Power Network Analysis

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Week-12

Lecture-57

Lecture 57: Stability analysis- More on equal area criteria, Basics of numeric Integration

Hello everyone, welcome to the second lecture of week 12 of the course Power Network Analysis. In this lecture, we will continue with the discussion on equal area criteria, which is an indirect method of doing transient rotor angle stability analysis conventionally for a single machine infinite bus system or two coherent or two non-coherent synchronous machines in a given power network. In this discussion, we will also briefly talk about a few numeric integration techniques that would be useful for solving the second-order swing equation that we have discussed while doing transient O-triangle stability analysis for a multi-machine system or trying to do practical transient O-triangle stability analysis. where equal area criteria cannot be used or will not give appreciably accurate results. In the last lecture, we talked about the synchronizing power coefficient, what it is, and it is actually an indirect measure of synchronizing torque; the more the synchronizing power coefficient, the lesser the chances of the power network undergoing transient root triangle in stable conditions. And it also helps in defining what are what could be the feasible initial steady state operating point in terms of rotor angle for a synchronous generator.

So we have started with the equal area criteria concept specifically for a single machine infinite bus system in which we have considered one synchronous generator that is connected to the infinite bus, the infinite bus representing part or the entire power network. And in a steady state condition, that is when there is no fault or disturbance, the generator is feeding some power into the infinite bus. That was the pre-fault condition. In the post-fault condition, the breakers which are represented by these square boxes, the breaker B gets closed and as a result of which the dead short circuit or a path for current or power to flow from the generator terminal or transformer terminal to the ground becomes feasible when the breaker is closed and as a result of which no useful power goes to the infinite bus as shown from the figure.

The entire current or path flows through this short circuit path, which connects the generator terminal or transformer terminal to the ground. So we were trying to understand what dynamics go into when this single machine infinite bus system (SMIB) undergoes a disturbance, to be specific, a dead three-phase short circuit fault by a closure of breaker B. And we had seen or understood the different points on the p delta curve or power delta curve, p on the y axis, delta on the x axis. The power angle points or curve followed a certain pattern in the pre-fault condition, which is the condition before the fault; during the fault, the breaker was still closed, and in the post-fault condition, this breaker, either at A or B, is now opened so that this path doesn't become available. That is what is meant by the post-fault point.

And we had seen five points on this P-delta curve, which are A, B, C, D, and E. And this was the analysis before, during, and after the fault condition. After the post-fault condition, the power angle points pertain to points E, A, and F. The equal area criteria suggest that areas A1 and A2 are going to be equal to each other, and areas A3 and A4 are going to be equal to each other, as a result of which we can conclude this without doing direct numeric integration or directly solving the classical conventional swing equation through usual integration techniques. We can get an estimate of delta X, which is the power angle for point E, or delta Y, or maybe it is delta F here, delta F rotor angle with the operating point as point F.

So, without doing numeric integration using Euclidean criteria, we can still get the estimates of these angles, and these angles would definitely depend on what the initial operating point is, which is delta naught in terms of rotor angle. The synchronous generator here is being represented through its internal EMF, whose voltage magnitude is constant because AVR dynamics are slower, so their dynamics are ignored. Only the rotor angle is changing because of different disturbances or short-circuit faults that are occurring. and the corresponding machine is also having its own transient direct access reactance which is considered in the value of Pmax which is shown over here. It is already incorporated through the transfer reactance value X.

To sort of summarize what is happening in these respective points, Maybe I'll show each of those conditions one by one. So let's focus on point A and for the time being ignore all other points. So let me erase this. In fact, I can go back and focus on point A. At point A, which was the pre-fault condition, what we are seeing is that the machine's electrical power output, which is PE, is the same as the mechanical input power, which is PM, as indicated here, and at point A since the rotor angle is delta naught, which we know from some analysis or a brief fault situation.

This is equal to Pmax times the sine of delta naught. So essentially, Pm matching or crossing this curve at point A indicates that Pm is equal to Pmax sin delta naught ($P_m = P_{max} \sin \delta_0$), which is also equal to Pe. And that's the reason. at point A which is the pre-fault condition. Everything is in a steady state. The rotor speed of the synchronous generator is the same as the synchronous speed. Rotor angle is delta naught. Electrical power is the same as mechanical power, which is input power. So the rotor is not experiencing any acceleration at all, which is essentially what is shown in this particular table. The initial operating angle is delta naught, electrical power is P max sine delta naught, since Pm and Pe are equal.

Therefore, acceleration power Pa is zero, and since rotor speed omega is the same as the synchronous speed, the relative velocity or relative synchronous speed omega r is perfectly zero, which is the first derivative of the angle itself. Now, if point A is clear, then let's move to point B. At point B, which is, so let's say if at t is equal to 0, the disturbance has happened. At point B, the time hasn't progressed significantly. It is just the immediate moment after the fault.

And since there is a dead short circuit fault because of which the machine terminal voltage is almost zero and as a result of which no useful power is going to the infinite bus from the synchronous generator. And that's the reason why, in the immediate next instance at point B, electrical power is becoming perfectly zero, which is what is shown over here. The rotor angle has still not changed, or maybe in a way, if I were to include a time column here, let's say time t. so at point A which was t is equal to 0 that is the instant of disturbance point B is being observed at a time 0 plus which is the next immediate instance after the fault has occurred and since it is a dead short circuit fault A and B are offering a closed path for the generator current to flow through and current always tends to flow through a path of least impedance. So the entire useful power current is flowing to the ground, and as a result, the feasible output power PE to the infinite bus is zero.

Which is what is mentioned here. And since P is becoming zero, the mechanical input power is constant because we are ignoring AGC, ALFC, or governor dynamics since its time constant is particularly larger compared to the transient time period in which we are going to see. So acceleration power Pa is becoming positive because Pm is again, as I have discussed, Pmax sine delta naught; so delta naught being positive and Pmax being positive means that Pm is positive and the acceleration power is positive. So from t is equal to zero plus instant the rotor is experiencing acceleration but since it is a rotating device it will take some time to gain extra kinetic energy that means it would take some time for the rotor speed omega to deviate from synchronous speed and hence at t is equal to 0 plus the relative velocity omega r is still 0. Now let us see what is happening after t is equal to 0.

The fault is persisting, which is indicated by this particular arrow. The rotor is accelerating because PM is acceleration power. So at, let's say, time T is equal to TC, which is the fault clearing time. The rotor is accelerating. Rotor angle has also increased.

Rotor speed has also increased. And that's the result of why, at point C, which is time TC. TC is the fault clearing time. Delta C is a function of delta naught, which is dependent on the result of the swing equation integration, so there is an additional gain in the rotor angle itself. The electrical power is still zero because the fault persists.

In fact, this is at the instant t, which is equal to tc minus just before the fault is cleared. And hence, the electrical power is still zero. Since electrical power is zero, acceleration power still remains as Pm. And omega r, as per the integration of the sine equation, results in a positive quantity. That means this is the additional speed that the rotor has gained compared to synchronous speed.

Point	δ	t	P_e	$P_a = P_m - P_e$	$w_r = \frac{d\delta}{dt} = \hat{w} - w_s$
a	δ_0	0	$P_e = P_m = P_{max} \sin \delta_0$	0	0
b	δ_0	0^+	0	P_m (+ve)	0
c	$\delta_c = \delta_0 + \frac{w_s P_m}{4H} t_c^2$		0	P_m (+ve)	$\frac{w_s P_m}{2H} t_c$ (+ve)

So, after the fault is cleared, which is at t equal to tc, the electrical power. So, this path is now closed because one of the breakers has been opened and electrical power, which was being fed to the infinite bus, immediately shoots up because the rotor angle has now increased from delta naught to delta c. So point D is being observed at T equal to TC plus just the instant after the fault has been cleared. Since the rotor angle has advanced, the corresponding electrical power immediately shoots up, and hence at point D, we have the rotor angle as delta C. So this is the time at T, which is equal to TC plus.

Electrical power, which is now recovered and is being fed to the infinite bus, becomes P max sine delta C. Since delta C is... More than delta naught because this term is a positive quantity, so the acceleration power, where pm was p max sine delta naught, delta naught being less than delta c, the overall pa becomes a negative term, so the machine or the rotor now starts facing the acceleration.

So it's, in a way, here it is all acceleration, and from here onwards it is all deacceleration in the first power angle curve. The rotor speed at t is equal to tc plus immediately hasn't changed. So, the relative rotor speed remains the same. Now after the fault has been cleared, since the machine is now facing deacceleration and the relative rotor speed is positive, it would take some time for the rotor to continue deaccelerating till omega R becomes zero which is what is marked by point E and that point E on the power angle curve is being marked by this point.

Ignore the em dash term here. The rotor continues decelerating and it would continue decelerating until omega r becomes zero, as a result of which the rotor angle again advances. It has become more than delta c now. So we reach an angle delta x, the value of which we don't know what it is going to be. But definitely delta naught x is going to be more than delta naught, so electrical power will be positive and it will be more than the mechanical power, so the machine will still be in a deceleration mode, but the rotor speed will become instantly zero at point E. After point E has been reached, the position of the power angle curve changes.

We have the second power angle curve, which is all about the post-fault condition. Since at point E the relative rotor speed is zero, the machine is still decelerating and the electrical fault has been cleared. So the machine would keep on decelerating until it reaches a point of no acceleration at all, which is marked by point A, and at point A we have the rotor angle as delta naught. PE is equal to PM, so instantly at point A during the fault condition, rotor acceleration is perfectly zero, and since the machine was decelerating. The rotor's relative speed, which was zero in the acceleration mode, omega R, will now become negative because, in the prior instant, P A was negative.

So rotor speed would be relatively negative compared to synchronous speed at point A. Since the machine is instantly facing a zero acceleration instant but the rotor speed is less than synchronous speed, it is a rotating device, so it cannot stop immediately because of its own rotating inertia; it would continue decelerating. Till it reaches point F, which is marked by this particular point, at point F, the rotor angle, since the machine is decelerating, the rotor angle would go down and delta F angle would be reached, which would be less than delta naught. And electrical power would be less than mechanical power. So from point A to F, if I were to say, from point A to F, the machine experiences acceleration, and as a result of this acceleration, the relative rotor speed, which was negative, instantly becomes zero.

And from zero again, the machine backtracks and reaches point A. So this sort of two-fourth oscillation or circular movement continues during the fault condition, and it is expected that with sufficient damping in the post-fault condition, the pre-fault delta naught angle would be experienced again. So I hope that with this sort of table and the power angle curve, the associated points and their importance become clear. What is still to be proven is that area A1 and A2 and A3 and A4 which are as marked in these curves, how do they become equal to each other. So let's see how these angles become equal to each other.

So with this table from the swing equation, we know that the points A, E, and F are those points where, at an instant for those conditions, the relative rotor speed is perfectly zero, which is what is marked by these zeros over here.

Point	δ	P_e	$P_a = P_m - P_e$	$w_r = \frac{d\delta}{dt} = w - w_s$
a	δ_0	$P_e = P_m = P_{max} \sin \delta_0$	0	0
b	δ_0	0	P_m (+ve)	0
c	$\delta_c = \delta_0 + \frac{w_s P_m}{4H} t_c^2$	0	P_m (+ve)	$\frac{w_s P_m}{2H} t_c$ (+ve)
d	δ_c	$P_{max} \sin \delta_c$	$P_m - P_{max} \sin \delta_c$(-ve)	$\frac{w_s P_m}{2H} t_c$ (+ve)
e	δ_x ($> \delta_0$)	$P_{max} \sin \delta_x$	$P_m - P_{max} \sin \delta_x$(-ve)	0
a	δ_0	$P_e = P_m = P_{max} \sin \delta_0$	0	(-ve)
f	δ_f ($< \delta_0$)	$P_{max} \sin \delta_f$	$P_m - P_{max} \sin \delta_f$(+ve)	0
a	δ_0	$P_e = P_m = P_{max} \sin \delta_0$	0	(+ve)

So if we recollect our swing equation, our swing equation can be written as two first order differential equations. And in the actual sense, it is two H omega S, d square delta by dt square equal to PA, which is PM minus PE. We are replacing omega r with d delta by dt, so both of our equations are first-order equations. We will focus on this equation for the time being and multiply both sides of this equation by the term omega r, which is the relative rotor speed with respect to synchronous speed and since omega r is equal to d delta by dt so after multiplication we have d delta by dt and the two term which is associated with h gets carry forwarded to two omega r and now if we sort of rearrange the terms we get two first order terms both on the left hand side and right hand side of this equation

$$w_r = \frac{d\delta}{dt} = w - w_s, \qquad \frac{2H}{w_s}\frac{dw_r}{dt} = P_m - P_e$$

$$\Rightarrow \frac{H}{w_s} 2w_r \frac{dw_r}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

$$\Rightarrow \frac{H}{w_s}\frac{d(w_r^2)}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

which after integration based on two initial operating based on the boundary conditions, one initial point and one final point, we would get this sort of an expression where delta 1 and delta 2 and omega r and omega r2 are two relative operating points on the power angle curve.

$$\Rightarrow \frac{H}{w_s}(w_{r1}^2 - w_{r2}^2) = \int_{\delta_1}^{\delta_2} (P_m - P_e)\, d\delta$$

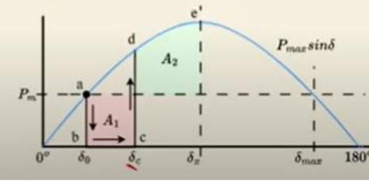
What is interesting to observe is that if omega r1 and omega r2, or their squares, become equal to each other, then suppose we figure out those two points, point one and point two, for which omega R1 is equal to omega R2. Then what would it mean? It would mean that for these two operating points, the corresponding LHS term becomes zero, which would also mean that this integration should become zero. Now, that is the idea, concept, or logic behind the equal area criteria.

So let's see what those two points are. So, the RHS term, as indicated here, is nothing but the area under the power angle curve for fixed PM. We have already seen the power angle curve in slide number 3 here. We have two power angle curves, one on the right hand side here and the other one on the left hand side here. So if we focus on this integral curve and omega r1 and omega r2 become zero or are equal to each other, it would mean that the area under the curve between those two points would also become zero. Now, if you recollect, we had points a and e in the first power angle curve where instantly the omega is the relative rotor angle speed or rotor speed with speed equal to 0.

So, omega r a and e are those points where the relative speed is 0. So, it would mean that the area under the curve from rotor angle delta naught to delta x should be zero. Equal to zero as per the integration equation that we have obtained. The last equation here can be further broken into two segments, one part being integrated from delta naught to delta c, where delta c is the rotor angle at the instant when the fault was cleared, and the other instance is delta 3 to delta x, which is essentially the green color area. So if I were to say pm minus pe integral, and I take this term on the right-hand side of the equation, then pm minus pe becomes pe minus pm.

$$\int_{\delta_0}^{\delta_c} (P_m - P_e)\, d\delta + \int_{\delta_c}^{\delta_x} (P_m - P_e)\, d\delta = 0$$

$$\int_{\delta_0}^{\delta_c} (P_m - P_e)\, d\delta = \int_{\delta_c}^{\delta_x} (P_e - P_m)\, d\delta \Rightarrow A_1 = A_2$$



And if I focus on the respective terms, then PM minus PE is essentially the area which is this pink color area from delta naught to delta C. So this becomes area A1. And the other term is PE minus PM integral with respect to delta, which is the area from, this is the PE value here. This is the Pm value here, and for area A1, this is Pm, and Pe is constrained to 0 because the fault was not cleared. So the second term here essentially becomes area A2, which is the green curve, and that's how area A1 and A2 match each other.

If we apply the same logic to our second power angle curve where we focus on points E and F, points E and F are those two post-fault power angle points where the relative rotor speeds are instantly zero, and that's how we can again get area A3 to be equal to area A4.

I hope that with this discussion, the idea or concept of why the equal area criteria work and how it can be proven is clear now. Now it's time to find out if we can determine delta x angle using this equal area criteria, which we still have not figured out, and correspondingly, for point F, can we find delta F without actually integrating the swing equation. So E here refers to the delta x angle and F refers to the delta F angle.

So yes, since the areas are equal. So, or maybe before I go into that, let me give you a brief analysis of the value of A1. Because we are talking about this integral, So, from the perspective of area A1, electrical power is perfectly zero. That means this term is an ever-present zero. PM is not changing. So, area A1 becomes PM delta-d. With respect to delta naught and delta C, Pm being constant in relation to our delta, this area is simply Pm delta C minus delta naught, where delta C is delta naught plus omega Sh by, I'm forgetting the term here, just give me a moment, it is omega S Pm by 4H Tc squared. Omega SPM by 4H TC is square. It is the result of integrating the sine equation twice. And TC is the time at which the fault has been cleared from T, which is equal to zero, the time of fault disturbance.

Similarly, we can find the area A2. Area A2 would be PM minus PE. Let's put PM in the function of delta. So it is P max sine delta minus P M. d delta integration from delta c to delta x. The second term again is constant, so the first term becomes p max, which is constant, sine delta d delta delta c to delta x minus p m. p m is constant, so it is delta x minus delta naught. The integral of sine delta between delta c and delta x would result in some cosine term, and if we equate a1 and a2, Pm is known, it's constant, Pmax is known, delta 0 is also known, delta c is a function of delta 0, the only unknown will be delta x, which will appear as delta x and the cosine of delta x as per the integral. So if we do that, then we can also find delta x without actually doing numerical integration; that's the idea. So if you look at area A1, area A1 depends on what is the value of delta C which in a way depends on what is the time taken to clear the fault and that's the reason why area A1 depends on critical clearing time. The more area value of area A1 is and A1 being equal to A2, A1 is actually also called as the accelerating area in which the rotor is accelerating and area A2 is called as the decelerating area. So basically, all the green ones are deceleration areas, and the pink ones or red ones are the accelerating areas.

The greater the value of area A1, the greater the value of area A2 must be in order to satisfy the equal area criteria condition. So if fault clearing is delayed, delta C would increase and area A1 would increase. By the equal area criteria, A2 should also increase according to area A1. And if it is prolonged, the fault is prolonged; it might happen that delta X becomes more than delta max. Now, what is delta max? Delta max is the angle that depends on delta naught, which in this particular curve is 180 degrees minus delta naught degrees; in terms of degrees, this is the value, or in terms of radians, it is pi minus delta naught radians.

So the more value area A1 takes, it may happen that delta x may become equal to or greater than delta dash or delta max value. Now, what is the problem with area delta x becoming more than delta max? So let's see. So after delta x, let's say it takes a value between delta max and pi by 2 radians or pi radians, sorry. Then in this particular zone, P E is always less than P M because P M is the straight line. So, it would mean that any value of angle beyond delta max or delta dash would result in the rotor going into an accelerating mode rather than going into a decelerating mode.

The areas being equal to each other, they match whatever energy the rotor has gained. The same is being lost again, and that's how the accelerating or decelerating area makes sense. If area A1 becomes large, A2 will also become large, but A2 has a dead end. After the delta max angle, the machine cannot decelerate. It will furthermore accelerate. So it may happen that the accelerating area or kinetic energy gained may become greater compared to the kinetic energy that could have been lost in terms of deceleration, and hence the situation would become an unstable condition. A2 will not increase beyond delta max area under swing curve is accelerating area. Rotor will keep accelerating, rotor speed will increase, and there is no point of return after the angle crosses, delta X crosses delta max value. So essentially, the machine might go into loss of synchronism, and rotor angle instability would happen. So using this logic, we can actually find the critical fault clearing time and the critical fault clearing angle, which dictate that if a fault is cleared in less than this fault clearing time, things are going to remain stable.

That's the essence of the entire conventional transient rotor angle analysis. So let's see. Let's find that particular fault-clearing time. So, by the equal area criteria, A1 is PM delta integral, A2 is P max sine delta integral, delta X has become equal to delta max, which in earlier slides was also delta naught dash, which is also the same as pi minus delta naught radians. And delta CR here is delta naught plus omega SPM divided by 4H TCR squared.

We want to find the critical fault-clearing time. If the fault is cleared after fault clearing, critical fault clearing time, it's a point of no return and system would go into unstable mode.

during and post fault

$\delta_x = \delta_{max} = \delta_0' = \pi - \delta_0$ $\qquad$ $\delta_{cr} = \delta_0 + \frac{\omega_s P_m}{4H} t_{cr}^2$

$$A_1 = A_2 \Rightarrow \int_{\delta_0}^{\delta_{cr}} P_m d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} sin\delta - P_m) d\delta$$

So if we equate these two, PM is equal to Pmax sine delta naught, delta max is pi minus delta naught, so we can use these two equations, equate them, do some sort of simplification, and get the delta CR value,

$$\text{As } P_m = P_{\max} \sin \delta_0, \delta_{\max} = \pi - \delta_0$$
$$\delta_{cr} = \cos^{-1}\{(\pi - 2\delta_0)\sin \delta_0 - \cos \delta_0\}$$

which is all in terms of delta naught, substitute this delta CR back into delta naught, and we can correspondingly find the critical fault clearing time also.

$$\delta_{cr} = \delta_0 + \frac{w_s P_m}{4H} t_{cr}^2 \Rightarrow t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{w_s P_m}}$$

In general, it may happen that a dead short circuit fault does not always occur at the terminal of the generator. There are faults in which some portion of power goes to the ground and some portion of the power may go to the infinite bus. But definitely, faults are detrimental events, so it is not possible that during a fault the power output of the synchronous machine would increase; it will only decrease, but it need not go down to zero.

So we take care of these respective factors that is fault need not always being a dead short circuit fault that is point number one and the second point is after the fault has been cleared it is not necessary that the post fault condition is same as the pre-fault condition maybe while clearing the fault some transmission line is intently has to be switched off in order to save the unhealthy healthy part from the unhealthy part so it is not necessary that the post fault power angle curve would be same as the pre fault power angle curve and these aspects they are taken care by these factors R1 and R2 where the black color curve is the pre fault power angle curve the The green color curve is the fault curve where, for a dead short circuit fault, R1 is perfectly 0. The red color curve is the post-fault curve where the power has definitely recovered, but it is not as good as the pre-fault condition. So that's the reason why R1 and R2 they will vary in the range of 0 to 1. R2 can never be greater than 1. R1 can never be more than 1, but it can definitely become 0. And the reason for that, as I have mentioned, is that faults are detrimental events. They don't help the system. They degrade the systems. So, under generic conditions, the equal area criteria will still hold true and how it would hold true. Area A1 is marked, A2 is marked, the corresponding initial angle delta naught is marked, and delta max is also marked. The difference here is that the delta max value, which was earlier in case R1, was 0; delta max was equal to pi minus delta naught. Here, if you see, delta max is a point on the red power angle curve. It is not a point on the black color curve. So the angle delta max would depend on the value of the angle of this rotor angle at the red line, let's say delta dash.

So, how do we find that delta dash? Delta dash or delta max is to be obtained from this particular point. How do we get that? The red color curve is marked by the expression r2 p max sine delta. Pe being equal to Pm at the initial point, so from this red color curve, Pm equal to R2 Pmax sine delta dash, let's say, would give us delta dash to be sine inverse of Pm by R2 Pmax. Since Pm is also equal to Pmax sine delta naught, the point on the black curve, so Pmax and Pmax get canceled, we have sine inverse sine delta naught by R2 as a delta dash value. And the difference between it and pi radians gives delta max. We put in those corresponding values, equate area A1 and A2; certain mathematical analysis is needed,

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{\sin\delta_0}{r_2}\right)$$

$$\delta_{cr} = \cos^{-1}\left\{\frac{(\delta_{max} - \delta_0)\sin\delta_0 + r_2\cos\delta_{max} - r_1\cos\delta_0}{r_2 - r_1}\right\}$$

but I have avoided those steps. I expect the viewers to please carry out the A1 and A2 exercises, find the area under the curve, and equate them. From there, you can find the critical fault clearing angle, and correspondingly, the critical clearing time can be obtained by using these boundary conditions for integration. So that's all from the equal area criteria perspective.

I'll briefly talk about a few numeric integration techniques. Since we can, we'll have to solve some equations for a multi-machine system and for practical transient stability analysis of triangles. So it's always better to handle first order differential equations rather than higher order differential equations so our swing equation it can be very well broken down into two first order equations which we have seen earlier also

$$\dot{x}_{1i} = \frac{d\delta}{dt} = w_{r_i} = w_i - w_s = x_{2i}$$

$$\dot{x}_{2i} = \frac{w_s}{2H_i}(P_{mi}^{o} - P_{ei}^{odp})$$

And we mark these corresponding variables that is x2 to be the relative rotor speed and x1 to be the rotor angle itself delta and these are our analogous state space form first order differential equations Let x dot, which is x dot, be nothing but dx by dt, the first time derivative, equal to some function, which is a function of x and t. In this case of the sine equation, t is not an exclusive variable; it's only a function of x. However, it may happen that generic differential equations may also have the time variable t appearing in themselves. So where x is the dependent variable and t is the independent variable, the objective is to find the numerical value of x rather than finding the analogous closed-form equation of x for different values of time.

So that's what we are trying to achieve. That with some initial operating point, t is equal to t naught, and x is equal to x naught, we are trying to find specific values of x at specific points in time. So the first very basic numeric integration technique is Euler's method, which is an explicit method in which the method takes use of the functional value of the slope itself. Since x dot is equal to f. So we equate x and t to be their initial points, find the derivative value, and this derivative, which is dx by dt, can be approximated as delta x by delta t.

$$\dot{x} = \frac{dx}{dt} = f(x,t); \left.\frac{dx}{dt}\right|_{x=x_0, t=t_0} = f(x_0, t_0)$$

So if we know the time step delta t, assuming that to be small h, then delta x is the increment in The value of x, so assuming h to be our small time step, we can find delta x, which is h f, add it to the initial value, get x1, and repeat this exercise to get x2 and so on.

$$\text{then } \Delta x = hf(x_0, t_0)$$
$$x_1 = x_0 + hf(x_0, t_0)$$
$$x_2 = x_1 + hf(x_1, t_1), t_1 = t_0 + h$$
$$x_{n+1} = x_n + hf(x_n, t_n), t_{n+1} = t_n + h$$

The problem with this Euler's first-order method is that in order to have sufficient accuracy in terms of x, the time step h has to be very, very small. Further refinements of the explicit method or Euler's method suggest that instead of using only the first derivative, which is a tangent point, we should also use derivatives at the initial point as well as the final point. So it's basically a predictive character method. In which we tend to first predict what the expected change in the value of X naught from the T naught perspective could be, and then take the average of both these functional slopes to get our actual corrected value.

So P here refers to prediction, while without P refers to the corrected value. And this sort of a two-step process for modified Euler's method is definitely better than the first order Euler's method and works well for higher order nonlinear equations.

$$x_1^P = x_0 + hf(x_0, t_0)$$
$$x_1 = x_0 + \frac{h}{2}\{f(x_0, t_0) + f(x_1^P, t_1)\}, t_1 = t_0 + h$$
$$\vdots$$
$$x_{n+1}^P = x_n + hf(x_n, t_n)$$
$$x_{n+1} = x_n + \frac{h}{2}\{f(x_n, t_n) + f(x_{n+1}^P, t_{n+1})\}, t_{n+1} = t_n + h$$

Euler's method can furthermore be made more rigorous by having modified Euler's method. So we call those methods Rangakatta methods or RK methods. The second-order RK method is the same as Euler's method, in which there are four averages or four slopes taken. Those four slopes are with respect to the first initial condition, the final initial condition, and two points taken between the initial and final steps.

The corresponding summarized equations K1 and K2 are marked over here. So with these known functions, we can get better estimates of higher order, second order Euler's method or fourth order Rangakatta method.

$$x_{n+1} = x_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_n, t_n), k_2 = f\left(x_n + \frac{k_1}{2}, t_n + \frac{h}{2}\right)$$

$$k_3 = f\left(x_n + \frac{k_2}{2}, t_n + \frac{h}{2}\right), k_4 = f(x_n + k_3, t_n + h)$$

The overall reason why they are called explicit methods is that while finding the new value of x, the entire RHS term is always known, and that's why it is called explicit evaluation. The problem with explicit methods is that they may face numerical instability, and it is highly dependent on the value of h and the values of the slopes that are taken. But it is easier to implement.

The getaway with these explicit methods is to make use of implicit methods. One of those methods is called the trapezoidal rule. The trapezoidal rule and implicit methods are numerically stable. The notion remains the same. Instead of using prediction correction or assuming everything to be known on the right-hand side, what implicit methods do is that, okay, we have to make use of some tangent functions to find our next incremental values of x, but let's not know them a priori.

$$x_{n+1} = x_n + \frac{h}{2}\{f(x_n, t_n) + f(x_{n+1}, t_{n+1})\};$$

$$t_{n+1} = t_n + h$$

The unknowns here appear both on the LHS as well as the RHS. And since these are all well-known functional values, the value of x or the function value of x itself is the form of x is known, The value of xn can be obtained then by applying any other method for solving nonlinear equations which we have seen earlier in context of power flow equations. So Newton-Raphson techniques are one common method used implicitly in implicit methods to improve the numerical stability and accuracy of the numerical integration techniques. In the next lecture, we'll talk about a few solved examples that will clarify how the equilateral criteria work and how numeric integration techniques work.

Thank you.