Power Network Analysis

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Week-12

Lecture-58

Lecture 58: Stability analysis- Solved examples

Hello everyone, welcome to Lecture 3 of Week 12 of the course Power Network Analysis. We are continuing our discussion on the last module, which is Transient Rotor Angle Stability Analysis, and in today's discussion, we will take up or discuss a few numeric examples that will help you better understand the aspects that we have discussed as part of this module. In the original template or the core structure, this discussion on solved examples should have been the next lecture, but for the better understanding of the students, I believe we should discuss the examples first in today's discussion and take up the discussion on multi-machine. Transient or triangle stability and the practical aspects involved, which sort of compensate for the assumptions in conventional transient or triangle stability analysis, will be discussed in the next lecture; this will be useful and will provide a proper picture or understanding of what the details are. So, that is how we will take up the two numeric examples in this discussion of Lecture 58. Until the previous lecture, we had discussed at length a direct approach to solving or analyzing transient triangle stability for SMIB or two coherent or non-coherent machines and their respective systems, which was the equal area criteria that essentially talked about certain areas.

Which should be equal to each other with respect to the power angle, power rotor angle, or power P delta curve for a given synchronous generator. And since equal area criteria will not be useful for a multi-machine system, we also talked about a few basic numeric integration techniques that would be useful in analyzing the swing equations, which would provide specific values of the rotor angle with respect to the initial operating condition for a given period of time. Numeric integration techniques do not provide analytical expressions of the rotor angle curve or the swing curve. And before I go deep into the details about the current lecture, I will talk a bit about the power angle curve that we have discussed.

I am sure the students who are going through this particular course would like to have this intuitive question in their mind that, in fact, I had thought that I would discuss this aspect while I was discussing the equal area criteria or the power angle equation of the curve, but somehow it was missed during the course of discussions; I thought I would compensate for or cover that aspect here. The question or the point that I am trying to make is when we started our discussion on the power angle equation for a given synchronous generator, which is represented by its internal constant, internal EMF, and the direct axis transient reactance, we focused only on the first half of this power angle curve.

$$P_e = \frac{EV}{X} \sin \delta$$

E is the corresponding internal EMF, delta is the corresponding rotor angle that coincides with the internal EMF angle, and V is the terminal voltage, infinite bus voltage, or some reference voltage based on which this angle is being measured. X is the corresponding transfer reactance that is present between the internal EMF voltage and the corresponding terminal or reference voltage. And since PE tends to follow this sine curve for fixed voltage and transfer reactance, obviously X can change as and when the system changes, whether due to changes in network topology or disturbances.

But because of the slow AVR loop E and V, their values would never change; the magnitudes would remain the same. And essentially, if we were to figure out the dependent variable in this expression of electrical power output of a synchronous machine, then delta happens to be the direct dependent variable. And since it is a sine curve, P is dependent on the sine of delta. We focused on this first half of the power angle curve where Pmax is eV by x, and the maximum angle appears at pi by 2 radians; 0 is the origin, and the maximum span we looked for was pi radians or 180 degrees. So, the question that might arise in the course of our discussion with equal area criteria is that, post-fault swing curve, it can also go below the origin, and that is where point F was considered.

And since the power angle curve can swing in the negative or the third quadrant of the PE delta, why did we not take that angle or curve into consideration when we started discussing power angle curves? So the answer is very simple. We are focusing on synchronous generators, to be precise. Although the analysis of transient or triangle stability is applicable for any synchronous machine—it could be a motor, a condenser, or a generator. But for the time being, we are focusing mostly on sources that are synchronous generators; that is point number 1. Point number 2 is for synchronous generators; the rotor angle is always positive, delta will always be a positive quantity, and since delta is always positive.

So, we are focusing on the typical values of delta that the synchronous generator might observe in steady state or during a disturbance, and that's the reason why the first quadrant of angle was considered. Now the question is: is delta always positive for synchronous generators, and can synchronous motors also contribute to transient stability or instability? What would the power angle curve be in the case of synchronous motors? Now, where in synchronous, this is the case for synchronous generators; for synchronous motors, delta would always be negative, internally enough angle always lacks the terminal voltage, and that is how the synchronous motor operates. The power angle curve of a synchronous motor shows that the expression of electrical power will remain more or less the same in the sense that for a generator, the machine is delivering real power, whereas for a synchronous motor, the machine would be absorbing real power. So, the expression remains the same; the sign of PE, or the orientation or direction of PE, would change, and hence the corresponding angle delta would also be negative. So, for analyzing transient motor angle stability for a synchronous motor, we would observe the plot of the power angle curve, not just the angle, but the absolute value of the angle.

The delta would increase or decrease accordingly as per the swing equation, and with the mod value of delta coming in, the corresponding mod will become a positive quantity. We can still use the similar power angle expression or plot for synchronous motors as well; that is one aspect. The other aspect is. The moment that is how the analysis for synchronous motors can be done for transient rotor angle stability analysis. The last part is during swings we observed that the rotor angle can go below zero, even for a synchronous generator.

Now remember the moment rotor angles go below zero, even for a synchronous generator designed to deliver real power with a negative torque angle, the machine would start absorbing real power. Ideally speaking, for a synchronous machine, this is not a problem, but practically, if a synchronous generator with a lot of swings in rotor angle starts absorbing real power from the grid instead of delivering real power, it is a Mishaps are waiting to become a disaster. Actual practical synchronous generators are not supposed to absorb real power at any point during their lifetime of physical healthy operation. So, that is the reason why we focused only on the typical ideal operating points on the power angle curve, wherein delta was always positive, and for the synchronous motor, delta will be negative. So, we will be dealing with PE versus the mod delta angle.

I hope that the explanation clarifies the point that I raised. So, coming to our numeric examples, we'll take up two examples in today's discussion. The first example, we have a three-bus power network in which bus two is a PQ bus, bus one is a PV bus, and as per the specification, if you recollect our discussion on load flow analysis, bus three can be treated as the slack bus or the angle reference bus. The network line parameters in per unit; their impedances are already given. Our load flow solution at this ideal healthy operating point is also given where all bus voltage magnitudes are known.

The reactive power injection from generator 1 is also provided by the load flow analysis. The question states that we will assume bus 3 to be an infinite bus. So the slack bus will be treated as an infinite bus. The parameters of the generator at bus 1, along with their corresponding tera-taxis transient reactance, are given. The operating frequency is given.

The inertia constant, in seconds, is also given. It's also given that the internal loss system in the generator is to be ignored. Their values are zero. So, in this healthy steady state condition, at this healthy steady state condition where rotor speed is exactly synchronous speed, the rotor angle is not known now, but the machine is definitely injecting some real power, which is given as 1 pu. A fault occurs, or a three-phase fault occurs, just on lines 1 and 2, very near to bus 1 itself.

So, we can safely consider the three-phase fault to be happening at bus 1 itself. The question says we have to determine the initial internal EMF of the generator at bus 1 before this three-phase horrid fault happened. That is bit 1. Bit 2 is that we have to determine the expression of the swing equation for the generator at bus 1 with respect to the infinite bus, which is bus 3. Where rotor angle delta is delta 1 minus delta 3, delta 1 is the rotor angle of the generator at bus 1, delta 3 is the rotor angle at bus 3, which is given as a reference, so it is 0 radians or degrees; that is bit 2.

So we have to find the swing equation; the first part is the internal EMF, and then the third part is using the trapezoidal rule of integration with a time step of 0.1 seconds. We have to find the relative rotor speed and the rotor angle after the disturbance has occurred at t equal to 0.2 seconds. So basically, at t equal to 0, this three-phase fault has occurred, and t is equal to 0.2 seconds. In the presence of a fault, we have to use numeric integration, specifically the trapezoidal rule, to find the relative rotor speed and rotor angle, which will be our third bit. So let's look at it bit by bit. So, in order to find or solve bit 1, we have to find the internal EMF of the generator at Bus 1. And for this internal EMF, we have to know what the current being fed by this generator is. If we don't know the current, then if you remember the internal EMF E1 dash delta 1, it's a generator, so it will be equal to the V1 phasor plus JXD dash into the current injection. XD dash is given as 0.1 PU already. So V1 is known from the power-flow solution. And so in order to find the internal EMF, we have to know what the current is. Fortunately, the power injections from the generator are already provided.

So PG1 is given as 0.1 PU. From the power flow, QG1 is 0.9222 PU. Voltage is known. The reactance is known, so we can find the complex power injection given by the generator at bus 1, which is 1 plus j 0.922 pu, and since we know the voltage phasor from load flow, we can also find the corresponding current injection, which is S1 conjugate by V1 conjugate.

Current injection from bus 1 to power network is $\underline{I} = \frac{S^*}{V_1^*} = \frac{(1+j0.9222)}{1pu\angle 0.28511°} = 1.3603135pu\angle -42.9674°$

So this gives us the current expression. Now, applying the basic KVL, we can then find the corresponding internal EMF as well.

Internal emf before fault is $E_1\angle\delta_1 = V_1 + jX'_d I = 1.9678pu\angle 4.946115°$, i.e., initial rotor angle is $\delta = 4.946115°$

So this basically completes part of bit 1. In bit 2, in order to find or go into bit 2, where we have to find the swing equation, we have to find what is Pm minus Pe, which is going to be equal to 2H by omega s d squared delta by dt squared. Omega is basically given as 2 pi f, so f is 60, which is known. H is already known. We have to know what PM is. Now that the machine was in a steady state before the three-phase bolted fault occurred near bus 1 or at bus 1, we have to find what the mechanical input is. That comes from the fact that the internal loss in the generator is perfectly zero. So we can essentially assume that in steady state, the mechanical input was the same as the electrical output, which was also 1 PU. When the fault occurs at bus 1, as we have discussed in the previous lectures, for a three-phase bolted fault at the terminal of the generator, the terminal voltage itself would become very close to zero, or perfectly zero, so the electrical output would also become zero.

Therefore, during a three-phase fault, the electrical power output is zero per unit. Since we now know what pm is, which is one pu h omega, we can find the corresponding swing equation during the three-phase fault. In this, t is equal to 0, which is the instant when the fault has occurred, and the initial operating angle delta naught from the power angle equation of internal EMF is 4.946, or 4.946115 degrees, which can be converted into respective radians.

$$\frac{2H}{w_s}\frac{d^2\delta}{dt^2} = P_m - P_e \Rightarrow \frac{d^2\delta}{dt^2} = \frac{w_s P_m}{2H} = 16.82996 rad/s^2 \qquad t=0,\ \delta_0 = 4.946115$$

So that covers the second part. The third bit is to use numeric integration, trapezoidal rule to be precise, to find the rotor angle speed omega r with respect to synchronous speed and the corresponding rotor angle at a time of 0.2 seconds from the instant of fault.

$$\frac{d\omega_r}{dt} = 16.82996, \qquad \frac{d\delta}{dt} = \omega_r$$

So what we do is first convert our several given differential equations into first-order differential equations;

$$\mathbf{x} = \begin{bmatrix} \omega_r \\ \delta \end{bmatrix} \text{ and } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} 16.82996 \\ \omega_r \end{bmatrix}$$

that is where x dot is equal to f(x) notation comes in; x dot is nothing but dx by dt. So if we represent our given differential equation as two first-order differential equations and treat x as equal to omega r and delta as given here, then f of x will simply be 16.83, which is essentially the term sitting here, and d delta by dt would simply be omega r.

So, given that we know fx, we know what x is; now let's apply the trapezoidal rule. The initial operating angle is already known, as I have mentioned. Since the machine was in a steady state, the rotor speed was exactly the same as the synchronous speed, and the initial value of omega r is perfectly zero, so we know what our initial conditions are at the time t equals zero when the fault or disturbance just occurred.

bus 1 ($E_1\angle\delta_1 = 1.9678pu\angle 4.946115°$), at $t = 0$,

$$\mathbf{x}^0 = \begin{bmatrix} \omega_r^0 \\ \delta^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.946115° \end{bmatrix} = \begin{bmatrix} 0 \\ 0.086326rad \end{bmatrix}, \text{ as machine}$$

So, given that the time step is 0.1 seconds by the trapezoidal rule of integration, which is an implicit method of integration,

$$\mathbf{x}^1 = \begin{bmatrix} \omega_r^1 \\ \delta^1 \end{bmatrix} = \mathbf{x}^0 + 0.5h\{\mathbf{f}(\mathbf{x}^0) + \mathbf{f}(\mathbf{x}^1)\}$$

The unknown here is x1, which is the value of x at the next time step, and in implicit methods, the unknown appears not only on the left-hand side but also on the right-hand side. Coincidentally, for the equation that we have, we usually require non-linear techniques like the Newton-Raphson method or the Gauss-Friedel method to further solve this implicit equation in the trapezoidal rule. But given the specific example that we have, we will likely not require any additional technique. The implicit method will automatically become an explicit method in some ways. X1 is the unknown that we want to know. X0 is known. The vector is already defined. It is also known, which is what was shown in the previous slide, but it is present here. So f is nothing but 16.83 and the latest value of omega r. So for x0, which is omega r0 and delta 0, f is omega r0, and f, the first term remains the same. The second term is omega r1 at the next time step.

$$\mathbf{x}^1 = \begin{bmatrix} \omega_r^0 \\ \delta^0 \end{bmatrix} + 0.5h\left\{\begin{bmatrix} 16.82996 \\ \omega_r^0 \end{bmatrix} + \begin{bmatrix} 16.82996 \\ \omega_r^1 \end{bmatrix}\right\}$$

So if we expand this equation, what do we get? The results are given on the next slide, but I'll clarify this here for further details. X1 is Omega R1. It consists of two equations. We're writing for the first part. Omega R1 is equal to omega R0 plus 0.5H, where H is given as 0.1 seconds. That is also known. Multiplied by 16.82996. Plus again 16.82996, that is the first half of the equation. The second half is delta1 is equal to delta0 plus 0.5H multiplied by omega r0 plus omega r1. So what we see here is yes, omega r1 is unknown, but since it consists of two equations, we can easily get omega r1 from here because all terms from this left-hand side are all known.

So, we can first get the omega r1 value; then, by plugging in this omega r1 value, we will be able to get delta 1, and that is how the process continues. Since the first step is h = 0.1 seconds, we solve both equations together: omega r1 is h times 16.829996, and delta r1 is again h squared.

The above is the solution at t equal to 0.1 seconds. For t equal to 0.2 seconds, we repeat the similar recursive step, and essentially we would get omega r squared and delta r two; we plug in t equal to 0.1 seconds, and this is our result at t equal to 0.2 seconds. The same process can continue for higher values or for several other time steps to follow, and essentially we will be able to get a discrete time plot of relative rotor speed and the corresponding rotor angle.

$$\Rightarrow \omega_r^1 = \omega_r^0 + 0.5 \times h \times 2 \times 16.82996 = h16.82996 rad/s$$

$$\Rightarrow \delta_r^1 = \delta_r^0 + 0.5 \times h(\omega_r^0 + \omega_r^1)$$

$$\delta_r^1 = \delta_r^0 + h^2 8.41498 rad$$

- The above is the solution at $t = 0 + h = 0.1s$
- For solution at $t = 2h = 0.2s$,

$$\mathbf{x}^2 = \begin{bmatrix}\omega_r^2 \\ \delta^2\end{bmatrix} = \mathbf{x}^1 + 0.5h\{\mathbf{f}(\mathbf{x}^1) + \mathbf{f}(\mathbf{x}^2)\}$$

$$\mathbf{x}^2 = \begin{bmatrix}\omega_r^1 \\ \delta^1\end{bmatrix} + 0.5h\left\{\begin{bmatrix}16.82996 \\ \omega_r^1\end{bmatrix} + \begin{bmatrix}16.82996 \\ \omega_r^2\end{bmatrix}\right\}$$

$$\Rightarrow \omega_r^2 = \omega_r^1 + 0.5 \times h \times 2 \times 16.82996$$

$$\omega_r^2 = h33.65992 rad/s$$

$$\Rightarrow \delta_r^2 = \delta_r^1 + 0.5 \times h(\omega_r^1 + \omega_r^2)$$

$$\delta_r^2 = \delta_r^1 + h^2 25.24494 rad$$

$$\delta_r^2 = \delta_r^0 + h^2 33.65992 rad$$

, at $t = 0.2s$,

$$\omega_r^2 = 3.365992 rad/s$$

$$\delta_r^2 = 0.4229252 rad = 24.23183°$$

The second example is a slightly detailed example. So what we have here is the single line diagram of a single machine infinite bus system connected through a set of two parallel transmission lines. Their operating frequency is 60 hertz.

The machine is delivering 1 PU of real power. That aspect is missing. So it is basically the real power that is being delivered by the synchronous generator. Basically, PE is 1 PU. The machine terminal voltage and infinite bus voltage magnitudes are 1 pu each. So here we have one at an angle of 0, assuming the swing bus to be the reference bus.

And here we have an angle Vt at an angle alpha, where alpha is the angle with respect to the infinite bus. And it's given that Vt mod is also 1 p.u. The line reactance is given on a common system base. The synchronous generators, transient axis, and direct axis reactors are also given. We have to find the power angle and the swing equations for the first bit, which is the steady state operating condition. So, in the steady-state operating condition, we have to essentially know, first of all, that in order to find the swing equation, remember which is 2h by omega s d square delta by dt square, where delta is e dash and an angle delta, and this delta is being measured with respect to the voltage angle at the swing bus. It would be equal to pm minus pe. If we have to find the sine equation, we have to know what Pm is and we have to know what Pe is So let's look at it in the steady-state condition. In the steady-state condition, if we need to know what Pe is, remember that Pe would be E dash multiplied by some voltage divided by X with respect to sine delta. Now this X is where the machine or the network parameters would come in, which we call transfer reactants. So the first process to find PE would be to find X because voltage magnitudes, in a way, can be obtained from the relative current expressions. So X is going to be obtained for the... between the terminal voltage and the infinite bus over here, which is at an angle of 1 and an angle of 0. So the corresponding transfer reactance X is basically a series and parallel combination of these two reactances, which comes out to be 0.3 PU. The terminal voltage is Vt at an angle alpha, where Vt is given as 1 Pu, and this terminal voltage is given with respect to the infinite bus. So we can use the real power expression of power flow from the terminal voltage to the infinite bus.

Since it is a lossless system, whatever power is generated by the synchronous generator, which is 1 pu, the same 1 pu power also has to flow through this point with J0.1 reactance, as well as the parallel combination of J0.4. So essentially, by using the real power expression, we can find the value of alpha, which is 17.458 degrees. So essentially, the terminal voltage here is Vt at an angle alpha. Vt is given as 1 p.u. Since the system is lossless, the same 1 pu real power has to flow from the terminal of the generator to the infinite bus, in between which there is a transfer reactance of j 0.3 pu. So we can set X as 0.3, VT as 1, and equate this to 1; from there, we would get this as the alpha value. Since we now know what alpha is, we can find the effective current flow from the terminal to the infinite bus, and the same current would be fed by the generator itself. So by KCL, we get the current, and then we apply KVL from the internal EMF to the machine terminal; we get the corresponding internal EMF. The magnitude of internal EMF remains the same as 1.05 PU. The delta value that we are getting is the initial delta angle, which is 28.439 degrees. So essentially, in order to find the swing equation in steady state, the overall transfer reactance should also consider the machine's transient axis reactance because, for the internal swing equation, you have to model the generator through its corresponding transient model. So essentially, X changes from 0.3 to 0.5 PU, which is the internal EMF bus that is also being considered. And in this case, if you find the real power expression PE, it would be E1 dash times 1, where 1 is the terminal voltage at the swing bus. X is 0.5 PU, so we get 2.1 sine delta as our PE expression, with the initial angle delta being 28.439 degrees. PM, since it is not given as lossless, we can easily choose PM to be equal to PE in steady-state condition, which is going to be 1 PU. So the corresponding swing equation turns out to be that H and omega S are given, resulting in this unique expression, which is the value provided here. This is the swing equation in a steady state. Now, suppose the next bit says a fault occurs at point P; the point P is actually the midpoint of the second parallel line.

So, if a three-phase solid fault occurs at point P, essentially the line is broken into two halves, and in this condition, we have to find the same power angle as well as the swing equation. So, how do we do it? The corresponding electrical diagram changes a bit here. The values that you see here are the corresponding reactances that are present between the respective buses and with respect to ground.

The internal EMF magnitude doesn't change, so 1.05 is retained. Delta, it's actually not delta naught; it's actually only delta. Delta value is being measured with respect to the infinite bus, and you might wonder how this J2.5 or J.5 came about. The answer lies in the fact that earlier we had J.4 impedance. If we convert it to the corresponding reactance or admittance, it would be 1 by J.4, which gives us minus J2.5. Similarly, if the fault occurs between lines 2 and 3, J.4 gets divided into J.2 on either half, whose inverse is J5. So that's how the corresponding admittances are determined J0.33 is basically the inverse of J0.2 and J0.1 combined together, which are the respective radiotoxic transient reactants and the transformer reactants. In this condition where the fault has occurred in the line, we have to find the corresponding swing equation again. For the swing equation, we would assume PM will not change because the governor dynamics are ignored. PE expression, however, could change because the network has experienced a change.

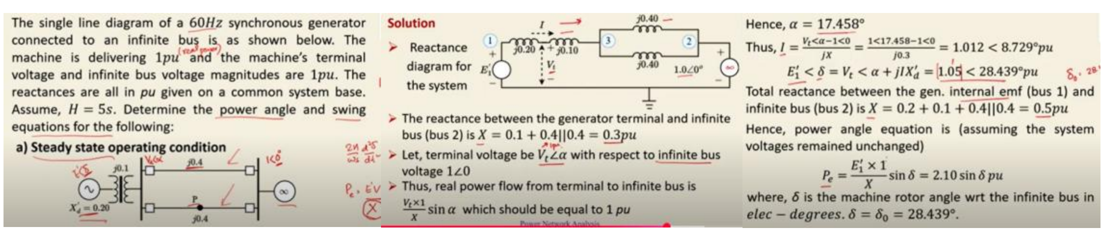
So corresponding X would have definitely changed. So, how do we do it? The answer lies in our ward reduction discussions. We find the corresponding Y bus for the respective three-bus network that is shown here. Buses three and four. Bus number 3 is not actually important. Bus 1, which is the internal EMF, and bus 2, which is the infinite bus, they are the only point of importance here.

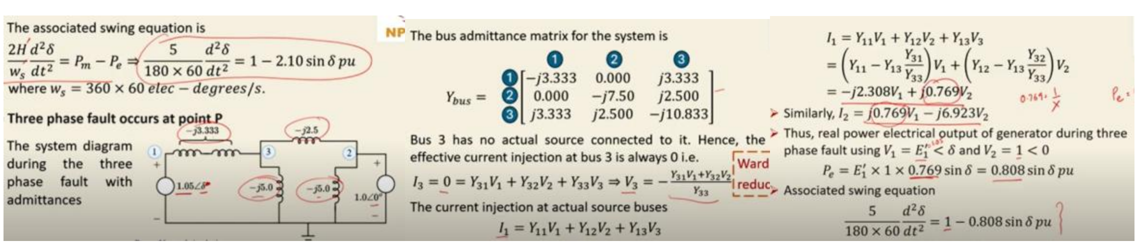
So we use WAD reduction. Bus 3 has no injection, so I3 is zero. We can convert V3 in terms of corresponding voltages and then substitute these expressions of currents and voltages back into the corresponding I1 and I2. So I turns out to be this expression. Similarly, I_2 turns out to be this expression. With these current expressions, if we try to find the PE value, then the corresponding terminal voltages would be E1 dash, which is 1.05 at an angle delta, and V2 is 1 at an angle 0. The correspondent transfer reactance is simply 0.769, as shown here, and hence that same value of 1 by X comes into this. Basically, these are admittance values which essentially indicate that 0.769 is the value of 1 by X. And PE is E minus E1 minus V by X sine delta, E1 is 1.05, V is 1, and 1 by X, or X is 0.769 inverse. So that's how we get 0.808 sin delta. The electrical power and mechanical power output don't change, so PM remains the same; the associated swing equation is given here.

If we want to find the power angle, given a specific point in time, we can use the corresponding numeric integration technique to solve this. The third bit is supposed to indicate that the fault is now cleared by opening the second line at either end. So the entire network boils down to the fact that the second line in this network is completely gone.

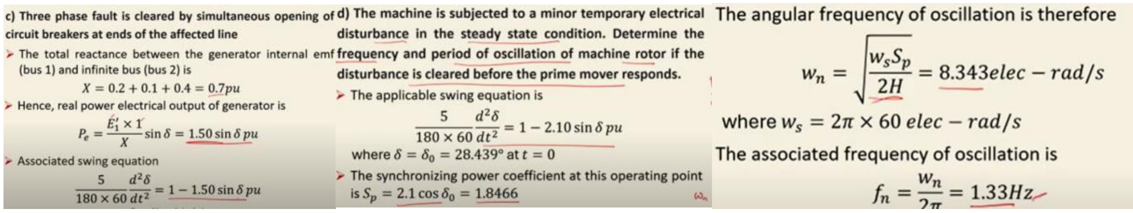
We have only J.2, J.1, and J.4 as our effective reactances present between the internal EMF and the infinite bus. So overall X again changes after the fault has occurred. Which turns out to be 0.7 pu, E1 remains the same, 1 is the same, so 0.7 pu, we get 1.5 sine delta. So after the fault is cleared, the swing equation form has changed again. The third and last bit is that suppose the machine is subjected to a minor temporary electrical disturbance in a steady state condition where Pm minus Pe was 1 minus 2.1 sine delta. We have to find the corresponding frequency and period of oscillation in the machine rotor if the disturbance is cleared before the machine, in the mechanical input, before the governor comes into the picture.

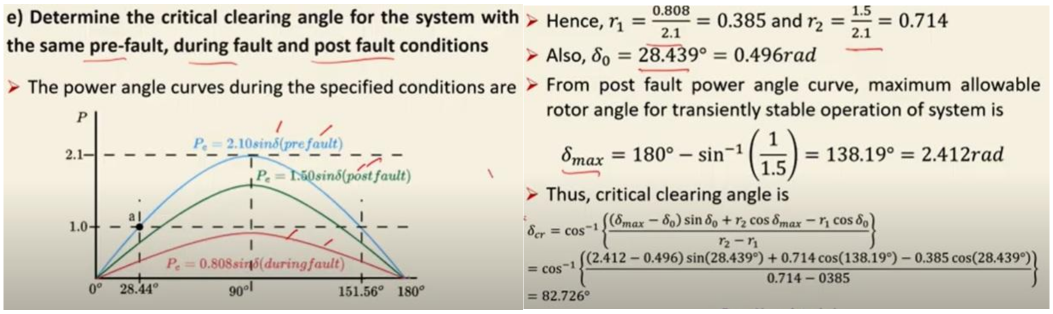
So essentially, we have this equation that we have to deal with; the initial angles are given. Synchronizing power coefficient at this angle is 2.1 cos delta naught, so it is 1.8466. Remember that the natural frequency omega n is dependent on the square root of the minus sp term; that's how this term comes in.And omega n is 0.8436. 8.343 electrical radians per second, whose corresponding frequency is 1.33 Hz. This is going to be the natural period of oscillation around the minor perturbations. The last bit is that we have to find the critical fault clearing angle or critical clearing time with the same pre-fault, during fault, and post-fault conditions, the equations of which we have already devised. So if we remember and go back or recollect our previous equations, we had three swing curves for pre, during, and post fault conditions. So we can apply the logic from the last lecture, where we can find the R1 and R2 values that are already given here.





Delta naught is given as the initial angle. So we can find delta max and use the similar expression that we saw in the previous lecture to find the corresponding critical clearing angle.





If one wants to find the corresponding critical clearing time, then some form of numeric integration with reverse technology or reverse engineering can be used to find the corresponding critical clearing time from this particular critical clearing angle. That's all for this discussion. In the next lecture, we will take up a generic discussion on how trans-neutral triangle stability can be achieved for multi-machine systems and what the practical aspects are involved in analyzing those multi-machine cases, which are actually more complicated and require a detailed understanding of power flow and a lot of other stability aspects.

Thank you.