Power Network Analysis

Dr. Abheejeet Mohapatra

Department of Electrical Engineering

IIT Kanpur

Week - 10

Lecture-49

Hello everyone, welcome to lecture 4 of week 10 of the course Power Network Analysis, in which we continue with the discussion on the Fault Analysis module. We will continue with the unbalanced fault analysis discussion that we had in the last lecture, in which we extensively talked about Fortescue's theorem and its application to understand the analysis behind evaluating fault currents for single line to ground faults and double line faults. For a single line to ground fault, we chose the specific case of phase a to ground fault. By the way, the single line-to-ground fault is also analogous to the phase-to-ground fault. So the notions of line and phase are interchangeable in the context of fault analysis. We had taken the case of phase a to ground fault where we chose phase a as a reference for sequence components.

And on similar lines, if one has to analyze phase b to ground fault or phase c to ground fault, then phase a can act as a reference for sequence components, but it is suggested that for phase b to ground fault, one should choose phase b as a reference for sequence components and phase c as a reference for sequence components for c to ground fault, the reason being that with the choice of phase a as a reference for sequence components, all three faults can be analyzed, but in b to g and c to g fault, it will be difficult to draw an equivalent circuit of how the sequence networks would be connected for b to g and c to g fault if phase a were to be chosen as a reference because the current relationships, specifically the sequence current relationships, become different. And that's what a simple suggestion is referred to in terms of choice of reference for sequence components. Similarly, for the double line fault, we took the case of the b2c fault, which is the fault between phases b2 and c, and we chose phase a as a reference for the sequence component. Similarly, if you have to do a c to a analysis, please choose phase b as a reference for the sequence component, or for an a to b fault, choose phase c as a reference for the sequence component.

The reason remains the same that if we choose this as our reference for the sequence component according to the respective faults, we would be able to visually draw the equivalent circuit of how the symmetric components would look. Given that in today's discussion we will start to understand the last category of asymmetric faults, which is the double line to ground fault or double phase fault, we will also understand the application of Thevenin's theorem in addition to Fortescue's theorem to analyze the occurrence of faults elsewhere in the network, which could be in a transmission line, bus, transformer, and not just at the terminal of the three-phase synchronous generator that we considered in the last lecture as well as in today's lecture. In this lecture, we will consider the analysis for the phase b to c fault at the terminal of the generator or a variant of the generator. So what we have here is a similar synchronous generator that is unloaded, and a phase b to c fault has occurred, which means a double line-to-ground fault with two phases or two lines and a ground also involved. These two phases are shorted to the ground through a fault impedance Z_f, and since the generator is unloaded, the corresponding phase a current is 0 .

For simplicity, we are ignoring the impact or effect of mutual coupling between the phase impedances of the generator; although if one has to consider the impact of mutual coupling, it can be done on similar lines as it was done for three-phase balanced load sequence impedance. Even if the generator were to be loaded, the level of change of current in phase A with respect to phase b and phase c would be very, very marginal. Hence, it is safer to have this sort of assumption where the phase a current is almost zero. After the fault has occurred, if we apply KVL between phase b and phase c to the ground, we get V_b equal to V_c equal to I_f into Z_f, and the fault current value is equal to the sum of phase b and phase c currents. So, the way we applied Fortescue's theorem in double line and line-to-ground faults, similar lines, we will also apply it for double line-to-ground fault on these three indicative equations.

So we choose phase a as a reference for sequence components, and since phase b voltage and phase c voltage are equal, in terms of the corresponding sequence voltages, that is V_b in terms of V^{a0}, V^{a1}, V^{a2}, because we have chosen phase a as a reference for sequence components, we can get a relationship in terms of sequence voltages based on the condition of KVL between phase b and phase c, which indicates that the positive sequence voltage would be the same as the negative sequence voltage. And remember, for the positive sequence circuit, maybe I should reiterate a bit: the positive sequence network consists of an active source, E^{a1}, which is the internal EMF of the generator, in series with the positive sequence impedance; the current being positive sequence current, I^{a1}, and the terminal voltage of this positive sequence network is nothing but V^{a1}. For zero sequence and negative sequence, the circuits are passive, which means they don't have any voltage source along with them, and for negative sequence current, we have I^{a2}, Z_2 as a negative sequence impedance, and the terminal voltage is V^{a2}. Similarly, for the zero sequence circuit, we have Z_0, I^{a0} as the outgoing current and terminal voltage being V^{a0}. So this is for positive, this is for negative, and this is for zero sequence.
For a double line-to-ground fault, what we are getting is that the terminal voltage of the

negative sequence circuit should be the same as the terminal voltage of the positive sequence circuit, which is an indication that the positive and negative sequence circuits for voltages are equal; these two circuits have to be connected in parallel, which is another clue that we would get as well. Phase a current is almost zero, which means the sum of positive and negative sequence currents is equal to the negative of the zero sequence current. So we have two facts: fact number one and fact number two. If we combine these two facts, from fact one we get that the positive and negative sequence circuits have to be parallel because their voltages are equal to each other. And if this KCL equation has to be true for the second fact, that means that positive and negative sequences are parallel to each other and are connected in anti-series in some fashion to the terminals of the zero-sequence circuit.

How that connection would look, we still haven't reached there. That's where we would use our third equation, in which the fault current is in terms of the zero sequence current. And I think there is a step that has been missed here; in a sense, that step is actually present here. The corresponding circuit was given earlier; pardon me for that. But this connection of $3Z_f$, how it is coming, we haven't arrived at that yet.

What we got was that the positive and negative sequence voltages have to be in parallel because of fact number one, which is this. And because of fact number two, these two circuits, when they are in parallel, have to be connected in anti-series to the zero sequence network. So that is what is giving us fact number two. How is $3Z_f$ coming in? It is included because of the discussion given in the next slide. By KVL, the phase b voltage or phase c voltage is $I_f Z_f$.

So if we decompose our phase voltages into their respective sequence components, this, in a way, would tell us that the positive sequence voltage is equal to some factor of a square plus, basically, if you recollect the property of a, where a was the complex operator at an angle of 120 degrees, then as per this definition, 1 plus a plus a square is equal to 0.1 plus a plus a^2 being equal to 0 , which would mean that a^2 plus a is equal to minus 1 , and that is how V^{a1} is equal to V^{a2}, which is equal to the negative of I^{a0} component. V^{a1} and V^{a2} are equal, so with this sort of combination, we are able to indicate that. If V^{a1} and V^{a2} have to be equal to each other, which is the voltage appearing across these two nodes, V^{a1} is equal to V^{a2}, and this has to be equal to the negative of the current, zero sequence current, with the corresponding zero sequence impedance and $3Z_f$. This would indicate that if I^{a0} is the current over here, then this voltage has to be equal to the voltage drop across this impedance as well as this impedance, and that's how the $3Z_f$ term is coming in.

$$\text{Since } V_b = V_c$$

$$\Rightarrow V^{a0} + a^2V^{a1} + aV^{a2} = V^{a0} + aV^{a1} + a^2V^{a2}$$
$$\Rightarrow (a^2 - a)V^{a1} = (a^2 - a)V^{a2}$$
$$\Rightarrow V^{a1} = V^{a2} \text{ as } a^2 \neq a$$

$$\text{Also } I_a = 0$$

$$\Rightarrow I^{a1} + I^{a2} = -I^{a0}$$

$$\text{Further, } V_b = I_f Z_f = 3I^{a0}Z_f$$

$$\Rightarrow V^{a0} + a^2V^{a1} + aV^{a2} = 3I^{a0}Z_f \Rightarrow (a^2 + a)V^{a1} = I^{a0}(Z_0 + 3Z_f)$$

$$\text{as } V^{a1} = V^{a2}, \Rightarrow V^{a1} = V^{a2} = -I^{a0}(Z_0 + 3Z_f)$$

$$\Rightarrow I^{a2} = -V^{a1}/Z_2, I^{a0} = -V^{a1}/(Z_0 + 3Z_f) \text{ and } I^{a1} = -(I^{a2} + I^{a0})$$

Therefore

$$I^{a1} = \frac{E^{a1}}{Z_1 + Z_2 \| (Z_0 + 3Z_f)}$$

$$\text{and } I_f = 3I^{a0}$$

So I hope that the explanation is satisfactory, and once we know the circuit arrangement of how the sequence networks would be connected, then in that case we can find the individual sequence currents I^{a2}, I^{a0}, I^{a1}, and eventually we can get the function of I_f, which is the fault current. So what we observe here is that in the case of ground faults, be it a double line-toground fault or a line-to-ground fault, the zero-sequence network comes in because whenever there is a ground fault effect, the zero-sequence current becomes live depending on how the zero-sequence current flows or doesn't flow, which means that zero being zero or infinity in a separate matter depends on how the transformer connections are. But if we have faults that don't involve ground, like a three-phase fault or a double-line fault, the zerosequence network or circuit would never come in. In fact, for balanced faults, it is only the positive sequence network that matters. For a double line fault, it is the positive and negative sequence networks that matter, as they are in anti-series to each other.

In line with ground fault as well as double line to ground fault, all three sequence networks come in, namely positive, negative, and zero; they are connected in a particular fashion. The connections are, however, different. So now, with this, if we have to extend this discussion for the occurrence of any of these four faults, balanced or the three unsymmetric faults, at any point in the power network, then how would you do that? So that is where the Thevenin theorem approach comes in. We have extensively talked about the application of Thevenin's theorem for balanced faults. Now for balanced faults, Fortescue's theorem need not be applied because it involves only one positive sequence network circuit, the analogous circuit that is usually present due to the inherent generation of positive sequence voltage and synchronous generators.

But for asymmetric faults, Fortescue's theorem is important, and that is where we need a combination of Fortescue's theorem and Thevenin's theorem to understand unbalanced or asymmetric faults at any point in the network; let that point be bus k , with the fault having some impedance Z_f. And since we are talking about faults occurring on the transmission network, which are usually balanced, that means they are inherently operating in the positive sequence voltage or positive sequence current indication. With the assumption that the pre-fault system is already balanced, to be specific, it is already operating in the positive sequence mode; that means the voltages are positive sequence voltages and the currents are positive sequence currents. Then, in that case, how do you go ahead with the occurrence of an asymmetric fault? So, what do you do? The way we try to find the Thevenin circuit in Thevenin's theorem application means we need the Thevenin voltage source, which is the pre-fault voltage source, and the Thevenin impedance, which was extracted from the bus impedance matrix that was basically obtained from the bus admittance matrix. So, we actually evaluate the positive-sequence and negative-sequence admittance matrices.

The process of evaluating these admittance matrices is exactly the same as we discussed in load flow analysis, but the difference here is that in power flow analysis, we never considered the generators' reactance to be specific; reactance was never considered for fault analysis while evaluating the admit-tance matrices. Remember that all our synchronous machines, to be specific, these can be generators or motors, are represented by their internal EMF, assuming that they are unloaded or loaded. In case the internal EMF is different from the terminal voltages, the sub-transient direct access reactances are present in series between the internal EMF and the terminal voltage. So, the point that I am trying to make is that for positive and negative sequence admittance matrices, one has to consider the presence of jX_d'' in addition to transmission line parameters and transformer parameters. For the zero sequence admittance matrix, the evaluation process would remain the same as discussed above.

But depending on the transformer configuration - whether it is star-connected, star-grounded, or delta-the one which we discussed in the last lecture, the zero sequence admittance matrix has to be carefully obtained, as it extensively depends on the transformer connection. As I mentioned, for a deltadelta transformer, the transformer leakage reactance or impedance will never come into the picture because there is no path for the zero sequence current to escape out of the terminals. For star-connected and delta-connected configurations, in star connection, zero-sequence current cannot flow because the neutral is never grounded. So, on the HV or primary side, if zero sequence currents cannot exist, zero sequence current would not exist here either. So, except in star-grounded transformers, we will have the transformer leakage reactance coming in while evaluating the zero-sequence admittance matrix.

Similarly, for generators that are ungrounded, if the neutral is not grounded, then the corresponding zero sequence impedance of the generator becomes infinite, and hence the corresponding admittance matrix will have a zero element in the corresponding evaluation. So these aspects have to be carefully kept in mind while evaluating these matrices. Once you know these matrices, you can invert them and the corresponding k and k elements. Why, k, k ? Because the fault location is bus k , to find the Thevenin impedance with respect to bus k and the fault point, it could be the neutral or the ground. Thevenin impedances would be k , k elements of the respective zero-sequence, positive-sequence, and negative-sequence impedance matrices.

And once these are known, we can, for the sake of convenience, choose phase A as the reference for symmetric components, and for a line-to-ground fault, to be specific, phase A to ground fault. The way we evaluated the corresponding sequence currents in terms of fault impedances, this expression is exactly the same as the expressions we saw in the last lecture; the only difference you would observe here is that instead of Z_{kk}^{0}, we have just the Z_0 impedance, which is the 0 sequence impedance of the generator. For Z_{kk}^{1} instead, we had Z_1, which was the positive sequence of the generator, and Z_{kk}^{2} instead of that, we had Z_2, which was the negative sequence impedance of the generator. Now, since the impedance position has changed, the fault location has changed. So, we are replacing these impedances with the respective Thevenin impedances.

$$I_k^{a0} = I_k^{a1} = I_k^{a2} = \frac{E^{a1}}{Z_{kk}^{0} + Z_{kk}^{1} + Z_{kk}^{2} + 3Z_f}$$

For a phase B to ground fault, you may wonder what the fault current or sequence current evaluation would be. The component or expressions would remain more or less the same; in fact, they would be exactly the same provided you choose phase b as a reference for the sequence component, and instead of phase a, the numbers coming in here would be I^{b1}, I^{b0}, and I^{b2}. Even if you choose phase A as a reference for phase B to ground fault, the expressions will remain the same; only the physical regulation of the circuit won't be possible. For a double line fault, specifically the b to c fault with phase a as a reference, the expressions remain the same.

For LL fault (b-c) at bus k with fault impedance Z_f

$$I_k^{a0} = 0, I_k^{a1} = -I_k^{a2} = \frac{E_k^{a1}}{Z_{kk}^1 + Z_{kk}^2 + Z_f}$$

There is no ground involved. There is no zero sequence involved. So, the zero-sequence current is zero, and the positive and negative sequence currents are in anti-series to each other, as we have seen in a double line fault. For a double line to ground fault, the expressions are again analogous to what we saw on slide number seven, or sorry, six here. If you compare these expressions with the expressions given over here as well as over here, then the numbers or formulas given in slide number nine exactly match the previous expressions; only here do we have the impedance matrix elements coming in instead of actual impedances. So the question then remains: having known the Thevenin impedance, what is the Thevenin source? The Thevenin voltage source is the pre-fault voltage source, which in the case of no load is the same as the terminal voltage of the bus itself.

$$I_k^{a1} = \frac{E^{a1}}{Z_{kk}^1 + Z_{kk}^2 \| \left(Z_{kk}^0 + 3Z_f\right)}$$

$$I_k^{a2} = \frac{-E^{a1} + Z_{kk}^1 I_k^{a1}}{Z_{kk}^2}$$

$$I_k^{a0} = \frac{-E^{a1} + Z_{kk}^1 I_k^{a1}}{Z_{kk}^0 + 3Z_f}$$

Since you may question or wonder, okay, we have been using E as the internal EMF voltage representation, how come it becomes the same as the pre-fault voltage at the bus where the fault has occurred? Okay, the explanation is very simple. Bus k , where the fault has occurred, may or may not have a generator connected to it. So, in case there is no generator connected to it, maybe it is just a load bus with no rotating load. So in that case, what is the internal EMF of that particular bus? Since there is no generator, there is no notion of internal EMF.

That is point number one. Point number two: when we were discussing Thevenin's theorem application for analyzing balanced faults, did we ever consider the internal EMF of the generator? No. We had determined the fault location to be the bus k fault. where a three-phase solid fault or a Z_f fault has occurred. And we chose bus k pre-fault voltage as the Thevenin equivalent pre-fault voltage. So if we combine points number 1 and 2 , in case there is a fault at bus k with no generator connected to it, then the internal EMF of the generator would be the same as the fictitious load bus or the pre-fault voltage bus.

And Thevenin's theorem clearly explains that the pre-fault voltage is to be used to find the corresponding fault currents. So that is the reason why, for the free fault condition, even if bus k has a generator or doesn't have a generator, we would use its free fault voltage condition to apply the corresponding Thevenin application combined with Fortescue. So, with this as our voltage, once we know the Thevenin voltage, we can find the corresponding sequence currents, and once we know the sequence currents, depending on the type of fault, we can use Thevenin's theorem to find all corresponding sequence post-fault voltages for the corresponding buses. Z capital Z_{ik} is the i , kth element of the respective zero positive and negative sequence impedance matrix. Once we know the post-fault voltages, we can use the A matrix transformation to get back our individual phase voltages at all the buses in the power network.

$$V_i^{a0,\text{ post}} = -Z_{ik}^0 I_k^{a0}$$
$$V_i^{a1,\text{ post}} = V_{ia}^{pre} - Z_{ik}^1 I_k^{a1}$$
$$V_i^{a2,\text{ post}} = -Z_{ik}^2 I_k^{a2}$$

On similar lines, the way we can find the phase voltages, we can find the corresponding phase-wise current injections using the capital A matrix because these currents are already known based on the corresponding fault that has occurred. So essentially, in the pre-fault condition where the load is not present, all bus injections would be zero except for the post-fault condition at bus k . Once we know the corresponding phase current injection at different buses in the load condition, we can also use this A matrix. How do I obtain the line currents? It's always suggested to use or do all the evaluations on a sequence basis first, and then transform the sequence components to the corresponding phase components, specifically in cases where you have transformers of dissimilar configurations. So it is always suggested to carry out the evaluations in sequence mode first, sequence components first, and then at the last end stage, use the capital A matrix transformation to revert to phase quantities.

$$I_{ij}^{a0,post} = \frac{V_i^{a0,post} - V_j^{a0,post}}{z_{ij0}}, I_{ij}^{a1,post} = \frac{V_i^{a1,post} - V_j^{a1,post}}{z_{ij1}}$$
$$I_{ij}^{a2,post} = \frac{V_i^{a2,post} - V_j^{a2,post}}{z_{ij2}}$$

The same notion is applied to the corresponding sequence currents on the lines where small z_{ij}^0, z_{ij}^1, and z_{ij}^2 are the actual sequence impedances of the lines, which are obviously known as per the transmission and parameter evaluation. And once the sequence currents are obtained, depending on whether the transformer configurations are similar or dissimilar. Please revisit the transformer module on per unit analysis, where we extensively talked about this phase shift configuration. In case there is a transformer of dissimilar configuration, let's say one side is delta and the other side is star, whether the star is grounded or ungrounded is not important here, because as long as the star-star configuration exists, phase shift won't happen between the primary and secondary quantities. P here refers to the primary side of the transformer, and S here refers to the secondary side of the transformer.

If we have evaluated our sequence currents, which are the currents shown over here, specifically with respect to positive and negative sequence current, in the case of a delta-star or star-delta transformer, if we have to convert, let's say, I... Let me choose the notation here: $I_{ij}^{a1,post}$ which I have evaluated on the secondary of the transformer, which is star connected, and I want to find the same current on the delta side of the transformer. Then, as per the star-delta phase shift, this current should have a 30 -degree lag on the secondary of the delta head of the transformer.

So, basically, the secondary side here would be $I_{ij}^{a1,\text{ post}}$ with an additional phase shift of minus 30 degrees. Similarly, if I have evaluated this current on the primary side, then if I move from delta to star, I will have a plus $30 -$ phase shift. This is only for positive sequence current. For negative sequence current, the reverse phase shift would happen because the phase shifts that we had discussed were based on the assumption that our network is operating in balanced conditions where inherently positive sequence currents and voltages exist. As the sequence current changes, the orientation of the phase shifts would also change.

So, basically, if let us say I have found $I_{ij}^{a2,\text{ post}}$ on the primary of the transformer which is connected in star configuration and I want to find the same current on the secondary delta side for negative sequence current, then this would be $I_{ij}^{a2,\text{ post}}$. With instead of having a phase shift of minus 30 , this would have a phase shift of plus 30 . That is what is meant by "reverse" being true for negative sequence currents, because the notion or orientation of sequence itself has changed. These phase shifts, as they have been drawn here, are applicable for a, b, c sequence, which is the inherent positive sequence nature of generation of voltages or current. For negative sequence current, this sequence is a, c, b in a clockwise direction.

So, if a, b, c leads to a phase shift of plus 30 from delta to star, the same a, b, c phase shift would bring in a phase shift of minus 30 from prime to delta to star; that is where the reverse nature is. And so this is all about the positive and negative sequence components. You may question, sir, what is there for the zero-sequence current? What is to be done with $I_{ij}^{a0,post}$? So, the answer is very simple. If I have been able to find a non-zero value of $I_{ij}^{a0,\text{ post}}$, which is the zero-sequence current in the post-fault condition, then please revisit the last-to-last lecture discussion where we were talking about sequence impedances of transformers for dissimilar configurations. If I have a zero-sequence current, which should not exist, first of all, that's what I said because here I have a star ungrounded condition.

So, most likely, in fact, not most likely, obviously, you would find a zero current on this side. This non-zero current may exist on the delta side of the transformer, but either way, if you are able to find the zero sequence current on either side of a transformer with dissimilar configurations, the zero sequence current would not be transformed to the other side because the terminals won't be able to experience that zero sequence current due to the dissimilar configuration. As I mentioned, only in the case of a stargrounded transformer, if I have found $I_{ij}^{a0,\text{ post}}$ on one side of the transformer, the same current would flow over here in the same quantum with no phase shift required. Otherwise, in delta-star, star-star, or delta-delta connections, the zero-sequence currents on the primary of the transformer won't be passed on to the secondary at all because the currents can't escape due to the similar configurations. So, after the incorporation of relevant phase shifts in positive and negative sequence currents, when you apply the capital A matrix operation, you will be able to get the actual phase currents, and that is where the sequence current relevance is important.

If you have a power network where such dissimilar configurations of transformers don't exist, then essentially you need not evaluate or do this step for transformers with dissimilar configurations, with similar configurations specifically star-grounded and star-grounded. If you try to evaluate the phase currents in the lines with respect to the phase voltages of the buses, you would get exactly the same current values as we have done in the corresponding phase. But in case you wonder, "Okay, I have a star-star transformer; can I still apply or find the corresponding phase line currents from the corresponding phase bus voltages?" You have to be extremely careful in doing that because, in this sort of configuration, zero sequence current will never pass from one side to the other. So it is always safer to evaluate the sequence currents of buses, the sequence components of voltages, and the sequence components of line currents. After you have done with all the sequence components, incorporate the relevant phase shifts if they are required, and at the end, try to make use of the capital A matrix to find the corresponding phase voltages, phase current injections, and phase line currents.

So that's all for this discussion. We will take up two numeric examples at length in the next lecture to further understand how we actually do the asymmetric fault analysis using Thevenin's theorem and Fortescue's theorem for an actual power network. Thank you.