Power Network Analysis

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Week-11

Lecture-55

Lecture 55: Stability analysis- Ward reduction, Single machine infinite bus system

Hello everyone, welcome to lecture 5, the last lecture of week 11 of the course Power Network Analysis, in which we continue our discussion on transient rotor angle stability analysis of power networks; in short, we refer to it as stability analysis. And in this discussion we will continue, we will take up a method of reducing power network buses into buses that only represent internal EMFs of the synchronous generators for transient rotor angle stability analysis. Because synchronous machines have to be represented by the internal EMFs in series with direct access transient reactance XD. So how do we reduce the power networks with different buses into simpler buses so that the corresponding static equations involved in analyzing transient neutral angle stability, how they can be reduced and that technique we will call as Ward Reduction. And then we will extend this discussion on trans-neutral angle stability analysis to a SMIB system, fully known as a single machine infinite bus system. The last lecture we had discussed at length various aspects of the swing equation, how swing equations exist for every synchronous machine in the power network, how with more renewable integration, the H value tends to likely go down, and the chances of stability or instability tend to go up. Details will obviously be discussed at length in the last lecture of this entire course or module. We also understood different aspects of reducing the number of swing equations based on the coherency or non-coherency aspects between this and different synchronous machines. So, in terms of WAD reduction, why do we need WAD reduction? Let us understand that a little bit. So, for transient ultra-angle stability analysis, every synchronous machine is to be represented in terms of its transient internal EMF E, which is in series with Xd dash and the corresponding terminal voltage Vt.

Now, what is VT? In terms of power flow analysis, we were only focused on the terminal voltage, specifically for generator buses and we used to treat these buses as PV buses in power flow analysis. For fault analysis, we had to understand what the worst-case fault currents are. So we went; we did not limit ourselves only to the terminal of the generator or the bus. We went inside the machine terminal and represented every synchronous machine in terms of its internal EMF E or in terms of E dash if I call it because E here represents the transient situation; E double dash could be for the sub-transient representation, and this internal EMF was in series with the sub-transient reactance XE double dash in fault analysis. Since in transient rotor angle stability analysis we have focused about how is this rotor angle in terms of the internal EMF for with respect to the terminal voltage.

How is that varying for different disturbances? So we essentially represent these different generators in terms of transient internal EMF and the corresponding transient reactance coming in. So essentially the focus here in terms of transient triangle stability is on these buses rather than the terminal buses and that's the reason why internal EMFs have to be represented because if internally if EMFs are not represented we won't be able to estimate what is the angle delta and we will only be able to get a information of what is this phase angle appearing on the terminal of the bus which may not be a correct representation of the swing curve or the rotor angle variation. And in conventional transient neutral angle stability analysis, the assumptions we considered were that our load dynamics were completely absent. We would represent all our loads as constant admittance or impedance-type loads. Dynamic equations representing rotary rotation are only used for analyzing transient neutral angle stability.

And in order to minimize the effort involved in doing exclusive power flow, because without power flow, we won't be able to get the phase or the terminal information and once we don't know the terminal information, it is possibly not possible to directly get the internal EMF having known the corresponding current injection from the generators into the different buses. So in order to find the initial equilibrium point before, which exists before a disturbance occurs for which transient two triangle stability analysis has to be done. Without exclusive power flow analysis, the corresponding initial operating point or initial value of delta cannot be known. So, is there a way of avoiding that? Classical trans-notor angle stability analysis allows for such a representation, and that representation or reduction is essentially known as Ward reduction, in which the entire network is reduced only to machine nodes, and these machine nodes refer to the constant internal transient EMF E dash, where the magnitude of E is not changing. What can change is only the corresponding angle delta.

And by representing the entire network only with these machine nodes, there is no need to perform power flow for the entire network. It's an assumption again. One can always counter question and argue why this assumption is necessary. I would recommend, for the sake of understanding the concept, that this assumption might not sound logical, but to understand or have a premise, it is useful for understanding that. In order to conduct more complicated analyses, specifically for multi-machine transient triangle stability analysis, war reduction may be completely avoided, exclusive power flow may be performed, and then the corresponding initial operating condition may be obtained, but with a warning that Transcendental triangle stability analysis is a limited-time period of

study.
It is not a steady-state period. So while doing our calculations, if the number of equations involved is so large that the time taken to solve them is so long, the event might come and go, and our analysis might still not give us any relevant solution, because of which the system's corresponding corrective actions to maintain stability or avoid or avert instability might be lost. That is the overall trade-off for power engineers to maintain corrective decisions in a timely manner with a limited amount of computational effort involved. So let's understand what is reward reduction. I hope I have been able to explain the importance of reward reduction.

The intent is to avoid power flow analysis being done to the extent possible to reduce our computational effort. So for every synchronous machine, we consider a fictitious generator bus, the bus which I have marked here to be present between the internal EMF, whose magnitude remains constant; the angle can change, and following that bus is the series transient direct axis reactance, after which we have this terminal voltage Vt. Similarly, for the load buses, we can have a similar representation, but since load dynamics are involved, load buses need not have these internal EMFs present with them. Considering these series transient direct access reactance and all network lines present in the power network, we evaluate the overall bus admittance matrix with internal EMF buses also represented, and transient reactance is considered. After doing that, we can obtain this value of Y dash.

The process of evaluating the bus admittance matrix remains the same. Since we are focused only on generator buses, so we can categorize the number of buses into set capital G and set capital L, where the respective current injections, which is the vector I, is associating the corresponding bus admittance matrix Y dash with respect to voltages at all generator as well as load buses.

$$I = Y'V$$

So if we break up this equation in terms of current injections at buses G and L, where capital G indicates the buses where internal EMFs are present for synchronous machines and L represents those buses where only loads are present, whose dynamics are ignored and are all modeled as constant impedance or admittance loads. Then we can break the corresponding current injection into two distinct equations. where the first equation represents the injection of currents at generator buses in terms of voltage at generator buses and voltage at load buses.

$$\mathbf{I_G} = \mathbf{Y'_{GG}V_G} + \mathbf{Y'_{GL}V_L}, \; \mathbf{I_L} = \mathbf{Y'_{LG}V_G} + \mathbf{Y'_{LL}V_L}$$

Now, what would Vg be Vg is essentially a vector of all generator internal EMFs. So, let us say E1 - delta 1, E2 - delta 2, and so on. Where the magnitudes are fixed at the constant, delta 1 and delta 2 can have variations, and similarly, VL is the generator bus voltage at all load buses. Since we are ignoring the dynamics at load buses, so load currents are not so important or current injections at non-generator buses are not important for classical transient neutral angle stability analysis, the dynamics are all ignored. So, we can assume that these currents are actually zero.

So, if the value of this equation becomes 0, then we can represent the load bus voltages in terms of generator bus voltages, and for this relationship to hold true, obviously, this inverse should exist; that is, Y dash LL should not be a singular matrix. y dash ll in order to not be a singular matrix there has to be a proper grounding present somewhere because of which the corresponding bus admittance matrix should not become a singular matrix.

$$\mathbf{I_L} = \mathbf{Y'_{LG}V_G} + \mathbf{Y'_{LL}V_L} = 0$$
$$\Rightarrow \mathbf{V_L} = -\mathbf{Y'_{LL}}^{-1}\mathbf{Y'_{LG}V_G}$$

Please recollect from the powerful analysis module where we were discussing the bus admittance matrix that if the network is not directly connected to the ground with respect to which voltage or potential is being measured, then the Y matrix tends to be a singular matrix. To avoid having such a situation, it is assumed that the entire power network, which has VG voltages at generator buses and VL at load buses, with VG being the internal EMFs, has some proper connection to the ground with respect to which these potentials are being measured. Assuming that the potential reference exists and this is the proper connection between ground and the power network, YLL dash will not be a singular matrix. YLL dash will definitely be a square matrix; only singularity is to be avoided with proper grounding. So, we can substitute VL in terms of VG and get an analogous equation that is completely in terms of currents and voltages at the internal EMF buses only.

$$\mathbf{I_G} = \mathbf{Y'_{GG}V_G} + \mathbf{Y'_{GL}V_L}$$
$$\Rightarrow \mathbf{I_G} = \left(\mathbf{Y'_{GG}} - \mathbf{Y'_{GL}Y'_{LL}}^{-1}\mathbf{Y'_{LG}}\right)\mathbf{V_G}$$
$$\Rightarrow \mathbf{I_G} = \mathbf{Y_{bus}V_G}$$

And essentially, the Y bus that we are getting, which is equal to YGG dash minus YGL dash YLL inverse dash YLG dash, is the effective reduced bus admittance matrix of the power network with only the internal EMF of the buses connected; transient direct axis reactances are already considered, and all network lines are also already considered. So if we focus on or reduce our network to the generator's internal EMFs, we can directly get an estimate of these load angles, which I was talking about here in terms of delta 1 and delta 2, without solving power flow equations exclusively. Obviously based on the assumption that network dynamics and load dynamics are either completely present or not present at all.

So, if we have understood that water reduction aspect coming to our powering angle equation which we were discussing in the last lecture also, we consider a case two basically where we have two synchronous generators represented by the corresponding internal EMFs E1 delta 1 and E2 delta 2 measured with respect to some common voltage reference. And we have already applied watt reduction, by which we have obtained the corresponding reduced Vibus. Since there are only two buses involved, the two internal EMF buses, we have a 2 cross 2 Vibus matrix after applying the watt reduction aspect to it.

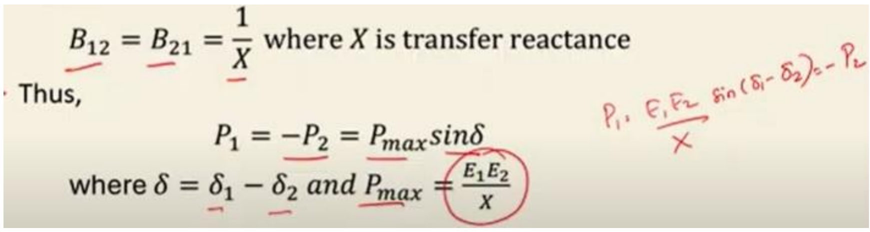
$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

And using this WAD reduction Y bus, we can express the power injections at these generator buses in terms of the load flow equations that we have already discussed and analyzed,

$$P_1 = E_1^2 G_{11} + E_1 E_2 \{G_{12} \cos(\delta_2 - \delta_1) - B_{12} \sin(\delta_2 - \delta_1)\}$$
$$P_2 = E_2^2 G_{22} + E_1 E_2 \{G_{21} \cos(\delta_1 - \delta_2) - B_{21} \sin(\delta_1 - \delta_2)\}$$

where G11, G12, and B12. They are essentially Y11 in this particular term, which is nothing but G11 plus JB11, and Y12 is G12 plus JB12; similarly, we have G21 and B21, which are the corresponding real and imaginary parts of Y21 and Y22, respectively. Now, with these as our power injection equations, which are not new equations, but are obtained from basic power flow equation analysis and the fibers that we have obtained after water reduction, recollect or remember that in classical triangle stability analysis, one of the assumptions that was involved was Network damping is completely ignored, which means network resistances are completely absent. They are not to be considered. Even if they are present, their values are not to be considered. So in such a case where the network is lossless in terms of resistances, recall in terms of DC power flow, the corresponding G values will all become 0. If the values of G become 0, that means these terms are all absent; they have no contributions, and P1 and P2 are effectively left only with minus B12 E1 E2 sine delta 1 minus delta 1, and P2 would be minus B21 E1 E2 sine delta 1 minus delta 2.

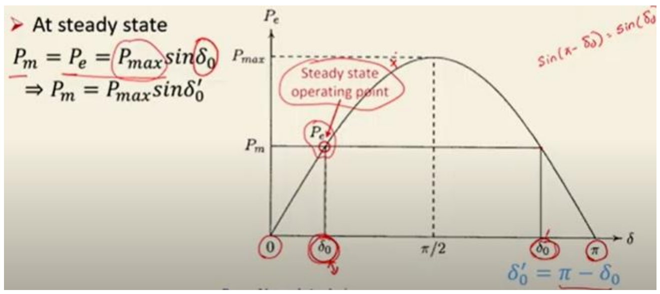
So if we focus only on such a network, for which resistances are neglected because damping is ignored, Y bus becomes a pure imaginary matrix, and B12 and B21 can be written in terms of X, which is a transfer reactance, and P1 becomes nothing but the negative of P2, where P1 is P max sine delta, P max is E1 E2 by X, as shown here in terms of E1 E2 and B1 B2. So delta becomes delta 1 minus delta 2, and the negative term here is incorporated inside; hence, delta 2 minus delta 1 is effectively minus delta. So we have P1 as E1 E2 by X sine delta 1 minus delta 2, which is also equal to minus P2.



Please compare this equation with the equations given on slide number 6. So effectively, what we are trying to understand is that since E1 and E2 are their voltage magnitudes of internal EMF, they will not change because AVR action is neglected.

A disturbance has occurred in the network, so what could be the possible changes that would bring about a deviation in P1 and P2? The answer to that lies in the value of transfer reactance X. Because of disturbances, the corresponding transfer reactance X value is changing, which is deviating the electrical power output of the two synchronous generators; correspondingly, the rotor angle delta might also have a deviation, and that is where the answer lies. To the analysis behind transient rotor angle stability. So, to give an example, during a fault, the machine's reactance itself varies. It varies from Xd double dash to Xd dash, and then in steady state, it becomes Xd, the corresponding saturated steady state value. And in terms of the equation that we have defined which is P1 equal to Pmax sine delta, if we plot So basically P max, it is varying under different disturbances. And if we plot P and delta for a fixed P max, we would essentially get the first half of a sine curve. There's a sine equation here. So we get the first half of the sine curve where zero degrees or zero radians and pi radians are marked here. And at steady state, since mechanical input is constant, it's steady state, so mechanical input, power is equal to the electrical power output so PM and PE they match rotor speed is exactly equal to synchronous speed the rotor angle with respect to synchronous rotating frame is also not changing and its initial value is delta naught which is decided or dictated by the loading present in the power network which is being fed by each of these individual machines delta naught is that Operating point or initial equilibrium point that we need to know before we understand the importance of transino-triangle stability analysis in terms of some disturbance, during which the Pmax value itself might change because the x value is likely going to change.

So the question now is, we will understand that in the next few lectures to come, but there is another interesting aspect here. If delta naught can be the initial operating point or equilibrium point before the disturbance has occurred, why are we saying that PE marked over here represents the steady-state operating point Analogously, or mathematically, there also exists an angle delta naught dash, which is pi minus delta naught. At which Pe is also equal to Pmax sine delta naught, because sine of pi minus delta naught is exactly the same as sine of delta naught. So, why can delta naught dash not be an initial equilibrium point for a synchronous machine, which is likely to undergo a disturbance for which transindo triangle stability analysis needs to be done? The answer to that will be discussed in the next lecture at length and there are well proven mathematical conditions which inhibit delta naught dash to be a steady state operating point. The steady state operating point will always be an angle between 0 radians and pi by 2 radians.



And following a disturbance, delta naught, the angle of delta, which is this angle here, will deviate from the delta naught value. And depending on the disturbance, the post-disturbance condition may or may not be the same as delta naught and PM value in themselves. The question still remains: my response to Delta naught dash, not being a strategic operating point, is still a pending response, and we will discuss that in the next lecture. We'll conclude this lecture with an example, specifically on wall reduction, in which a three-bus network is given. The network is all lossless. The corresponding line reactances in per unit are all provided. Generators one, buses one and two, they are connected by, they are having two synchronous generators whose respective transient access generators data taxes transient reactances are also mentioned and for transient low triangle stability analysis we have to use WAD reduction where only the internal EMFs of synchronous generators remain connected.

So essentially, instead of this synchronous generator, we have J 0.10 PU, which is a transient direct axis reactance of machine 1, and this machine has an internal EMF E1 dash with an angle delta 1. Similarly, bus 2 or the generator at bus 2 is represented by its corresponding direct axis transient reactance, whose value is J 0.05 PU, with the internal EMF being E2 dash and an angle delta 2, where delta 1 and delta 2 are measured with respect to some voltage angle reference. And E1 dash and E2 dash are remaining constant.

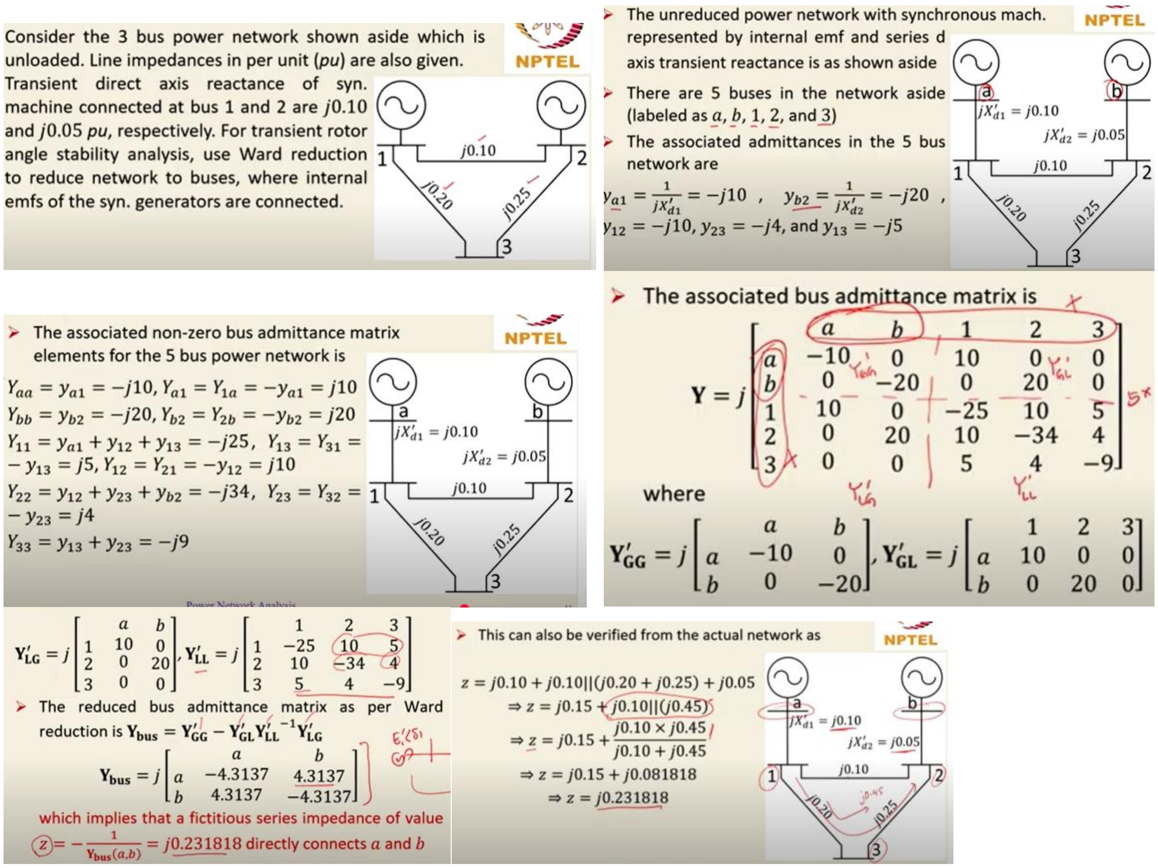
So, our intent or idea is to reduce this entire power network where only these two buses or fictitious buses remain connected; the rest, buses 1, 2, and 3, and the respective lines are all removed, obviated, or completely nullified. So, how do we do? As I explained in the previous slide, here in slide number 9, I have marked those fictitious buses as buses A and B, and I know how these different buses are now connected. So, I evaluate my usual Y bus matrix for this 5-bus network where the buses are A, B, 1, 2, and the evaluation of this Y bus process is exactly the same as we discussed for power flow analysis. We try to find the corresponding admittances of the respective lines A1, B2, 1, 2, 2, 3, and 1, 3, and then correspondingly find the 5 by 5 admittances.

Element Y bus matrix. The corresponding evaluations are all shown here. So, if we go by the usual discussion of the Y bus evaluation, the overall Y bus with A, B, 1, 2, and 3 as the bus, and the row and column labels as per the bus numbers, this is my 5 by 5 Y bus matrix. Now, in this 5 by 5 bus matrix, I want to retain only buses A and B as row and label columns. Buses 1, 2, and 3 in a row and labeled columns should be removed. So essentially, capital G here, which is the set of all buses having internal EMFs, is A and B, and L index or L buses are 1, 2, and 3.

So in a way, if I bifurcate or break this 5 by 5 Vibus matrix, then this 3 by 3 lower half is YLL dash. The lower half present here is YLL. The upper half here is YGL dash, and here I have YGG dash, which are also shown over here as per the corresponding row and column labels. So now, since YLL dash, we know it's a three-by-three matrix and it is not a singular matrix. As we can see the corresponding rows they don't add up to zero in each of the three rows so it's perfectly not a singular matrix only row three and column three they have addition zeros it still is a full rank matrix that means there's a proper ground reference present somewhere coincidentally it happens to be this line or the transient reactance which acts as the ground reference because with respect to terminal voltage internally emf e1 and e2 those are those two are being measured So, YLL dash inverse exists, and if we plug in those corresponding values as per the definition of Vibus, this is our 2 by 2 Vibus matrix, which also indicates that there exists a fictitious series impedance between internal EMFs E1 dash delta 1 and E2 dash delta 2.

Whose impedance is Z, and this impedance can be evaluated from the corresponding transfer reactance element, which is 0.231818. So this is what the odd reduction aspect is. Can there be a way to verify that this actually is a strategic series impedance? Yes. Instead of doing the Y bus evaluation, one can use the usual network series-parallel combinations of impedances.

So if we see that we want to focus only on buses A and B, we wish to remove buses 1, 2, and 3. So if we focus on, let's say, buses 1, 3, and 2, it appears that J 0.2 is in series with J.25, so effectively the impedance present on this path is J 0.45. J 0.45 happens to be in parallel with J.1, which is essentially what is mentioned here in this particular term. And then we have J 0.1 and J 0.05 in series with the parallel combination of J 0.1 and J 0.45. So, if we apply the series and parallel combination of these different impedances, the Z value turns out to be the same value of 0.231818, which we also got from the wall reduction evaluation of retaining only buses A and B.



So that, in a way, is physical evidence, okay, that word reduction actually helps in reducing the network, even though this is confirmed by the series-parallel combination of impedances. That's all for this lecture.

In the next lecture, we will Understand the reason behind why delta naught dash cannot be a steady state operating point, and the answer to that lies in the concept of defining synchronizing power coefficient. We will also understand an important way of avoiding numeric integration for analyzing transient stability analysis of the SMIB system or two-machine systems that are either coherent with each other or not coherent with each other. And that criterion or technique is known as the equal area criterion.

Thank you.