Dynamics of Physical Systems
Prof. S. Banerjee
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 19
The Bond Graph Approach – VII

Now, while deriving the differential equations you have noticed that the second question was referring to only the integrally causal storage elements. The question was what do the integrally causal storage elements receive from the system. Why the integrally causal storage elements only, because only those ones those storage elements would create state variables, because they are able to vary their state variables independently. Now, what to do with differentially causal storage elements, we have seen that that can arise in certain cases; for example, if you have a motor.

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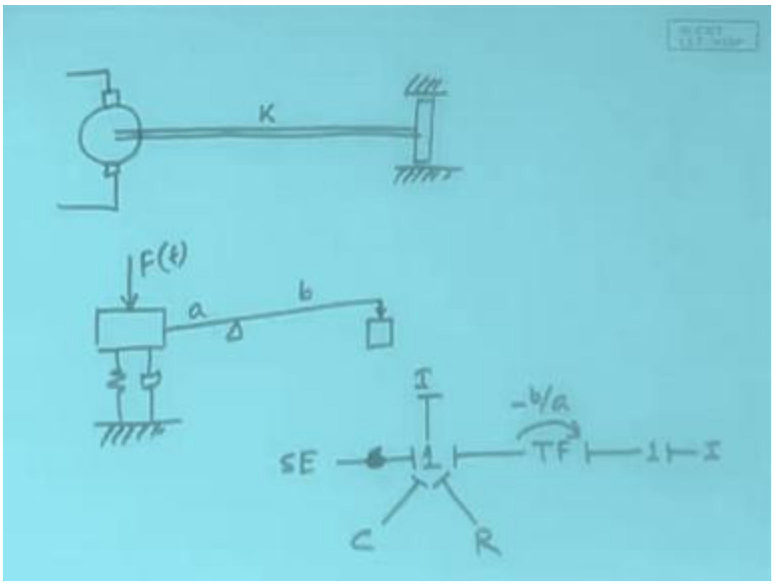

This side I am not drawing because you know what it was here suppose, there is a shaft and here there is a mass doing something. Now, if you assume that the shaft is rigid then what will happen this mass and that mass will not have any independent degree of freedom as a result of which, one will give rise to a differential causality. It is understandable then that one mass, one of the two will give rise to a state variable and therefore, you have to ask that question concerning that specific one, but what to do with this because it is after all there.

Now, while discussing the problem of differential causality I have said that one possible way out might be to assume some kind of a small amount of tortional springiness, so that you will assume some spring constant k. So, that this part and that part have a bit of degree bit of freedom; however, small it may be that it suffices. Imagine the situation where you had a mass with a spring damper arrangement here and well, anything there and then there was a, now suppose you have a mass directly connected here then what happens.

These two masses then though they apparently are located quite different places, they do not have their own degrees of freedom. Why because if this fellow goes up and this fellow will have to go up and down exactly in the same way, so they do not have any independent degree of freedom. So, if you if you draw the bond graph for this system, it will be 1 junction, here you have the I element, here the SE element and here C, here R, then there will be a transformer representing what minus b by a, then you have what.

If you have only this then how do you represent it, this is after all a velocity point, so there will be a 1 junction connected to an I element. So, in that case you would notice let us just do this one again to refresh our memory, how the differential causality comes. Suppose, we start from here the SE elements receives flow and gives effort, now this 1 junction this will be integrally causal, so this brings in the flow information, the other ones must distribute the flow information.

A transformer has received the flow information therefore it must give out 1 junction is receive the flow information therefore, it must be give out. Now, here we find that this is not properly causal, differentially causal, because it is receiving the flow information, so that was the point, now how to overcome this, one way to overcome is to assume a bit of springiness in this long lever shaft.

So, if you assume a bit of springiness you will have to connect a bit of spring there as a result of which, here you will find that it becomes a properly integrally causal, but this way of overcoming the problem has a practical difficulty. The practical difficulty is that you are after all assuming a very rigid shaft with a bit of springiness; that means, it is almost like a rigid shaft; which means, that even though this and that or other elements in the system for example, here there was a inductance

So, there would be these inductances and the masses would have their own characteristic frequencies while this shaft will have a very high characteristic frequency, so they will tend to oscillate at quite, quite different frequencies. Here also if this fellow has a very rigid shaft; that means, you will assume a capacitance here, but that capacitance will be set to a very high value as a result of which this will tend to vibrate at a very high frequency, much higher than the characteristic frequency of the other elements.

What is the practical difficulty with that, the practical difficulty is that when you are trying to solve it, solve the differential equations by some numerical means the step length will be very difficult to choose. Such systems are called stiff systems, stiff means the ones that have widely different characteristic time lengths within the system, they will have they will be very difficult to solve numerically and they are called the stiff systems.

Normally, if you try to solve it we will talk about the different routines for the numerical calculation later, but probably you have gone through some kind of numerical algorithms course and you have seen that these are normally solved by Runge-Kutta method. And probably you have learnt the Runge-Kutta method related to a fixed step length, but if you see a problem like mat lab, you will find that they are variable step length depending on whether it is going straight or whether it is getting get taking very sharp turns, it adjust the step length.

In such systems, such a program will have difficulty in really deciding the value, so it will have difficulty in deciding that value which has to be set in the step length, so in such cases somehow we will have to figure out how to solve systems with differential causalities. Let us see, so you have suppose a system like this and our objective, now is in spite of the fact that it has a differentially causal element we will try to obtain the differential equations, let us see how to do it.

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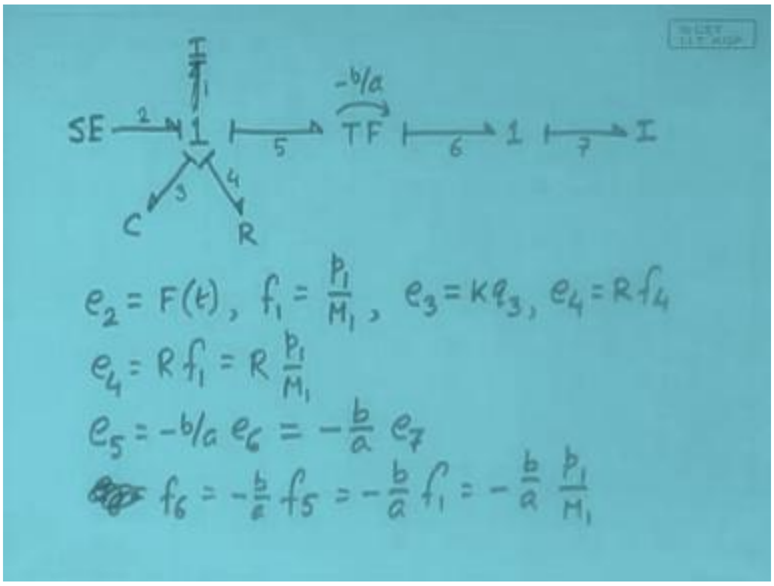

Let me just copy this bond graph here, and then we will set of SE two way 1 junction to an I is it visible yes to C to R, then transformer to a 1 junction to I, so the power directions will be like this, the causalities will be like this, that we have already seen, now let us give numbers 1 2 3 4 5 6 and 7. Now, let us try to obtain the differential equations for this system, you would notice that it would require a bit of different treatment to obtain the differential equation, but it is possible.

First, let us ask the first question related to all the elements, so first question this is e 2, so it gives e 2, e 2 in this case there was a force acting here that is F of t, what does this element give f 1 is p 1 by M 1, what does this element give e 3 which is related to this spring, so it is K q 3, this element gives R, R gives what e 4, e 4 is R f 4, R f 4 is equal to what, the information is coming from there. So, e 4 is equal to R f 1 is equal to this R p 1 by M 1.

Now, we need to ask this question related to the transformer, what does it give to the rest of the system through this side, it gives e 5, e 5 is equal to this was minus b by a, so it is minus b by a this side e 6, now e 6 comes from, e 6 has to come from e 7 because it is a through junction. So, it is minus b by a e 7 as if we do not know what is e 7, so leave it at this stage, now let us what does the transformer give to the other side, it gives f 6, it gives f 6 is equal to minus b by a f 5 and f 5 comes from here, so minus b by a f 1 is equal to minus b by a p 1 by M 1.

So, far there was no problem, but now we also have to ask this question related to this element and this is the differentially causal I element. Now, there we have to treat it a slightly differently, let us see how it is, but I need to keep it displayed while we do it, what does it gives to the system, e 7, so you have start from e 7.

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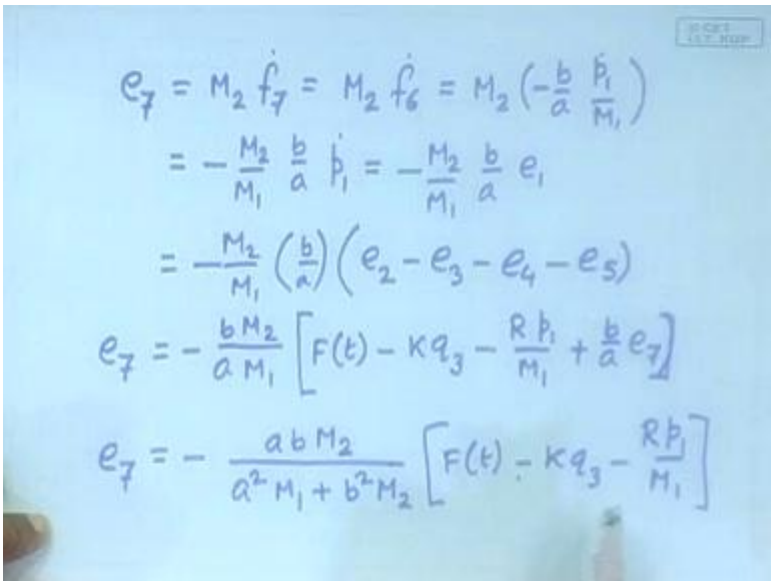

So, it gives e 7 is equal to no, notice the line of logic it gives e 7, but that should be related to its property and its property is that effort is equal to mass into acceleration, effort is equal to mass into acceleration, so e 7 is M 2 f 7 dot. So, wherever we are asking this question, we are trying to relate it to the character of that system that particular element only. So, it is given e 7, but that is depend on its own character, that is M 2 and force is equal to mass into acceleration that is what you written here.

So, we have got a dotted term in the right hand side, but we will we will try to take care of it, f 7 comes from f 6, so is equal to M 2 f 6 dot. Now, f 6 dot, f 6 we have already learnt what it is, so here. So, it is M 2 into minus b by a p 1 dot by M 1 because of this dot, we have just substituted f 6 is this, so f 6 dot would be this is constant, so p 1 dot here, so let us make it tidy minus M 2 by M 1 b by a p 1 dot. Now, what is p 1 dot, p 1 dot is nothing, but e 1.

So, that is how we get rid of the dot is equal to minus M 2 by M 1 b by a e 1, so now we have to find out where does this e 1 information come from. e 1 is nothing, but since it is a effort summing junction it is e 2 minus e 3 minus e 4 minus e 5. So, we have minus M

2 by M 1 b by a e 2 minus e 3 minus e 4 minus e 5. What is e 2, e 2 is f applied force, e 3 is known, e 4 is known, e 5 is not completely known because e 7 appears in the right hand side.

So, let us substitute and see what ultimately is obtained is equal to minus b by a M 2 by M 1, now all these we will substitute. It comes to be F of t from for e 2 minus e 3 is K q 3 minus this is R p 1 by M 1 minus, this fellow was minus b by a e 7, so this becomes plus b by a e 7. So, when you have this, you would notice that in the left hand side you have e 7 and in the right hand side also you have e 7, so you can easily extract this value.

Now, you just do this a bit of algebra, you get it in the left hand side and then divide. So, that e 7 is obtained independently, it will become minus a b M 2 divided by a square M 1 plus b square M 2 F t minus K q 3 minus R p 1 by M 1, any way it is obtainable no difficulty. So, there was no great conceptual difficulty, only thing that we did was we asked the question related to this element and then linked it with the character of this element by force is equal to mass into acceleration equation.

Then we went along the bond graph, to find out where this information comes from finally, we reached a place where we know the variable that is p 1 dot, we made it e 1 and then we could substitute the rest, but such a system always would lead to the same term appearing in the left as well as in the right. And then there has to be a bit of algebraic manipulation involved in order to calculate this variable, but it is doable, that is the point.

So, even if you have the difficulty it is not always desirable or necessary to put in additional dummy variables because that those additional variables would in many cases give rise to a lot of problems in actual computation. So, once you have obtained this, you ask the second question.

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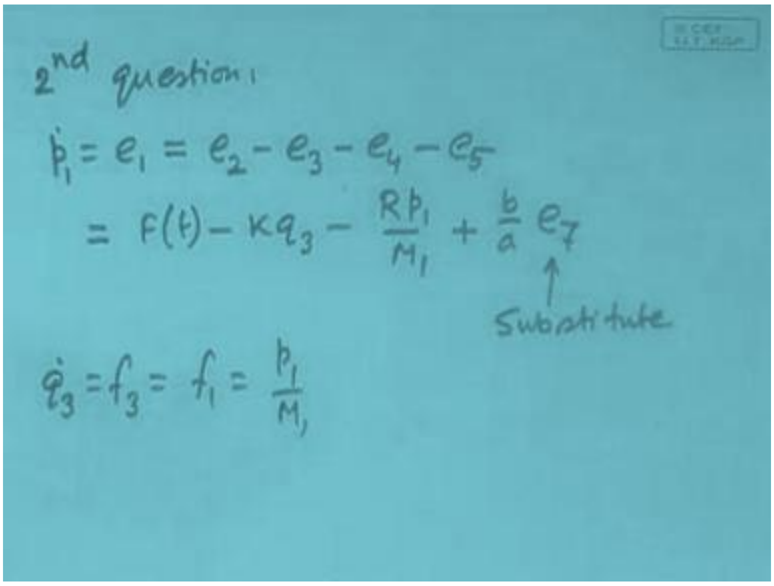

Where is it here you ask the second question, related to the integrally causal storage elements only, so first this is it receives e 1 which is p 1 dot. So, we will write p 1 dot is equal to e 1 is equal to this whole lot e 2 minus e 3 minus e 4 minus e 5, which we already have substituted once and therefore, this whole lot is this. So, we can easily substitute it here and get F of t minus K q 3 minus R p 1 by M 1 plus b by a e 7. Now, this e 7, now we know all these, just put it here that does it.

So, the second variable was the other integrally causal storage element was this, it was receiving flow so f 3 was q 3 dot is equal to f 3 is equal to f 3 came from f 1 is equal to p 1 by that is it, so these two will be the differential equations. So, notice that it is possible to obtain the differential equations if there are integrally causal storage elements no difficulty, only thing is that there is a bit more algebraic manipulation involved, but it is doable.

Now, I will give you some problems to solve, so that you can get some practice and then we will go to the computer demonstration of this software. First, note down the problems.

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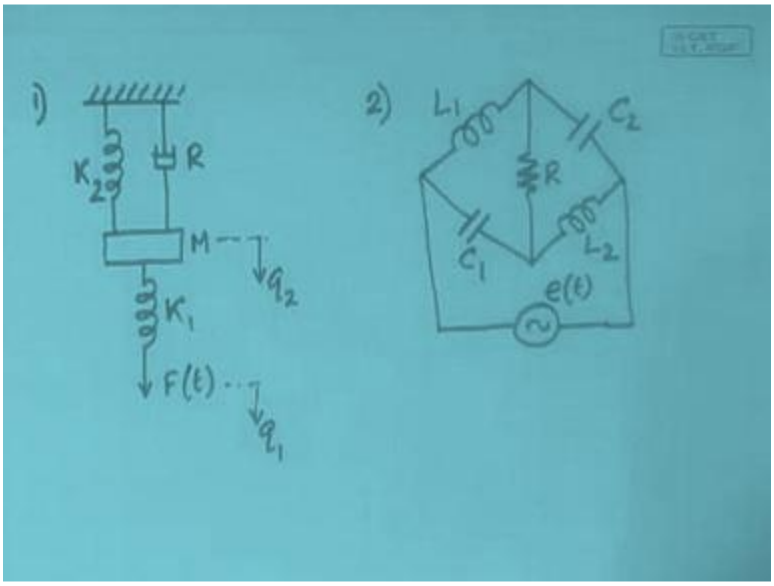

Two I gave earlier, but still I will repeat here, so that you can note down and you might assume the position variable here and here as a two variables which are q 1 and q 2, that is problem number one, problem number two obtain the equations for this system, this is voltage source. Have you noted it, fine.

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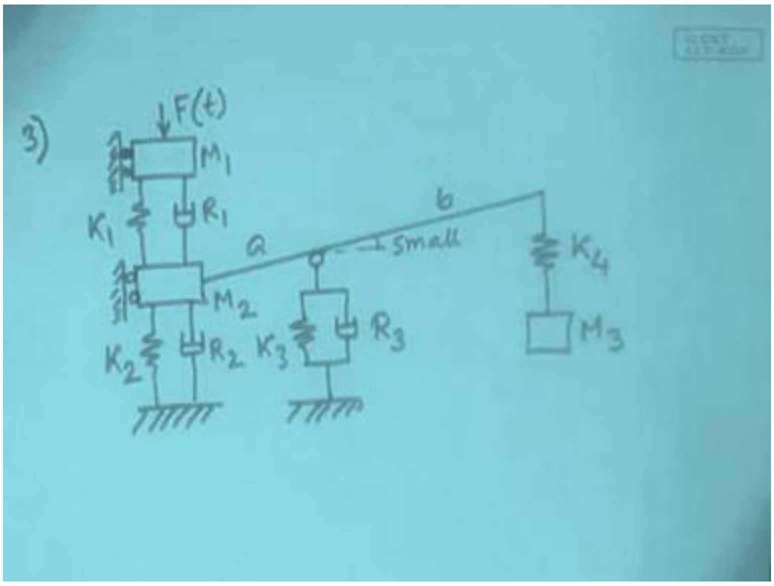

Problem three, there is one mass here acted on by a force F of t, here is a mass here is a spring damper arrangement to another mass, remember these ones are allowed to move only in the vertical direction with the help of these, I am just showing that they are able to move only in the vertical direction and here it is ground. Now, this point is connected to a lever, but then the suspension here it is not fixed, here it is connected by a spring damper arrangement to the ground, so this point is also able to move and then you have got a spring mass.

This is M 1, this is M 2, this is M 3, this is K 1, K 2, there is there is another K 3 and K 4 R 1, R 2, R 3 a b fine, but the point is that while this one goes up and down, if you assume that this is you know inclined then the problem will become very complicated. So, assume that this is small this angle is small, which is not a very bad assumption, so then it will be practical problem. The rest are simple, only you are to figure out what to do with this. The rest I have already done, so the additional complication that I have introduced here is this, how to include this. Next problem also you have seen, but nevertheless I will give.

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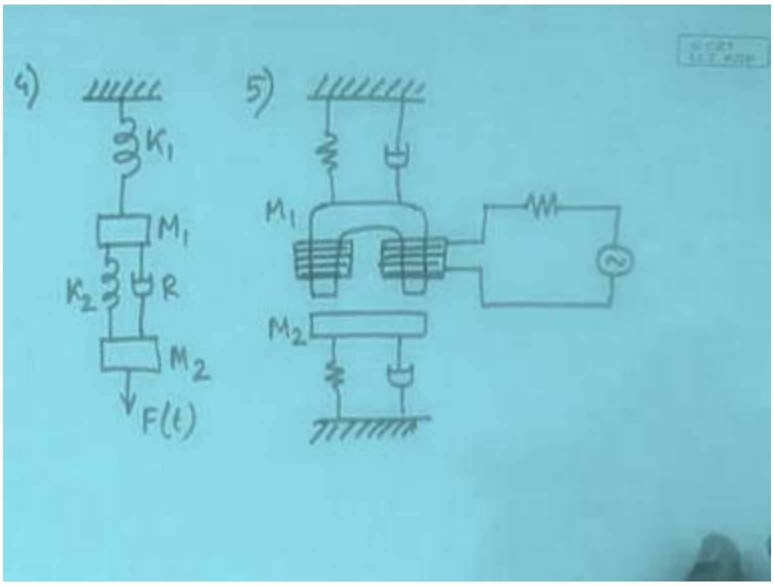

So, that your tutorial copies are complete, here is a mass, here is a there is a spring then a mass then another spring with a damper and a mass and it is being pulled. Five, this problem I gave you the last time, but nevertheless let me give it because it was not displayed in this spring, yes there is a force here, there is a force pulling it, here is a spring damper arrangement connected to a horse shoe magnet and the horse shoe magnet magnetization coils are like this.

And these are supplied through a resistance and a source of effort and; obviously, this will have some inductance that you have to consider. Here there is a bar of iron, this one has a mass M 1, this one has mass M 2 and that is supported by another spring damper arrangement on the ground. So, you notice that as this one allows a variable current to flow, its magnetization will go up and down as a result of which it will be pulling this mass with different forces, there will be vibration of this mass and that is what you have to capture with the help of the bond graph. This one probably you have already done. Let us give a problem involving algebraic loop this will be six.

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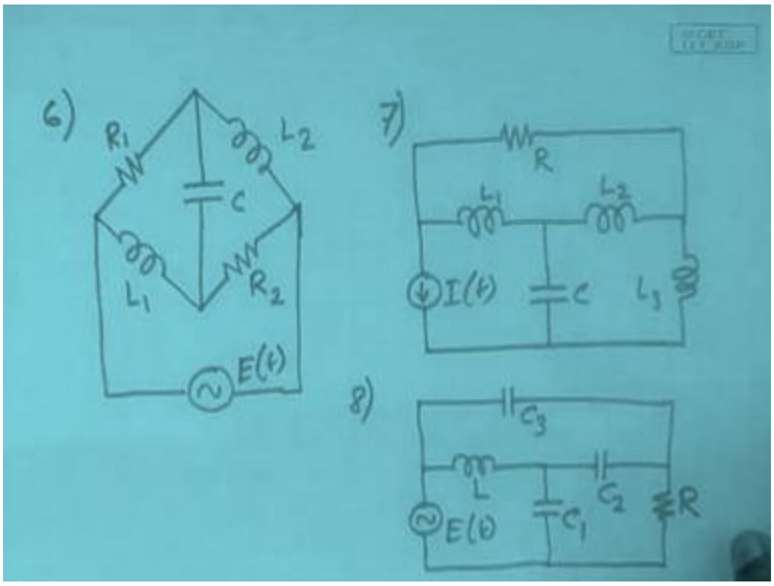

Let us consider this circuit in the earlier problem I have only changed the positions of the resistance and the capacitances. If you solve this you would notice that that will give rise to an algebraic loop problem here, so do you remember what algebraic loop is, if you write the bond graph, how to write the bond graph of a system like this, I have already shown.

But when you want to put the causality, you will find that these two will become difficult to causal, you have to assume a causality here, then this will be causal or you have to assume a causality here, then this will be causal and such a system will be giving rise to a differential causality. So, obtain the differential equations for this system then some more electrical circuits to give you practice. Will it give rise to differential causality, think, have you noted it down.

Similarly, let me give another problem here, similarly this will probably give rise to, so in this cases obtain the differential equations. Take another motor problem with a feedback loop. Let us see because you have learned how to handle feedback loops.

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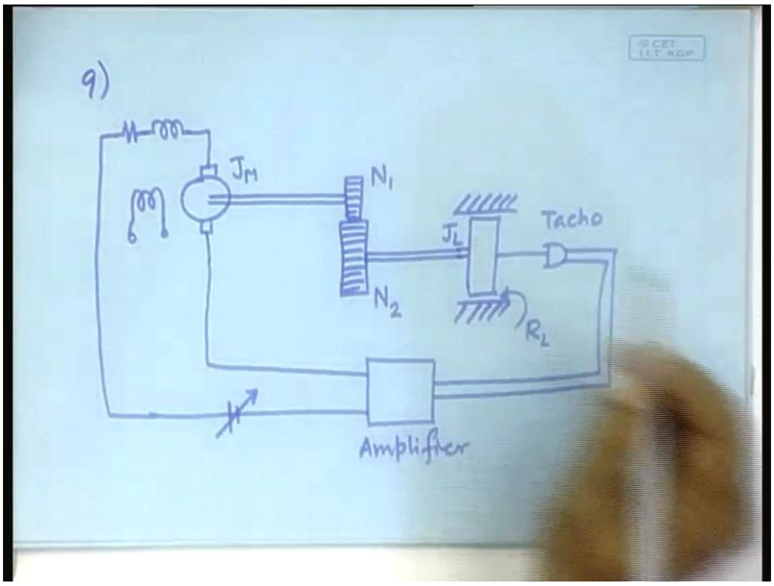

A DC motor with the armature connected in the usual way to the armature inductance, armature resistance, the field is constant, now this fellow is driving a gear box, so this is a gear box can you see that, the tooth ratio is N 1 by N 2 or speed ratio is N 1 by N 2. Then this is connected to a mass, which rotates and I have to show the ground against with it rotates, so at this point you will have the mass J of the load. Here there is a J of the motor then at this point there will be also the R of the load friction fine, but now this is connected to a tachometer.

The tachogenerator is giving a voltage, now that voltage is fed back like this, so you would need an amplifier and that is connected to the input side, obviously you need a power source here, so let us put a power source, variable power source. So, you would notice that here there is a observer, the tachometer is an observer, observer of what the flow, it is a flow observer. The flow information is coming in, but effort information is not coming, so this will give rise to an activated bond.

This is where, it is the flow will be converted into an effort information to be added to the external force, external voltage, so that is what we give the input voltage here, this is a very reasonably practical system, electro mechanical system. What was problem number, it would be nine, one problem involving mutual inductances, so that you get some practice.

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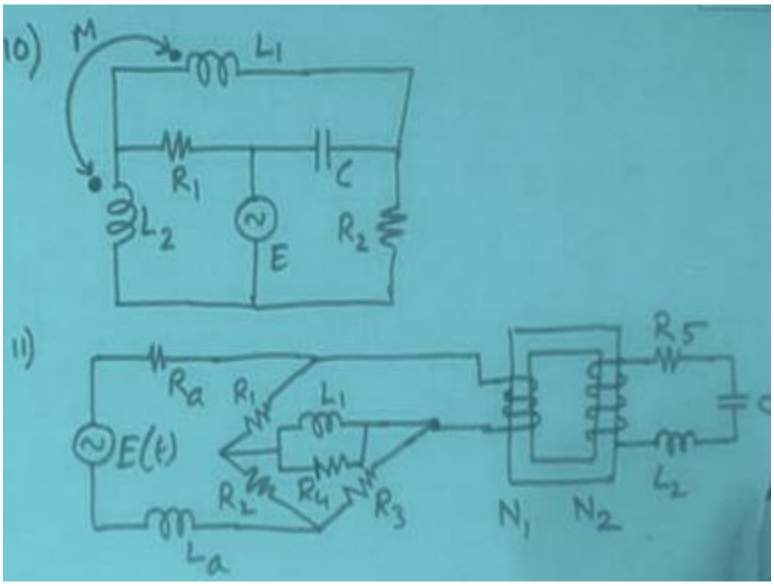

Ten, now one more problem here involving a physical transformer. Suppose there is a power supply going through a resistance to a network that network contains a resistance here, a resistance here, a resistance here and in between this you have an inductance and a resistance combination connected to this point and say there is nothing here that is also perfectly all right. So, here after this you have got this and that going to how do I draw transformer core.

So, you have and the other side you have, there is a resistance, inductance and a capacitance, enough complicated, so you are happy. So, it is say E of t you have got the different R's L a this is the network R 1 R 2 R 3 R 4 L 1, this is L 2 R C R 5 there is no other C, there has to be a trans ratio defined. You have to show, here because people are now noting down this, have you copied, a couple of more problems may be.

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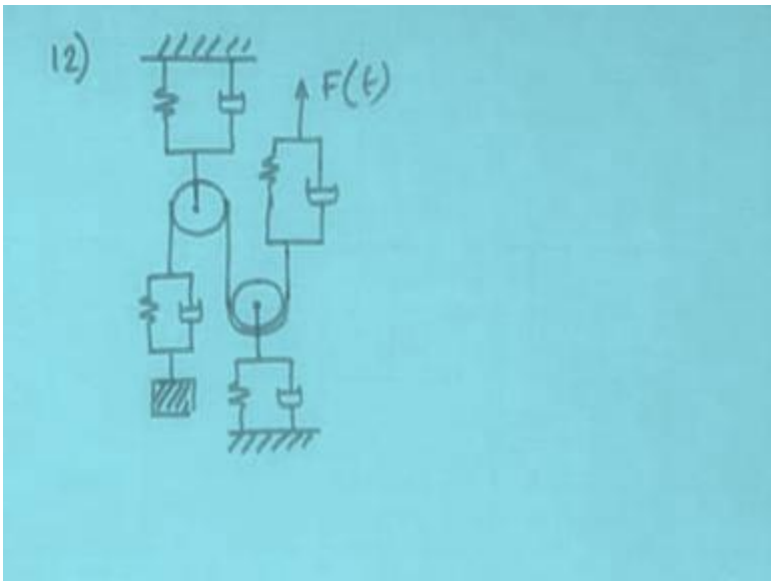

You have got the a resistor a spring damper arrangement connected to here, but this is from here there is a pulley suspended and the pulley is connected by means of a spring damper arrangement to a mass. The other side of the pulley is connected to another pulley and this suspension is also connected to the ground. Now, these, this side is connected to by means of another spring damper arrangement to and this point is being pulled with.

So, do you see the arrangement, this point is being pulled there is a spring damper here and on a spring damper this particular pulley rests, It goes through, this chord is connected to another spring damper and this is also suspended by. So, as you go moving this with a force, there will be all sorts of dynamics inside and that is what you have to model. So, that is enough that is enough, enough for practice.

Let us now turn our attention to the computer, let us turn our attention to computer. Now, as I told you all these bond graphs once you have obtained the bond graph that can be conveyed to the computer and then the computer can do the rest. And for this there is a program called symbols that was written here in IIT, Kharagpur, so I will just Google search it symbols shakti, that was the name of the program. So, it shows the first one is high-tech consultants that is a company floated with in IIT, Kharagpur, it is in this step.

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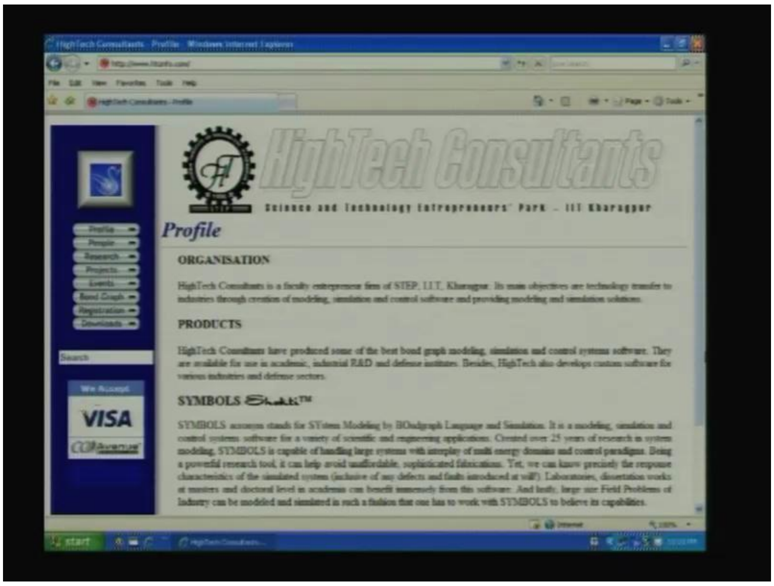

So, you can see that this is the company it was the program was written actually by, started by the students here, later it was developed further by professors and staff, here ultimately it is commercialized, so it is now available as a commercial software. In the left hand side you will see a downloads, so if you press the downloads then it shows the specific features of the program, but ultimately at the end of the page, as you go to that end of the page, you will find this downloads symbols shakti dot exe, just download it.

This is a regular version of the program, which is free there is a price diversion which is used for industrial scale systems and these are actually used to design say the whole motor cars for example, if the car is negotiating a bump for say one foot high, how much will they the passengers feel, all these need to be designed and these are actually designed by the this kind of programs. So, this program you download and install in your computer then all these, whatever I am showing will be doable by you.

But, I am not doing it now because I have already done it, so let me close it, on this computer I have already loaded. So, it is available in the programs, symbols shakti and there are different modules available out of which one is called bond pad that is what I will open, now that is where in that pad we can enter different bond graphs. So, which one shall we start with, let us start with, do you remember any of the bond graphs that I have drawn earlier, say this one.

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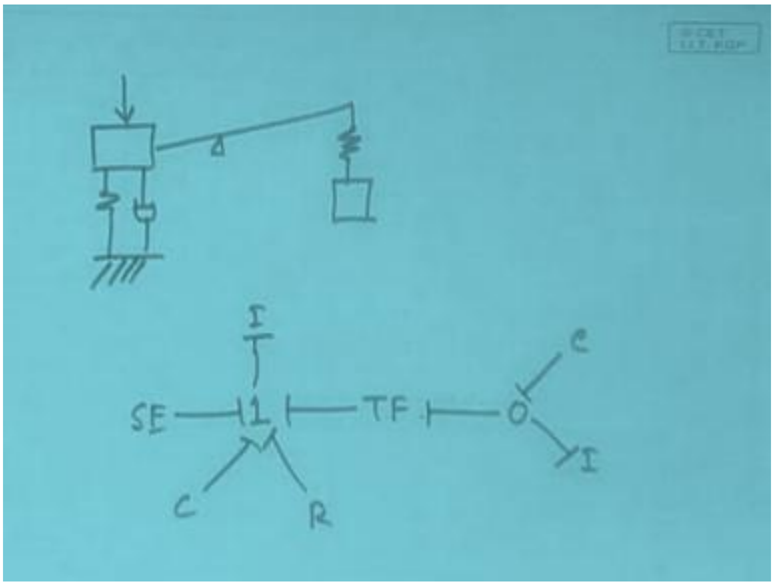

We have already done it, we are very much use to it, so let us just do it. I will not put a differential causality for this, it is bond graph will be 1 to I to SE to C R transformer and then it is 0, C and I. Put the causalities, this will be receiving it, so all the others will be taken this way 0, so I will be this way, so these has brought in the effort information and these others are taking out the, so this is done. So, let us put this, so in order to put this can you see on the computer, now can you see on the computer now. Would you please show on the computer, would you please display the computer, could you please display the computer. So, you would notice that here I start a new one which produces a grid.

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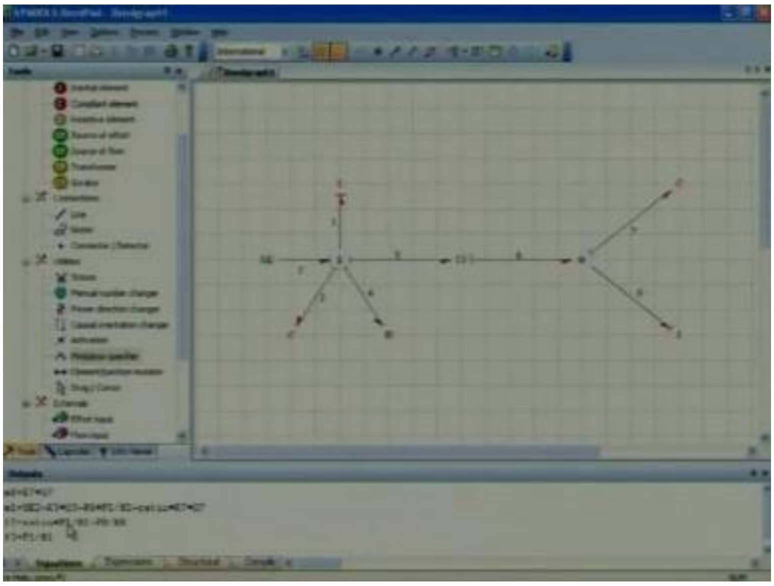

Now, in the left hand side you find the different icons 1 0 I C R A C S F T F G Y all these are very well known elements. So, we will start by taking them, and then 1 comes here, then you have the SE to the left, you have the I up, you have got a C here, you have got a R here, then you have got a transformer here, to the 0 junction this side, to a C element there and an I element here. Now, we have to connect, so for connecting we connect with the line.

There is a line icon here can you see that, so we connect with the lines, so the basic bonds are drawn up, now we what we normally do on the paper is to put the numbers. Now, this numbering can be automatically done, so you at the top you will find an icon for numbers. The moment you click it, all the numbers are given arbitrarily and you can change any of these numbers for which there are icons like this is, if you press this it will allow you to change the numbers.

Now, here is a power direction that can also be assigned automatically by the computer, so press it, it gives the power directions can you see, is it visible. So, all these power directions, just check if they are correct, AC gives out all this get in power, transformer has to pause to go through, these are all right. Now, this is a icon for causality that can also be done in a algorithmic way, so I press it, it has already given the causalities, probably because these are all circled, so I will hide the circle view is hide circles, so you can see that this is properly causal, this stroke is here.

SE is receiving the flow, C is receiving the flow, R is receiving the flow, flow is going this side, flow is going that side exactly as you have done on the paper, it has causal that way. Now, you need to check whether it is correct or not; that means, the consistency has to be checked for which there is an icon here, check integrate it and click it, it checks, it says that modulus for transformer is not declared; obviously, it has to be declared otherwise it does not know what is it.

So, here is a icon called as modulus specifier, so I go on the transformer and click it, it asks whether the direction is 5 to 6 or 6 to 5; obviously, because whatever you are specifying it could be this way or that way, I say 5 to 6, then I simply write ratio. So, that is the ratio between this side and that side just fine, so it has gone this side, it has, it is already taken it. Now, I check the integrity is fine 0 errors, 0 warnings.

Once I am there, it is possible for me to generate the equations, here is the generate equation icon I press it and in the bottom you will find that the equations are generated. I can you read them, they may be too small right, but if you do it yourself on your computer, you will find that these are pretty much readable. But let me write, what it has already derived, I will write on the paper, so please display the paper, so it says first.

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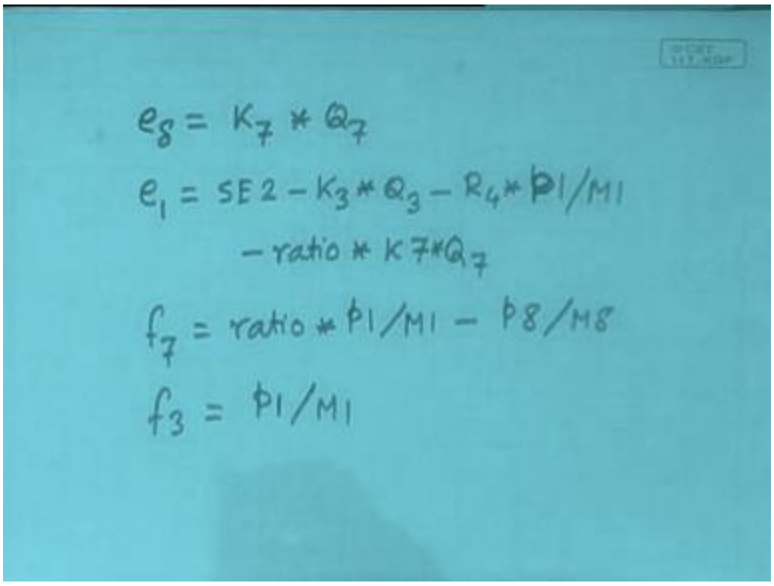

e 8 is equal to K 7 star Q 7, E 1 is equal to SE 2 minus K 3 star Q 3 minus R 4 star p 1 by M 1 minus ratio times K 7 star Q 7 check., you have already obtained the differential equations for this check, whether they are doing it correctly. f 7 is equal to ratio times p 1 by M 1 minus P 8 by M 8, f 3 is equal to p 1 by M 1. Check it is the correct, this will be correct of course, because this has been written in a very robust way, so that it always obtains the equations correctly.

Now, after this is done then these equations have to be solved and the program has the facility of solving the equation and doing all sorts of things on it. You can define the initial condition, on the basis of that it solves. And finally, solves means it plots the various variables, you can plot against time, you can plot one of the variable against the other, you can plot the Fourier transform and all that. All that are now possible once did the differential equations are solved.

We will come to the issue of solving differential equations starting from the next class, but this is how it works, which means that; however, complicated a system can be the moment you can put it on to the computer by means of the bond graph, the rest of things can be handled by the computer. So, that is why yes you will have to practice obtaining the equations by yourself, but then you have keep in mind that all that can later be taken care of by the computer.

So, it will be helpful for you to download this program and get used to it. All the bond graph that we have obtained that can be put in, check what equations it gives and when I give you the tutorials, all these equations you can also obtain by this, so that will be one advantage with you, now that if you do mistake you can easily be able to check that it is doing in the different way. So, that is all for today, we will come to the next class.