

Power Network Analysis

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Week – 01

Lecture-05

Hello everyone, welcome to lecture 5 of week 1 which is going to be the first discussion on transformers and per unit analysis. Before we go deep into what exactly is per unit analysis, we would like to spend some time on understanding how our transformers operate, what is the basis of their operation. And I'll try to keep the discussion very simple. I'll start with the very basic minimal single phase two winding transformer so that it helps you in recollecting what all aspects are involved in understanding the operation of transformer. If we talk about the previous few lectures that we have been discussing in this first week, we have essentially talked about evolution of power networks, how they have evolved, what are the issues in power networks, and then we also touched upon the basic circuit principles where in detail we talked about phasor diagrams, single phase circuits, three phases circuits, and how essentially star and delta connections play a role in evaluation of the relevant equations from current voltage power evaluation. So if we talk about a transformer, what essentially is a transformer? A transformer does not have a typical transformer I would say does not have any rotating device or rotating part and hence often these transformers are static devices which essentially help in transfer of electrical power from one voltage level to another voltage level in two different circuits and these circuits are often isolated in the sense these circuits are not electrically connected but they are magnetically coupled. So, essentially the transformers typical transformers job is to transfer electrical power from let us say voltage A to voltage B where voltages A and B are not electrically connected, but these two circuits are two different voltages they are interacting among each other through some sort of a magnetic coupling.

And depending on the type of transformer, it would have at least two windings because these two windings would represent two distinct circuits which are at different voltage levels. Let's say voltage level A, voltage level B and transformer has to transfer power from these two different voltage levels. At least two different voltage levels have to be there. And that's the reason why a basic conventional transformer should have at least two windings for it to behave as a transformer.

Depending on how many circuits or how many voltage levels exist in coherency, the number of windings or number of circuits involved in transformers may be more than two also. Let's say we have three-phase transformers, we can have three-phase windings. And all these three windings or two windings or n number of windings where n is more than equal to 2, they're all wound on some magnetic device or magnetic core which we call as ferromagnetic core with certain laminations on it. Now why these laminations are there, we'll probably discuss or come back to this when we go into details of what transformers are. So the reason why we need transformers, the answer is pretty simple.

We have been emphasizing about it in the last few lectures, is that typical generation happens at very low voltages. And if power was to be transmitted at such low voltages, the distance to which this power can be transmitted would be very small, because the corresponding I^2R loss would be very high at such low voltages. To reduce these I^2R loss, the power is amped up or the voltage is amped up to few hundreds of kVs so that the corresponding I^2R loss is minimized and the power generated at generation can travel a lot of distance, few thousands of kilometers and then eventually reach the industrial area or residential area where the problem is that typical loads are not designed to operate at few hundreds of kVs or few 1100s of kVs so there also at the consumption level we also need transformers which should step down the power at high voltage to acceptable voltages which is 415 volt three phase typically for Indian system or 230 volt to 20 volt single-phase system. So again at the consumption level or distribution level, step-down transformers are needed and then only the power is sort of consumed by the different loads. So depending on the type of operation or application, I would say these transformers to be power transformers.

Power transformers, their efficiency is very high, close to typically around 95 to 98 percent and the voltage regulation is almost zero. I would request people who are unaware of what voltage regulation is or how do we figure out efficiency of transformers, I would request them to please refresh the relevant discussion or relevant material on transformers where you can find discussions on what efficiency is and what voltage regulation is. But from the operation perspective, power transformers they have very high efficiency close to 98-95% and the voltage regulation is almost negligible. And depending on the transformer, we are talking specifically about power transformers here for our discussion. Transformers can be of different types.

We can also have certain transformers which are used for protection perspective We have talked about the protection aspect in the first lecture itself where we mentioned about instrument transformers, current transformers, potential transformers. These transformers, they are all collectively called as protection transformers. There's a

difference between protection and power transformers in terms of the construction and it all depends on what is the acceptance or what is the expectation level from a transformer. So power transformers, if I were to operate or mark the operating points of this transformer on a typical BH curve where B is on X axis and H is on the Y axis, then the corresponding operating point which typically resembles initial few linear operation and then beyond a certain point the core tends to saturate in transformers. For power transformers the operating point is close to the knee point of the BH curve whereas for Protection transformers or instrument transformers, the operating point is very close to the initial linear half of the BH curve.

The reason why this is there or what implication would this have on the construction of transformers, reason being power transformers are not meant to provide isolation. Whereas protection transformers serve two purpose, one to provide isolation to the low voltage, low current relay circuit and to also properly replicate the fault currents, fault voltages during faults and for it to replicate properly the operating point should be in the linear half, then only the corresponding replication would happen. So anyone who is interested can figure out, can go into details of protection transformer and accordingly get to know more about these differences in the corresponding operating points. Coming to our discussion of premise of transformer as I said any transformer would at least have two windings and these two windings would be wound on some magnetic or ferromagnetic core which is typically laminated and The diagram here it shows the typical single phase two winding transformer and the corresponding electrical representation is shown in this circuit. Now in this particular diagram or circuit there are two windings and as one can see these windings have got certain dot marking or polarity marking on them.

The importance of these dot markings, these dot markings are very similar to the dot markings marked over here. The importance of these dot markings is that these dot markings or polarities help in understanding or anticipating how would these windings carry currents. If they carry currents, then what would be the corresponding implication on the core in terms of the magnetic flux? So in order to sort of summarize what these dot polarities mean, they would mean that if currents were to enter these dot markings, so let's say on this particular side, the current I_1 is entering this particular dot through this winding, and on similar lines, if current were to also enter this particular dot on the N_2 number of turns side. Then in that sense, both these currents, the corresponding flux that they would generate in this core, they would be additive to each other. Now how do we figure out the core or the flux orientation in this core? One can always make use of the right hand thumb rule or I would say right hand rule in which the right hand rule says depending on the flow of current if the fingers accept the thumb of the right hand they curl in the direction of the current in which the current direction is flowing then the

thumb would point towards the direction of the flux or magnetic flux which this solenoid or winding is creating.

So if we talk about the side which is having N_1 number of turns as the current I_1 flows through this winding in the direction of this current is in the right hand side curl of the right hand. So essentially the thumb would point towards the direction of the magnetic flux. Similarly if the current on the N_2 number of side enters this particular dot then the current would actually be having a direction of this order and in this way if I apply my right hand rule then the right hand rule says the finger should curl in the direction of the current, the thumb which is pointing downwards essentially would indicate the direction of magnetic flux which is being induced in the core. Since both these arrows they are additive so the corresponding flux get added in the corresponding core. That's the notion of dot polarity or dot marking or polarity marking.

Again anyone who is interested in details of what these dots are, I request them to please go through the relevant references or textbooks. Typically the side to which the power supply is connected, we call it as the primary winding, so among these windings N_1 and N_2 number of turns, N_1 refers to the number of turns on the primary side probably, N_2 refers to the number of turns on the secondary side. The side to which supply is connected is called as primary winding. The side to which load is connected is called as secondary winding. Depending on step-up or step-down transformer, the operating voltage will accordingly vary.

A step-up transformer steps up low voltage to high voltage. So primary winding is LV or low voltage on step-up. And similarly for step-down transformer, it steps down high voltage to low voltage. So primary winding in step-down refers to the high voltage windings. One thing I missed in this particular slide, there is a term which is mentioning here or called here as ideal transformer and this equivalent circuit that we have drawn is actually applicable only for an ideal transformer.

So let's say or let's talk about what are these attributes for an ideal transformer. There are four common attributes. The first attribute is, as the term ideal says, the first notion which can one get from the term ideal is, oh this transformer doesn't have any losses at all. So the forms of losses which can occur typically in electrical circuits are heating loss, copper loss, leakage loss, etc. So the first attribute which is true for a real transformer is that the windings or the wires which are wound on these cores over here and here, They are pure conductors, their resistances are all negligible, so there is no copper loss at all in an ideal transformer.

Whatever magnetic flux is induced by these currents on the primary side or the currents

on the secondary side, all this flux is confined to the core. There is no flux which is escaping out of this particular solenoid and not flowing through the core. There is no flux which is flowing or passing through this solenoid. It is not flowing through the core but is flowing through the air gap in between the winding and the corresponding core. So in an ideal transformer, there is no leakage flux.

All flux is confined to core and windings. The third and fourth attributes, they are interrelated. The third attribute talks about the property of core. The core here is a perfect magnetic medium, means the permeability is infinite. There is no external force required for this magnetic core to magnetize itself.

That means there is no external MMF, which is also known as magnetomotive force. There is no need of any external MMF needed, no magnetization needed for this ferromagnetic core to become magnetized. It's a perfectly ideal or perfect magnetic material, it's always highly magnetic and hence the corresponding current or flux which is needed to magnetize the core is not necessary. So we'll come back to this particular current in a moment. The corresponding reluctance is zero.

And similarly, the core is also not having any form of heating loss or copper loss. So essentially the core loss is negligible. I will come back to this current in a moment. I'll talk a bit about why lamination is needed, specifically when the transformer is not an ideal transformer. In fact, practical transformers are mostly non-ideal transformers.

They are highly impractical. All these attributes, four attributes, which we have discussed here, they're not applicable for practical transformer. And lamination or core lamination essentially helps in minimizing the losses that occur specifically because of attribute four, which is all about core losses are not negligible. They can be made negligible by dividing up the core into few laminated sheets. And instead of having a single solid core wherein the core might get heated up. If we break up the corresponding core into two laminated sheets, the possibility of eddy currents being induced would be minimized and hence the corresponding core loss can also be minimized.

That's applicable for practical transformer. For time being we are discussing about ideal transformer, so lamination or no lamination of core is not important in ideal transformer. So with these four attributes, if we go ahead with the discussion that let's say what happens when the ideal transformer is realized and it is made to operate. So if we combine points three and four, there's a current which is known as excitation current. For the ideal transformer, excitation current need not be there.

It is perfectly zero. So if we talk about this ideal transformer, which is for time being not

loaded, that means this load Z_L , it is not getting any supply because the switch here is open, and inherently I'm assuming that there is a source present here, single phase source, which is connected to these terminals here. The load is still open, and we have to understand how this transformer actually operates. So the source voltage V_1 It induces time varying flux in the core as per Faraday's law and Lenz law. And since this winding here is a perfect conductor, there is no voltage drop or there is zero voltage drop in this particular winding. So essentially whatever is the applied voltage, the same is the voltage which is also appearing across this particular solenoid or turns of windings.

So that's the reason why E_1 , which we call as induced EMF, is same as the applied voltage. And these two voltages, they are equal to N_1 , where N was the number of turns on the supply winding or primary winding, $d\phi$ by dt , and this is because of Faraday's law. The same equation if we try to attribute to the secondary side then let's say on the secondary side there is some voltage V_2 which is appearing on the secondary side we don't know what that V_2 is but for sure this V_2 is not supplying any current to the load here because it's still open. For some voltage V_2 which is appearing on the secondary side again this conductor is a perfect conductor so there is no voltage drop as a result induced voltage on secondary is same as the output voltage on secondary. And again by Faraday's law, $N_2 d\phi$ by dt is the expression which is equated to E_2 and V_2 . Here ϕ is the flux or the flux which is present in the core.

This flux is again having no leakages. So whatever flux is associated to the primary and secondary, all of it is linked to the core itself. There is no leakage at all. So now if we compare our equations, let's say equation A with equation B, we get one unique result. The result is E_1 by E_2 is equal to V_1 by V_2 , which is also equal to N_1 by N_2 .

$d\phi$ by dt is common in both the equations, and that gives us our definition of turn ratio. We often associate small a I mean that's the notion which I follow others can have different notation. The turn ratio of a transformer is nothing but the ratio of primary voltage divided by the secondary voltage. If I know what is my primary voltage and I know the number of turns physical turns on the transformer I can also find the corresponding secondary voltage which would appear at the load. Now suppose this transformer is loaded, that means the switch is now closed.

Now the switch is closed, there is a voltage which is appearing V_2 because of V_1 , V_2 is same as V_1 by a , where a is N_1 by N_2 . If there is a non-zero voltage on the secondary and the switch is now closed, there is a finite load which is present over here, it's obvious that I_2 will no more remain zero. Current tends to follow the path of least impedance, so because of voltage appearing or induced, current would flow. Now when current flows, over here if we look, again going back to the definition of dot polarity or dot markings, I_2

is now the current which is leaving the particular dot. If I_2 is the current which is leaving this particular dot, that means the currents in these windings they are in this particular arrow and if we now apply a right hand rule, the corresponding flux which is being produced or corresponding MMF which is being produced because of I_2 current, it is in upward direction.

The quantum of that MMF or flux manodomotive force basically is $N_2 I_2$ which is opposing the core flux. In order to balance out this core flux in the core of the transformer, the corresponding information on primary side should also get conveyed. So, essentially to balance out the flux in the core which is ϕ $N_1 I_1$ MMF will now be produced or additionally be produced on the primary side. So, that the corresponding flux which has now deviated from ϕ being now equal to ϕ minus $N_2 I_2$. The corresponding support from primary sides would also come in so that the overall flux $N_1 I_1$ MMF would be in direction to the actual flux.

This should again become equal to ϕ . So from here we get that $N_1 I_1$ should be equal to $N_2 I_2$. That is essentially what is mentioned over here and beautifully seeing the ratio of primary and secondary currents is nothing but inverse of the turn ratio. And it is through this MMF balancing how the primary gets to know any change which has happened on the electrical side of the secondary of the transformer. Remember, these two circuits, they are not electrically connected. They are magnetically connected through this MMF balancing act, which is inherently there in transformers, specifically under steady state conditions.

The MMF balancing act helps let the primary know that if electrically things have changed on the secondary side, the primary side quantity should accordingly change. That's how the balancing part happens. MMF balancing lets primary know of any change in the secondary current. As long as the secondary side is unloaded, there is no secondary current, so there is no need of any primary current to there. The core is also, perfect ideal magnetic material so by attribute 3 and 4 there is no need of any excitation current so as a result the overall current drawn at the primary level of a ideal transformer is going to be zero as long as the secondary of the ideal transformer is unloaded.

And under all conditions if we try to figure out the corresponding apparent power they're always going to be equal and that's the reason how through magnetic coupling power can get transferred from one end to the other end while having perfect power balance. There is an ideal transformer there is no loss at all so whatever is the power input on the primary side the same quantum of power is being transferred on the secondary side at a different voltage level which depends on what is the value of the corresponding turns ratio which is A which is N_1 by N_2 . If N_1 and N_2 are equal to 1, the

transformer does not serve any purpose, there is no need of having a transformer. power is transferring from the same voltage to the same voltage level. So, if we have to draw a phasor diagram which explains how the corresponding quantities look like in an ideal transformer.

If we take flux in the core as a reference then as per Faraday's law induced emf is $N \frac{d\phi}{dt}$. So, from here we would typically get E to be equal to $N \phi_{max} \omega \cos \omega t$ and if we recollect our discussion on phasor diagrams sine wave and cos wave they are phase apart by 90 degrees. So, if I choose ϕ which is the MMF or the core in the flux as a reference for my phasor diagram the corresponding induced EMF on the primary and secondary side for a ideal transformer they would all lead the corresponding flux. If the ideal transformer is loaded by an inductive load, which would mean that the current lags the corresponding applied voltage, then we would also have this corresponding impedance angle or phase angle. The quantities in secondary and primary, they would be linked as per the turn ratio.

The voltages are in proportion to the turn ratio, whereas the currents are in inverse proportion of the turn ratio. Before we go into the attributes for a practical transformer, there's one also specific example or application for transformers which happens in context of maximum power transfer theorem. Transformers are also used to match the impedance between the source and the load. That's what this example is all about. So suppose we have a transformer which is having a load Z_L on the secondary of the transformer and if I define this Z_L in terms of the secondary voltage and secondary current then by basic KVL this is the situation.

Now if I were to measure what is the equivalent impedance being sensed on the primary side. So let's say the primary side impedance is Z_1 , then as the load impedance definition is true for secondary side, Z_1 would nothing be but equal to V_1 by I_1 , V_1 by I_1 , V_2 by I_2 here could refer to RMS values, they could also refer to corresponding impedances of the time varying quantities and by definition of turn ratio V_1 is nothing but A times V_2 whereas I_1 is nothing but I_2 by A which results in that Z_1 is equal to $A^2 \frac{V_2}{I_2}$ and $\frac{V_2}{I_2}$ is nothing but Z_L . So essentially if there is a impedance Z_L connected on secondary of the transformer, the impedance which would appear on the primary of transformer is A^2 times the actual impedance. And that's what is essentially given over here. In order to ensure maximum power transfer from the source end, we also have certain load matching transformers where the corresponding turn ratio of this transformer can be regulated, modified.

so that for maximum power transfer, as for maximum power transfer, the source impedance should be conjugate of the load impedance, or other way speaking, the load

impedance should be conjugate of the source impedance to ensure this balancing for maximum power transfer. The turn ratio and load matching transformer can be regulated so that the corresponding maximum power transfer can happen from the source perspective. In real transformer, the four attributes that we saw for ideal transformer, they are no more applicable.

We have non-zero copper loss. We have actual leakage. All the flux is not assisted to the core. There is always leakage in terms of flux. The core is not a perfect ideal magnetic device. Its permeability is finite. So it would necessarily need some non-zero MMF in order to magnetize itself and no matter how much lamination we have the core loss which is component of eddy and hysteresis loss this also exists.

Now if these four attributes practically exist That's the essence of how the nature operates, that's how the conductors are designed, that's how we can have our operation in place. So what are the implications of these attributes of real transformer on the circuit of the ideal transformer? So let's look at that. If the winding resistances are not negligible, that means V_1 and E_1 which were earlier equal to each other or V_2 and E_2 which were earlier equal to each other they would definitely not be equal to each other because there would be a voltage drop across this non-zero winding resistance. So essentially we need to have some resistance model incorporated here. Leakage flux happens, that means some form of inductance or leakage inductance also needs to be incorporated.

Permeability of core is not infinite, so we also need some form of inductive representation to manage or showcase the permeability of the corresponding non-ideal core. And similar to copper loss, if core loss exists, then there also needs to be some form of a representation for RC which is the equivalent resistance for core losses. So that's the summary which is present over here. Lumped resistance in series has to be there in the windings. Inductance in series has to be there to represent the leakage reactance.

The third and fourth point essentially mean that the excitation current is not zero anymore. In fact, in practice, it is usually five to 10% of the actual primary current, which is I_1 . And for large power transformers, this current can be significantly high. So if I were to incorporate all these non-idealities this is how the equivalent circuit diagram would look like. In between here I have my beautiful well-defined ideal transformer.

On the primary side I have winding resistance R_1 which is because of non-ideal condition of the primary winding. Similarly I have R_2 which is the binding resistance of the secondary winding. Similarly to account for flux leakage I have corresponding flux leakage reactance on primary and secondary windings. The winding here or reactance here indicates that some portion of the core flux is getting leaked out into the air between

the primary winding and core. Similarly this particular reactance indicates some leakage of flux between the core and the air and the secondary winding through the air medium.

And the non-ideality of the core is represented by two such parallel legs. One is the core loss component, which is RC part, and the second part here, or the parallel part here, is because of the non-zero MMF needed to magnetize the corresponding core itself. So that in a way summarizes what or the circuit of how a real or practical transformer looks like. Now since we have the different elements in place, there are ways of how these R_1 , R_2 , X_1 , X_2 can be evaluated.

We will probably talk about it briefly in the next lecture. But having known these quantities, now we can also define certain relationships for this real transformer. Those relationships specifically for an unloaded case are shown over here. Depending on whether the current exists or not, V_2 and E_2 , they will not be equal because of the voltage drop across the winding and the leakage reactance. E_1 and E_2 , the induced EMF, they are still interrelated as per the turn ratio. The secondary current which is I_2 is reflected as I_2 by A in terms of I_2 dash and I_2 dash is not equal to I_1 because the core is not ideal.

So that's the relationship of induced secondary current from the perspective of primary. Then because of non-ideal core we have the core loss current and the magnetization current which were discussed initially in the slide talking about attributes of ideal transformer and both these currents they are vectorially added to give us the excitation current which is usually non-zero and excitation current gets added to the reflected secondary current to give us our actual primary current. Again there is a drop of voltage between induced EMF and the applied voltage so there is a voltage drop appearing over here. If I were to represent all these phasors on a common diagram for secondary current being equal to zero, this is how the corresponding phasor diagram would look like for a real transformer at no load. In the next lecture, we will further talk about few more basics of transformers and try to understand the philosophy behind operation of three-phase transformers because we are moving to discussion of three-phase AC systems.

So single-phase two winding transformers don't actually exist for three-phase systems. So we'll have to understand how three-phase transformers work and that's what would be part of discussion for the first lecture of week two, which is on three-phase transformers. Thank you.