Power Network Analysis

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Week-02

Lecture-06

Transformers and per unit analysis-Basic and 3Φ transformers

Hello everyone, welcome to first lecture of week 2 on the course power network analysis. In today's discussion which is the first discussion of week 2, we will continue our discussion on transformers and per unit analysis. In the previous lecture we had taken up the discussion on basics of transformers wherein we briefly touched upon how does a single phase two winding transformer operate or what is the modeling aspect behind it, what are the attributes of a ideal transformer and how a practical transformer differs from ideal transformer. Continuing with that we will talk about more about of how the single phase two winding transformer electrical equivalent circuit would look like when it is loaded when it is unloaded and then we will also take up the aspects of basics of three phase transformers. So if you recollect our previous lectures discussion we had ended up with the exact equivalent circuit of a practical single phase two winding transformer in which in which we also have the ideal part represented as it is. And then the non-idealities they are represented in terms of R2 and R1 which represent the non-zero winding resistance indicating there would be copper loss in the windings.

Then we have X2 and X1 which account for the flux leakage between the primary and the secondary windings with respect to core. Then we have two parallel circuits. The first parallel circuit which is being encircled represents the fact that the permeability of a practical transformer, the core of the practical transformer is not a perfect magnetic material. It does have finite permeability.

So it requires certain magnetomotive force or certain magnetization current. And that excitation current or magnetization current IM is represented to this parallel branch. And then we have another parallel branch which carries current IC, which in a way indicates the non-zero core loss happening in a practical transformer in terms of eddy and hysteresis losses. And we had also looked at the phasor diagram of this practical two winding single phase transformer for the case when the secondary of the transformer is unloaded. And the relevant equations which come in the ones which are being

highlighted here they are as per the KVL applied on the secondary and primary of the transformers.

Exact Equi. Circuit: Real Transformer

➢ For no load, primary current is same as excitation current

$$I_2 = 0 \text{ (unloaded)}$$
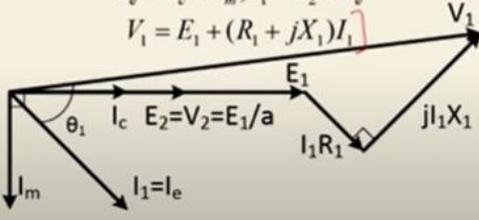$$V_2 = E_2 - (R_2 + jX_2)I_2$$
$$E_1 = aE_2, I_2' = I_2 / a$$
$$I_c = E_1 / R_c, I_m = E_1 / jX_m$$
$$I_e = I_c + I_m, I_1 = I_2' + I_e$$
$$V_1 = E_1 + (R_1 + jX_1)I_1$$

Phasor diagram for no load

And then we have the current conversion and the voltage conversion as per the turn ratio of the transformer where a is the turn ratio. And then IC which is the core loss component and IM is the magnetization current component which comes as per the KVL across the parallel branch of the single phase two winding transformer. And the excitation current is nothing but sum of core loss component and magnetization component and eventually I is the excitation current which for a practical transformer is around 5 to 10 percent of the primary current where the primary current is I1 and then this equation comes in by KCL at the point of common node which is this node. which is combination of reflected secondary current on the primary and the excitation current which results in the primary current. So now if this transformer is loaded which is the case when we have a load ZL which is of some magnitude ZL with an phase angle theta L which is connected across the secondary of the transformer.

So by KVL across the load the secondary current in phasor domain would nothing be voltage drop which is appearing across the load which is a result of output of the transformer divided by the corresponding impedance with other equations remaining as they were for the previous case where the transformer was unloaded. So in such a diagram look like? Phasor diagram in the sense, a phasor diagram in which all the corresponding phasor quantities, be it voltages, be it induced EMF or applied voltage, and the corresponding currents, they can be represented on that common phasor diagram. So that is a little.

.. not so obvious that is little tricky. So, how do we approach it? Now one clear way of approaching or finding out or drawing the phasor diagram would be to start with the

relevant equations which are going to remain the same no matter whether the transformer is going to be loaded or unloaded. So if we see the equations which would remain invariant whether the transformer is loaded or unloaded one of the important equations would be the relationship between the induced EMFs on the primary and secondary which is defined by the corresponding turn ratio. So if we choose this as our equation of importance rest all equations if we focus or look at all other equations apart from the one which has been highlighted or sort of encircled or taken up in a square box, all other equations will have currents in them. And depending on whether the transformer is loaded or unloaded, the corresponding equations can vary except the equation which is shown over here.

 So, probably it is a good idea to start with the equation which remains invariant with the transformer is unloaded or loaded depending on the relationship. So, we start with this equation wherein we first mark what is the induced EMF on the primary E1 and a assuming it to be a scalar, although in certain cases, the turn ratio A could also be a complex number. which we are avoiding for the case of our discussion to keep make it simple. But probably in power flow analysis module, we will also see a case where A can also be a complex number, it need not remain a real number. Assuming A is a scalar or real number, if E1 is the induced EMF on primary, then corresponding induced EMF of secondary can be marked by the corresponding red line and I'm assuming here that A is less than 1 for the corresponding case under consideration although a similar diagram can also be drawn when we choose A to be greater than equal to 1 which we are not considering as of now but the discussion that we are going to have going to remain the same in more or less similar manner.
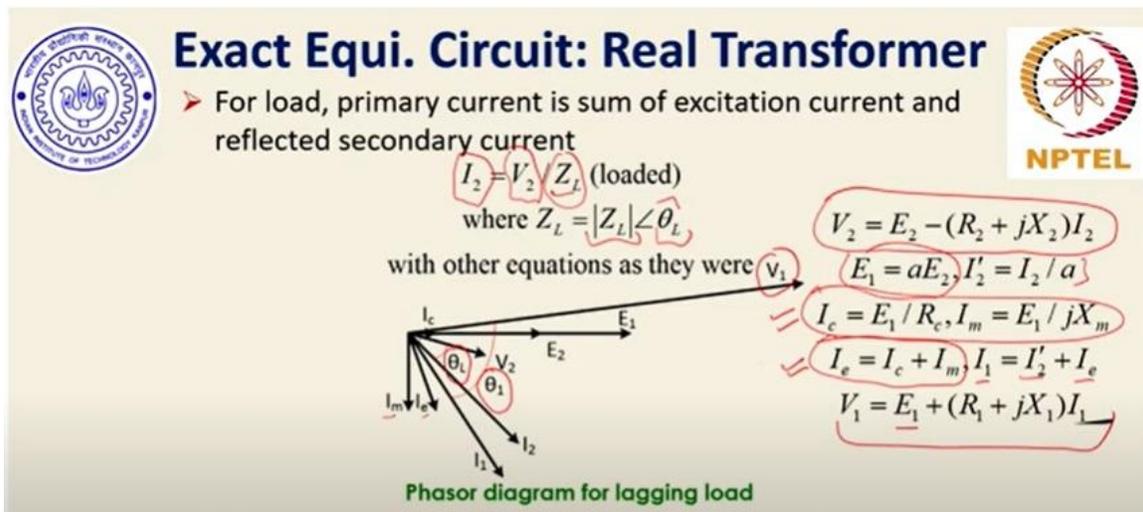
 So having identified E1 and E2 now let us look at the other equations which would depend on E1 and E2. So the next set of equations if we look at then we can easily figure out that no matter whether the transformer is loaded or unloaded For a practical transformer we would have a non-zero core loss component of current. We would also have a non-zero magnetization current component and these would be there no matter whether I2 is zero or non-zero. So maybe the next step would be to mark these currents and IC is nothing but E1 by RC. So basically RC being a resistive component for a resistor or a pure resistor current and voltage they are in phase.

 So IC is in phase with E1. Whereas for a pure inductor where in we have Jxm, it would correspondingly lag the corresponding voltage by 90 degrees. So we have Im marked here in this manner and then the next equation which is being represented or noted here is the corresponding KCl which is the addition of Ic and Im vector to get the excitation current. So now we have marked our excitation current also. And then the next, so basically we have been able to identify or mark these equations in our phasor diagram so

far.

Now the remaining equations if you see, there is no other way to mark them until and unless we know what is the current I2 on the secondary side. So now let us say we choose some current I2 or mark some current I2 although the marking of current I2 on the phasor diagram would also have a logic which would become evident in a moment of time. So let us say we mark I2 and once we have marked I2 we can correspondingly apply the KVL equation and probably figure out what is the applied or voltage which is appearing on the secondary of transformer which is basically V2 and V2 and I2 they have a phase angle of theta L which is the angle in the across the impedance and we are assuming here it is a lagging load so basically current is lagging the corresponding applied voltage or voltage at the secondary. A similar diagram may also be drawn for a leading load aspect. Now once we know I2, we can also find the corresponding reflection of current on the primary and having known I2 dash and IE, now we can mark what is I1 which is the primary current.

Having known what is primary current and known what is E1, we can then apply this corresponding equation and get the corresponding voltage V1 which is being the applied voltage on the primary. So now if we see there is a unique distinction or difference which comes in that the load which was applied on the secondary had a corresponding phase angle of theta L between the applied output voltage and the corresponding current whereas the same load when it is reflected on the primary gets appeared as a phase angle of theta 1. So the phase angles change and hence The power factors at the primary and secondary side, they may be different even for the same load appearing on the secondary side. And the reason for this different power factor or different phase angles comes in because of all these non-idealities which exist in a practical transformer. In an ideal transformer, phase angle differences won't exist as long as there are no losses and there is no excitation current.

Exact Equi. Circuit: Real Transformer

➤ For load, primary current is sum of excitation current and reflected secondary current

$$I_2 = V_2 / Z_L \text{ (loaded)}$$

where $Z_L = |Z_L| \angle \theta_L$,

with other equations as they were

$$V_2 = E_2 - (R_2 + jX_2)I_2$$
$$E_1 = aE_2, I_2' = I_2/a$$
$$I_c = E_1/R_c, I_m = E_1/jX_m$$
$$I_e = I_c + I_m, I_1 = I_2' + I_e$$
$$V_1 = E_1 + (R_1 + jX_1)I_1$$

Phasor diagram for lagging load

The next slide here shows a simplification of the equivalent circuit which we had seen in the initial slides. So what is happening here? All the quantities which are in red, that is the quantities which are primarily referring to the primary side, They have all been reflected or transferred to the secondary side. So, if we basically go back by the voltage transformation ratio or current transformation ratio of a transformer having turn ratio a which is basically the voltage ratio V1 by V2 which is approximately same as E1 by E2 under certain assumptions or conditions. then this turn ratio essentially helps in reflecting all my primary side quantities onto the secondary side in a way the corresponding ideal transform which is sitting in between that can be avoided. A similar representation can also be done for quantities reflected to the primary side entirely all secondary quantities they are being reflected on the primary side and in a way this circuit or the previous circuit it helps in analyzing the steady state model of the transformer.

Although there is no strict restriction which side should be converted. There is also another circuit known as approximate equivalent circuit in which the primary winding resistance and secondary winding resistance as well as the leakage reactances they are all clubbed together and the corresponding parallel part branch is separately considered This is because of the fact that our transformers are highly efficient devices and have very low voltage regulation. So the way transformers are designed, it is such that the applied voltage is almost same as the induced voltage on the primary side. So corresponding voltage drops are pretty low and under such approximation or assumptions, we can also make use of this approximate equivalent circuit for simply analyzing the steady-state model of the transformer. although there would always be a trade-off between the accuracy and the complexity involved in analyzing the corresponding circuit.

So most of the situations this approximate equivalent circuit would be very suitable enough model for understanding the performance analysis of a transformer. Coming to the nameplate rating of a transformer, all electrical equipments or elements by now you would pretty well understand and appreciate that each of those elements typically always have a power rating which is the volt ampere rating and a voltage rating which depending on whether it's a single phase element or a three phase element could refer to line to line or phase voltages. For typical three phase transformers or single phase transformers let us say if I have a transformer of 10 kVA with voltage ratio defined of this form wherein the primary side probably or the HV side winding has 1.1 kV voltage rating and the secondary side has 110 volt secondary voltage rating in terms of a single phase transformer then it essentially means That this transformer has maximum of two windings, one winding operating at 1100 volt, the other one operating at 110 volt and each of these windings can have maximum power handling capacity of 10 kVa. In practice, the voltages that are mentioned, they are actually the rated induced EMFs, but

for performance analysis, we can also consider them to be equal to terminal voltages, as long as the transformer has a low voltage regulation.

And the turn ratio is approximately 10 is to 1 because this is essentially coming from 1100 volt by 110 volt, assuming these to be the induced EMFs or terminal voltages. And from the current ratings, from the power ratings and voltage ratings, you can also find corresponding maximum currents that can flow on the HV side and LV side. So HV side winding they would essentially carry low amount of current whereas the LV side winding would carry a higher amount of current because power capacity remains the same and the voltage levels accordingly go up or go down in HV and LV windings respectively. In practice there would also be some additional excitation current along with the secondary current which is in a way assumed to be negligible for this name plate rating discussion. In practice excitation current itself could be some few 5 or 10 percent of 9.

09 amperes or 90.9 amperes depending on which side it is being calculated. In order to sort of understand or how to use approximate equivalent circuit model, one has to first identify how do you figure out the parameters, be it the parallel branch part or be it the series branch part. So basically I mean to say we want to know what is R1, R2, X1, X2, Xc and Rc which Xc or Xm or Rc which we have seen in the approximate equivalent circuit, how do you figure out those values because a priori it is very difficult to know what these values would be. So there are two simple standard tests which help in figuring out these elements. The first part or group of sets, it can be figured out by doing the short circuit test The first set of branch which is finding out the series elements which is R1, R2 or X1, X2 and the corresponding parallel branch part, they can respectively be found by doing open circuit test or short circuit test on the typical single phase two winding transformer.

 I will briefly talk about what these tests are and why do we conduct them on the HV side and LV sides respectively. Open circuit test or no load test essentially says that the transformer is unloaded and essentially we want to find or conduct certain test by which we will be able to find the corresponding parallel branch parameters. Basically open circuit test helps in identifying what are XM and RC values whereas short circuit test helps in figuring out what are the corresponding series branch elements. Now in open circuit test or no loads test or be it short circuit test, one important part which is bottom line or underline for all electrical experiment basically is safety. So as long as the current levels are low, voltage levels are low, the person who is conducting these experiments, his or her safety would not be at stake and in open circuit test as the name suggests, the HV side winding or HV winding is kept open and on the LV side just sufficient amount of voltage is applied so that the HV side has rated voltage appearing across it in a no load condition And you can imagine if HV side winding is let's say of 1100 volts, the

corresponding LV side supply typically may be of 110 volts with respect to the previous or previous to previous slide where we talked about the nameplate rating.

 It's obviously safer to work with a low voltage source compared to a high voltage source. So that's the reason why the supply is given on the low voltage side and just sufficient voltage is applied so that Rated voltage appears on HV side so basically 110 volt will be applied on the LV side and corresponding power loss current ratings are all measured and from that particular readings we can essentially do some calculation to find a corresponding parallel branch elements. Similar logic is also applicable for short circuit test in which the LV side is shorted because the current capacity on the LV side is going to be higher. the experimenter would typically operate or work on a low current operation which is going to appear on the HV side and just the sufficient amount of supply is given so that LV side carries rated amount of current and corresponding measurements are made in terms of wattmeter, power loss and voltages and corresponding calculations are done to figure out the series elements. More discussions or details can be found out in the relevant textbooks where these tests are discussed at length.

Moving on to three-phase transformers, typical operation of three-phase AC circuits or power networks. It generates, transmits, and distributes power in three phases. So it's essential to have three-phase transformers, which would essentially serve the purpose of either stepping up the voltage or stepping down the voltage. and the way single phase transformers operate, more or less similar philosophy is applicable for three phase transformers. There can be two ways of realizing a three phase transformer.

 One common straightforward way is to have three separate single phase two winding transformers, one transformer for each single phase. So we would have essentially three single phase transformers, each phase transformer pertaining to a particular phase. Or the other way could be to have a single transformer core across which three pairs of bindings are wound on a common magnetic core. And usually the second option is preferred industrially because There would be substantial savings in terms of the core component. There will be substantial saving in terms of insulation requirement.

 Essentially, the weight, cost and space of the corresponding transformer would substantially reduce if we have a three-phase transformer being realized by a single transformer core and all the three-phase windings, pair of windings bound across the transformer core accordingly. The operation remains more or less similar as single-phase transformers and there can be four typical possibilities. If you remember when we started discussing three-phase transformers, we talked about two typical configurations of three-phase circuits. One is star connection, the other is delta connection. There can be other combinations possible but to keep our discussion simple we will focus only on star and delta configurations.

So for three phase transformers the way these windings are wound or the way the windings are connected to each other there can be four possibilities. The first possibility of configuration could be star star, the other possibility could be star delta, next could be star grounded. star grounded and the last possibility common possibility could be delta delta connection both on the primary and secondary side. One can also think of oh why not delta and star that is also very well possible, but whatever discussion we would have for star delta could be applied for delta star also and we can also have star-star configuration whether the neutral is grounded or not. So probably this combination which when we discuss the same could be applied here also.

So in the forthcoming slides we will talk about each of these configurations and look at how the voltage transformation happens or the turn ratio can be defined. So what we have here is that we have a three-phase transformer and this three-phase transformer has three pair of windings. One pair is let's say P1 which is the primary side of P1 and correspondingly the corresponding parallel winding which is S1 here, these two windings which are in parallel they orient or refer to one pair of winding. Similarly we have winding P2 on the primary side and correspondingly parallel leg is S2 here and lastly we have pair 3 whose corresponding secondary is S3 on this side. So if we apply let's say rated voltage which is VLL, the line-to-line voltage on the primary side of the single three-phase transformer across any two phases, phase A and phase B, and define these turn ratios which is number of turns, NP1, NS1, by this relationship which is A and the same turn ratio is also being absorbed across the other two pair of windings.

Then going by the relationship of voltages, line to line voltage and phase voltage in star configuration, if line to line voltage is applied here, the corresponding phase voltage would be 1 by root 3 times of the line to line voltage with 30 degree lag. and since this voltage appears across this pair of winding a similar voltage would appear on the secondary winding also difference would be the factor of turn ratio division by A and from there we can also try to find that if this is my phase voltage across one phase of star configuration the corresponding line to line voltage would be VLL by A. So basically we are trying to shift from the phase calculation to line to line calculation in which we have to multiply this phase quantity by root 3 and lead the corresponding phase voltage by 30 degree which nullifies minus 30 and hence we get simply VLL by A. So essentially the line-to-line voltage applied on primary of star-star configuration simply gets reflected as VLL by A on the secondary side.

That's one observation. The other observation is because of the way windings are connected, there's a simple change of terminal on the secondary side. So while operating with three-phase transformers, one need to be very careful about this change of phase terminal. The change of phase terminal, as I said again, I will repeat again, is all because

the windings which are in parallel to each other, they correspondingly refer to the corresponding terminal. So if phase B is appearing on this terminal here and it is associated with P2, the corresponding secondary winding is S2 and hence this terminal should refer to small phase B on the secondary side. Then we have star delta configuration wherein the star side remains the same.

On the delta side there are differences in the sense going by the same discussion we have our individual turn ratios defined by small a and the corresponding secondary voltage which appears across one phase of delta is 1 by root 3 VLL at an angle minus 30 divided by a. And if we remember recall at our delta configuration discussion the line to line voltage is same as phase voltage and delta configuration. So, the same quantum of voltage which is appearing on the phase of the delta winding or delta connection is appearing on the line to line aspect also. And hence that is a substantial difference the factor of 1 by root 3 is appearing.

A phase angle shift is also appearing. The terminals, they change the way star-star configuration was done. Similar analysis is also applicable for delta-star. Here if the applied voltage is VLL, then here we have a factor of root 3 appearing with a phase lead off at an angle 30 degree. And for delta-delta configuration, it's exactly similar to star-star discussion. The difference here is the terminals here do not change.

Because if this is my phase B terminal which is linked with P2 and P3 and if I see on the secondary side the corresponding P3 refers to S3 and P2 here refers to S2. So, if this terminal is capital B then it is intuitive that this terminal would be small b. So, it depends on how the corresponding windings are drawn or connected and as per this particular drawing there is no change in terminals of windings for a delta-delta configuration. So to summarize, when we have similar configurations of three-phase transformers, the three-phase turn ratio which is essentially the ratio of, so basically if I have to define three-phase turn ratio, then I essentially measure the line-to-line voltage appearing on the primary divided by the line to line voltage is appearing on the secondary.

$$N_{p1} = N_{p2} = N_{p3} = N_p, N_{s1} = N_{s2} = N_{s3} = N_s, a = N_p / N_s$$

So, for similar configuration this three phase turn ratio is exactly same as the single phase turn ratio whereas for star delta and delta star we have factors of root 3 and 1 by root 3.

$$Y - Y \text{ or } \Delta - \Delta \rightarrow a_{3\phi} = a,$$
$$Y - \Delta \rightarrow a_{3\phi} = a\sqrt{3}$$
$$\Delta - Y \rightarrow a_{3\phi} = a/\sqrt{3}$$

Additionally when we have dissimilar configuration we also have this 30 degree and minus 30 degree phase shift. So, how do you keep track of this phase shift? There is a very simple standard which has been given by ANSI. It states that for dissimilar configurations of three phase transformers no matter whether it is the voltage or the current depending on from primary to secondary whether the primary is delta connected and secondary is star connected. As we move our quantities from primary side to secondary side evaluation factors of root 3 with plus 30 degree should come in and if we are having primary winding connected in star and delta connected on the secondary side, the quantities from primary side should get shifted by a factor of 1 by root 3 with a phase lag of minus 30.

That's the simple standard. And if we keep track of this particular standard defined by ANSI, then our overall analysis becomes very simpler. Why this phase shifting happens? The corresponding reason for the same has been very well explained in these few slides. Slide number 13 and 14. So that's all for this lecture. We will discuss about the next part of this particular model which is per unit analysis and try to understand how per unit analysis can simplify analysis of three-phase transformers specifically when there are dissimilar configurations.

 Thank you.