Power Network Analysis

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Week-11

Lecture-54

Lecture 54: Stability analysis- Swing equation - Part II

Hello everyone, welcome to Lecture 4 of Week 11 of the course Power Network Analysis. We are continuing with our discussion on the last module, which is transient rotor angle stability analysis; in short, I have intentionally kept it simple as stability analysis. And in this discussion we will continue with the aspect of swing equation which is analogous Newton's second law of motion for rotating synchronous machine specifically from the rotor perspective and this is going to be the second ongoing lecture in terms of sin equation. We have discussed the basis of the swing equation in the previous lecture. So what we discussed or understood in the previous lecture was that every synchronous machine the associated rotor rotates at synchronous speed under steady state condition during which the input mechanical power and electrical output power they both match each other they balance out each other assuming the losses to be negligible. So, the associated acceleration torque, which is the difference in acceleration power, which is the difference between P E and P M, turns out to be 0.

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_e\ (W)$$

And hence under steady state condition the associated rotor angle which we inherently coincide with the internal EMF of internal angle of the internal EMF of the synchronous generator with respect to terminal voltage that corresponding double derivative tends to be equal to 0 under steady state and hence the rotor rotates at synchronous speed with respect to the stationary axis on the stator or alternatively The rotor speed and the rotating stator MMF in the air gap have a relative velocity of zero because both tend to rotate at the same synchronous speed, as dictated by the operating frequency f and the number of poles in the synchronous machine. And we have understood in the synchronous generator module that the number of poles induced on the stator and the number of poles induced on the rotor correspondingly match each other due to the corresponding MMF. And we concluded our last discussion. With the assumption or with analysis.

In fact, there are several assumptions involved in the transient root triangle stability analysis. One of the assumptions is that network dynamics or network damping is neglected. The change in mechanical power input, mechanical power, or input torque does not exist because the associated turbine and governor dynamics are very small compared to the transient period of study in which we want to analyze the stability problem. And additionally, we also had this angular momentum term defined as M, which is actually the product of J, which is the angular momentum of the rotor and turbine combined together, multiplied by omega S, which is actually the rotor speed in terms of the rotating reference axis. We had assumed this M to be a constant quantity, and hence we assume that J is equal to M, which is equal to J omega SM, where omega SM is the synchronous mechanical speed of the generator, which is a constant for a given frequency and number of poles.

Whereas, on the other hand, the actual swing equation consists of the term j omega s instead of m, and this assumption of choosing omega s to be equal to omega sm in terms of defining this constant m works well for transient rotor angle stability analysis and helps in analyzing our actual effects on the rotor more effectively, although. Practically speaking, if one does not want to have this constant m assumption and chooses j omega s instead of m, one needs to have a detailed analysis because then we will have different multiple variables appearing on the right-hand side where omega s will also be a variable and delta m will also be a variable. So the corresponding numeric equations to solve those equations may be different. We will not discuss those details in length. We will keep our discussion simple because the intent of this module is to apprise all my students of the basic concepts of how trans-neutral triangle stability analysis can be done in the first place.

There is an alternate constant defined instead of M. We call that constant the H constant, also known as the inertia constant, which is the ratio of stored kinetic energy in megajoules at synchronous speed in the synchronous machine divided by the individual rating of the machine in MVA. and its unit turns out to be in seconds. Why? Because if you see the ratio of H, H is energy divided by power, whose units are MVA or megawatts. And as we all know, power is nothing but the rate of change of energy, or let's say E is the energy and T is the time.

$$H = \frac{\text{stored KE in } MJ \text{ at syn. speed}}{\text{machine rating in } MVA} \text{ (seconds)}$$

$$H = \frac{0.5 J w_{sm}^2}{S_{mach}} = \frac{M w_{sm}}{2 S_{mach}} (s)$$

Then the corresponding ratio of H would turn out to be E by P, which is also analogous to E by E by T, which is only left with a dimension of time T, and hence its unit is in terms of constants. If we correlate the stored energy in terms of the angular momentum J and the actual synchronous speed of the machine defined as omega SM. S-MAC is the machine rating in MVA. We can define M, since M is nothing but JωSM, as we have discussed here. So inherently, M is equal to 2H times machine rating S-MAC divided by omega SM.

So we can substitute M in terms of H where the machine rating is already known. Omega SM is the actual synchronous speed of a machine, which is also known for a given machine. And H, in a way, is a representation of M, which indicates how much kinetic energy is stored or can be stored during disturbances in relation to the machine rating. So if we represent M in terms of H, this becomes our analogous swing equation in per unit because on the right-hand side we have power, whose unit is Watt, and then we also have S Mac, whose unit is also Watt, I mean the multiplier obviously, so those multiplier terms can be neglected. And basically, watt by watt is essentially nothing but per unit.

$$\frac{2H}{w_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{S_{mach}} \Rightarrow \frac{2H}{w_s}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ (pu)}$$

So, maybe towards the end of the lecture of this particular module, in fact, the last lecture of this module, we will understand at length the importance of this inertia constant H, specifically in terms of renewable energy integration. But just to apprise you of or highlight the importance of H in terms of different generating resources that are connected to the power network from the transient rotor angle stability perspective. If we have resources which are non-rotating in nature, for example solar PV plant, solar PV panels, solar PV plant, battery energy storage systems and related static devices, then Then what essentially is happening from the power network perspective is that from the overall inertia constant of the entire power network or at a point of interest where H constant is to be evaluated for a given node or given point in the power network. If more and more generation capacity is pushed into the network, what happens is H being proportional to the stored kinetic energy divided by the generation capacity or machine rating in general. With more and more generators being pushed in, the overall generation capacity of the system tends to go up, but with no rotating devices present specifically for solar PV and battery energy storage systems, there are no rotating parts.

So, these generators, in a way, cannot contribute anything to the kinetic energy that is already present in the network. So what happens is that with more and more static or non-conventional generating resources with no capability of directly storing kinetic energy, only the generation capacity tends to increase; kinetic energy more or less remains the same, and as a result, with more and more renewables being pushed in, the H value tends to go down, the overall effective H tends to go down, and with the reduction in H, what could be the problem that one can anticipate? The answer to that lies in this particular question. That let's say we have a unique disturbance which is happening in two scenarios for a power network. In one scenario, the power network doesn't have many renewable energy resources connected to it. Specifically, solar PV and battery energy storage systems tend to reduce the overall H.

That is one scenario. And in another scenario, our power network has a high penetration of solar PV and battery energy storage systems in which the effective value of H tends to go down. So if the same disturbance is happening at some point and we are trying to find the overall transient rotor angle stability analysis specifically from the perspective of this rotor angle delta, which is being measured with respect to the rotating reference frame in the synchronous generator or an analogous synchronous generator, then since the disturbance remains the same, the RHS quantity will more or less remain the same in both scenarios, whether with low penetration of renewables or high penetration of renewables. With high penetration of renewables, the problem is that the RHS term or the disturbance term, because mechanical power is not changing, so it will remain constant. The only term that would change is PE.

With the same disturbance, the RHS effective term would remain the same. But with high renewable penetration, as discussed here, the value of H would decrease. So it is likely that the corresponding double derivative term sitting over here will take on a higher value. In the case of higher renewable integration, specifically solar PV and energy storage systems, compared to a scenario where renewable integration, specifically solar PV and energy storage systems, is lower, the corresponding H value would be higher, and with the same disturbance, the RHS term more or less remains the same, so the LHS products would balance each other out, and with a lower value of H in the case of high solar PV and energy storage. The corresponding double derated term will tend to take a higher value, which increases the chance of having more worrisome cases in terms of delta angle having more deviation or lesser deviation compared to a case where the H value is higher in width.

In terms of lower renewable energy integration. So, we will discuss this aspect specifically in the last lecture of this module. So, please have patience with that, but I just wanted to highlight the impact of renewable energy integration on the value of H and the corresponding issues that might arise in terms of transient triangle stability. Coming to our equation here, I have reduced omega SM or rewritten omega SM in terms of omega S here where I am intently removing the notation of M here basically it should be M here because omega SM or omega SE they pertain to respective mechanical or electrical synchronous speed of machine and since we are talking about The equations are in terms of per unit. So, both omega sm and omega se in per unit will turn out to be the same values.
So, we have removed the intent of the notation of m indicating that omega s is the corresponding synchronous speed irrespective of the mechanical or electrical domain. And delta is the associated rotor angular displacement with respect to the synchronously

rotating stator MMF. So, in terms of the given frequency, as I mentioned, omega s is nothing but 2 pi f. If we substitute omega s as 2 pi f here, then our corresponding swing equation is in terms of h and f, and the remaining terms remain the same. Interestingly, every synchronous machine or every bus in the power network can have this emulation of the corresponding swing equation.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ (pu)}$$

Specifically for synchronous machines, we will keep our discussion simple. Every synchronous machine connected to the power network will have such a swing equation and this swing equation which is a second order equation can be broken down into two individual first order equations

$$\frac{d\delta}{dt} = w_r = w - w_s\ (rad/s), \frac{2H}{w_s}\frac{dw_r}{dt} = P_m - P_e \text{ (pu)}$$

where d delta by dt is defined as omega r, where omega r is a relative angular velocity of the rotor speed with respect to the synchronous speed omega s. Omega is the actual rotor speed with respect to the rotating axis in the synchronous machine. And the double derivative d delta squared by dt squared will become d omega r by dt. So the solution of these swing equations, be it the double derivative form or the first-order differential equations, if we solve these equations for fixed PM and varying PE, we will get expressions for delta and omega R.

In terms of time, this swing equation essentially represents the variation of rotor angle with respect to time, and that is why we also refer to this plot, or analytical plot, as the swing curve of the machine. Inspection of this swing curve for different machines indicates whether these machines would remain in synchronism, which means the rotor angle differences, whether they will retain the same steady state value or whether they would increase or decrease with respect to a particular disturbance, sort of premised by an initial operating condition. So, to analyze these different swing equations for respective machines, the way we converted all our electrical calculations with respect to a common system base, the conversion of these H values to a common system base is also important, and how this change happens or how these base quantities match, the answer to that lies in the definition or in the The definition of H itself, so H and SMAC are inversely proportional. To maintain the same system base, if I have to convert the machine H in terms of a common system base MVA S base, then I have to essentially maintain the product of HMAC and SMAC because H and S are inversely proportional, so the product should remain uniform, and that's how this conversion works. H can be made for the respective machines on a common system base.

$$H_{base} = H_{mach}\frac{S_{mach}}{S_{base}}$$

For a large system with many such synchronous machines, the number of swing equations would accordingly be more and more for large synchronous machines so can there be ways of reducing the number of swing equations so that the computational time or effort involved can be minimized. So yes, there are different ways of reducing such a number of swing equations. The first set of reductions comes from the aspect that there will likely be synchronous machines that tend to swing with respect to each other, which means their outer angle variations or their respective swing curves after a disturbance follow a similar pattern with respect to time or follow a similar trajectory. And we tend to call those machines coherent machines or swing machines because their variations or their response to a disturbance are more or less the same. So if we can identify such swing equations on machines, there are ways to identify such coherent machines.

There are different studies involved in that. We will not discuss those in length. But suppose we identify such a set of coherent machines where these coherent machines are maybe 2, 3, or 4 in number. We will consider the case of two such machines that are incoherent with each other. And each of those machines, machine 1 and machine 2, has its respective coefficients.

$$\frac{2H_1}{w_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}\ (\text{pu})\ \&\ \frac{2H_2}{w_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}\ (pu)$$

Swing equations are respectively defined, and the rotors are mechanically responding together. So, we can reduce these two swing equations because the rotor angles are deviating or varying in the same manner. So, we can combine these two swing equations; basically, we can add these two equations to get an analogous swing equation that comes together, replicating both individual swing equations,

$$\frac{2(H_1 + H_2)}{w_s}\frac{d^2\delta}{dt^2} = (P_{m1} + P_{m2}) - (P_{e1} + P_{e2})\ \text{where}\ \delta = \delta_1 = \delta_2$$

and that is one way of reducing two swing equations to one single swing equation. If the number of coherent machines is more than 2, then we will have additional terms coming in, such as H3, H4, H5, and as many equations as we can get for the respective HS, similarly for the respective PMs and their electrical power outputs. So swing equations can be reduced for coherent machines into a single equation, but one has to identify these coherent machines a priori.The other set is non-coherent machines, where the rotor angle variations of two synchronous machines following a disturbance are completely different. Their swing curves are completely dissimilar. And we would call those sets non-coherent machines. Suppose we have two such non-coherent machines; then there is a way of reducing these two non-coherent swing equations into one single equation. How are we doing? We write, first of all, the respective swing equation of every such non-coherent machine and take the difference of These two equations are on their respective RHS and LHS terms.

$$\frac{d^2\delta_1}{dt^2} = \frac{w_s}{2}\left(\frac{P_{m1} - P_{e1}}{H_1}\right) \ \& \ \frac{d^2\delta_2}{dt^2} = \frac{w_s}{2}\left(\frac{P_{m2} - P_{e2}}{H_2}\right)$$

$$\frac{2H_{eq}}{w_s}\frac{d^2\delta}{dt^2} = P_{meq} - P_{eeq} \text{ where } \delta = \delta_1 - \delta_2$$

So basically, the delta term is delta 1 minus delta 2. So we are basically subtracting the second equation from the first equation, and we can write one common swing equation for two non-coherent machines. Where HEQ, PMEQ, and PEQ are the respective H input mechanical power and electrical power output equivalent terms as defined in terms of the individual machine constants H1, H2, PM1, PM2, PE1, and PE2.

$$H_{eq} = \frac{H_1 H_2}{H_1 + H_2}, P_{meq} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2}, P_{eeq} = \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2}$$

So essentially, swing equation, what we have discussed that in order to understand transient triangle stability analysis, we have to first of all solve these swing equations.

The more these number of swing equations are, the more time can be involved. So there have to be ways of figuring out how this number of equations can be reduced because these are differential equations. So, essentially, if the machine or the network tends to remain stable, what we would observe is that, irrespective of whether the machines are coherent or non-coherent, the rotor angle differences would tend to decrease or tend to remain finite following a disturbance. In case instability occurs, this quantity or this aspect would not be true, which means the rotor angle deviations or differences would tend to increase. If they tend to go down, they will not remain finite.

And essentially for a respective machine, if one has to understand the transient rotor and integral stability perspective, then it is basically a relative property associated with the dynamic behavior of other coherent and non-coherent machines. So, to keep our discussion simple, we will consider three cases essentially; or, in fact, we'll start with two cases. We'll first try to understand the transient rotor angle stability of a single machine with a finite H constant with respect to the infinite bus. The notion of the infinite bus remains the same as we discussed in terms of the context of power flow analysis as well as in the context of the economic dispatch perspective or synchronous module perspective. So we'll consider three cases, starting with case number one, where we focus on transino-triangle stability of a single synchronous machine and a rational network being represented as an infinite bus.

Then, having understood case one, we'll try to move on to case two, where we will try to understand the stability phenomena between two non-coherent machines. With finite edge constants. They can be coherent or non-coherent. So I have written about the case of swinging with respect to each other. The discussion that we will follow for the second case can also be applied to two non-coherent machines.

And lastly, we will take up the most complicated case of multi-machine stability analysis, where we will definitely have to use some numeric integration technique. The beauty or importance of the first two cases is that there are ways to avoid this numeric integration perspective. Why is numeric integration necessary? Because there are differential equations involved, these differential equations will be solved through numeric integration. Specifically for case three. For the first two cases, the concept of numeric integration need not be understood or applied.

Without numeric integration, cases 1 and 2 can still be analyzed, and any stability or instability can still be identified. So we will start with a simpler case and try to complicate or go to further complicated cases as the cases progress. So in terms of the swing equation, there is also another important equation known as the power angle equation. So, for a synchronous machine, from the transient neutral angle stability perspective, mechanical input power would remain constant as the transient time period is comparatively smaller than the response time of AGC. The governor's dynamics are all ignored or neglected; AVR action is also neglected.

At steady state, both these powers, input power and output power, are the same, so the rotor rotates at synchronous speed. As disturbances happen, specifically electrical disturbances, the corresponding electrical output power would vary. This would lead to a non-zero value of the acceleration torque irrespective of the PM remaining constant. If PA is positive, the rotor would tend to accelerate because d squared delta by dt is proportional to PA; positive PA means the rotor would accelerate.

For negative PA, the rotor would decelerate. And this change in electrical power as I mentioned is due to disturbances happening in the power network because of which the electromagnetic transients would exist in terms of variations in rotor angle with respect to time also known as in terms of variations of the swing curve. For this analysis, as already discussed in the assumptions of transient ultra-angle stability analysis, every synchronous machine will be represented by its internal EMF in series with the direct axis transient reactance xd dash. So we won't be considering the buses at which terminal voltages were considered. So basically, in terms of PV buses in load flow, we'll have to go inside the machine terminal and figure out a fictitious bus at which this internal EMF is connected, similar to the way we did for the fault analysis module, where instead of Xd dash, we have been making use of Xd double dash, the sub-transient reactance. Since it is a transient study, the corresponding transient reactance value comes into the picture.

We'll conclude this lecture with a simple numerical example where there is a synchronous machine given whose direct axis transient reactance is provided and the armature resistance is negligible. The machine is connected to an infinite bus whose voltage magnitude is fixed at one per unit. The generator or the machine is delivering a real power of 0.5 per unit at a 0.8 power factor lagging. Under this condition, we have to determine what the internal EMF or the voltage behind the transient reactance is while neglecting the rotor saliency effect, which means the machine we are considering is a cylindrical rotor machine. So how do we do this? Since we know that the machine is connected to an infinite bus whose terminal voltage is fixed at 1 per unit, if we can find the corresponding current which is being fed by this synchronous machine which is responsible for this 0.5 per unit real power then probably we can correspondingly find the internal EMF E which is being measured with respect to the terminal voltage of the synchronous generator. So, how do we do that? We first of all find out the corresponding power factor angle and since it is a lagging power factor angle, the corresponding power which is being delivered by the generator is actually being fed to an inductive load and hence the corresponding armature current is lagging the associated terminal voltage or the infinite bus voltage by an angle 36.87 degrees. How do we get the corresponding current? V is given as 1 per unit; we are choosing V at an angle of 0 or the infinite bus voltage as a reference for all phasors, so the corresponding armature current is now known. With saliency neglected, V being known as the reference voltage at the infinite bus, we can actually make use of the transient axis, transient reactance, direct axis reactance, and find the corresponding internal EMF E dash, which is leading the corresponding terminal voltage. Remember that generating action delta will always remain positive, and that is evident here as well. In the next lecture, we will take up the interesting discussion on how to reduce the power network buses to only those retained with internal EMFs of the synchronous generators for Transcendental Triangle Stability analysis, which we will call WAD reduction. And we will also take up the first case of the Transcendental Triangle Stability problem, specifically for the single-machine infinite bus system.

Which, in short, we also call SMIB, with S standing for single, M for machine, I for infinite, and B for bus.

Thank you.