Power Network Analysis

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Week-11

Lecture-53

Lecture 53: Stability analysis- Swing equation - Part I

Hello everyone, welcome to the third lecture of week eleven of the course Power Network Analysis. In this discussion, we will continue with our last module on the stability analysis of power networks, specifically the transient rotor angle stability analysis of power networks. And in this discussion, we will understand the premise of the equation or the numeric aspect of how the transient rotor angle stability of a power network with several synchronous generators connected can be analyzed. And as the term transient rotor angle stability analysis mentions, by transient rotor angle, we are interested in the transients that are happening in the rotor angle of the synchronous generator, the rotor angle being, on an assumption basis, the angle of the internal EMF with respect to the terminal voltage of the synchronous generator. In case of a disturbance, how is that angle varying? What are the disturbances or repercussion effects on this angle? and whether the synchronous generator will remain in a stable operating condition following this disturbance. If at all it tends to remain in the same condition with respect to an initial equilibrium point, then how much have the transients or variations been, and if it is completely different from the previous initial operating condition, what is that new operating condition in which the system tends to be in a stable condition? And obviously, when it is unstable, the rotor angle will tend to keep on increasing or keep on decreasing with respect to other synchronous generators in the power network, and eventually, there will be a loss of synchronism, which we have discussed in detail and at length in the previous lecture on the basics and classification of power system stability problems, where we started with rotor angle stability and then understood the small signal aspect.

Transient rotor angle aspect, voltage stability, frequency stability, and different forms of these classifications in different time domains under different disturbance sizes and initial operating conditions. So essentially, the point that we can all conclude or understand is that if the system tends to be stable, then the relevant variables—be it rotor angle, phase angle, voltage magnitude, or frequency—tend to remain bounded and follow a particular trajectory. If the system tends to be in an unstable condition following a disturbance based on some initial operating condition, then it is also important to understand what this variable is likely to take as its value and whether certain measures can be taken into consideration that can minimize the impact of these unstable conditions. So, today's discussion is on the swing equation.

Swing equation, let us see what that swing equation is. So before we go into the swing equation, which is the governing equation for transient rotor angle stability analysis, as well as for small signal rotor angle stability analysis. We have discussed the variations in a typical synchronous machine at length in the synchronous generator module as well as the previous module in fault analysis. If a disturbance happens, it could be a load change, line tripping, generation outage, excitation drop, and so on, or even the occurrence of a fault. Then the oscillations that appear in the rotor angle, frequency, or rotor speed, as well as the transients that occur in these different variables pertaining to the synchronous machine or generator.

They get some sort of damping, which is provided by the armature, the armature winding resistance, the network resistance, as well as the rotor damper windings. The notion of this damping comes into play as a damper; it tends to minimize the overshoots and undershoots that appear in these different variables, and network resistance happens to be a good damper in itself, which will tend to minimize these variations. These rotor windings are usually shorted, or specifically, rotor damper windings are usually shorted by a large resistance. which carries current only when the rotor speed differs from the synchronous speed. As long as the rotor speed is the same as the synchronous speed, the rotor would never experience any slip or any variation; in terms of induction machines, we call it slip or associated frequency as slip frequency.

When the rotor speed is the same as synchronous speed, we call the slip 0, and hence the damper windings present on the rotor do not have any induced EMF, as a result of which there is no current flowing in the corresponding rotor damper windings as long as the rotor speed is the same as synchronous speed. If rotor speed is different from synchronous speed, then the rotor damper windings tend to carry a current at a frequency known as the slip frequency. For more details, please refer to the induction generator discussion in relevant textbooks and other chapters wherever you would find the relevant context. And the last part about synchronous machine parameters, specifically the reactance, is that it tends to vary after a disturbance, which leads to different time responses for a typical synchronous generator, and this response, as we have seen in length. can be divided into a sub-transient period in which we essentially carry out the fault analysis study.

The last part is the steady state period in which we usually tend to carry out our power flow analysis, and essentially for power flow analysis one does not need to go inside the machine terminal; only the terminal information is sufficiently enough, and hence usually the corresponding reactance or steady state synchronous reactance never comes into the picture. The intermediate stage is the transient period that lasts after the sub-transient period before the steady-state period and follows for a few cycles in which we would emphasize our current module or topic, which is transient rotor angle stability. We would try to understand how these transients are happening, and the corresponding transient reactants should be considered for analyzing the stability with respect to synchronous generators. So the purpose of transient rotor angle stability is to check whether, following a disturbance with respect to some initial operating condition, the synchronous generator—be it a motor or generator—can remain in the conventional synchronized manner; following a disturbance, there are four conditions for synchronism. Voltage magnitude should match the terminal voltage of the incoming bus, frequency should match, phase angles should match, and phase sequence should match.

So, this synchronism is mostly from the transient rotor angle stability perspective; it is more to do with checking whether the rotor speed is equal to the synchronous speed following a disturbance. If it is not, then how much is the deviation, and can the rotor speed regain its synchronous speed value again? And as I was mentioning, it is also a check to see whether the rotors of the machine return to their constant speed operation, which is the usual case following a disturbance. If there has been a deviation in the rotor speed, then how much has the deviation been, and can something be done to bring the rotor speed back to the actual synchronous speed so that the conditions of synchronism are well satisfied and maintained? So, in order to do this analysis or understand this transient roto-triangle stability or instability analysis, obviously, any analysis or study goes by with some assumptions. I will just explain those assumptions, and I would call them the assumptions associated with the conventional transient roto-triangle stability analysis. The assumptions do have some basis, but it is not necessary that, from the practical network perspective, these assumptions should always hold true.

In case these assumptions do not hold true and one has to go beyond the conventional transient row triangle stability analysis, then appropriate considerations must be taken into account. So, in conventional transient root triangle stability analysis, we will not consider the dynamics or changes that are happening in the automatic load frequency control (ALFC) or the automatic generation control effect, which, in a way, tends to regulate the mechanical input of a synchronous generator given the load change increase or decrease. And as a result, since the dynamics of ALFC or the corresponding turbine control are much slower compared to the transient period, we would be interested in understanding the aspects of rotor angle stability or instability. So we would consider those dynamics to be not present or not considered. And hence the bottom line is that the mechanical input of a synchronous machine, in a generator it is the input; in the case of a motor, it is the output; it would always remain constant.

The machine and system damping and the voltage regulation effects are also neglected again from the conventional perspective, but from a practical perspective, this may not be applicable, and the synchronous machine or generator is represented by a constant internal EMF similar to the fault analysis module. In series with a direct-axis transient reactance. In fault analysis, we had the direct axis sub-transient reactance, which was Xd double dash; for transient rotor angle stability, we will focus on using the direct axis transient reactance because the direct axis transient reactance is a dominant reactance, while the corresponding quadrature axis reactances are negligible. On similar lines, network transients are also to be neglected, that is, transmission line resistance, the corresponding load damping, load modeling, or load effect on load sensitivity towards voltage and load sensitivity towards frequency, which we will discuss a bit towards the end of this particular module. We also talked about these different load models when we discussed the load power flow analysis module.

So we will briefly touch upon those practical models that tend to deviate from the conventional transient load triangle stability analysis. For conventional sake, network transients are neglected, and the line is modeled by a static lumped pie model. The entire network is reduced only to the machine nodes; by machine nodes, I mean buses or nodes that are directly connected to the internal EMFs of the synchronous generator. These internal EMFs refer to the constant internal EMF to which I was referring in the previous slide. And we will make use of a special technique known as Ward reduction, which is also a generalized or a specific form of Cron reduction that we will discuss at length in a particular lecture to come.

And the benefit of all these assumptions is that one need not do extensive power flow analysis, and one might wonder why power flow is needed. The purpose remains the same; when we did fault analysis, we had to have an estimate of what our initial pre-fault voltage was. For any stability problem, we also have to understand what our initial equilibrium point or initial steady-state operating condition is. And without doing power flow or state estimation, we won't be able to get those estimates. One should practically do power flow to get these initial operating conditions, but then from a conventional perspective, one can make use of water reduction with some assumptions on these internal EMFs to make the process faster.

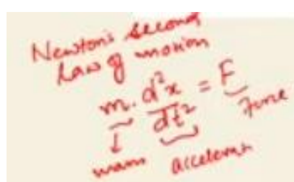
And avoid power flow entirely. Load dynamics are, as I mentioned, neglected, and they are all modeled as constant impedance or admittance type models. No load dependency on voltage or frequency is considered, which should be considered practically otherwise. And the biggest assumption, as I mentioned in the initial few slides of this lecture, is that the machine's internal EMF angle coincides with the machine's rotor angle. By internal EMF angle, I specifically mean the angle of internal EMF EF, which is, let us say, at an angle delta measured with respect to the terminal voltage of the synchronous generator, which we are choosing as the zero reference for phasor measurement.

Now this angle delta is usually positive or should be positive for generating action, and it is negative for motor action. By rotor angle, we are assuming that this delta angle itself pertains to the machine's rotor angle, and it is a direct measure of that particular angle deviation that we are trying to measure. The rotor saliency aspects are neglected; we would be consistently considering the cylindrical machine-based rotor or cylindrical pole rotor for our discussion, although from a practical perspective, some countermeasures have to be considered to conduct this transient rotor angle stability analysis for salient pole machines. In general, we would be using some static power flow equations, specifically some power flow-based equations; I won't say the usual power flow equations, but rather some power-based equations for representing the reduced network obtained by Ward reduction, along with certain second-order differential equations that represent the rotor's rotational variation. Which together we would be using to understand this module of interest.

So, essentially, we will be dealing with some differential equations in addition to some static power-based relationships between power and the phase angles. So, coming to the swing equation, which is basically the defining or governing equation behind the rotor dynamics of a synchronous machine during a disturbance, that swing equation is a second-order differential equation of this form.

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \ (Nm)$$

Before I go deep into what this equation means, let us try to revisit or recollect what Newton's second law of motion is. Newton's second law of motion states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. Newton's second law of motion is mathematically represented by an equation of the form mass times d square x by dt square, where x is distance or displacement, t is time, and it is equal to some force.



Now, here m is mass, as I mentioned; the double derivative of distance or displacement with respect to time is acceleration, and f is the net force acting on this mass m. Now, what does the second law of motion indicate and represent? It represents that if a force or a non-zero force is applied to a mass m, which may be stationary or non-stationary before the force is applied. It will eventually experience an acceleration that would tend to increase its distance or displacement and corresponding speed or velocity. If the same set of analogies is to be applied to the rotors of our synchronous machines, then since the rotor is a rotating body and has some kinetic energy and mass associated with it, it is not a massless body. So, in case the rotor tends to experience some sort of torque or force in a rotating medium, then the corresponding dynamics observed in the rotor angle, which is basically theta m, is indicated by this similar Newton's second law of motion analogous equation, which we call the Shing equation from a power network perspective.

So in this equation, the swing equation J refers to the total moment of inertia of the rotor combined together in kg meter squared, and theta m is the absolute rotor angle displacement with respect to a stationary axis on the generator stator measured in mechanical radians. T is time, small t is time here as I mentioned, and Tm is the mechanical input torque or shaft torque supplied by the prime mover with negligible losses considered. So, its unit is the Newton meter, which, if applied, tends to bring a positive effect on the torque and accelerates the rotor with theta m increasing over time. TE is the counter electromagnetic torque produced because the machine is delivering electrical power output, and because of the armature reaction effect, the stator also tends to get a push back that, oh, let us not only accelerate; let us also bring in a counterbalance between mechanical input and electrical output. The corresponding counter torque is known as electromagnetic torque, which arises from the electrical power output of the synchronous generator and tends to decelerate the rotor if Te is non-zero; hence, it is accelerating the rotor in the negative theta m. Ta is the net acceleration torque, which is measured in Newton meters. Now, under steady state, as I mentioned, as long as there is a balance between the electrical mechanical input to the generator and the electrical output of the generator, considering negligible losses for the sake of simplicity in our discussion. Then T A under steady state would be perfectly zero, which would mean that this entire term becomes zero, indicating that the rotor is not experiencing any acceleration or deceleration under steady state conditions. And as a result, during steady state, which is the usual case, rotor speed is the same as the constant synchronous speed, and the rotor and turbine are happily together in synchronism with no variations.

$$0 = T_a = T_m - T_e$$

The rotor speed or the corresponding rotor angle. Now, in terms of mechanical torque or shaft torque, since we are neglecting the effect of AGC control and turbine control because their dynamics are smaller, or their initial time constant is smaller. So, for our study of transient rotor angle stability analysis, we would consider TM to be constant. The electrical electromagnetic torque Te can change because of the disturbance occurring in the generator electrically from the transient rotor angle stability perspective. And hence the corresponding changes that can happen for transient rotor angle stability would all occur from the perspective of electrical power output or electromagnetic torque, which in a way accounts for power transfer between the rotor and the stator through the air gap.

Now, if we talk about our angle theta m, which is being measured with respect to a stationary axis on the stator. So let's focus on the steady-state condition to be a precise case. So in a steady-state condition, what we observe is that since the acceleration torque is 0, d squared theta m by dt squared would be 0, because j cannot be 0; it refers to the moment of inertia of a physical device, so j cannot be 0. And hence the rotor acceleration is effectively 0, which would mean that if I integrate with respect to some initial condition, then dθ/dt is basically a constant C. And which, if I further integrate, then theta m is some c of t plus k, where c and k are integration constants depending on some time condition.
So, which would mean that even when the rotor is not experiencing any acceleration or deceleration that is perfectly zero in a steady state. Theta, on the other hand, is continuously varying with time, and mathematically speaking, it looks all okay, but from a power network or an engineering perspective, we are actually interested in the transient ultra-angle stability perspective. In understanding the third aspect, which states that if the rotor speed has deviated, we are not actually interested in the absolute change in the rotor angle with respect to some stationary axis. So, can something be done for easier analysis while the understanding of the swing equation remains intact? Yes, what we can do is, as I mentioned, we are interested in the rotor speed with respect to synchronous speed and not the actual rotor speed in a practical essence perspective, because we know for sure that the system is stable and the rotor speed would be the same as the synchronous speed. So what we can do is change the reference with which this rotor angle theta m is being measured; instead of measuring it with respect to some stationary axis on the stator, can we not measure it with respect to some rotating axis, again with respect to the stator, and the rotating axis can coincidentally happen to be the Rotating stator flux in the air gap of a synchronous generator that rotates at synchronous speed under steady-state conditions.

So, what we are interested in is, as I mentioned, measuring the rotor angle position with respect to a reference axis, which is also rotating at synchronous speed, and that can happen to be the rotor—not the rotor because the rotor is undergoing dynamics. This reference axis can be the stator air gap flux, which is rotating in the air gap at synchronous speed. So,

$$\theta_m = \omega_{sm} t + \delta_m$$

what we are doing is trying to change the reference for theta m, and the associated terms that come in are omega s m, where omega s m is the synchronous speed of the machine measured with respect to the same stationary axis of the stator based on which theta m was being measured. t is the time, and delta m now happens to be our analogous rotor angle, which is being measured with respect to the synchronous rotating reference axis. With this change in our reference point, any deviation occurring between this synchronous rotating reference axis and the stationary axis is being compensated through this omega SM angular speed.

So delta M can serve as our new analogous transient rotor angle, which will help us analyze rotor speed with respect to synchronous speed. So what we do is try to find the first and second derivatives of theta m so that we can get the corresponding expressions in terms of delta m.

$$\frac{d\theta_m}{dt} = w_m = w_{sm} + \frac{d\delta_m}{dt} \text{ and } \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

When we take the first derivative of theta m, it is nothing but omega m. Now, omega m is the actual rotor speed measured with respect to the same stationary axis with which omega sm and theta m are being measured. So the corresponding first derivative is omega s m.

Plus, the first derivative of delta m and the second derivatives are perfectly identical to each other because omega sm happens to be a time-independent quantity. Now, what is the situation when things are in a steady state? When things are in a steady state, we know for sure that these two terms will be equal to 0. If these two terms are equal to zero, then it also means that d(delta m) by dt would also be zero, and that is where omega m is equal to omega sm when the relative motion between the rotor and synchronous rotating reference speed is zero, which means that under steady state when the rotor speed is the same as The synchronous speed of the motor, omega SM, which is basically the synchronous speed of the machine, and omega M, which is the rotor speed of the machine, both being measured on the same reference axis, in steady state, rotor speed would be the same as the synchronous speed. So, omega M has to be the same as omega SM, and under that condition, when there is no deviation, d delta m by dt would also be 0. And in case there is any disturbance or deviation that results in a change of this expression not being equal to 0, under that condition d delta m by dt would also not be equal to 0.

- Swing equation $J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \ (Nm)$
- Rotor speed wrt syn. speed is of interest, hence

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e \ (Nm)$$

$$Jw_m\frac{d^2\delta_m}{dt^2} = (T_m - T_e)w_m \ (Nm - rad/s)$$

So, essentially, the swing equation in terms of theta m can be rewritten in terms of delta m, and often, as electrical engineers, it is convenient for us to deal with power rather than torque. So, we can also multiply both sides by omega m, which is the actual rotor speed with respect to some stationary axis on the stator. So, the corresponding torque to speed is nothing but electrical power, which is measured in watts. So, this becomes our analogous swing equation, which we can use further for analyzing the transient rotor angle stability of synchronous generators.

$$J w_m \frac{d^2 \delta_m}{dt^2} = P_m - P_e \ (W)$$

Pm now becomes the input turbine power, the mechanical input power to the generator Pe is the electrical power output crossing the air gap in watts. J ω m is the angular momentum of the rotor and the turbine considered together. We also define another term constant known as capital M, which is j omega SM and not j omega M. Omega SM, as defined on slide number 8, is the synchronous speed of the machine measured with respect to a stationary axis, and for a machine that is already designed and operational, synchronous speed is a function of frequency and the number of poles in steady state. Under steady state, frequency won't change, the number of poles in the machine won't change, so omega SM is actually a constant for a given machine, and hence capital M is also a unique constant for every synchronous generator because J is, again, the moment of inertia of a physical device, rotor, and turbine combined together.

Omega SM is a fixed quantity for a given generator. So, capital M is the inertia constant of the machine angular momentum of the rotor and turbine considered together in kg meter squared radian per second. So, what we do for the sake of simplicity is we tend to replace jωm in the equation shown here with capital M, and hence this becomes our analogous swing equation,

$$M \frac{d^2 \delta_m}{dt^2} = P_m - P_e \ (W)$$

which we will discuss further in the next lecture. It is true that capital M is not the actual angular momentum of the rotor, and there might be deviations or variations in the transient triangle stability analysis results. But the inertia constant m, actually not being a constant, does not make much of a deviation; this sort of assumption tends to go well with transient rotor angle stability analysis.

So that is all for today's discussion; in the next lecture, we will continue with our discussion on the swing equation. In which we will at length talk about how this equation can be analyzed under different disturbances and what can be done regarding the stability or instability problems that might arise under different disturbances through the use of this equation that we will discuss in the next lecture.

Thank you.