Power Network Analysis

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Week-02

Lecture-07

Lecture 07: Transformers and per unit analysis- per unit analysis

Hello everyone, welcome to lecture 2 of week 2 on the course power network analysis. In this particular discussion or lecture we will continue our discussion on the module transformers and per unit analysis. In this particular discussion we will specifically talk about per unit analysis, reason being In the previous lecture, we concluded with our discussion of steady state models or equivalent circuit models, approximate equivalent circuits for single phase transformers and then star and delta configurations for three phase transformers and there was one Common observation in three-phase transformers, which is enumerator shown in this particular slide, is that when we have three-phase transformers of similar configurations, then the corresponding voltages on the primary or on the secondary, their line-to-line ratios, they go by whatever is the individual phase ratio. Whereas when we have dissimilar configurations of three phase transformers like star, delta or delta star, factors of root 3 or 1 by root 3 also come in which in a way reflect additional to the individual voltage ratio of turn ratio of a. And additionally, these dissimilar configurations also lead to inherent phase angle shifts between voltages and currents on the primary and secondary side. So for example, when the primary side is delta connected in three-phase transformer and secondary side is star connected, the voltages and currents of the delta side, they would be led by 30 degrees on the star side.

Similarly, if the primary side is star connected and secondary side is delta connected the quantities on the primary will have a phase lag of 30 degrees when they move or when they are evaluated on the secondary side in addition to factors of root 3 and 1 by root 3 coming in as per the ANSI standard. So, the question which remains now is. Suppose we have a three-phase AC network in which we have these lot of three-phase transformers because generation, transmission, and distribution, a common mode is to do at three-phase level. And when these three-phase transformers, their primary and secondary quantities are going to have these additional factors of root three and phase angle shifts apart from the per-phase turn ratios, how do we keep track or monitor of these quantities specifically when analyzing the power network? So there is one way, one way could be if we recollect our discussion on basic principles when we were discussing three-phase circuits.

For balanced three-phase AC networks, probably the analysis could be done on single-phase basis, and then the quantities can be amped up or multiplied by three or root three, depending on whether we are evaluating the power calculations, or line-to-line voltage current calculations which are affected by root three factor for phase quantities. But now apart from those single phase or three phase conversions, now in three phase transformers we have these additional phase shifts of 30 or plus 30. So probably the analysis or the overall simple KCL KVL application won't be so straightforward. And that is essentially what is enumerated here in this particular slide. any single pair of primary and secondary winding in a three phase transformer they behave very similar to primary and secondary winding of a single phase transformer and as long as the operating conditions are balanced and the transformers three phase transformers have similar configurations we can use the per phase analysis wherein we do all calculations on per phase basis and then amp up those quantities by three or root three depending on the power or current calculations, but the bottleneck here is analysis of three phase transformers with dissimilar windings is not possible on a per phase basis reason being the phase to phase and line to line quantities they are highly interconnected, they are highly complex and hence per phase basis could not be done.

Because let us say when you have a delta configuration transformer on the primary side The terminals could be A, B, C, but on the phase winding it's very difficult to identify which particular phase is corresponding to phase A or phase B inside the delta configuration. It could be possible to do for a star configuration transformer on the secondary or primary side, but definitely not possible to do on a delta side. So, per phase distribution itself goes out of window. So, what is the solution available to us? There's a very simple solution, which is known as per unit analysis. It dramatically simplifies these complex calculations, remembering consideration of factor of three, root three, phase shift 30 plus 30, et cetera, et cetera, and it essentially gives the same answers.

It does the same analysis, which would have been done if we were to retain our three-phase modeling for three-phase transformers with dissimilar windings. So the purpose of this discussion today would be to understand how this works, what this per unit analysis is, and how it can simplify or bring in such beauty or magic in analyzing three-phase AC circuits. So let's dig deep into what is per unit analysis. As the term per unit says, it is essentially re-representing the power network quantities which are essentially going to be either voltage, current, power or impedance which often have their units as volt, ampere, watt volt ampere etcetera etcetera or impedance in terms of ohms instead of representing these electrical quantities is there a way to represent them as dimensionless quantity that is without dimension. So, yes it is possible and that is where per unit system comes in per unit as the term says there is going to be some ratio of a voltage with respect to a corresponding quantity which is also in actual units and hence when you take ratio of two voltages the corresponding ratio or unit gets nullified and hence the relevant quantity which is going to be evaluated let us say alpha it becomes dimensionless.

Same is also true for current by current the units gets cancelled, ohm by ohm the unit gets cancelled and so on. So, essentially alpha is the quantity which is to be represented in per unit which is a dimensionless quantity. This alpha could be voltage, current, power or impedance. So, mathematically it is just a ratio. Per unit quantity or PU in short is ratio of actual quantity divided by the base quantity.

This base quantity should be of the same unit as that of the actual quantity that is only possibility for defining per unit quantity. If we define per unit quantity with ratio on numerator being or numerator of the ratio being voltage and base being current it does not remain per unit it basically becomes a impedance. The same unit notion is important here for defining PU and let's say we have V representing voltage, I representing current, S representing power, Z representing impedance and the corresponding bases are let's say VB, small b in the subscript here indicates the base quantity. So if we know these base quantities VB, IB, SB and ZB then each of these individual voltages, current, power and impedance they can be represented in corresponding per unit sensors.

$$V(pu) = \frac{V}{V_B}, I(pu) = \frac{I}{I_B}, S(pu) = \frac{S}{S_B}, Z(pu) = \frac{Z}{Z_B}$$

Now the question then here is a minimum of four base quantities, one each for voltage, current and power and impedance are necessary to define a complete per unit system for a power network. Coincidentally these entities or basis they are not independent as we all understand and know power is nothing but product of voltage and current impedance is nothing but ratio of voltage and current. So, essentially if we know any of these two base quantities the remaining two base quantities can be easily derived. So, the question now is which of these four elements or options which is voltage, current, power or impedance among these four which two should be considered as the base quantities? The answer is pretty simple. We have talked about this or we had have a discussion on specifications for AC networks in our previous discussion specifically in the module of basic circuit principles. wherein it is pretty common or very convenient to have electrical specifications of elements in terms of power and voltages.

So, inspired by that same notion that most of our electrical elements they have at least these two specifications which indicates what is the rated capacity or operating capacity or consumption of this device in terms of power. and at what rated voltage it should operate its performance is not detrimental given this notion of power and voltage specifications that inspires that for power networks we can choose base voltages and base power as a reference quantities for per unit system. So, essentially we define base for power and voltage and suppose we know ratio of power base power and base voltage the corresponding current would essentially be some ratio of this base power and voltage and corresponding base impedance would be ratio of the known base voltage and the base current which would be evaluated as ratio of power and voltage. So essentially let us say for a single phase system If we define or know the base power, which is, let's say, SB, which is in mega volt ampere, MVA, and the base voltage is in, let's say, KV, so the corresponding base current IB would, since it is a single-phase system, The current is nothing but ratio of power by voltage. So we have a factor of 1000 sitting here because our base power unit is MVA whereas we have chosen base voltage in terms of small k and V.

If we choose base power as volt ampere and voltage also in volt then the factor of 1000 need not be defined or need not be there. Consequently the base impedance is nothing but base voltage divided by current. Now remember the base voltage unit is in kilovolt whereas if you see the base current unit it's in ampere. So a factor of thousand is important to retain here and if we substitute base current expression in the denominator here the thousand factor goes away and our base impedance is simply Vb square by Sb where Vb is in kilovolt and Sb is in MVa. This is one such uniqueness about defining base powers in megavolt amperes and base voltage in kVs.

$$\text{base current } I_B(\text{A}) = \frac{1000 S_B}{V_B}$$

$$\text{base impedance } Z_B(\Omega) = \frac{1000 V_B}{I_B} = \frac{V_B^2}{S_B}$$

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If base power and base voltages are simply volt ampere or voltages then also the same formula would be applicable for defining base impedances. This is all about single phase system things become little different for three phase systems. Now in three phase systems one should understand or remember the power specifications do not refer to single phase power they refer to three phase power. So, the factor or the number three is appearing here and similarly the base voltage which is referring to line to line voltage. is necessary to point out and hence the number 3 is appearing here.

So unlike single phase systems where the base power was single phase power and voltage was single phase voltage, here S3B is typically the three phase power and likely V3B is going to be some line to line voltage so with that thing in mind if we know what are our base power and voltage base power often it is convenient to choose for typical power networks it is often convenient to choose 100 MVA as the base typically for transmission systems. For distribution systems the same base power could be few hundred of kVA or according quantity and if we know what is our base power which is unit is MVA and base voltage whose unit is in kV then our base current would be ratio of the single phase power corresponding to corresponding single phase voltage. Now how do we find the corresponding single phase power? Remember, if you recollect the discussion which we were having some moment ago, S3B here is referring to three phase power. So if I have to find what is the corresponding single phase power, it should exactly be S3B by 3. assuming things are all balanced and if I know what is this V3b, it is definitely some line to line voltage because that's how power specification or voltage specification for three phase systems work.

$$\text{base current } I_{3B}(\text{A}) = \frac{1000S_{3B}/3}{V_{3B}/\sqrt{3}} = \frac{1000S_{3B}}{\sqrt{3}V_{3B}}$$

The corresponding phase voltage from here in terms of magnitude would simply be V3b by root 3. Now if you compare or correlate these formulas with the term sitting over here they are more or less similar except the fact that the factor of 1000 is coming in. The factor of 1000 is coming in because base power unit is MVA whereas base voltage unit is KV and that is how the base current definition comes in. It essentially by convention refers to rated phase current in a star connection. Having known the base current, we can now also find the base impedance or phase impedance.

The definition of base impedance or phase impedance is little different. If you recollect the principles which we discussed in basic circuit module, there we said or talked about that unlike balanced currents and balanced voltages, for balanced load, a balanced load would only appear when the same quantum of load appears across each phase. So that's how the notion of base impedance is also little different. It doesn't refer to any line to line impedance or phase to phase impedance. And hence it has got no physical implications specifically line to line impedance.

Only the phase impedance is important. Phase impedance would nothing be ratio of phase voltage by phase current. We know what is our phase voltage here. We have also known what is our phase current or base current because this phase current or base current is referring to a conventional star configuration. So if you put in those numbers here, here also magically the base impedance turns out to be V3B square by S3B, similar to the single phase discussion.

$$Z_B(\Omega) = \frac{1000V_{3B}/\sqrt{3}}{I_{3B}} = \frac{V_{3B}^2}{S_{3B}}$$

So, that is all about the mathematical understanding of what per unit system is and now let us look at why do we need per unit. We have talked about it in a brief, but then we will also enumerate certain advantages of per unit system. Often manufacturers when they manufacture a particular machine. So, there can be different manufacturer for a generator, there could be a different manufacturer for a transformer. the manufacturer for industrial loads, induction motors, they could be different.

So each of these manufacturers at the time of manufacturing these elements, they probably don't talk to each other and it's not necessary that they should talk but probably there is no way of how they can communicate to each other. So what these manufacturers do is when they design or manufacture a particular electrical element like a generator or a transformer or a motor, They have a nameplate rating specification in which definitely the three-phase power and the corresponding line-to-line voltage if it is a three-phase element is there. In addition to it, they also mention certain machine parameters. For example, for three-phase transformers or single-phase transformers, manufacturers also provide the value of what is the typical winding resistance, corresponding winding leakage. leakage reactants, core parameters, etc, etc.

Now how do they specify these parameters? If they were to specify these parameters in actual units, then probably the nameplate rating specification, the space available would become small. So what these manufacturers do is they provide these details of specific elements in terms of per unit and the reason why they provide in per unit because the corresponding numbers tend to be between 0 and 1 and that is one beauty which per unit system provides. So, essentially different elements they could have different machine parameters under different basis and when these different elements are connected in a common power network be it generators or transformers. It is important that all these impedances of different elements, they are scaled down to a common rating or a common base. So how do we ensure that these elements they are all being referred to a common base is what is going to be the discussion here.

So basically we need to reconvert or recalculate our per unit system which was there for individual elements all scaled up to a common system base. We'll talk more about this particular aspect when we take an example in the upcoming lectures. So please bear with me if there is any confusion or doubt in this regard. So let's say We have a machine whose nameplate specification impedance was ZPU old and this machine power was SB old and VB old. So once we know this machine whose power ratings and voltage ratings they are defined as per the nameplate, we can actually evaluate what is the actual impedance of this element in ohms.

And if you recollect the definition of per unit, per unit is nothing but actual by base So essentially this tells us that if I have to find the actual quantity, then basically I multiply the base quantity with the per unit value. So ZPU old is given in the machine nameplate specification and now I have to find what is the base impedance for this particular machine. Since I know what is the corresponding base power and base voltage, if I revisit or re-look at the previous slides, ZB old is nothing but this particular ratio. So from here I can find the corresponding actual impedance. Now if this actual impedance is to be redefined on a new base or a new per unit system whose base power is SB new and base voltage is VB new.

So essentially what I do I again go back to my actual definition of per unit. I need to redefine my base. So for impedance if I have to redefine my base then the new base impedance would nothing be vb new square by sb new where vb new and sb new are known for the new per unit system and I define the new per unit. as ratio of actual impedance divided by the new corresponding base. And that's how if I resubstitute all these quantities, this is the generic expression of how per unit quantities can be exchanged among each other on different basis.

$$Z_{pu}^{new} = \frac{Z}{Z_B^{new}} = \frac{Z_{pu}^{old} Z_B^{old}}{Z_B^{new}} = Z_{pu}^{old} \left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \frac{S_B^{new}}{S_B^{old}}$$

So we'll again come back to this when we take a particular example in this particular regard. Please note that in the previous formula which was the one which is shown here in all these expressions and that is one uniqueness of or that is one uniqueness of per unit systems. In per unit evaluations the phase impedances or the phasor impedances or sorry the phase magnitudes are the ones that only change. The corresponding phase angle of any phasor be it voltage, current, power or impedance it remains unaffected. Our base quantities, they are defined as scalar quantities, they are not complex numbers. And advantage of per unit system is, which we will see in today's discussion, the effect of transformer's turn ratio on impedances is completely avoided. It appears that in per unit system, this actual transformer doesn't remain another transformer in the corresponding equivalent modeling. The equipment impedances, they tend to vary in a very narrow range, so gross errors can be easily identified and rectified. Different voltage levels in different parts of the network, generators operating at few tens of kVs, transformers operating at high voltages, loads operating at very low voltages, all those different voltage levels, they are simply removed. The entire network in per unit sense appears to be operating at a common voltage level.

The circuit laws KVL, KCL, power calculations they are still valid and one important part is this complication of remembering when to consider factor of root 3 while transforming line to line to phase or converting per phase power to three phase power by considering factor of three, all that problem is nullified and resolved in per unit analysis. These considerations need not be remembered. The life becomes very easy, analysis becomes very simpler. So in today's discussion, we'll emphatically talk about the first pertinent advantage, and through example, we will focus on the remaining three aspects, which is probably going to be part of the upcoming lectures. So let's say we have a beautiful, usual, single-phase two-winding transformer whose supply voltage is V1.

The quantities here are all in phasor. They're not time domain quantities. And for simplicity of discussion, we have neglected the magnetic branch part, which is the parallel branch part. We are only considering only on the ideal part of the transformer and the corresponding primary impedance and secondary impedance, which consists of winding resistance and leakage reactances. We are focusing only on that. The discussion, however, would remain the same if we also consider the minorizing branch part.

The discussion won't change dramatically. So what we have here is a single phase 2 winding transformer whose supply voltage is V1 phasor the output voltage is supposed to be V2 phasor and then you also know the turn ratio a which is ratio of individual turns on the primary and secondary side and if we have to write KVL of this particular single phase two winding transformer in actual units then let us say we write KVL between node R1 and node R2 then KVL across node R1 and R2 it results in the fact that V1 minus I1 Z1 is nothing but the EMF($E_1 = V_1 - I_1 Z_1$) which is induced on the primary side of transformer and then the induced EMF on the secondary which is E2 is nothing but E1 by a which is a turn ratio means it is V1 I1 Z1 by a and if we write KVL across R3 and R4 where R3 is a node across which E2 EMF is being induced then KVL across R3 and R4 indicates that V2 should be equal to E2 minus I2 Z2 which if we write in terms of all actual quantities then it is essentially V1 minus I1 Z1 by a minus I2 Z2 and then you also have the KVL across the load which says that V2 should be not equal to I2 into the corresponding load impedance ZL where ZL is a complex number.

$$E_2 = \frac{E_1}{a} = \frac{V_1 - I_1 Z_1}{a}$$

$$V_2 = E_2 - I_2 Z_2$$

$$V_2 = \frac{V_1 - I_1 Z_1}{a} - I_2 Z_2 = I_2 Z_L$$

So now if we look at this KVL, it has got a turn ratio appearing across it, then there are primary quantities, secondary quantities, et cetera, et cetera. Can per unit analysis simplify this into a common equation where no ratios appear in, no turn ratio appears in? So let's see that. So this is the essential equation which I was deriving at, and the same is mentioned over here. All quantities in the above equation are phases and not time domains.

They're all being measured across common reference. So now to understand per unit system, we have to define the per unit base, the corresponding base power and base current. Now if you remember or recollect, for the single phase two winding transformer, the power specification on the primary as well as the secondary side, they are the same. A transformer is a passive device.

It doesn't generate its own power. So essentially whatever power is input on the primary side, more or less similar quantum of power if not similar little less similar amount or less than amount of input power should appear on the secondary. So, power specifications on both primary and secondary side they are the same in a transformer, the difference however is the operating voltages on primary and secondary they are different. So in terms of base definition for transformers, we define one common base MVA or base volt ampere, which is common for both primary and secondary sites because transformer is a passive device. And if we choose the voltage base on primary as VB1, We are considering here a single phase transformer, so we don't have to worry about three phase single phase connection. It's a single phase two winding transformer, so VB1 is the ratio on the primary side.

The trick is the secondary side voltage should be chosen such that the turn ratio of the transformer is always respected. So, if we know what is VB1 as per the name plate specification of transformer the corresponding waste voltage on the secondary should always be as per this definition. If this is not ensured the advantages of per unit system are lost. This is a mandatory or a necessary exercise to be kept in mind while defining per unit systems for three phase networks specifically for single phase transformer. Now if we know these voltages, base voltages and base power, can we not convert or rewrite this expression in terms of quantities? Probably yes.

So let's see how do we define that. We can also, having defined the corresponding base powers and base voltages on primary and secondary side, we can also use the definitions for base currents and base impedances and define the corresponding basis for primary side and secondary side. The uniqueness here is that the secondary side base current is a times of primary side base current whereas the secondary side base impedance is 1 by a square times of the primary side base impedance. And over here if you remember recollect the turn ratio a is reflecting or converting primary quantity to secondary quantity in a similar way the load impedance matching transformer discussion was done.

$$I_{B1} = \frac{S_B}{V_{B1}}, I_{B2} = \frac{S_B}{V_{B2}} = aI_{B1}$$

So, now if we know our bases then probably we can again go back to our usual equation here all these quantities are actual quantities and remember each actual quantity can be rewritten in terms of per unit multiplied with the corresponding base. So, if I have to rewrite V2 in terms of per unit and the corresponding base then probably V2 is going to be V2 per unit into the corresponding base which is VB2.

VB2 is as per the definition given over here and likewise I can also write V1 as V1PU into VB1 and so on for current and impedances. So if we plug in all these values we get our first equation, which is defining all my actual quantities in terms of per unit and the corresponding base quantities. And having obtained this, now if we start simplifying or plugging in the relationships of base currents, base powers, base voltages, what we see here is that VB1 it is nothing but product of IB1 and ZB1. Why? Because that's how our base impedance was defined. So from in this expression we can bring VB1 out and similarly on this particular side IB2 into ZB2 is nothing but VB2.

$$Z_{B1} = \frac{V_{B1}}{I_{B1}} = \frac{V_{B1}^2}{S_B}, Z_{B2} = \frac{V_{B2}}{I_{B2}} = \frac{V_{B2}^2}{S_B} = \frac{Z_{B1}}{a^2}$$

Why? That's how the base impedance on secondary was defined. and gradually if we start simplifying what we see here VB1 by a is nothing but VB2 and now if we compare this equation we have VB2 common both on the right hand side and left hand side and we know for sure that VB2 is not a zero quantity because it's a base voltage. So we are left with this equation wherein we see that the actual equation which had this ratio of a in it, the converted equation in per unit doesn't have a turn ratio at all. And it appears as if It is a single element having two series currents, I1PU and I2PU, with a common voltage level appearing across it.

That's the beauty of per unit voltage. That's how the turn ratio of transformers doesn't appear in actual analysis while using per unit equation. Turn ratio is completely absent, different voltage levels will completely disappear. We can also try to find relationships between the per unit currents on primary and secondary side. So, if I have to find the per unit primary current it is nothing but actual current by base current wherein if I substitute the relationships in terms of secondary quantities and turn ratio what I see here is that the primary current in per unit is same as the primary current on the secondary side. Actual currents in transformer they have this inverse ratio of a.

In per unit system transformer single phase two band transformer the ratio of A goes away in currents. Similarly, if we try to find the corresponding So if I1PU is same as I2PU, we have this one single equation wherein it appears that earlier there were two series currents, I1PU, I2PU, coincidentally they are same. So now I have a single element or a single circuit where the input voltage is V1PU, common current is I1PU, which is creating voltage drop across primary impedance per unit and secondary impedance per unit, and eventually I'm getting an output of V2PU. So it's This can be equivalently represented as a circuit like this where the input voltage is V1 pu and there is a series impedance Z1 pu plus Z2 pu across which I1 or I1 pu current is flowing and the output terminal is experiencing a voltage of V2 pu.

$$V_{2pu} = V_{1pu} - I_{1pu}Z_{1pu} - I_{2pu}Z_{2pu} \Rightarrow V_{2pu} = V_{1pu} - I_{1pu}(Z_{1pu} + Z_{2pu})$$

If I compare this circuit with the circuit which I had actually considered for the single phase transformer, this ideal part is completely gone in per unit analysis. That's the beauty of PU systems and essentially I can also try to find what are the corresponding impedances in per unit evaluated from the primary and secondary side. If I evaluate that Coincidentally, they also turn out to be same no matter what side I am trying to evaluate my primary or secondary impedance in per unit. In actual transformer primary and secondary impedance will have different ratios. Some will have 1 by a square term, some will have a square term. So the numbers might be different on primary and secondary and actual ohms whereas in PU system the numbers exactly match with completely remaining dimensionless.

That's how we conclude our today's discussion. In the upcoming lecture, we will take a numeric example and understand more of how power analysis can be done or evaluations can be done in PU while avoiding factors of three, root three, how circulars are still valid, et cetera, et cetera.

Thank you.