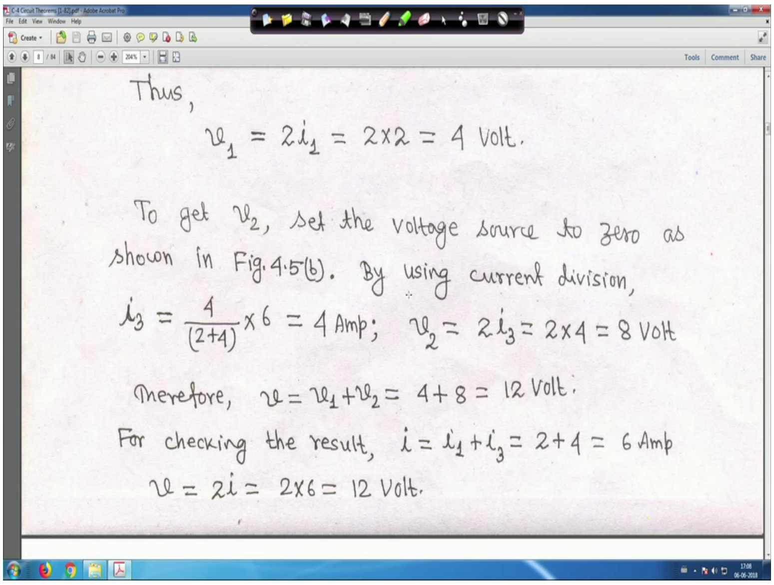

So, all this things are now, for a check you solve this your i 1 plus i 3 is equal to 6 ampere, and v is equal to 12 volt. So, checking the result you can easily find out your this thing what you call, you solve the circuit right and see that i here you will get i is equal to 6 ampere, because when solving here while solving here look here we got i 1 is equal to your 2 ampere right and while solving here, here we got for this circuit we got i 3 is equal to 4 ampere right.

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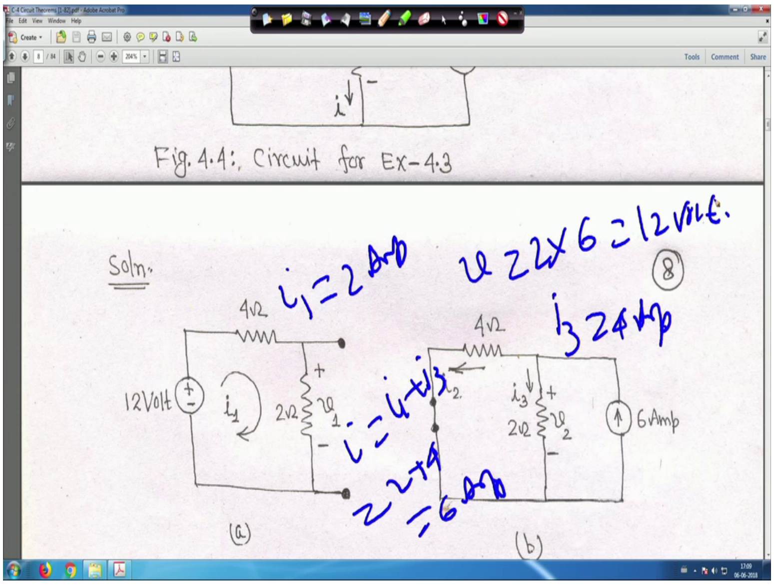

So, i is equal to superposition theorem, that i is equal to actually i 1 plus i 3 is equal to 2 plus 4 is equal to 6 ampere right and if it is i is equal to 6 ampere, then the resistance was 2 ohm. So, 2 into 6 is equal to 12 volt right.

So, let me clear it and; that means, this circuit you solve you find out I will get i is equal to 6 ampere and v is equal to 6 into 12 volt. So, all this things we have done it so; that means, your what you call here it is given v is equal to 12 volt. So, problem is solved right superposition theorem and your as a whole if you solve the circuit.

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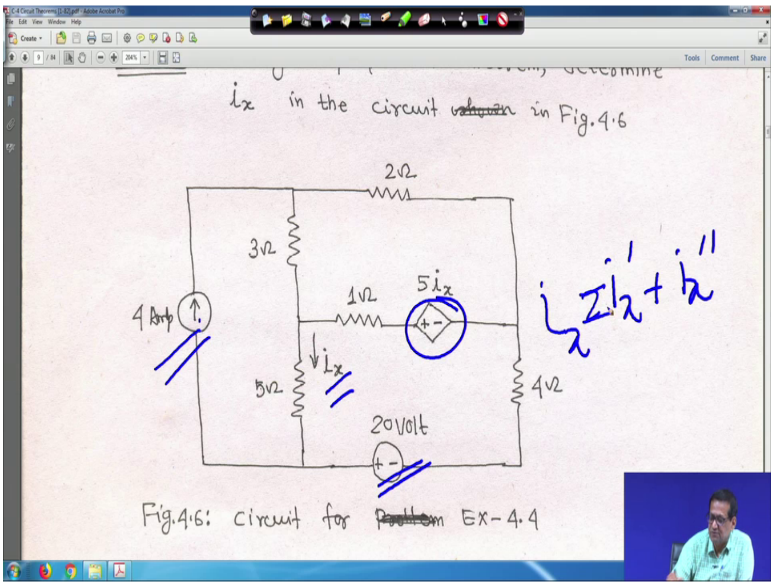

Now, next one is using superposition theorem determine i x in the circuit in the figure 4. So, in this case you have to find out what is the i x using the superposition theorem; that means, this i x you have to find out and one dependent voltage source is there 5 into i x; and as I told you that dependent voltage source you cannot it should be intact in the circuit, you cannot remove it, you cannot turn it off right.

But one independent current source is there 4 ampere and one independent voltage source is there 20 volt, all others are eh resistance values are given, we have to find out here what you call i x using superposition theorem right. So, in that case what we what will do, will consider one source at a time, once will consider this voltage source or will consider the current source right. When you consider the voltage source this will be open, when will consider current source this will be shorted at that time each case i x will change. So, i x basically say for example i x will become say i x dash plus i x double dash right, this is equal to right.

So, how let us see just hold on.

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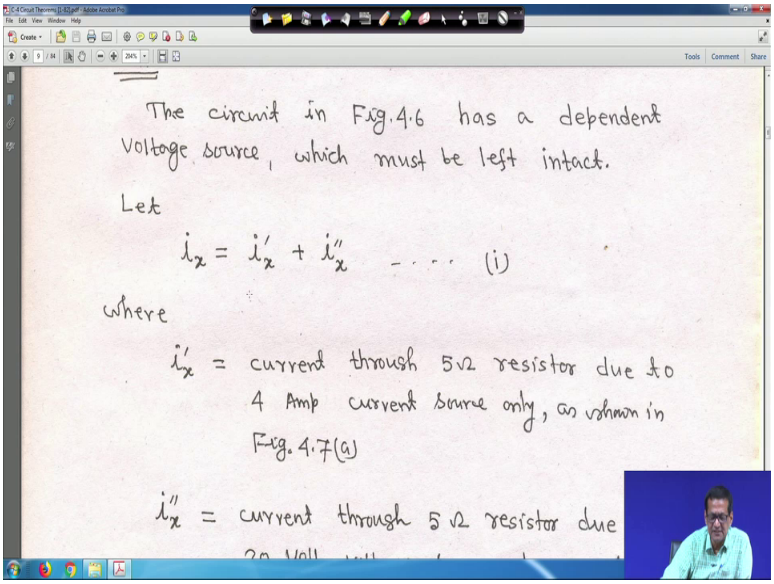

So, in this case that the circuit is that I told as the dependent volt with must be left intact that is i x should be is equal to i x dash plus ix double dash. Now i x dash is equal to current to 5 ohm resistor due to 4 ampere current source only as shown in figure 4.7.

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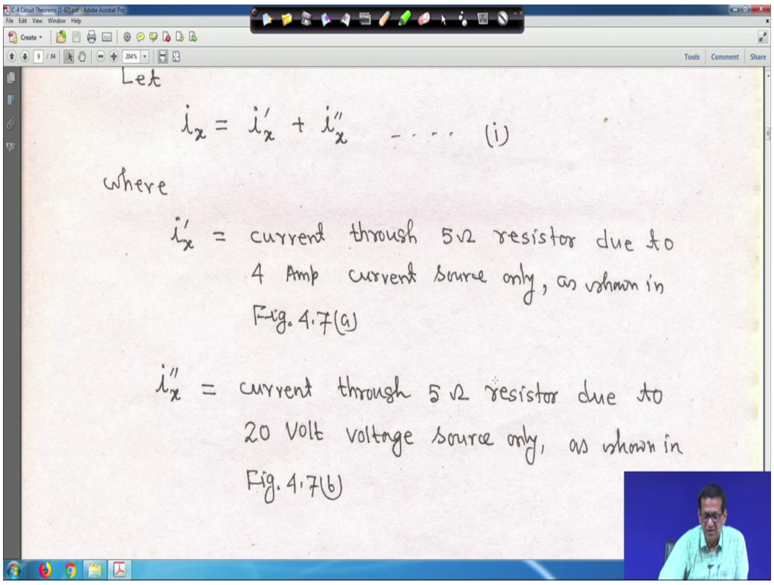

Now superposition we are applying. So, this voltage this voltage source is shorted here, voltage source is shorted here right here.

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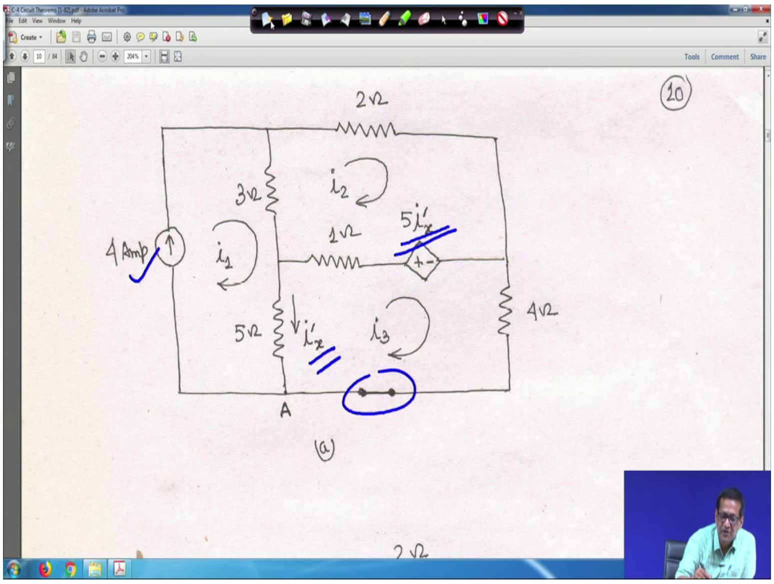

So, voltage source is shorted here we have shorted only current, when 4 ampere current source is only there at that time this is the current i x dash right. I told you i x is equal to i x dash plus i x double dash right and similarly and as per all the mesh your what you call that i 1 i 2 i 3 is shown, and here also the dependent this dependent it are here also it will be 5 into i x dash right. So, it is not i x now it is i x dash similarly when voltage source is there right.

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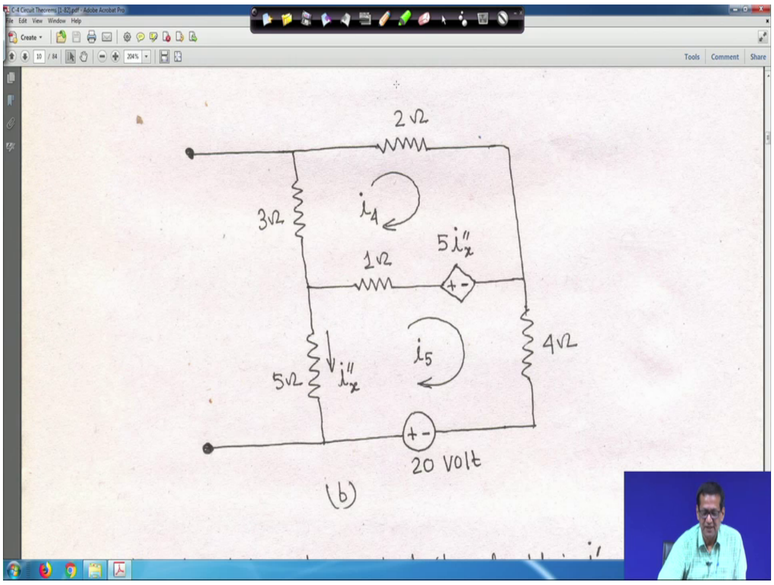

When voltage source is there current is current source is not there, look current source is not there it is open right at that time it is i x double dash right and here also it is your 5 into i x double dash.

So, that means, this circuit and that circuit you have to solve for i x dash and i x double dash, then you add it up and as a whole you add as a circuit you will get the same answer right. So, let me clear it. So, that means, first you have to solve this circuit right, you have to solve this circuit. One thing is there that all this things are written that there is a node a right. So, this 4 ampere current is going like these.

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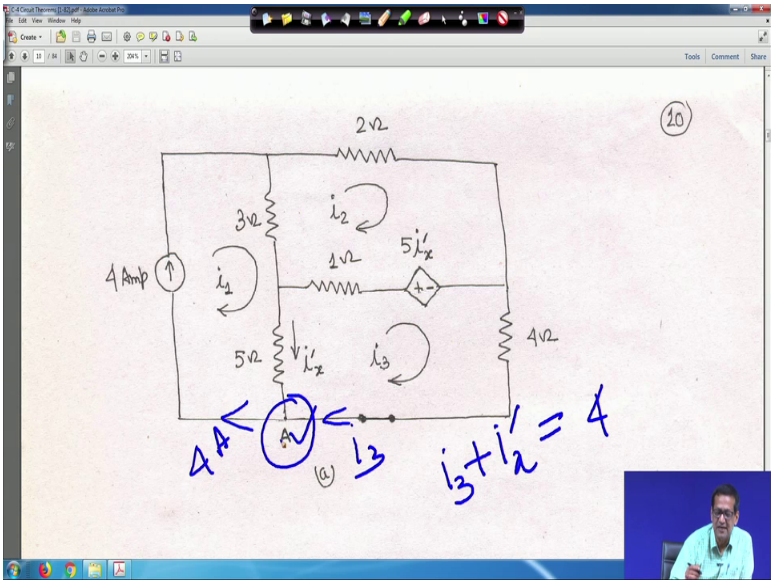

So, this will be your 4 ampere right and this i 3 current clockwise we have taken. So, this i 3 also entering into the node, and i x node also entering into the node. So, if you apply KCL at node A so it will become your i 3 plus i x dash both current are entering into the node and 4 ampere is leaving and this is your 4 right.

So, in this case what you call a this KCL is needed, for solving this circuit. So, let me clear it. So, for this one your similarly for your this i 1 is equal to 4 ampere, if you see this mesh one this i 1 is equal to 4 ampere because here where you are moving like this so, this 4 ampere in this upward direction. So, i 1 is equal to 4 ampere right.

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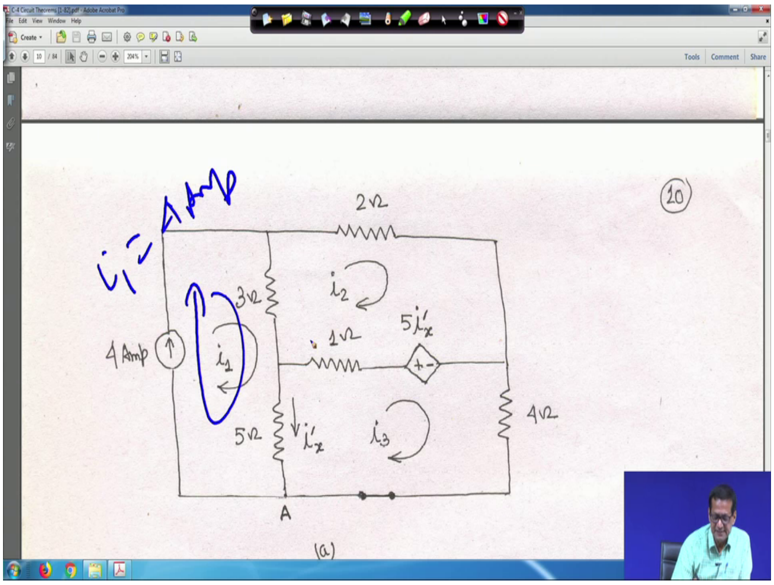

So, rest you can easily apply your what you call k your what you call that your KVL, and KCL also at this node I told you and accordingly you can solve the circuit. So, all this things are a given. So, if you then this is your what you call circuit 4.7 a 7 a and this is circuit 7 b.

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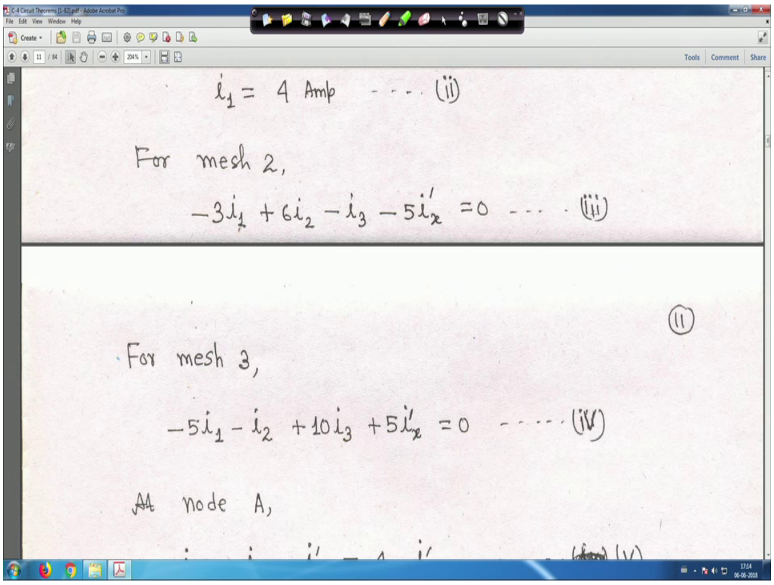

So, accordingly for mesh 2 when you apply KVL, you will get the equation 3 here it is equation 3 you will get. Now you please apply because all this KVL, KCL you have gone through right.

Similarly, for mesh 3 you will get this your what you call minus 5 this another equation for i x dash, and at node a I told you apply your k your KCL that is 4 minus i x dash is equal to 5. So, substitute equation 2 and 5 in equation 3 and 4 right. So, after giving 2 simultaneous equations it will get that is your 3 i 2 minus 2 i x dash is equal to 8 and i 2 plus 5 and i x dash is 20, if you solve this 2 equation you will get i x dash is equal to 52 by 17 ampere right.

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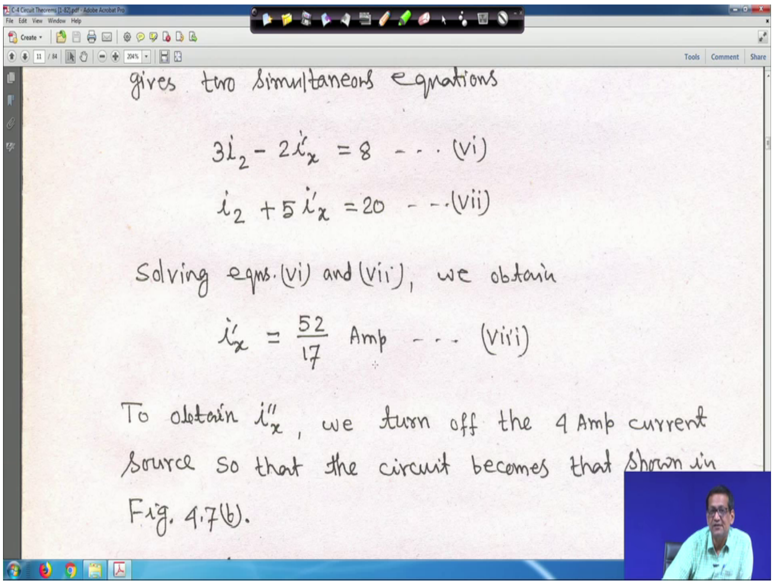

Now, to obtain i x double dash again you turn off the 4 ampere current source we have seen so, that circuit becomes that shown in figure 7 b that also we have shown. So, here also for mesh 4 you apply KVL, here where I am taking this circuit this is you are calling mesh 4 right, this is your mesh 4, this is mesh 4 and this is mesh 5 you apply KVL here right both the cases and just solve it. So, all these equations now you can write right.

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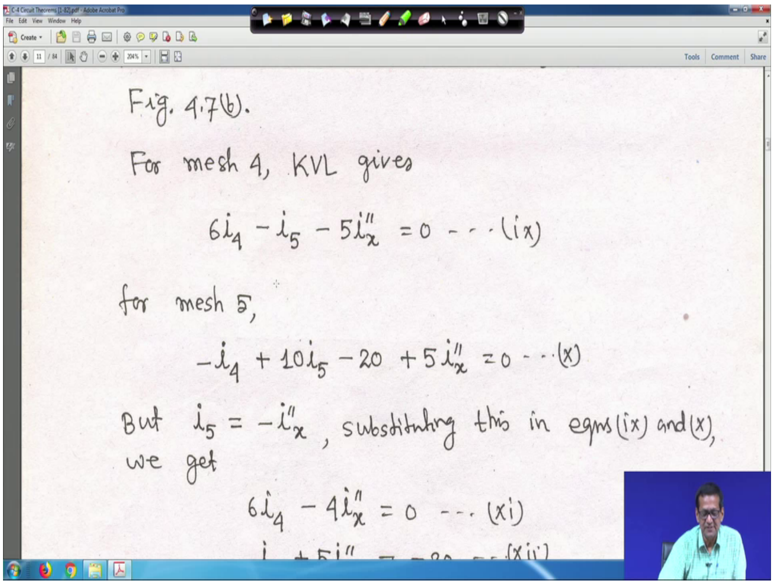

So, if you do so, mesh 4 it will become 6 i 4 minus i 5 minus 5 i x double dash equal to 0, similarly mesh 5 you write the equation and look at that mesh 5 it will become i 5 is equal to minus i x double dash, because direction will just see upward and downward right. So, substituting this equation 9 and 10 you will get this equation that 6 i 4 minus 4 i x dash is equal to 0 right and this one is equal to minus 20. Solve this you will get i x double dash is equal to minus 16 upon 17 ampere.

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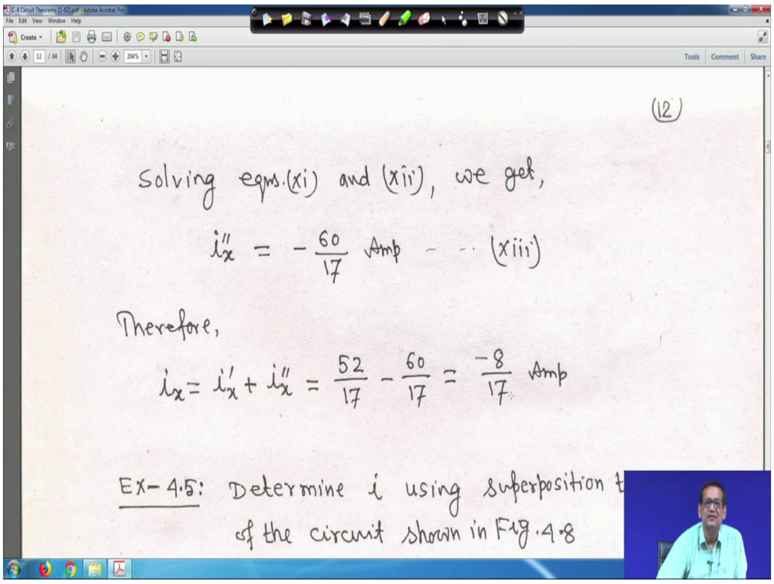

Therefore i x is equal to i x dash plus i x double dash is equal to minus 8 upon 17 ampere. If you solve this one this circuit you will get the same answer minus 18 upon 17 ampere right just one here see one more simple example, determine i using superposition theorem of the circuit shown in figure; there are 3 sources 2 independent voltage sources one independent current sources right.

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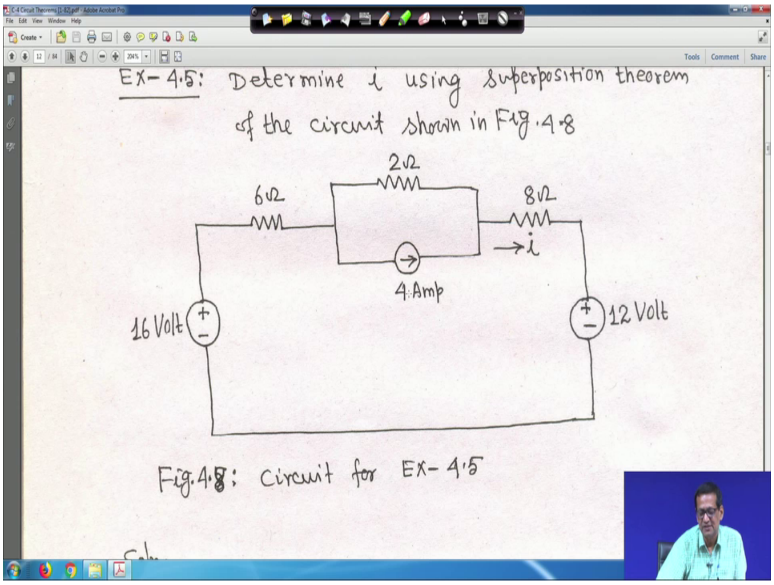

One independent current source so take one at a time.

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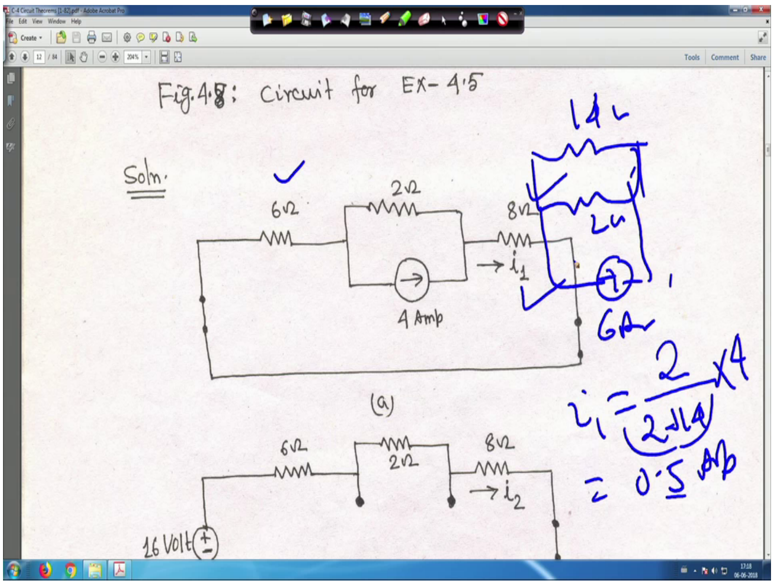

So, first you 2 voltage sources are shorted only current source is there. So, this is a simple thing right because 6 ohm and 8 ohm both are in your what you call your series, if you look into that both are in series. So, 6 plus 8 is actually 14 ohm, but any way you have to find out the current i 1. So, this circuit 8 ohm this thing and when the first case the current is i 1 because i is equal to i 1 plus i 2 plus i 3 right.

Similarly, when I mean when this 16 volt source is taken, but current source is open and when another voltage source is shorted. Similarly for the third one the 12 volt source is there current source is open and another volt is shorted right. So, this way one source at a time rest will be turned off. So, this way of you find out very simple here i 1 is equal to right.

So, if you try to find out what will be your this thing i 1. So, this is this is your 8 ohm and 6 ohm basically they are in series and the and with that and that means, this is your 14 ohm right and this is your 2 ohm, and with that this current source is there, this current source is there 6 ampere basically this all this 3 things are in parallel right.

So, then what will be i 1? So, it is current division method only. So, if you try to find out i 1, i 1 will be is equal to 2 divided by 2 plus 14 right into your 4 ampere right. So, basically it will become 0.5 ampere 16, 64 by 16 so, it will be up so 2 into your 8, 8 by 16, 0.5 ampere.

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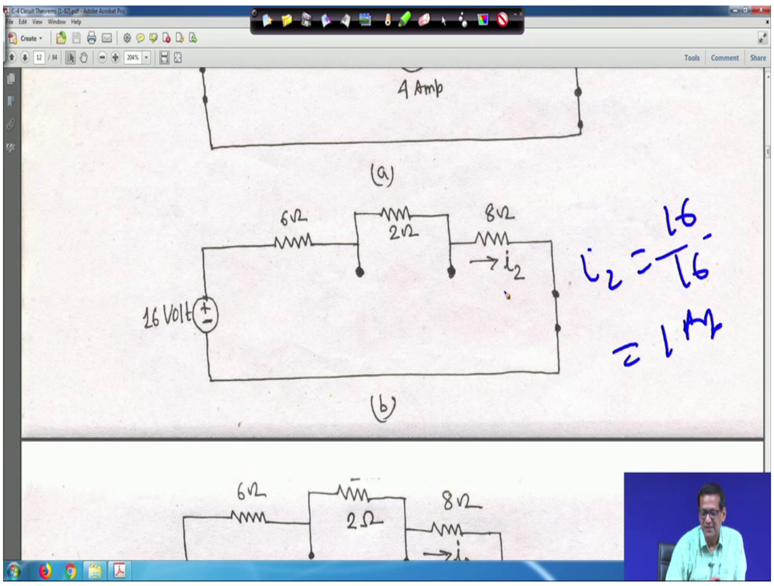

So, i 1 will be 0.5 ampere right so, let me clear it very easy. Similarly here directly you will get it what will be i 2; i 2 will be your basically if you add 6 2 and 8 all are in series. So, 16 your 16 ohm so your i 2 will be is equal to 16 by 16 is equal to 1 ampere in this direction right. So, this is your 1 ampere so, that is your i 2 and then your i 3.

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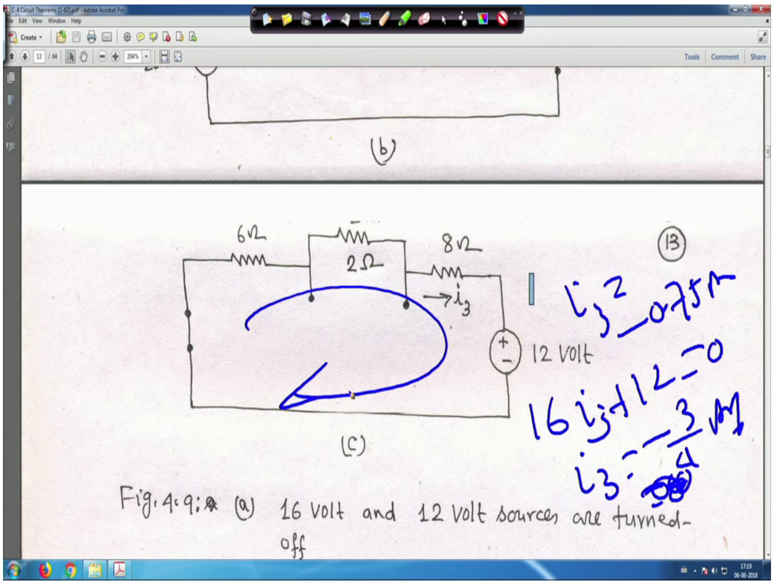

So, i 3 in this case, here in this case this 6 this 16 ohm your what you call is in series. So, basically if you sorry if you apply your KVL, it will be 16 into i 3 then encountering plus terminal plus 12 is equal to 0 therefore, i 3 is equal to minus 3 by 4 ampere. So, minus here I am making it i 3 is equal to minus 0.75 ampere right.

Because here it is plus terminal encountering plus if you apply KVL like this so; that means, i is equal to superposition theorem i 1 plus i 2 plus i 3.

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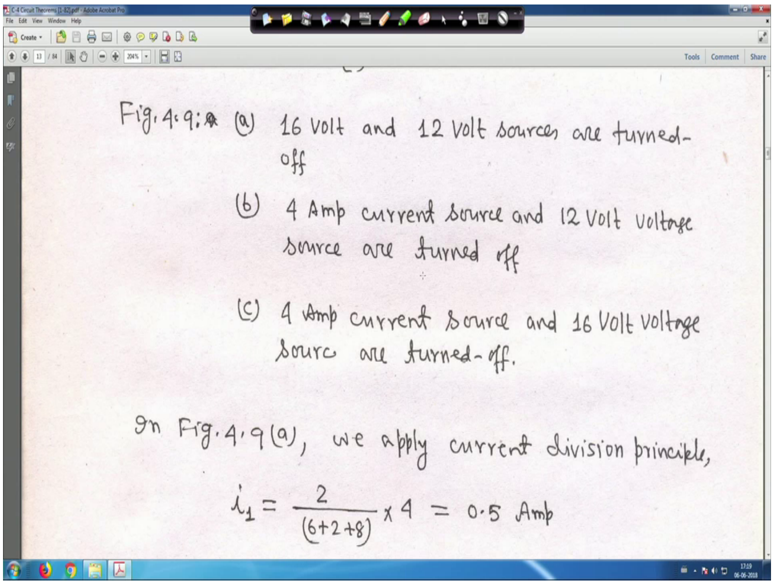

So, in this case your; all this things I have told all this things at that circuit a b c everything I told right and i 1 calculation also given i 2 I told i 3 also I told minus 0.75.

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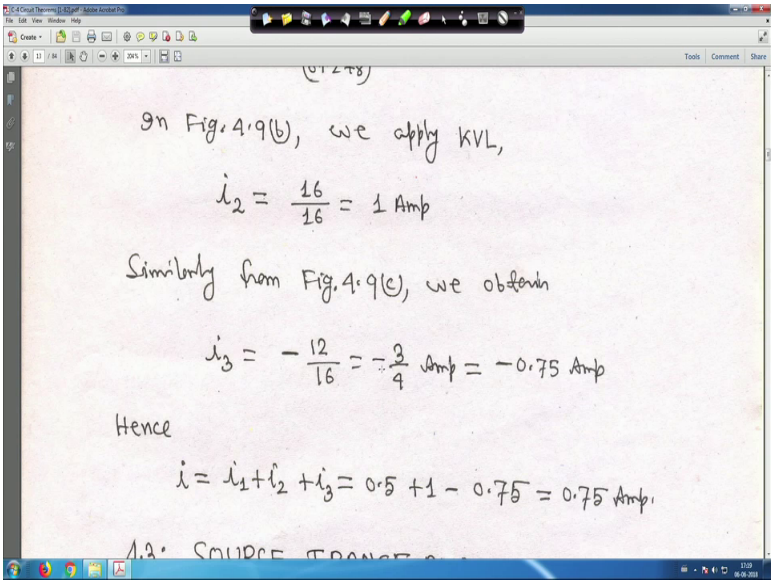

If you apply a super position, i 1 plus i 2 plus i 3 it be 0.5 plus 1 minus 0.75 is equal to 0.75 ampere that is i right. I suggest that you solve this circuit as a whole this considering all this sources you solve this circuit right and see that you i you are getting same; sorry i you are getting the same value whatever using superposition we have got with this.

Thank you very much we will be back again.