

Power Network Analysis
Dr. Abheejeet Mohapatra
Department of Electrical Engineering
IIT Kanpur
Week-02
Lecture-08

Lecture 08: Transformers and per unit analysis-example

Hello, welcome to lecture 3 of week 2 in which we will continue our discussion on transformers and per unit analysis. In this discussion today, we will understand how per unit analysis offers its advantages through a numeric example and the advantages of this per unit analysis we had discussed in the previous lecture. wherein polynomial analysis essentially converts the voltage, current, power, impedance in a given AC network be it three-phase or single-phase into certain dimensionless quantities wherein only the magnitudes of these phasor quantities get affected the phase angles remain more or less similar under certain special conditions the phase angles might also change specifically for voltages and currents when there are dissimilar transformer configurations. And the advantages which this PU system offers essentially it converts all the corresponding equipment impedances which could be on different basis on a common per unit base wherein any gross errors in the equipment impedances can be easily identified. The effect of turn ratio of usual transformers that is avoided the quantities the currents and voltages they all appear to be for a single circuit without having practical transformers this happens because the voltage levels they are all avoided all the network impedances as if appear to be connected to a single source and more importantly The basic circuit laws like Kirchhoff's current law, voltage law, all network theorems they are still applicable and valid and the beauty is that the ratio or the factors of root 3 and 3 involved in three phase circuit evaluation specifically from power and voltage perspective, all those need not be considered in per unit analysis. It is as if the per unit system is equivalent single phase system with all quantities referred to a common base.

So few typical steps that, generic steps that one can follow for per unit analysis of a given three phase power network or a single phase power network. The same are enumerated here. The discussion here is specifically for a three phase power network. And a three-phase power network, as we all understand, there could be different types of generators, different types of transformers, different loads operating at different voltage levels, different types of transmission lines.

So all these quantities, their impedances, power calculation, voltage current, they are all to be converted to a common base first. So for the first important step in PU analysis is essentially to choose a common base volt ampere, base apparent power for all the elements. And this common

base power, it can typically refer to three-phase power rating of a given generator or a transformer. If not, then conventionally 100 MVA can be a good choice for base power for transmission systems. And it is important to note that whatever choice of base MVA or volt ampere we make, it need not precisely match the volt amperes of individual machine or devices.

Certain machines or devices could have different volt ampere ratings. and the same ratings need not be same as the common base MVA which is chosen for PU analysis. Having defined the base apparent power, the next step is to choose the corresponding base voltages. Now while choosing the corresponding base voltages, one preliminary step to be kept into consideration is that depending on whether the network has transformers in it, specifically three-phase transformers, which could have similar configurations, could have dissimilar configurations. By similar configurations, I mean star star or delta delta, and by dissimilar configuration I mean delta star which has its own peculiarity depending on these transformers depending on the types of transformers the configurations and the operating voltages the network can be divided into different segments where each of these segments might be operating at different rated voltages and this segregation is important in terms of choosing a corresponding base voltage. So, what we do is we choose one of the segments it could be any particular segment operating at a particular voltage in the power network. We focus on that one particular segment choose a corresponding base voltage and then having defined this base voltage which could refer to one of the rated voltages of a generator or a load or a transformer winding present in that segment, we can choose any of these voltages to be a common base voltage. Again, it need not be same as the base voltage of generator load or transformer, it could be some other convenient number. that number which is chosen as base voltage should be close enough to the rated voltages present in that particular segment for example if a particular segment rated voltage is chosen to be 22 kV let's say which is a specified voltage then it doesn't make sense to choose base voltage for 22 kV rated to be 100 kV the numbers or the ratios in PU could be dramatically small we can at best choose 22 kV as common pu voltage or to be a number which is convenient to multiply or divide with we can choose a number somewhere between 20 to 25 kV as the base voltage for this 22 kV rated section.

Choosing 100 kVA might lead to inferior results and there could be other issues. The point is that in per unit system we want our quantities or phasors to be as close to one that is the bottom line idea. Having defined one of the base voltage for one segment, how do you choose the base voltage for other segments? That's where the role of turn ratios of transformers come in. The base voltage of other segments, they should be chosen while respecting the turn ratio or line to line voltage ratio of the intermediate transformer. Specifically for case where we have transformers with dissimilar connections, there is special consideration to be made while evaluating the actual quantities which we will see in the next slide.

But whether it is dissimilar configuration or similar configuration, the turn ratio should always be respected while choosing the base voltages for different segments. Having defined the base voltages for different segments, one can then evaluate the corresponding base impedance and corresponding base current if required. Convert the corresponding voltage power impedance of a given segment with respect to the base values of a given segment into per unit values. Analyze the circuit in per unit by using Kirchhoff's current law, voltage law or any other network theorem

which is convenient or applicable and once the analysis is done if needed the actual PU quantities can be referred back to actual quantities again by multiplying the corresponding PU quantities with the corresponding base quantities of a given respective section or a segment. Now specifically for transformers with dissimilar configuration as I mentioned where you can have delta star or star-delta configuration for three-phase transformers, a special consideration needs to be made.

For transformers with similar connections or configurations like star-star or delta-delta, this special consideration need not be taken into account. This is only applicable for dissimilar configuration. And why this special consideration needs to be there? The special considerations comes from the fact that the quantities as per ANSI standard if we recollect our previous discussion in the previous lecture. As per ANSI standard quantities which are being calculated on the delta side the same when they are reflected on the star side a factor of root 3 or 1 by root 3 comes in and correspondingly there is also a phase shifting involved of plus minus 30 degrees depending on whether we are converting quantity to star or star quantity to delta. Now the factor of root 3 or 1 by root 3 need not be considered as part of per unit analysis that is the beauty of per unit system, but special consideration is needed to take into account this factor of plus minus 30 degrees which come in while converting quantities from delta to star or star to delta vice versa.

So that's where this particular circuit diagram comes into picture. What we have here is a transformer which is of dissimilar configuration. Both the cases are given. Side 1 is the quantity given over here whereas side 2 is the quantity given over here. side 1 could be delta, side 2 could be star or side 1 could be star, side 2 could be delta.

Depending on which side or configuration these transformers are Z effective PU is essentially the effective per unit impedance of this particular transformer which is evaluated on any one side. We have discussed this in the previous lecture that impedance evaluations for a transformer, they remain the same in per unit irrespective of whether the quantities are being evaluated from the primary side or the secondary side. So Z effective PU is the effective impedance of the transformer in per unit. The special consideration here is with respect to this phase angle θ , where this θ should be 30 degrees if the side one is delta and side 2 is star or this angle should be minus 30 degrees when side 1 is star and the corresponding side 2 is delta. This special consideration of angle again comes from the fact that as per ANSI standard be it current or voltage on primary side when it is reflected to the secondary side the phase angle should be considered as part of the natural principle of operation of three phase transformers.

So as long as this one simple step is considered specifically considering this phase angle while converting quantities in per unit on the side one side to side two side are per unit evaluations be it for dissimilar transformers or similar transformers will result in the same answer as we do without per unit analysis. So let's understand bit more about this particular special consideration through this particular example wherein there is this power network shown in figure 1. It has three distinct sections. One section probably refers to the section which is consisting of generator 1. Let us say I call it as segment 1 which consists of the generator which is rated at 60 MVA And rated voltage is 20 kV line-to-line RMS.

Its corresponding reactance, which we will talk about in detail when we discuss module 3 and the last two modules. The generator's reactance is given to be 9%. And remember, the reactances which are given for generators or for individual transformers they have been given by the respective manufacturers on their respective apparent power or voltage rating specifications. So, 9 percent is the reactance of the generator with respect to 60 MVA and 20 kV line to line voltage. So, segment 1 consists of generator of 60 MVA and 20 kV and segment 1 also consists of the star side of the transformer which is rated at 25 kV line to line, Y here refers to the star and for a transformer input power and output power have to be similar.

So, 50 MVA is rated input power of transformer and the star side of the transformer is also part of segment 1. Segment 2 here specifically can refer to this transmission line. which is, whose impedance is given as absolute value 120 plus J 200 ohms. It's a RL circuit, inductive resistive circuit. The resistance is 120 ohms and 200 ohms is the inductive reactance for this line.

And segment two consists of this line. It also consists of the delta side of transformer T1, which is rated at 50 MVA and 250 kilovolt line to line RMS and it also consists of the delta side of transformer T2 as given in the nameplate specification here which is rated at 240 kV with power as 50 MVA and lastly segment 3 consists of this motor which is 50 MVA at 18 kV line to line RMS and it also consists of the star side winding of transformer T2 which is operating or rated at 24 kV line to line. The base power is given for this system. It's also mentioned that 22 kV can be chosen as the generator base voltage. And the question here is to transform this entire circuit into a common circuit in per unit.

$$G : 60MVA. 20KV, X = 9\%$$

$$T_1 : 50MVA. 25YKV/250\Delta KV. X = 10\%$$

$$T_2 : 50MVA. 240 \Delta KV/24YKV. X = 10\%$$

$$M : 50MVA. 18KV. X = 8\%$$

Then given that the motor draws 45 MVA at 0.8 power factor lagging, we have to determine what is the current in this particular transmission line which is this question and lastly for this operating condition we have to also find the internal voltage of generator in kilo volt. Now if this example or circuit would have had transformers T 1 and T 2 which were having similar configurations like say star star the overall analysis would have been very simple and probably there was no need of doing or having this per unit analysis since these transformers have dissimilar configurations per unit system will definitely help in analyzing this circuit so let's see how per unit system helps in understanding that Now given that the base power is 100 MVA and the base voltage of generator is to be 22 kV that means we inherently know from the question that the base quantities for segment 1 have been defined. that's essentially what is mentioned here in the first statement for segment one base power and generator side voltage are given base MVA or base power should remain same irrespective of what segment the network is being considered for and while choosing the base voltages we have to respect the corresponding line to line voltage

ratios of the transformers so between Segment 1 and segment 2 we have transformer T1 whose line to line voltage ratio is given as 25 kV by 250 kV. That means if the primary side is operating at 25 kV, 250 kV would be the voltage on the secondary side.

irrespective of whether it is star or delta we focus only on the line to line quantities here and if 22 kV is the base voltage on generator which means the corresponding base here is 22 kV it's likely that for the base voltage on the transmission line side one should always satisfy this turn ratio which means the base voltage for segment 2 should be 22 into 250 by 25 which essentially is 220 kV line to line that's essentially what is mentioned over here and once we know segment 2 base voltage we then can find the base voltage for segment 3 where we have to consider the turn ratio of transformer T2 which is 240 kV by 24 kV again a turn ratio of 10 so essentially the base voltage on segment 3 becomes 22 kV Now, remember the base voltage that we have chosen 22 kV, it is nowhere matching the individual base voltages of any of the quantities that are given in this particular network here. So, the next step involved is to convert the corresponding machine reactances given at individual bases to a common base which is 100 MVA 22 kV for segment 1 or 100 MVA 220 kV for segment 2 or 100 MVA 22 kV for segment 3. So the next few steps essentially indicate that we evaluate the base impedance of segment 2 which is the line impedance, base impedance and we convert the corresponding actual line impedance into a per unit impedance here. Similarly, we convert the individual impedances of all the elements given in this particular network to the common base MVA, which is 100 MVA 22 kV or 100 MVA 220 kV as per the relationship for converting actual PUs from an old base system to a new base system. So that's how these impedances are evaluated.

$$S_B = 100MVA, V_{gB} = 22KV$$

$$V_{lB} = V_{gB} * \frac{250}{25} = 220KV$$

$$V_{mB} = V_{lB} * \frac{24}{240} = 22KV$$

$$Z_{lB} = \frac{V_{lB}^2}{S_B} = 48.4\Omega$$

Hence line impedance in pu is $Z_{lpu} = \frac{(120+j200)}{Z_{lB}} = 0.2479 + j0.4132$

$$Z_{gpu} = j0.09 \times \left(\frac{20}{V_{gB}}\right)^2 \times \frac{S_B}{60} = j0.1240$$

$$Z_{T1pu} = j0.10 \times \left(\frac{25}{V_{gB}}\right)^2 \times \frac{S_B}{50} = j0.2583$$

$$Z_{T2pu} = j0.10 \times \left(\frac{24}{V_{mB}}\right)^2 \times \frac{S_B}{50} = j0.2380$$

$$Z_{mpu} = j0.08 \times \left(\frac{18}{V_{mB}}\right)^2 \times \frac{S_B}{50} = j0.1071$$

Once we know these impedances, we can draw the impedance diagram marking all those impedances in per unit given over here and thanks to per unit system it appears that there are no transformers considered at all or there is no transformer involved at all it's given that the rated terminal voltage of motor is 18 kV so essentially the question here says that the motor is operating at rated terminal voltage which is 18 kV so we convert 18 kV on 22 kV base which is 0.8182 and we choose this as our phasor reference for all the phasors that we will evaluate in this particular impedance diagram of the power network. Once we know the base the phasor reference for all phasor quantities we can convert the base power or the operating power of the synchronous of the induction motor which is operating at 45 MVA and 0.8 power factor lagging this is essentially what it is mentioned so 0.8 power factor lagging when we convert into corresponding real and reactive power it would become 0.8 plus j 0.6 multiplied by 45 the base power is 100 MVA so we get this as our base power and remember we have defined our complex power as nothing but product of voltage and conjugate of current. So using that relationship, we can find the complex current of the induction motor, which is operating at 0.55 minus 36.87 degrees PU. Once we know this current, it is imperative that if this is the current, which is IM as evaluated in the next slide, then this current has only one possible path, that possibility is through this transformer, through this transmission line, through this transformer and the source of this current as generator.

$$S_m = \frac{45(0.8 + j0.6)}{S_B} = (0.36 + 0.27)$$

Thus the motor current $I_m = \frac{S_m^*}{V_{mt}^*} = 0.44 - j0.33 = 0.55 \angle -36.87$

$$I_{lB} = \frac{S_B}{\sqrt{3}V_{ln}} = 262.432A$$

So when we evaluate the corresponding current on the line side, the corresponding base current needs to be first evaluated on the line side and in per unit analysis, the phasor would remain the same only thing which would change is a 30 degree phase shift specifically minus 30 degree phase shift because motor side is star connected and the line side is delta connected so there has to be a phase shift of minus 30 while converting this motor current to line current and that's how this IEM with the base current because base current for line is different from the base current for motor with minus 30 degree phase shift is the actual current in amperes running through this transmission line.

$$I_l = I_m I_{lB} \angle -30^\circ = 144.3376 \angle -66.87^\circ = (56.6985 - j132.7351)A$$

Now once we know this transmission line current which is evaluated in terms of per unit remember when we are bringing in this phase shift this is only important while finding the actual line current. If you look at the circuit diagram again The motor side was star connected and the

generator side is also star connected. So if there is a phase shift of minus 30 happening over here in the current then the same quantity when it has been again transformed from delta to side will again have a phase shift of plus 30 degree. So essentially this plus 30 in the current and minus 30 degree in the current gets nullified and essentially we can only make use of this entire single circuit apply KVL from where we would get that the internal emf voltage is nothing but terminal voltage of motor plus all impedances multiplied by the motor current this motor current is same as the motor current evaluated over here because motor side is star delta and generator side is again star delta so star star effect eliminated the delta configurations don't affect at all and hence the current remains the same and then we can find the corresponding base operating voltage in pu on the generator side We multiply it with the corresponding 22 kV base voltage which was given, we can find the corresponding actual currents.

$$E_g = V_{mt} + (Z_{g_{pu}} + Z_{T1_{pu}} + Z_{l_{pu}} + Z_{T2_{pu}})I_m = 1.3220 \angle 16.3851^\circ$$

The line-line internal voltage is $E_g V_{gB} = 29.0845 \angle 16.3851^\circ KV$

One can also verify these calculations by evaluating or understanding this circuit analysis from actual network perspective. We can probably also convert this circuit into equivalent single phase circuit, but then the difficulty would be considering how the star delta and delta star of transformer be replicated into a single phase corresponding equivalent circuit. That's where the beauty of PU system comes in. That's all for today's lecture. In the next lecture, which would be the beginning of our next module, we will start with basics of three-phase machines, specifically from synchronous generators perspective.

Thank you.