Power Network Analysis

Dr. Abheejeet Mohapatra

Department of Electrical Engineering

IIT Kanpur

Week-12

Lecture-56

Lecture 56: Stability analysis- Synchronizing power coefficient, Equal area criteria

Hello everyone, welcome to the first lecture of week 12 of the course Power Network Analysis. We are in our last week of discussion, and this particular week is the last week we will be talking about the last module, which is transient two-triangle stability analysis. In this discussion, we will understand or continue with the discussion that we had in the last lecture, which was all about the power angle equation for a single machine infinite bus system, in short also known as SMIB, and we also discussed a way of reducing. The entire power network, the buses of the entire power network, is represented by buses where internal EMFs have a constant magnitude of voltage; the sources are represented with those corresponding internal EMFs in series with their transient axis and direct axis reactants, which are considered part of this reduction known as WAD reduction, and such a reduction helps in. Avoiding extensive power flow analysis, which would otherwise be necessary to find the initial equilibrium operating condition, after which, if a disturbance occurs, the corresponding transient rotor angle stability analysis would be discussed. So, in today's discussion, we will continue to discuss more about the power angle equation for the SMIB system, and specifically, we will understand the importance of a very important coefficient, which is the synchronizing power coefficient, which, in a way, is a measure of the synchronizing torque available in the power network with regard to the transient row triangle stability analysis.

If the synchronizing power coefficient is higher, which would mean that the corresponding synchronizing torque would be higher, then the possibility of having transient rotor angle instability would likely be less. And this coefficient would also help us understand that what could be a feasible operating point in terms of rotor angle for a corresponding synchronous generator. And lastly, we will discuss a direct approach to analyzing transient row triangle stability, which in a way involves solving the swing equation, which is a differential equation without using numeric integration. So that direct approach would be called the equal area criteria.

And, as the term suggests, it involves some aspect of the area under the curve. To be specific, the area under the curve pertains to the P delta plot that we discussed in the last lecture, which we also call the power angle equation. So, let us discuss more about this. So, in the last lecture, we considered the case of a single machine infinite bus system, which indicates that there is a synchronous generator represented by its internal EMF with constant magnitude, voltage magnitude, and the corresponding rotor angle delta, whose initial operating value is marked as delta naught, and this machine has its direct access reactance Jxd dash, which is connected to an infinite bus having some fixed terminal voltage. And the essence was that in case so during steady state, when the synchronous machine is feeding power to the power network, which is represented by this infinite bus or the entire power grid, then assuming that the losses are negligible, the input mechanical power to the synchronous generator would be the same as the output electrical power.

Which can be numerically marked on this P-delta plot refers to the electrical power output as the point encircled by this red circle, which is called the steady-state operating point. Numerically, if delta naught is the initial operating point, which otherwise would be obtained from exact power flow analysis or by the use of word reduction, then depending on the power output of the machine, we can directly make use of this expression to find the delta naught value from the corresponding inverse. Numerically, at steady state, there also exists another angle, which is delta naught dash, which is pi minus delta naught radian. At that point, the mechanical input power also matches the corresponding power output that would otherwise be obtained at the delta naught dash angle. And we had discussed in the previous lecture that we call delta naught delta, which is equal to delta naught, the initial operating point, to be the stable operating point or steady-state operating point.

Whereas delta is equal to delta naught dash, it otherwise appears to be mathematically a possible steady state operating point; however, in practice, a synchronous machine will never have this as its steady state initial equilibrium operating point. The operating point itself is an unstable operating point, and to define or justify that is where the importance of the synchronizing power coefficient term comes in. So, following a disturbance in a SMIB system, the disturbance is entirely electrical in nature; whatever disturbances or perturbations are occurring, they are happening on the power network side. Input mechanical power remains constant; it doesn't change. The variation, if at all, will occur in PE, which indirectly comes from the change in the value of Pmax because the corresponding transfer reactance might change when disturbances happen in the power network following a disturbance. The swing equation needs to be analyzed or solved for which delta naught will serve as the initial operating condition.

$$P_m = P_e = P_{max} \sin \delta_0$$

$$P_m = P_{max} \sin \delta_0'$$

So, as I was mentioning, mathematically, delta naught and delta naught dash, which is equal to pi minus delta naught radians, both can serve as the steady-state operating point. However, in practice, it is not true. A feasible operating point is a point at which the synchronous machine, specifically the generator, should not lose its synchronism; that means the rotor angle remains bounded, and the voltage, frequency, and phase angle all remain bounded even for small temporary changes if they occur in the electrical power output of the synchronous generator because of disturbances happening on the network side. And this notion of not losing synchronism is again being made with respect to some reference; that reference could happen to be the infinite bus in the case of the SMIB system, which was case 1 that we discussed in the last discussion, or it could be with respect to a nearby coherent synchronous machine or a non-coherent synchronous machine, which was case 2 in the previous discussion. So, for a fixed mechanical input power, We have seen that this is going to be our second-order swing equation,

$$\frac{2H}{w_s}\frac{d^2\delta}{dt^2} = P_m - P_{max} sin\delta$$

where the second term P max sine delta is nothing but P e, and changes are happening in the electrical power output of the machine. H is the inertia constant in seconds; omega s is the rate of synchronous speed of the machine with respect to a stationary axis. Delta happens to be the internal EMF angle with respect to terminal voltage, which, for classical transient neutral angle stability analysis, also happens to be the rotor angle. So suppose we have some delta equal to delta OP, OP referring to the operating term. Let's say that this synchronous machine with respect to SMIB and the infinite bus or a nearby synchronous machine is operating at some steady-state operating point at which the initial outer angle is delta OP, and around this delta OP, we are trying to see the effect of this small temporary change or perturbation that could happen.

So, suppose some perturbation has happened as a result of which the rotor angle has also been perturbed by an angle delta delta which we do not know as of now. To find delta delta, we have to integrate this equation with this as our initial operating point. Because other terms are all known, P m is fixed, H omega s is fixed, and P max can be determined from the change that has occurred in the steady state as well as the post-disturbance condition. So, without going into much of the complications involved in analyzing or trying to find a solution for delta delta, let us understand that some perturbation has happened in the electrical power output, because of which the rotor angle also has some perturbation, delta delta. So if we substitute this sort of perturbation into this second-order differential equation, then what we would get is that I can substitute delta, which is appearing over here and also appearing over here, in terms of delta OP and delta delta, and I am assuming that delta delta is a pretty small perturbation. around which system linearization can be done. So let's see what that perturbation or linearization is. So what I have done is, since PM is fixed, I am not perturbing PM at all, because governor dynamics are much larger or the time period is much larger compared to the transient period of importance for triangle stability analysis. So we are only trying to see the impact of this perturbation on the corresponding angles where delta comes in. So delta, if we substitute in terms of Pe, which is P max sine delta, where delta is now delta OP plus delta delta.

$$P_e = P_{max}\sin(\delta_{op} + \Delta\delta) \approx P_{max}sin\delta_{op} + P_{max}cos\delta_{op}\Delta\delta$$

And since delta delta is very small, we can make use of the Taylor series expansion. Remember, for Taylor series, if there is a function f of x, which can be expanded at some initial operating point x_0, then this is nothing but $f(x_0)$ plus (df/dx) at $x = x_0$ multiplied by $(x - x_0)$ plus (1/2!) (d^2f/dx^2) at $x = x_0$ multiplied by $(x - x_0)^2$. So, for perturbation being small where the delta delta value or delta x value, which is x minus x naught, if it is very small, then we can truncate the Taylor series expansion to the first order and assume all higher order terms to be 0 because the perturbation is very small. If the same logic of truncating the Taylor series for small delta x is applied to this term, then essentially we get P max sine delta OP, which is analogous to fx naught because fx here is P max sine delta and the first derivative of delta with respect to f of x, or let's say if I say here x is analogous to delta, then the first derivative of sine delta would be cos delta, which is the first derivative term multiplied by the corresponding delta-delta that is happening. So, we can linearize by using the Taylor series expansion for a small delta-delta angle in this particular order And if we substitute this into the actual swing equation, which we have seen here in this particular slide, then what do we observe? We observe two distinct terms here. Let's write those terms, or those two equations, to be specific. This entire equation can be broken down into 2h omega s d delta squared by dt squared at delta equal to delta op equal to pm minus pmax sine delta OP; that is one part of the equation, followed by terms like 2H by omega S on the LHS, d delta squared by dt squared, and then on the right-hand side we have minus P max. $\cos(\delta) * \Delta\delta$. Now, if we focus on the relevant terms that are present on the left-hand side and right-hand side of the equality equation, equality symbol.

Then the upper half that I have written is exactly similar to the swing equation that is mentioned here for angle delta, which is equal to delta OP. Remember, before the perturbation or disturbance happened, the machine is assumed to be in a steady-state condition. This means that at this initial operating point, this swing equation is also holding true because the machine is operating in a stable condition. At delta is equal to delta op, we would essentially get 2h by omega s d delta square op by dt square. Op is not the right term here, but at delta is equal to delta op should be equal to p m minus p max sign of delta OP because the machine was in a steady-state condition before the perturbation happened in PE or delta delta.

So with this as our initial condition, what can we see? We can see that these two terms, in a way, cancel each other out because of the validation of the sine equation at the initial operating condition, and essentially we are only left with the equation, which reduces to $\frac{2h}{\omega_s d \delta^2} \frac{d^2 \delta}{dt^2} +$ this is a minus sign here, so we have brought it to the left-hand side; it is $P_{max} \cos(\delta_{op}) \delta = 0$. This is the resulting equation that I get after linearizing around the perturbed initial operating condition. So, since this is true at the operating point, we are left only with this particular equation, which is what I have mentioned here, and P max cos delta OP is assigned a value or a symbol known as SP. This SP is nothing but a measure of the synchronizing torque, also called the synchronizing power coefficient, and as per the definition of the synchronizing power coefficient, SP is nothing but the slope of the power angle equation or the P delta curve at a given operating value or rotor angle value. So SP is basically the measure of the tangent at delta equal to delta naught, at delta equal to delta naught dash, or at any other point.

So, if I take the derivative of this P delta plot and mark it with respect to delta on the x-axis, I will get an S P versus delta curve, and the S P value will definitely be different because the tangents are different at different values of delta. So the synchronizing power coefficient is the slope of the power angle curve at a given value of delta. So let's see what the value of the synchronizing power coefficient is when delta is equal to delta naught. We have two conditions. And at delta is equal to pi minus delta naught.

which is delta, not dash, also. So before going into that, if we see this equation, which is of importance, it basically represents some form of equation that is analogous to the simple harmonic motion of an ideal pendulum,

$$\frac{2H}{w_s}\frac{d^2\Delta\delta}{dt^2} + S_P\Delta\delta = 0,\ S_P = P_{max}cos\delta_{op}(pu)$$

and in general form, this value of delta delta as a function of time, the closed-form solution of delta delta as a function of time can be represented as a function of exponents,

$$\Delta\delta = A(e^{wt} - e^{-wt}), \text{where } w = \sqrt{-\frac{w_s S_p}{2H}}$$

where omega is the In a way, the angular frequency, which depends on the sign of synchronous speed, depends on the sign and value of the synchronizing power coefficient and the corresponding inertia constant. Typically, for a synchronous machine, omega s and h would be positive quantities because synchronous speed depends on frequency and the number of poles. h is the inertia constant, which is the measure of the ratio of stored kinetic energy divided by the rated MVA, so it has to be a positive quantity.

But the value of SP can change its sign depending on which angle we are looking at. So at delta is equal to delta naught dash pi minus delta naught. So, if you take the cosine of delta OP at pi minus delta naught value, remember that the cosine value for angles greater than 90 degrees or pi by 2 radians tends to be a negative quantity. So, this is, in a way, minus cos of delta naught. So, the SP value, which is nothing but P max delta cos delta OP, is irrespective of what delta naught is, which is typically going to be a value between 0 and pi by 2 radians. Because of delta OP, the synchronizing power coefficient will be negative, which means that the argument inside the square root of the angular frequency omega determines the period of the perturbation delta delta with respect to time. The overall term would become positive, which would mean that omega will become a positive quantity, and as time progresses, e to the power of omega t would tend towards infinity, whereas e to the power of minus omega t, as t tends to infinity, would tend towards zero. So essentially, in steady state, the rotor angle swing would become unbounded because of the first term, and in a way, even for a small perturbation happening on the electrical side of the synchronous machine, the corresponding operating condition in steady state would become infeasible.

$$\Delta\delta(t) = A(e^{wt} - e^{-wt})$$

If there are no disturbances, if it is a perfectly ideal condition, yes, pi minus delta naught can serve as an initial operating condition, but at that initial operating condition, the rotor angles would all become unbounded, even for a small perturbation, even if it is tending to zero; the perturbation can be tending to zero. Because of the corresponding positive angular frequency, which in a way would result in delta delta tending towards infinity as t tends to zero.

And hence, the corresponding angle π minus δ_0 is not a feasible operating condition before a disturbance. Otherwise, at delta naught, sp is positive because p max cos delta is opposite to angles less than pi by 2 radians. From 0 to pi by 2, cos becomes positive; thus, if cos is positive, omega in this case would become an imaginary quantity. When we substitute an imaginary value, let's say omega is j beta, where j is Root over minus from the complex operator. The corresponding exponential functions recollect our hyperbolic functions where omega, if it is imaginary, then these expressions all tend to become sine or cosine functions, and omega n becomes the angular frequency, which is omega n equal to the square root of omega s sp by 2h.

$$\Delta\delta(t) = j2A\sin(w_n t) \text{ where } w = jw_n$$

$$\text{Where } \omega_n = \sqrt{\frac{\omega_s S_p}{2H}}$$

The minus sign has been taken care of in this complex operator. With sufficient damping present, which is ever-present and is intently neglected in classical transient rotor angle stability analysis, rotor angle swing would remain bounded even for small perturbations happening at the initial operating point where delta is equal to delta naught, and hence the operation would remain feasible. So that is the reason why delta is equal to delta naught is a feasible operating point, whereas delta is equal to delta naught dash is not a long-term feasible operating condition under practical conditions. Coming to the important point of the equal area criteria, the swing equation, as we have all seen, is a second-order differential equation following a disturbance. The swing equation needs to be solved, and often depending on what type of electrical change has happened on the electrical side, it may or may not be possible to always find closed-form expressions of delta using basic integration techniques.

Because it all depends on what sort of RHS term comes in, depending on the electrical power output. Hence, the only straightforward way possible is to make use of numeric integration techniques, where we can get specific values of a rotor angle at specific points in time when closed-form expressions cannot be obtained. At this point in time for this lecture, we will avoid discussing numeric integration techniques. We will take it up in the next lecture or the one after that. What we would understand or try to explain is whether there are ways of avoiding numeric integration techniques, and that is where this equal area criterion comes in.

It's a direct method of analyzing transient dot-triangle stability for the two initial cases that we discussed in the previous lecture. Case number one is for the SMIB system, and case number two is for two coherent or two non-coherent machines, where, eventually, be it for two coherent machines or two non-coherent machines, we can get one single swing equation. So let's see what this equal area criterion is and what this area is that we are talking about. It's a very beautiful concept. But it requires a little bit of understanding of how this area case happens.

So we'll take up the discussion first for case 1, and the same logic would be applicable for case number 2, which is for two coherent or two non-coherent machines. So for case 1, we have a synchronous machine that is connected to a power network through some transmission lines, and eventually, the rest of the network is represented through this infinite bus. At the steady state point in the pre-disturbance situation, circuit breaker A is closed and breaker B is open, as mentioned here, and breaker B is directly connected to the ground. Therefore, in steady state, there is no path for current or power to flow from the synchronous generator to the ground because breaker B is open, and whatever power is being transferred from the synchronous machine is all being fed. To the infinite bus through these transmission lines and the transformers that are shown over here.

There is no flow of power in the circuit between points A and B, so in steady state, everything is absolutely fine. So rotor speed is the same as synchronous speed, and hence omega r, the relative rotor speed, is perfectly zero. This steady-state operating point can be denoted by this point A, where delta not is the initial operating value of the rotor angle, depending on what electrical power output is being fed to the infinite bus. Since everything is in steady state, the electrical power output is the same as the input mechanical power, which is equal to P max. Sine of delta naught, P max is the maximum value appearing in this power angle equation P, and delta, I apologize for that; P max is the associated value that is observed when delta is equal to pi by 2 radians or 90 degrees.

So, we know for sure that A is our initial operating condition. Now, as I mentioned, everything is in a steady state, so this equation is applicable, as I have mentioned here. Now suppose breaker B is closed instead of being open; now suppose it is closed, and breaker B leads to a short circuit from the synchronous generator bus to the ground. So current always tends to follow the path of least impedance or least resistance, so what would happen? As nodes A and B are shorted, the terminal voltage of the synchronous generator would almost fall to zero, which would mean that whatever useful power was being transferred is all flowing to the ground. There is not sufficient power or significant enough power flowing to the infinite bus to the transmission lines.

So what is happening is that since the terminal voltage has now gone down to almost zero, the useful electrical power output of the infinite bus is also zero. And remember, acceleration torque is PM minus PE. PM is not changing because the governor dynamics are constant. PE has become zero. So, acceleration power is now positive. If acceleration power is positive, it would mean that the corresponding double derivative term becomes positive. So, the rotor would start accelerating, and this short circuit that has happened can be marked by point number B, which I have yet to show what that point B is, and this sort of acceleration keeps on happening. It will continue happening until what point? It would happen until the fault is cleared. How will the fault be cleared? Earlier, breaker B was closed. Either we open breaker B or open breaker A to clear that particular short circuit path, and the time spent from the appearance of this disturbance with the short circuit until it has been cleared is called the fault clearing time, which is TC.

Until the fault clearing time or until the fault has been cleared, electrical power would remain 0, so PE remains 0. But since the rotor is now accelerating, the corresponding rotor angle is now increased from delta naught. So P, since it is perfectly zero, the swing equation boils down to this. We can solve this simple differential equation very well.

$$\frac{2H}{w_s}\frac{d^2\delta}{dt^2} = P_m$$

$$\frac{d\delta}{dt} = w_r = w - w_s = \frac{w_s P_m}{2H} t \text{ and } \delta = \delta_0 + \frac{w_s P_m}{4H} t^2$$

So, the rotor angle is also positive. Relative rotor angle is also positive. The corresponding rotor angle has also now increased from delta zero. And we'll mark that point as point C. The fault clearing time at t is equal to tc, which is the fault clearing time; we can get exact specific values of this rotor angle at point number c and the corresponding relative rotor angle change that has happened, which is again going to be positive because omega s, pm, h, and tc are all positive. So if we mark those points on the power angle curve, A was our initial operating condition; at A, there was a short circuit in the SMIB system.

$$\delta_c = \delta_0 + \frac{w_s P_m}{4H} t_c^2$$
$$\frac{d\delta}{dt} = w_r = w - w_s = \frac{w_s P_m}{2H} t_c > 0$$

It was shorted, so PE immediately became zero, but the delta angle did not change, so this is our point number B. The power output remained zero until point C, when the fault was cleared, at which the delta has now increased to delta C, as defined by this particular equation. And after the fault has been cleared, what happens the moment the fault is cleared? Let's come back to this situation. Suppose I open up breaker A or breaker B; either way, the path which was available for short circuit current to flow down will not flow. The synchronous generator power would again be restored, which means Pe would again become non-zero.

Once Pe becomes non-zero, what happens? After point C, the fault has been cleared. What we would observe is that since the rotor angle has now increased from delta naught to delta C, after the fault is cleared at point C, the power output PE would be similar to the power angle equation, which is P max sine delta, but now at point C the angle is delta C and not delta naught. That means P max is sine delta C, which is the power output at point number C, and since delta C is more than delta naught, sine delta C would be more than sine delta naught. Power output has increased; mechanical input has not changed. So, with mechanical input remaining the same, the power angle has increased because at point D, this is the electrical power output that is being observed since the rotor angle has accelerated.

What would happen at point D once the fault is cleared? The machine, which was accelerating where power acceleration was positive at points B and C, instantaneously, when the fault is cleared, PA would become negative because electrical power output is more than the mechanical power input, so the machine starts deaccelerating, and it would definitely happen at point number D since the rotor speed is still positive. The machine would take some time to regain a relative speed equal to zero because, after point D, the machine has started decelerating. Whatever kinetic energy it had gained when the relative rotor angle speed was positive would start to wear off, and at point E, omega r instantly becomes zero, even if omega r is zero. It doesn't mean that the rotor's acceleration has become zero. At E, again, PE is more than PM, which means the rotor would keep on decelerating.

And hence, after the machine has sort of decelerated, it reaches point E. It starts swinging back and traverses all the way to point A. And the angle at point E, we would call it delta x, which can be obtained by solving the swing equation through differential methods or by using the equal area criteria. But the story doesn't stop here. The beauty of this analysis is that at point E, the relative rotor speed is zero, the acceleration and electrical power output are still greater than the mechanical power input, so the machine is still decelerating, and instantaneously it would reach point A, where at A, PM is the same as PE, so instantly at point A, the acceleration quantity is zero, but the rotor speed will not be zero at omega R at point A, so the machine again starts accelerating. P E becomes less than P M; the machine starts re-accelerating until it reaches a new point F, which is still not shown on another power angle curve, and this sort of swing continues from point F. It again comes back to point A, goes all the way back to point E, and basically the traverse or swing curve traverses at operating points E, A, and F. in the deaccelerating mode and in accelerating mode it all goes all the way from FAE. So this sort of acceleration and deceleration continues, and it is expected that with sufficient damping, eventually the swings happening from point A, which was the initial operating point, will keep on diminishing, and eventually, at steady state, the initial operating point, which is point number A, will be regained. So in the presence of sufficient damping, the rotor angle oscillates and gradually settles to point A at omega speed.

The point F that I was talking about refers to another point on the power angle curve, wherein the traverse happens from E to A to F in the deceleration mode, where relative speed is in a way negative, and from F to A to E, acceleration and deceleration continue. So this is where the two swing curves or power-angle equations come in. One is a power angle curve for the case when before the fault, during the fault, and after the fault. Before the fault points to point number A. During the fault, in case of a pre-fault, it is point number A. During the fault, the points are B and C. Post-faults are points D and E. We have one power angle curve here. The other power angle curve is the post-fault curve where E, A, F, F, A, E are all corresponding to post-fault curves.

And we can mark dedicated areas under these curves. We call those areas A1, A2, A3, and A4, as marked here on this slide as well as on this slide, A1, A2, A3, and A4. Equal area criteria say that a3, a4, and a1, a2 would always be equal to each other no matter what the condition is; the initial point has to be a stable operating point or initial operating point, and using this criteria, we can indirectly find delta x and delta f without ever integrating the swing equation or using numeric integration. So, more on this, we will see in the next lecture, which is where we would first prove why these two areas have to be equal and how, by using these two areas, we can get these angles without even looking forward to numeric integration or solving differential equations in a classical integration way. So, that's what we would discuss in the next lecture, and towards the end, we will start with the basics of numeric integration, which is definitely needed for multi-machine systems because case 1 and case 2 are very idealistic conditions, case 1 being an SMIB system and case 2 being two non-coherent or coherent machines swinging with respect to each other. So, numeric integration has to be used for multi-machine cases, and that's what we would see in the next lecture.

Thank you.