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NPTEL
ONLINE CERTIFICATION COURSE

On Industrial Automation and Control

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Topic Lecture – 14
PID Control Tuning
(Contd.)

Good morning right so this is this is a rather simplistic way of doing things and depends on so many things like you know more often than not people may not be able to express their model behavior exactly like this or may not be exactly able to give a reference model so such a simple procedure may not always be applicable but what people do.

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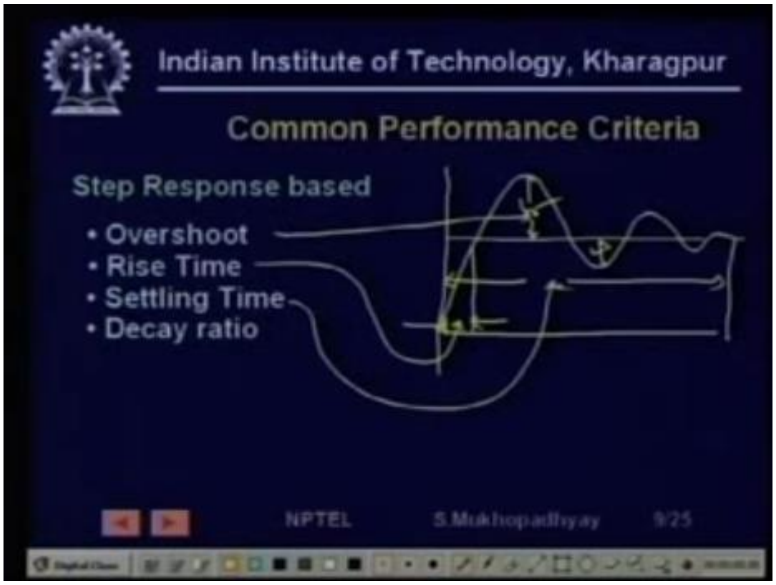

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Common Performance Criteria

Step Response based

- Overshoot
- Rise Time
- Settling Time
- Decay ratio

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Is that they do state certain rather than stating a complete reference model they often state some behavioral parameters of that reference model for example they can state that in the step response of the closed-loop model I want an overshoot which is generally stated as lower than something maybe I want less than ten percent overshoot or you can say that I want less than so many seconds or minutes of rise time which means that if I make a step change it will quickly rise to that value you may say that I want less than so much of settling time which means that after that after the up of the settling time will get nearly zero error or you can say sometimes you can say that I want a certain amount of DK ratio now what is DK ratio.

DK ratio is so you see that if you have a step response typical step response closed-loop plants are typically have exhibit and under damped response so our requirements may be based on terms of the overshoot this is the first one it may be based on some ten percent to ninety percent rise time which is this time it may be based on a settling time which is this time or it may be based on a DK ratio which says that what is the ratio of the first overshoot to the second overshoot that is this divided by this second overshoot by first overshoot now if the DK ratio is small it means that even if you have a even if you have a first overshoot which is likely large but it quickly comes down.

So the ratio of 2nd to 1st will be let us say if you require 25% that will be one-fourth then third to second will again be one forth so which means that your error is rapidly converging to zero so in some sense it is good so you see that such things are often required otherwise.

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Common Performance Criteria

Step Response based

- Overshoot
- Rise Time
- Settling Time
- Decay ratio

Stability based

- Gain Margin
- Phase Margin

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In various other controller designs obviously stability is a consideration and we have our well-known gain margins and phase margins which are basically protections that the that the loop will not oscillate even if the even if the process gain changes by drill bit or the or the overall loop phase changes by drill bit because process models based on which we are going to calculate the controllers they will not be exact so these are you know some of the common simple response parameters and stability parameters which are often considered in designing controller settings. So let us see how so one example.

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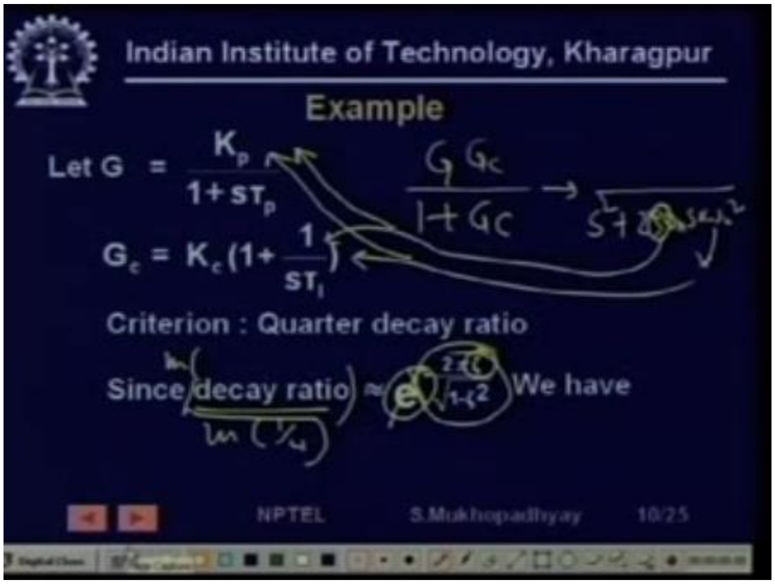

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Example

Let $G = \frac{K_p}{1+sT_p}$

$G_c = K_c\left(1+\frac{1}{sT_I}\right)$

$\frac{G\,G_c}{1+G_c} \rightarrow s^2+2\zeta\omega s+\omega^2$

Criterion : Quarter decay ratio

Since $\ln(\text{decay ratio}) \approx e^{\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$ We have

$\ln(1/4)$

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For example suppose I have a plant which is K_P by 1+ ST_p I am considering very simple examples if you have more complicated examples they actually the solution process becomes more complicated and you need numerical procedures for actually solving out but in essence the method remains the same and suppose I am considering a PI controller I already selected a PI controller and I want to set the value of K_C and TI suppose my criterion is quarter DK ratio that means the second overshoot the ratio of the second overshoot to the first overshoot will be .25 it will be right.

So it turns out that DK ratio will be given is actually given by this formula so this value should be less than 0.25 which means that the we can we can just simplify matters of solutions by saying that the thing that the log of the decay ration we can take log on this side so we can have you know L^m of the decay ratio is approximately equal to this is gone then approximately equal to this or equal to let us say so now so this I want to be equal to one-fourth right so this becomes equal to l^m of one forth.

Now what should be the value so what I have to find out essentially is what should be the value of this ζ such that this function evaluates to be less than one forth now what is θ so if I compute

GGC/ 1+ GGC then I will get in the denominator I will get some parameter in the form S $S^2 + 2\zeta$ ω S + ωn^2 so it will turn out that ω n is also a function of these parameters and ζ is also another function of these parameters so this ζ is going to be some function of K_c and Ti so that is what we are saying that if you try to solve this finally.

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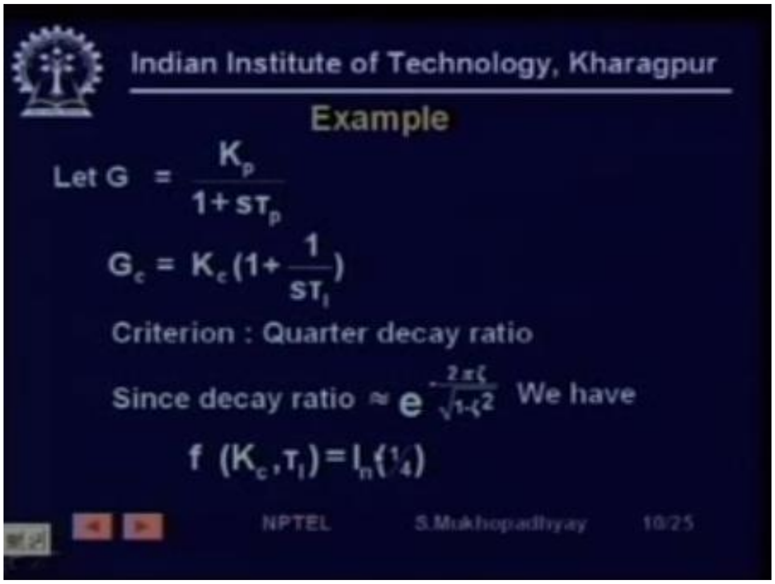

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Example

Let G = $\frac{K_p}{1+s\tau_p}$

$G_c = K_c(1+\frac{1}{s\tau_I})$

Criterion : Quarter decay ratio

Since decay ratio ≈ $e^{-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$ We have

$f\ (K_c, \tau_I) = l_n(1/4)$

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It turns out to be an equation of the type we will get an equation f of function of K_C and T_I is equal to l_n of ¼ so now we got one equation but we need to find out two parameters so obviously we cannot find the unique solution there are actually many solutions so which means that to be able to find unique solutions we need to have some other criteria so we can actually accommodate some other criteria for example if we consider in addition.

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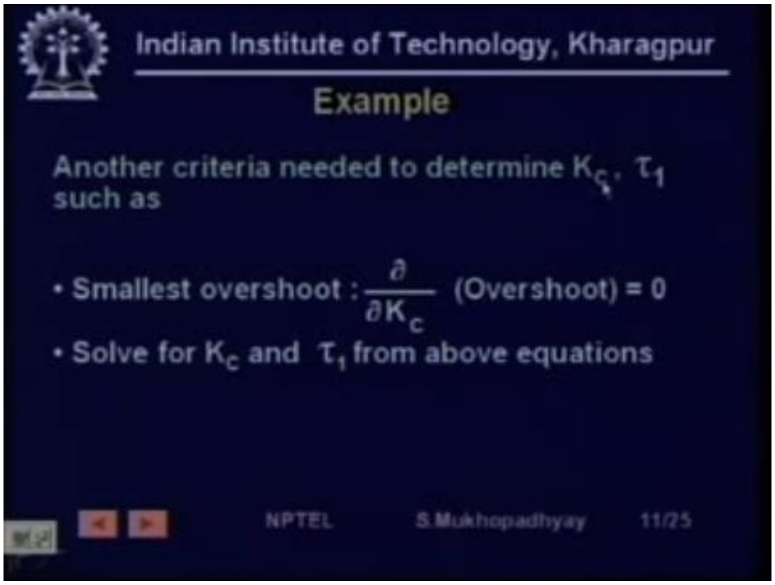

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Example

Another criteria needed to determine K_c, τ_1 such as

- Smallest overshoot : $\frac{\partial}{\partial K_c}$ (Overshoot) = 0
- Solve for K_c and τ_1 from above equations

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So we have another criteria needed to determine K_C and. I am sorry about this one so if I so I mean another criteria and then suppose I could use various kinds of criteria so for example 1 our criteria I could choose is that smallest overshoot I will choose that value of K_C which for a given value of T_1 gives me the minimum overshoot so I am I not only that the overshoots will decay fast I also want the first overshoot to be small so that I will reach settling time will become even smaller right.

So if I want to do that then I will I will take the overshoot function and then I will differentiate it with respect to K_C and then set it to 0 that will give me another equation so now I have two equations and I have two unknowns so now I can solve for k K_C and T_I so you see so it is essentially stating properties of the closed loop function then expressing them in terms of the unknowns that is the gain various times of the PID setting and then solving those equations this is the this is the essential procedure.

So these are you know direct solving procedures and now we see that it is not always that we are going to state them in terms of you know this kind of overshoot or settling time or decay ratio there are various other ways that we can that we can specify control parameter performance and essentially numerically we have to solve out the various parameters is generally not we will not be able to solve them analytically in such cases.

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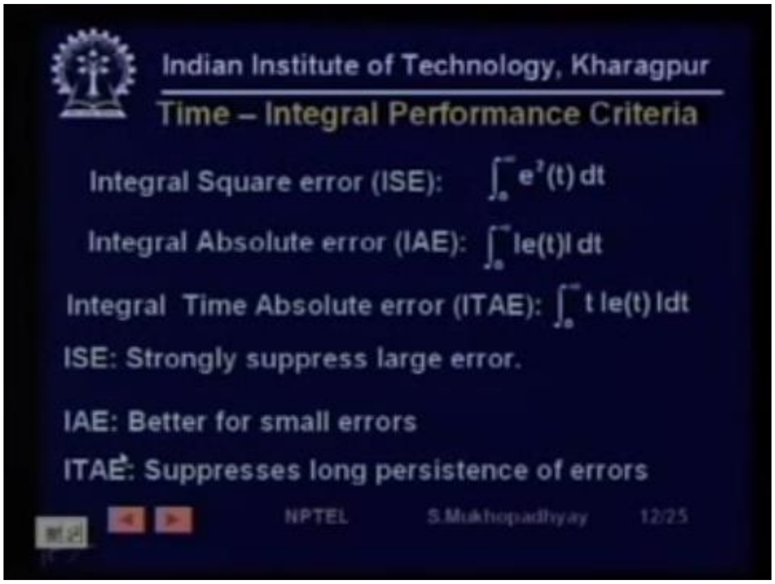

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Time – Integral Performance Criteria

Integral Square error (ISE): $\int_0^\infty e^2(t)\,dt$

Integral Absolute error (IAE): $\int_0^\infty |e(t)|\,dt$

Integral Time Absolute error (ITAE): $\int_0^\infty t\,|e(t)|\,dt$

ISE: Strongly suppress large error.

IAE: Better for small errors

ITAE: Suppresses long persistence of errors

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Accepting some rare cases for example we can select the integral square error that is after a step response is generated there will be error so if I integrate the error the error will eventually come to zero so if I integrate the error the square of error then I should choose such values of gain such that this function is going to be the least now why should I choose a square of error y naught cube of error why not whole to the power 4 of error generally square of error is actually very popular because in what because of two things.

First thing is that in square of error we are able to wait higher errors that is we are saying that if you commit an error of two you are four times better than if you commit an error of four so double errors if the error magnitude goes double then the control performance criterion which is to be minimized goes four times up if you go three times it will go nine times up which means that we are heavily parallelizing large errors.

So when we want to heavily penalize large errors we generally choose powers of E and square is chosen because the square criteria generally leads to very you know nice solution equations because by differentiating square you get linear equations which are very easy to solve but in some cases as we shall as we have seen that in the case of the PH control of the water discharge system we may be interested in and if you are not interested in not interested in penalizing high errors too much then we may choose integral of absolute error sometimes we may like that fine let there be error initially but let the error not persist.

So in which case I can choose what is known as an integral time absolute error when you can see that if an error occurs earlier after the step change it is not penalized so heavily but if it occurs later as time goes high the same amount of error is more and more heavily penalized so what we want is that if you want to have errors even large errors have them in the beginning but after the step change within certain times the errors must quickly converge to 0 so the errors must not persist for long times.

So this is what we are saying so that is what this says that the integral square error strongly suppresses large errors the integral absolute errors are better for small errors because in small errors e^2t becomes even smaller so the small errors are not penalized and in ITAE what we want is that we want that quickly errors converged so we won’t suppressed long persistence of errors

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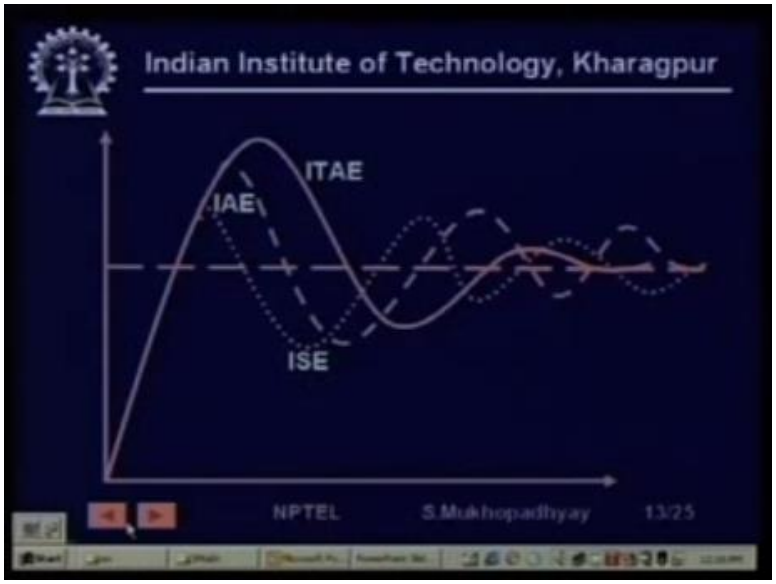

So for example if you took a typical this is a typical wave form that if you took a step if you took a control loop which is designed based on ITAE it may so happen that you will get that you will initially get a high overshoot but it will probably die down faster because that is what is penalized heavily on the other hand if you take an IAE if you if you compare between IAE and ISE you will find that in IAE error magnitudes are going to be less than IAE because in I because in ISE has lot bigger errors are actually heavily penalized.

So the error will not rise very much but it but it will it may not fall so fast also that is when it reaches a slow which is a low level it may persist for long times so this is what happens if you so depending on your choice of your performance criterion you can get different kinds of transient performance in the loop.

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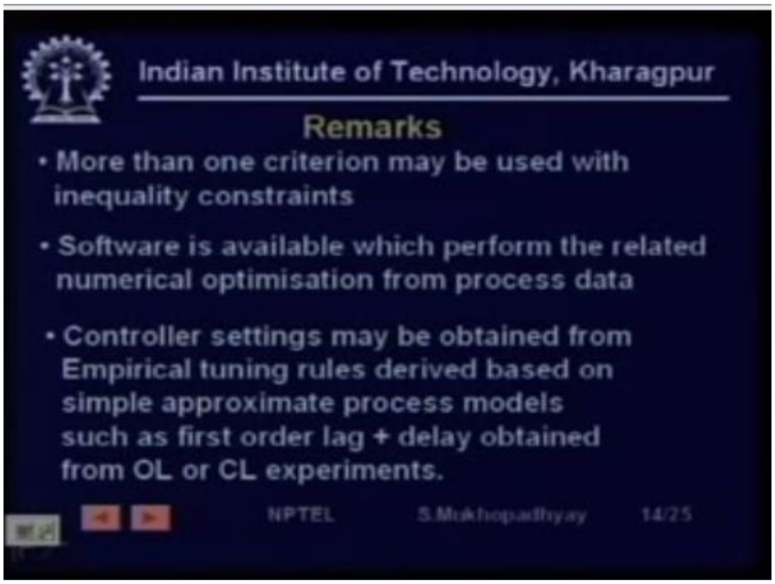

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Remarks

- More than one criterion may be used with inequality constraints
- Software is available which perform the related numerical optimisation from process data
- Controller settings may be obtained from Empirical tuning rules derived based on simple approximate process models such as first order lag + delay obtained from OL or CL experiments.

So in the remark we are saying that sometimes you have more than one criterion sometimes criterion may be conflicting also if you want to increase if you want to cut down on rise time you generally have to increase the gain too much if you increase the gain very heavily then you tend to have very large overshoots so having a small rise time and having a small overshoot generally conflicting criteria so you have to strike a compromise in many cases what will happen is that you are criteria may be stated in terms of inequality.

So you actually have a feasible region in which you can you can operate and in many cases so when you have complex processes that you have complex performance criterion the calculation of the gain may require large amounts of you know numerical trial and error over you know using sophisticated optimization algorithms which search for a good value of the parameter over or some feasible space and therefore you need software and various kinds of tuning software actually sell from the manufacturers.

Now all these that we have so far talked about generally employees assumes that that model forms are available but in many cases you may not exactly be able to obtain a the form of a model and then you must model the process using some simple model structure and based on some experiments so you actually perform an experiment see the response measure certain properties of that response and then design your controller based on that right. So typically you can have either open loop response open loop experiments or you can have closed loop experiments so we are going to look at these two.

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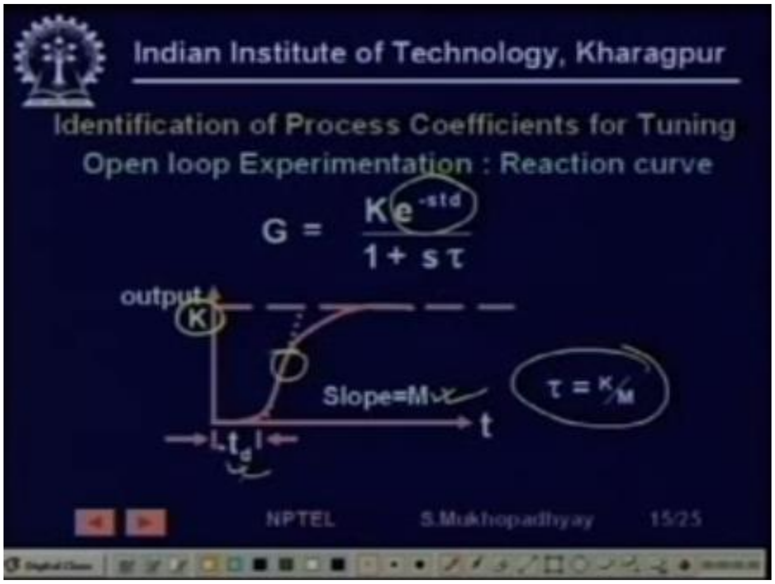

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Identification of Process Coefficients for Tuning

Open loop Experimentation : Reaction curve

$$G = \frac{Ke^{-std}}{1 + s\tau}$$

$\tau = K/M$

So for example if you a typical very popular way of open loop experimentation is the reaction curve that is you take the open loop plant which is typically of this assume to be of this shape but it is not directly in the in the in the experimentation this form is not used so you do a step response experiment so the output is going to write this for the open loop plant so the output is going to rise like this so now numerically you find out that typically in this model if you consider the step response of this model.

Then this step response will actually rise after a time td because of this e^{-std} term and then it will rise just like a first order system k/1+ s τ so / τ or rather τ is equal to K/M so perform an experiment assume that the plant is of this form which is with the reasonable approximation for actually many plants and then measure out what is the value of t_d and then what is the value of what is the slope M and based on the slope M and the final settling value which will give you the value of K.

So now after the experiment you know this t_d you know this K and you know this M so now you then based on this people have various kinds of empirical rules empirical rules means process control experts have actually done through their experience by many simulation and experiments actual experiments they have given some thumb rules that if you find that K is of so much value and M is of so much value and t_d is of so much value then use if you are using a P controller use this value of gain if you are using a PI controller use this value of K and this value of T_I

So there are certain rules which are not analytically derived but are empirically derived so empirically means just by doing experimentation from experiments can they cannot be explained.

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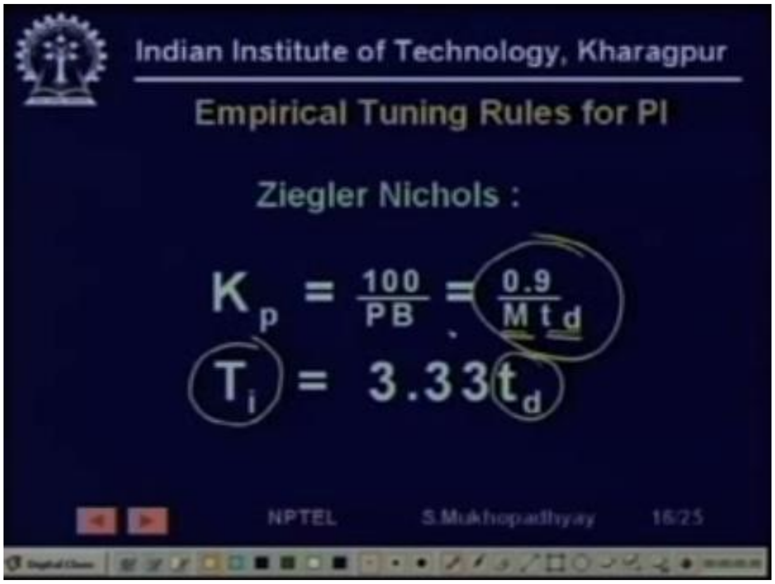

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Empirical Tuning Rules for PI

Ziegler Nichols :

$$K_p = \frac{100}{PB} = \frac{0.9}{M t_d}$$

$$T_i = 3.33 t_d$$

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So very famous rules which are Ziegler Nichols so the Ziegler Nichols tuning rule says that if you do this experiment and if you are designing a T_I controller then the value of the gain K_P should be this much so you see in terms of t_d and M and the value of the value of T_I should be given in terms of t_d this is what Ziegler Nichols found works well now obviously it will not work well everywhere so various people will say various so we have various other kinds of rules also for example we can have a we can have another rule which is known as the coherent cone rule.

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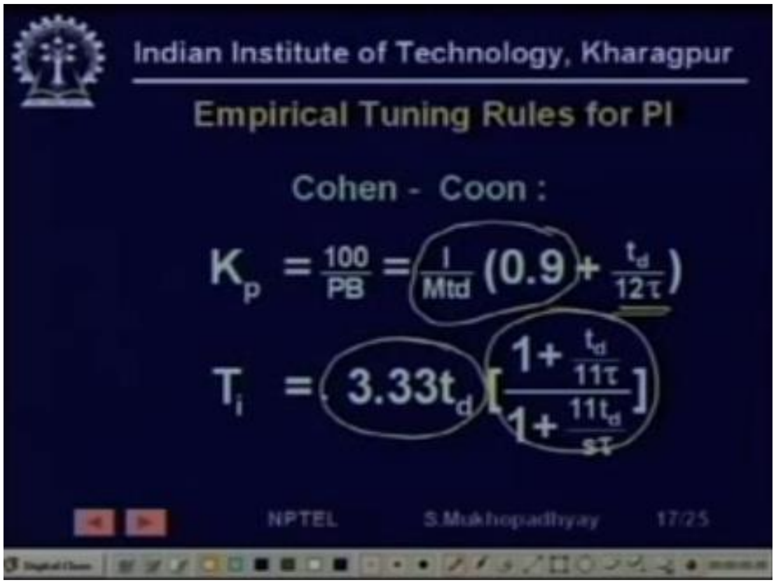

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Empirical Tuning Rules for PI

Cohen - Coon :

$$K_p = \frac{100}{PB} = \frac{1}{Mtd}\left(0.9 + \frac{t_d}{12\tau}\right)$$

$$T_i = 3.33t_d\left[\frac{1+\frac{t_d}{11\tau}}{1+\frac{11t_d}{5\tau}}\right]$$

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Which looks like this so you see that there they are actually fairly is fairly close only thing is that Ziegler Nichols said that K_P should be this much now Cohan and Coon actually adding another term so here also you see that in case of he is adding apart from this t_d he is adding a factor here right so you see that various kinds of empirical rules are given and then you can apply so actually what happens one must realize this that one must realize this that actually what is going to happen is that say.

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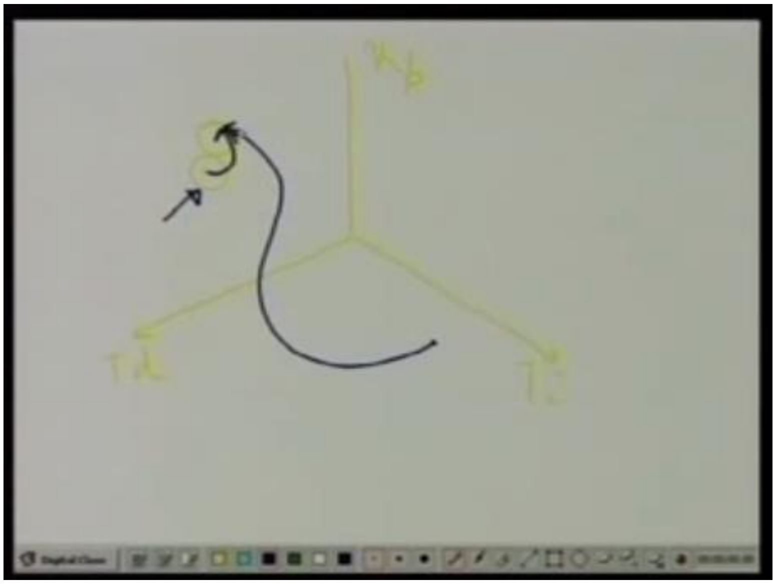

Look at this that you are looking for you are searching for some good parameter let us say in the let us say in the K_P t_d and T_I space right now suppose they the real best value of parameter actually lies here so now you have to you can exhaustively search the space and maybe finally you will actually locate that that is the best value of parameter which is very expensive so the idea is that if you can if you can search if you can by some simple rule if you can locate a value which is actually which is likely to be very close to the real true optimal value then you then your search becomes easier you actually quickly converge on you actually quickly converge on you can quickly converge on to the optimal.

Well if you if you do not have the benefit of this rule which is given by this Ziegler Nichols Cohan and Coon a kind of rules then you may search start search from here and then after long amount of search you may reach here so it is basically these rules should be treat taken in that spirit that they generally give you a good first guess for the values and then you have to you may have to do some amount of tuning to actually be able to get these real good parameters controller parameters for your process right.

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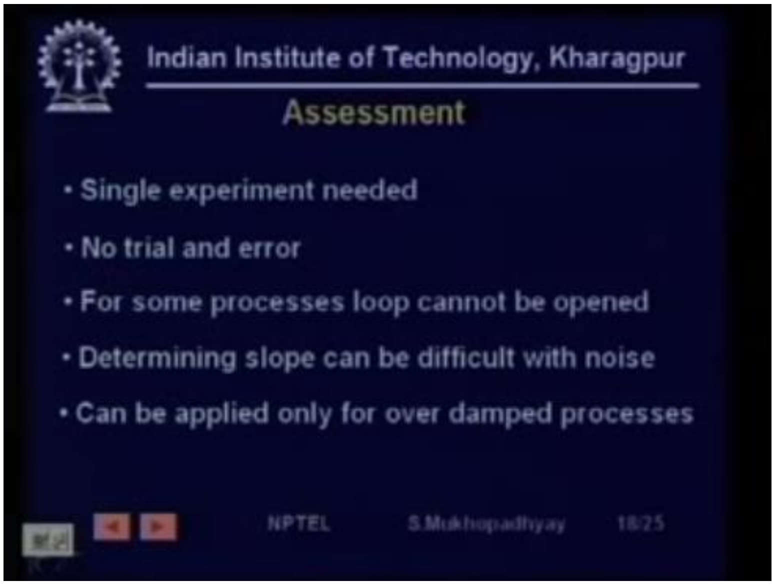

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Assessment

- Single experiment needed
- No trial and error
- For some processes loop cannot be opened
- Determining slope can be difficult with noise
- Can be applied only for over damped processes

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So an overall assessment of this of this open loop process is that you need only a single experiment just once give a step input and collect the values no trial and error is needed and for some processes loop cannot be opened one this is a big problem that that mean here we are we want to do an open look experiment now some processes are there already in closed loop you cannot open them nobody will allow them allow you to open the loop and do a do an open loop experiment so in such cases this is not applicable and sometimes determining the these slopes etc..

Can be actually very difficult especially when you have noise when you have noise finding numerically the slope from data can be very complicated and you have to use some good approximation procedures and finally note that this procedure is actually applicable only for over damped processes not for under damped processes, so you cannot use taken under damped open loop under damp process and then use this procedure although you may be able to find out some value of KTDM so we need a closed loop procedure.

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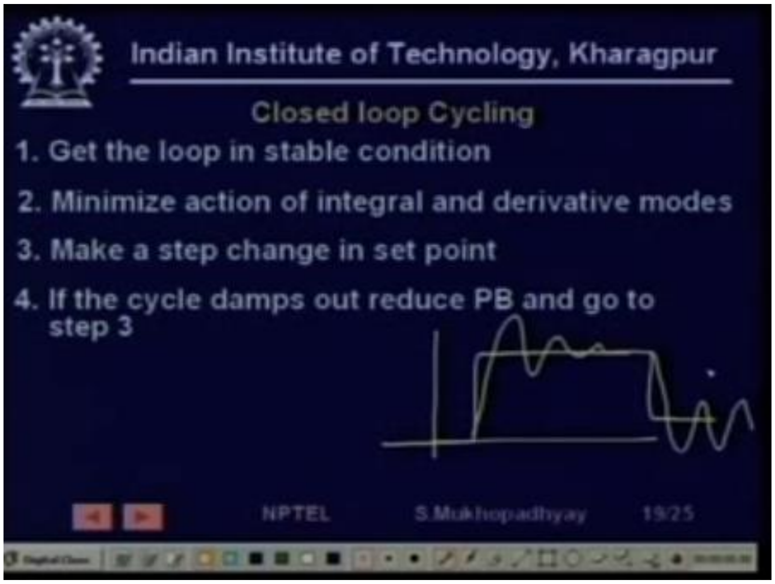

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Closed loop Cycling

1. Get the loop in stable condition
2. Minimize action of integral and derivative modes
3. Make a step change in set point
4. If the cycle damps out reduce PB and go to step 3

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And the closed loop procedure goes like this it is essentially an iterative procedure that is it involves trial and error so what it does is that first get the loop in a in a stable condition probably the loop is already in a stable condition operating then minimize action of integral and derivative modes reduce those integral and derivative gains if necessary reduced the K_P also so that the loop is stable now make a step change in the set point right so make a check so if you make a step change in the set point if you make a step change in the set point so you make a step change in the set point.

So the process loop rises which lies down if it dies down then if the cycle damps out then you reduce PB means increase the gain so if you increase the gain your gain margin will reduce which means that the process will be closer to one oscillation so maybe when you reduce the gain next time you give another step so you go to step 3 and then make another step change maybe the negative direction.

So then it may happen that this loop will tent to persist for longer term still it damps out so still it terms of again go to step 3 and then reduce gain little bit and make a step change in set points so go on doing this till the loop oscillates.

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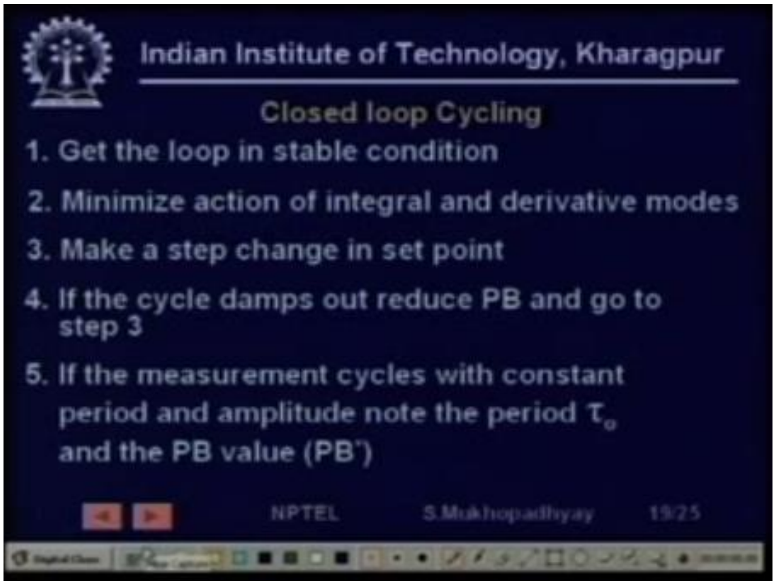

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Closed loop Cycling

1. Get the loop in stable condition
2. Minimize action of integral and derivative modes
3. Make a step change in set point
4. If the cycle damps out reduce PB and go to step 3
5. If the measurement cycles with constant period and amplitude note the period τ_o and the PB value (PB^*)

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In the measurement cycles with constant period and amplitude note this period τ and the PB value that is the gain which has made the planned oscillate this is called the ultimate gain right so now you have got two values one is this PB start which is the ultimate gain and another is the period τ_0 so now based on these there are again I am sorry based on this then again empirical formula.

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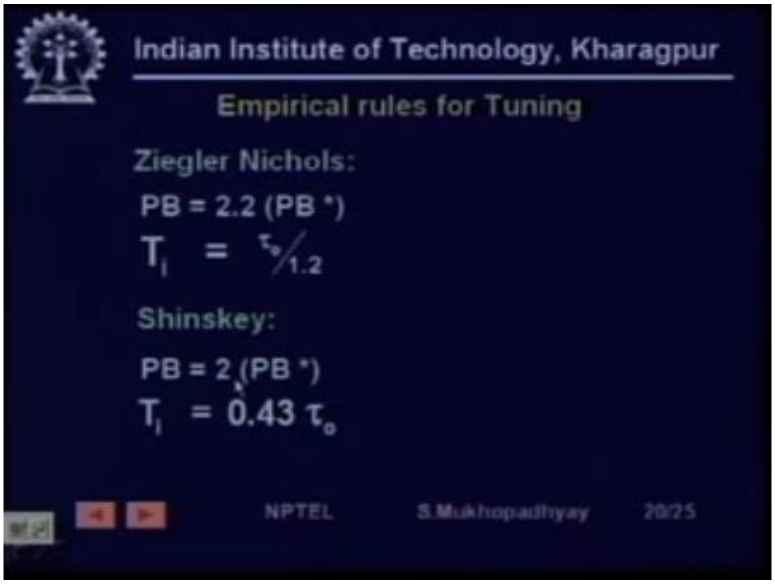

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Empirical rules for Tuning

Ziegler Nichols:

PB = 2.2 (PB *)

$T_I = \tau_0 / 1.2$

Shinskey:

PB = 2 (PB *)

$T_I = 0.43\, \tau_0$

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So once so for example Ziegler Nichole says that if you have found a particular value of TV start and found a particular value of τ_0 then the then for a PI control set the proportional band value as 2.2 times this value and set the T_I is this one again empirical purely empirical found so called by experiments similarly some other process control expert Shinsky says that no choose these two values of gain.

So obviously you are going to have difference in these difference in these properties for example here you have a larger gain and here you have a larger time constant to here you have a smaller gain here you have a no here you have a smaller gain and you have a smaller time constant here you have a larger gain and you have a larger time constant right larger gain and a smaller time constant so you will get different properties so what is the assessment of these closed-loop methods that some trial and error is needed.

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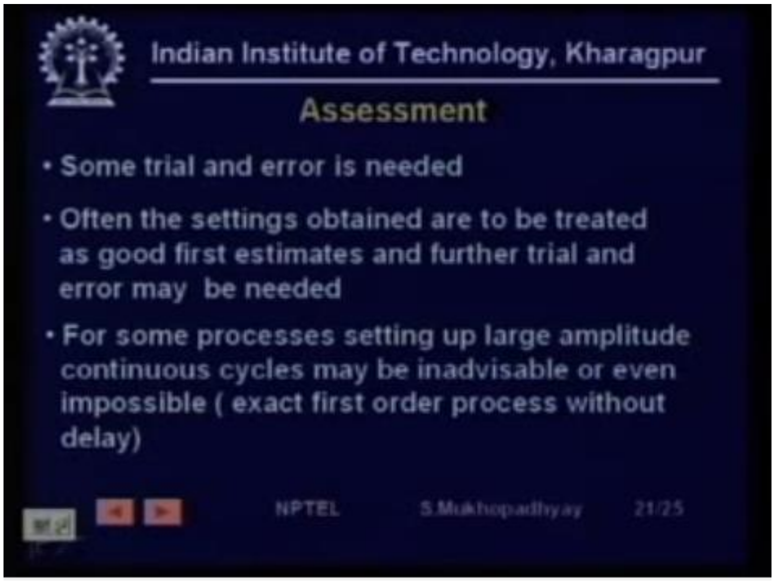

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Assessment

- Some trial and error is needed
- Often the settings obtained are to be treated as good first estimates and further trial and error may be needed
- For some processes setting up large amplitude continuous cycles may be inadvisable or even impossible (exact first order process without delay)

And often the settings obtained so as I have already told that the settings obtained are to be treated as good first estimates and further trial and error after around that you may like to tune little bit and see whether you are getting still better performance than should select those values. Now for some it so happens that this cycles when you when you want to make the loop cycle by continuously increasing PB the cycling amplitude there is the amplitude of the cycle may increase it is it is actually not in your control when a process oscillates it is cycling amplitude and period are actually not in your control.

So now but then that may be disastrous because a process may not be allowed to cycle with a with a very large amplitude, so then what are you going to do so in such a case there is a nice procedure which says that.

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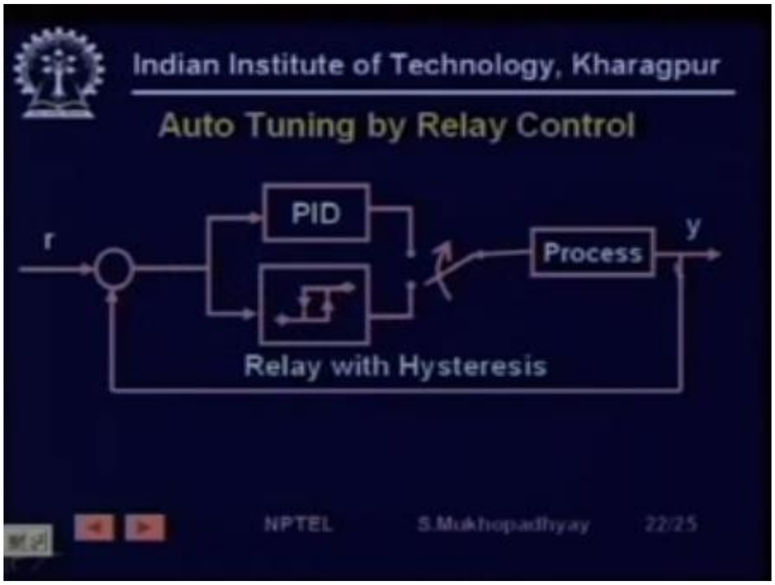
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Auto Tuning by Relay Control

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In set up the control loop like this so you have the normal PID controller and you have a relay with hysteresis here so what is the what does it mean that if the error goes positive then the relay will switch with switch in one direction if it is negative it will switch in the other direction so now what will happen is that the width because of this relays you now stick the switch and then try to control the plant with this relay then what will happen is that the relay will continuously toggle so the relay will give you an oscillating input.

So what will happen is that the plant will actually oscillate immediately it will start oscillating that is there will be no trial and error needed and the amplitude of oscillation cannot can now be controlled by the properties of the relay. So now you have got a nice situation where.

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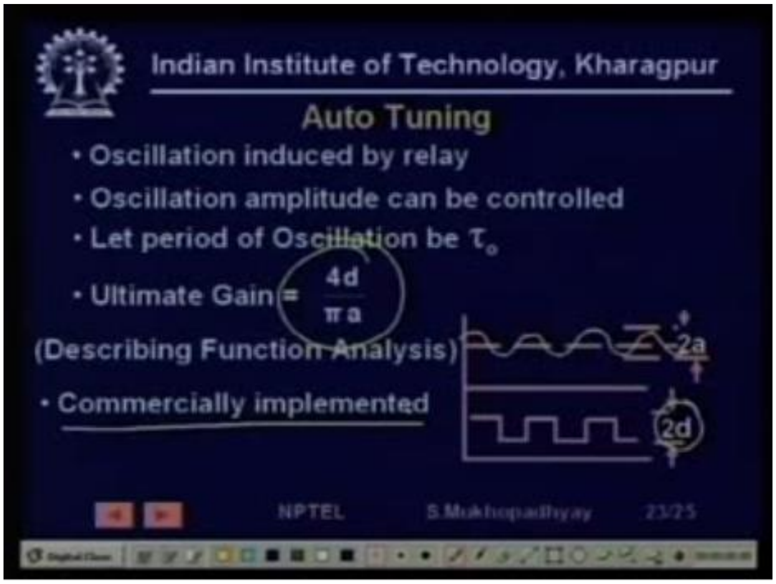

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Auto Tuning

- Oscillation induced by relay
- Oscillation amplitude can be controlled
- Let period of Oscillation be τ_o
- Ultimate Gain = $\frac{4d}{\pi a}$

(Describing Function Analysis)

- Commercially implemented

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The so that the oscillation can be directly induced by the relay without trial and error and its amplitude can be controlled so you can have induces small oscillation and it let them and then note the period of this oscillation and it turns out by analysis is called which is called the describing function analysis in nonlinear control.

That if you have if you have if you have an oscillation of this kind in the process which is of amplitude which can be controlled and if the relay switching's are off of amplitude 2d then the ultimate gain which you want to which you wanted to find out to be able to apply the closed-loop tuning formula can be given by this formula so you see you are still finding out that same gain TV start but now using an experimental which lets you set up an oscillation atone shot of a controllable amplitude so that is the big advantage and then now you can you can easily apply those formerly and then get the gain.

And it so happens that people have actually implemented this commercially in the PID controller so that whenever the operator feels that the process needs to be retuned then the process can be momentarily flicked on to the relay and immediately an oscillation will be setup and then by noting the amplitude of oscillation and the period of oscillation the gains can be setup in very quick quickly so no separate experimentation necessary and then automatically the controller can be tuned just by the flick of a push-button therefore it is called auto tuning right.

So here the here the whole tuning process has been automated so this is the end of our lesson and what we have basically reviewed are the various performance criteria for control loops.

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Lesson Review

- Performance Criteria for Control Loops
- Direct Computation of Controller Settings
- Empirical Tuning Rules based on Open-loop and Closed Loop Methods
- Auto-tuning

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Then we saw a method of direct computation of controller settings based on a process module open-loop plant model and the reference model then we found out some empirical tuning rules which are based on open loop and closed loop experiments that you can perform on the plant and finally we saw a procedure called auto tuning where we could quickly and automatically do an experiment and find out the controller settings.

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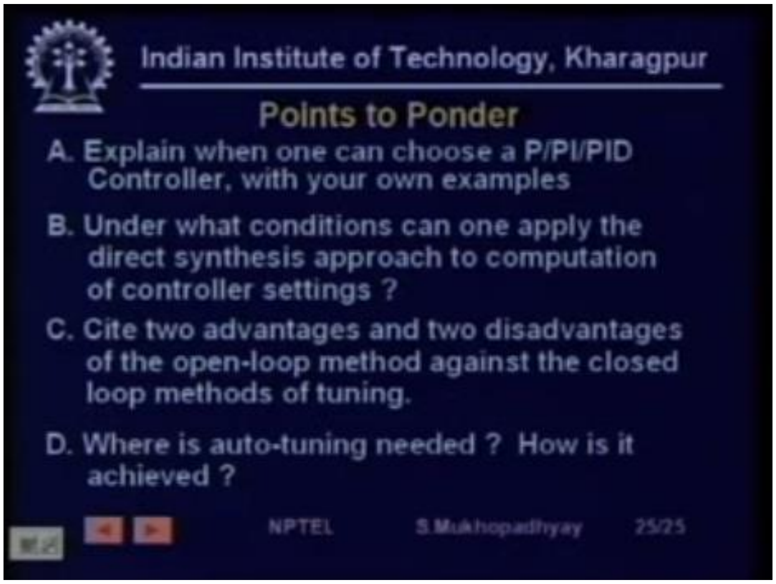

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Points to Ponder

A. Explain when one can choose a P/PI/PID Controller, with your own examples

B. Under what conditions can one apply the direct synthesis approach to computation of controller settings ?

C. Cite two advantages and two disadvantages of the open-loop method against the closed loop methods of tuning.

D. Where is auto-tuning needed ? How is it achieved ?

So some of the points to ponder for you or is that you can try to find out in what situations P/PI/PID controllers are to be used all the answers are in this lecture only and you can try to find out your own examples of processes and then under what conditions you can find under what conditions one can apply a direct Synthesis procedure.

Let us say in your application processes whether you can apply or not or if you can apply why you can apply or why you cannot apply all these things you can also site two advantages and disadvantages of open loop method against the closed loop method and finally you can you can find under what situation and an auto tuning feature will be needed and how is it achieved how is it how an auto tuning is achieved one procedure is already given in the lecture so here we end thank you very much.