Power Network Analysis

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Week - 10

Lecture-47

Hello everyone, welcome to lecture 2 of week 10 of the course Power Network Analysis, in which we continue our discussion on the second-to-last module, which is fault analysis. And in this discussion, we will understand a very interesting theorem given by C. L. Fortescue in 1918, pardon me, where he came up with this indigenous concept again named after himself for the analysis of faults in three-phase networks, specifically for unbalanced faults or asymmetric faults. The associated discussion that we had yesterday, not in the previous lecture which was on balanced fault analysis. The same analysis, if it were to be applied to unbalanced or asymmetric faults, means that the associated equations one has to deal with become highly complex; they become coupled and eventually unsolvable.

I mean there is a way to solve them, but then the associated burden would be very high. So C.L. Fortescue came up with this concept, which he named after his own name, known as Fortescue's theorem, in which, in 1918, C. L. Fortescue stated that any n-phase network or any n-phase systemspecifically, when I say n-phase system, I am referring to n phasors where n can be greater than or equal to 2 . Obviously, n belongs to the set of integers, positive integers to be precise. For n equal to 2,3,4, and 5 , for a 3 -phase system, n is equal to 3 . He stated that all these n -phase systems, where we deal with n voltage phasors or n current phasors, could refer to phases or line-to-line quantities.

All these n phasors can be represented as n systems of balanced phasors. That means each phasor in this n-phase system can be represented as n individual components, and each of these components would pertain to a system of balanced components. So with this, what happens is that CL Fortescue came up with this transformation, and in this transformed way of analyzing, the unbalanced or asymmetric faults become easier because the associated equations become decoupled; they are independent. And these systems of balanced phases are essentially what is known as sequence components or symmetric components. So let us see in detail what it is all about.

So asymmetric faults or unbalanced faults are all faults except for the three-phase fault. In a three-phase system, for n equal to three, the possibilities of unbalanced faults in the power network can pertain to single line-to-ground faults, line-to-line faults, or double line-to-

ground faults. So just to give you an idea of what I mean by a single line to ground fault. If I have a transmission line with phases a , phase b , and phase c , and I have to understand what a single line-to-ground fault is, then probably in one such case, if phase c gets shorted to the ground through some fault impedance Zf, I would call this particular single line-to-ground fault a phase c-to-ground fault, referring to ground. There could be two other possibilities: instead of phase b and c getting shorted, phase b might get shorted to the ground through a fault impedance, or the other alternative, the last possibility, is that instead of phase b, phase a itself might get grounded while the other two phases remain healthy and carry the normal load current.

So in a single line-to-ground fault, we can have three possibilities: phase a, b, and c , respective to the grounds. For line-to-line faults, the ground does not come into the picture; any two phases are getting shorted through each other due to some fault dependence. Lastly, the third category is double line-to-ground faults, which involve two phases and the ground that are shorted through some fault impedance Zf. So basically, we would have nine such asymmetric or unbalanced faults. And why are they called asymmetric? They are called asymmetric because the corresponding post-fault voltages and currents that we would observe have still not been figured out in terms of how they are to be obtained; they would inherently remain balanced regardless of whether the system in the previous condition was balanced or not.

Now, what do I mean by balanced or unbalanced? We have briefly talked about it in the basic circuit principles module, in which we say that a balanced phase system is specifically for a three-phase system. If I am talking about a balanced three-phase system, then in that balanced three-phase system, the voltage phasors for each of the phases have equal magnitudes and are spaced 120 degrees apart, corresponding to the orientation in which these three phasors of voltages can exist. We associate them inherently with the positive sequence voltage because our three-phase synchronous generators coincidentally generate this three-phase positive sequence voltage. So positive sequence voltage is one such concept of a balanced system. By balanced, I mean here specifically that the corresponding phasors that existed in the network before the fault occurred correspondingly pertain to positive sequence voltages or currents.

Positive sequence voltage or current in which all three phases of a particular voltage or all three phase voltages have equal magnitude and are 120 degrees spaced apart with positive sequence orientation. There can also be another balanced phase system that could be called a negative phase system in which the magnitudes of the phases are the same, the phase angles are 120 degrees apart, but they are in a reverse orientation compared to the positive sequence orientation. And lastly, the third balanced system that can exist in three-phase systems is the zero-sequence component, which coincidentally can exist as a common system for any

n -phase system where n could be 2,4 , 5,6 , and so on. So the unbalanced faults or asymmetric faults inherently produce a post-fault unbalance, no matter whether the pre-fault condition was balanced or not. And the problem with unbalanced analysis, as I mentioned at the beginning of our discussion, is that because of coupling, which arises from the associated equations, solving those equations becomes difficult, and hence sequence components or symmetric components play a very important role.

For balanced systems or balanced faults, we have understood the application of Thevenin's theorem, and since everything remained balanced in both pre-fault and post-fault conditions for balanced faults, the use of a single line per unit diagram was sufficient to apply Thevenin's theorem. The same Thevenin's theorem, although with a little bit of extension, may be used for unsymmetric or unbalanced faults, but then the bridge to applying Thevenin's theorem has to come with the application of the symmetric components or sequence components for unbalanced faults. Phase-based analysis for Thevenin's theorem application for unbalanced faults won't be feasible or realizable because of the coupling effect. So, as I mentioned Fortescue's theorem, it essentially results in resolving a set of unbalanced three-phase systems. This three-phase system could pertain to voltage phasors or current phasors.

Any unbalanced voltage or current phase system can be resolved into three distinct systems, where each of these systems would inherently remain balanced no matter what the level of unbalance present in the actual network is. In a three-phase network, those components are called positive, negative, and zero. We have briefly discussed this in the basic circuit principles as well. And there is a dedicated proof that exists according to Fortescue's work, which states that this n-phase system can always be resolved into n systems of balanced systems, and the beauty of such a transformation is that these systems would all remain decoupled. That means a positive sequence element or component will never appear in a negative sequence circuit, and a zero sequence component or circuit element will never appear in a positive sequence circuit, and that's how these three components or systems are perfectly decoupled or independent of each other, which aids in analyzing our unbalanced or asymmetric faults.

As decoupled as I mean the system to be, each system is independent of the others; balanced phases, as I mentioned, are each of the phases in a three- or n-phasor system, and they are to be of equal length with equal angles among each other. These components would be called symmetric or sequence components. In physicality or the real world, these components or systems don't exist. They are just a mathematical premise for creating a bridge between the actual unbalanced quantities and the corresponding fault analysis that we would conduct. Although there are ways of measuring this, if one were to specifically pinpoint an element where a negative sequence current flows, please identify it for me.

It's difficult to pinpoint. Although negative sequence current flows in its own way, there are ways to measure it. But in reality, these components do not exist in the actual network. That's what I mean to say. For example, if someone asks me or asks you to identify the part of the synchronous generator where negative sequence currents can exist, identify it for me.

They can exist in the network as it is, but it is very difficult to pinpoint or identify that, okay, in this particular winding, negative sequence current is flowing. That's what I mean by their fictitiousness and lack of physical presence in reality. These components can, as I said, be in terms of currents and voltages, and on similar lines, sequence impedances can also be defined, although the definition of impedances is a little bit different from sequence voltages and currents. So we will first understand what we mean by these sequence voltages as defined by Fortescue's theorem, and then we will try to see what I mean by the sequence impedances and what the differences are. So, for a three-phase system, three-phase power network, three-phase power network.

Fortescue's theorem says that if n is equal to three, a system of independent balanced systems can exist, and those systems are nothing but positive sequence, negative sequence, and zero sequence in terms of voltages. These phasors can also be represented in terms of corresponding currents. These currents could refer to line-to-line currents, line currents, and phase currents. In terms of voltages, they could also pertain to phase voltages or line-to-line voltages. Positive sequence voltage is the inherent form of operation or natural form of operation of a balanced power network because, as I mentioned, synchronous generators inherently produce these voltages as per their design.

Negative sequence and zero sequence voltages may turn up mathematically, or negative sequence or zero sequence currents may turn up mathematically under certain unbalanced conditions. As long as the system is balanced, we would have only the presence of positive sequence voltage for balanced operation. For balanced operation of this three-phase network, these components will never exist, and we will see that as we progress in these slides or in the next lecture slides. That is one aspect. The other aspect is that Fortescue's theorem only says that for n equal to 3, there can be 3 distinct components: positive, negative, and 0.

Fortescue's theorem also added one more definition that states if I have, I mean don't confuse these with Vc, Va, and Vb. I am just going to write a specific equation. Suppose I have a phase A voltage pertaining to the output of a three-phase synchronous generator, let's say the terminal voltage, which is, let's say, Va. I mean, this is a common bus, but these buses might be different. I have Va, Vb, and Vc as the three terminal voltages, the phase voltages of the three-phase synchronous generator.

And I want to understand how these three-phase voltages, Va, Vb, and Vc, relate to the corresponding sequence components: positive, negative, and zero sequence components. So Fortescue stated that if I have to denote phase A voltage in terms of the corresponding sequence components, which for phase A correspond to Va1, which is the positive sequence phase A voltage, Va2, which is the negative sequence phase A voltage, and Va0, which is the zero sequence phase A voltage. Then Va is nothing but Va1 plus Va2 plus Va0. This is again as per Fortescue's theorem, and a well-dedicated proof for this sort of analogy already exists in the literature. Similarly, for phase b voltage, if I have to express it in terms of the respective sequence components, which are Vb1, let me write those as Vb1.

So actually, this cross here pertains to Vb1, Vb2, and Vb0. So phase V voltage is Vb1 plus Vb2 plus Vb0, and similarly, phase C voltage would be Vc1 plus Vc2 plus Vc0, where Vc1 is the phasor defined over here, Vc2 is the phasor defined over here, and here we have Vc0. Now this is again as per Fortescue's theorem, and the beauty of this co-relationship is that at the outset it might appear that on the right-hand side of these three equations there are nine variables that need to be identified, three for every such phase. But in practice, according to Fortescue's theorem, there are nine variables at the outset that need to be identified for determining the sequence components. But according to Fortescue's theorem, if we identify only three variables, then the remaining six variables remain known.

So there are three variables in reality. If we know these three variables, then the other six terms or all nine terms in the RHS component will become known. What would those three variables be? Those three variables correspond to the sequence component of any one phase. So let's say I choose phase a as my reference phasor or suitable phase reference, and if I identify the positive sequence voltage of phase a, the negative sequence voltage of phase a , and the zero sequence voltage of phase a , I can also find Vb1. What would Vb1 be? Vb1, as per the positive sequence diagram, is lagging Va1 by 120 degrees.

So Vb1 would be Va1 at an angle of minus 120 degrees, which is precisely defined over here. Similarly, if I have to find the negative sequence voltage of phase C, then as per the negative sequence diagram, Vc2 is lagging Va2 by 120 degrees, so Vc2 is Va2 at an angle of minus 120 degrees. As per the zero sequence diagram, where phase voltages are again equal and the phase angle is zero degrees, Vb0 is the same as Va0, which is the same as Vc0. So in a way, if I know what these three sequence components are, I can identify the remaining six components. So that's where the uniqueness comes in.

And in practice, this sort of equation becomes solvable because I have three variables in reality that pertain to any phase sequence component. And I also have three equations from which these three variables can be solved. It's a win-win situation, n equations to be solved

for n unknowns. And that's how Fortescue's theorem can be physically realized. So with this, let us go a little deeper into what the mathematics involved are.

I hope this understanding and this discussion are clearer to all the viewers. So, to sort of simplify our discussion, we will define a complex operator a, which is a phasor of magnitude 1 and a phase angle of 120 degrees. And as per the diagram of positive sequence voltage, if I have to express Vc1, which is Va1 at an angle of 120 degrees and having defined small a, please do not confuse this a with the phase a reference; this is a different operator which is being defined as a at an angle of 120 degrees, then Vc1 is nothing but small a Va1. Similarly, Vb1 is actually leading in a way; if I were to say it is leading Va1 by 240 degrees, where a square would be one at an angle of 240 degrees, then Vb1 would be a square of Va1. You could also say that a square is also equal to a conjugate.

Yes, a square will also be equal to a conjugate because a conjugate is 1 at an angle of minus 120 degrees, which is the same as 1 at an angle of 240 degrees. So, a, the complex operator that we define here, becomes very critical, and this value of a satisfies these two equations. Please verify; it can be easily seen why this is verified. So if a is one at an angle of 120 degrees, a square, as I mentioned, is one at an angle of 240 degrees, and a cube would be one at an angle of 360 degrees. 360 degrees is equivalent to 0 degrees, which is the same as 1 , and that is why a cube is equal to 1 , and a square plus a plus 1 is equal to 0 , which is well observable through the respective positive sequence or negative sequence diagrams.

If I associate this as 1 at an angle of 0 , then on this side I have a and on this side I have a square. some of these three phases, the resulting component is always going to be zero and that's why this equation is valid. Using this a operator, I can define phase a voltages in terms of the corresponding sequence components with respect to phase a again. Va0 and Va1 are sequence components of phase a, and I can define this capital A matrix, which is in terms of 1 and the small a operator. The inverse of this matrix is also a beautiful similar matrix compared to the original matrix, and this equation tells us that I can find the phase components and symmetry components using the inverse of the phase voltages.

$$V_a = V^{a0} + V^{a1} + V^{a2}$$
$$V_b = V^{b0} + V^{b1} + V^{b2} = V^{a0} + a^2 V^{a1} + a V^{a2}$$
$$V_c = V^{c0} + V^{c1} + V^{c2} = V^{a0} + a V^{a1} + a^2 V^{a2}$$
$$\Rightarrow \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V^{a0} \\ V^{a1} \\ V^{a2} \end{bmatrix}, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$
$$\Rightarrow V_{abc} = A V^{a012}$$

Similar relationships also exist for the currents. For balanced systems, by balanced I mean that inherently I am working with positive sequence voltage, which in this case particularly

refs to positive sequence operation because that's an inherent nature of the operation of a power network. As you can see, Ia, Ib, and Ic, if they are all positively sequenced, arranged negative and zero sequence currents won't exist. So these three phasors would eventually add up to zero, and that's the reason why zero sequence currents can never exist in balanced positive sequence operation networks. For unbalanced systems when they are ungrounded, the zero sequence current will still be zero.

The answer lies in the notion of the part being ungrounded. If I have a star-grounded connection and this neutral point, let's say if this is a load, the three-phase load is carrying currents Ia, Ib, and Ic, and this load here is star-connected, but it is not grounded; the neutral point is still open. If I apply KCL at this neutral point, what I would observe is that the phase currents Ia plus Ib plus Ic, their sum will always be zero because of the KCL at the neutral point. So no matter whether phase A , phase B , or phase C currents are balanced or unbalanced, for ungrounded systems where the neutral points are not connected to the ground, the zero-sequence currents, which are again the average of the individual phase currents, would again be zero because of KCL at the neutral point. For unbalanced grounded systems, the zero-sequence current need not be zero because, if it is now grounded, a neutral current can also exist, which would be three times the zero-sequence current.

In terms of power, the three-phase powers can be expressed in terms of this matrix product, which again pertains to three types of individual sequence-based powers. This is basically zero sequence power, positive sequence power, and negative sequence power. If the components themselves do not exist, these power definitions and notions are also fictitious quantities, just mathematical definitions. Coming to impedances, the notion is a bit different. The notion here is that for a given positive sequence circuit, if there exists a positive sequence voltage that is leading to some positive sequence current, then the corresponding impedance offered to the flow of this positive sequence current in the positive sequence circuit would be called positive sequence impedance.

The way sequence components are extracted from three-phase voltages or currents, the same method of extraction won't hold true for impedances; it is just that the impedance offered to positive sequence current would be called positive sequence impedance, negative sequence current would have the impedance it is experiencing called negative sequence impedance, and similarly for zero sequence impedances. So no direct implication, or I would say these don't; I mean they do depend on the phase impedances, but their relationship in terms of sequence voltages or currents, the same notion doesn't hold true for impedances. So let's see what those impedances look like. So what I have considered here is a very generic three-phase star-connected or star-grounded load, and this load is balanced; that means each phase has the same impedance observed across it. These three-

phase balanced loads might also have mutual coupling along with them, and the neutral point, which is this point over here, is grounded in terms of a neutral impedance Zn .

So if I have to understand what the corresponding positive sequence impedance, negative sequence impedance, and zero sequence impedance of the three-phase balanced load, which has self-impedance Zs per phase, Zm as the mutual impedance, and Zm as the neutral impedance are, how do I proceed? It is always good to start with the basics in terms of analyzing
this three-phase balanced load; what would we do? We would apply KVL and KCL to get the corresponding equations for the given knowns and unknowns. We'll solve the corresponding equations and then understand how this load actually behaves. So, let's do a similar exercise here. If I apply KVL for phase a voltage, which is, let's say, the circuit over here, if I apply phase KVL between phase a and the ground point, then I would eventually traverse the path along phase a, come to the neutral point, and reach the corresponding ground point. So if I write the corresponding equations as I am starting with phase A terminal, I will have Va minus Ia into Zs, which is the voltage drop across this self-impedance.

Remember these impedances are also mutually coupled; the dot polarities have not been marked, but assuming that the dot polarities are all additive, then I would also have a voltage drop because of other phase currents, which would be IbZm and similarly IcZm. That leads me to this neutral point. After the neutral point, I have Zn impedance, so I have minus InZn, and then eventually I reach the ground point of the neutral point, which has a potential of zero. Now, what is the expression for this neutral current? By KCL at the neutral point, the neutral current is nothing but Ia plus Ib plus Ic. So instead of In, I can put these three currents over here.

Similar equations can also be written for phase b and phase c. I'll have the three-phase system, which is essentially what is shown or marked here. Since the neutral current In is Ia plus Ib plus Ic, every current, the 3×3 matrix of the Z matrix consists of the Zn component, and I know for sure that this 3×1 matrix Vabc is nothing but the capital A matrix assuming phase a as the reference for sequence components which are Va0, Va1, and Va2. Similarly, the phase Iabc currents are capital A assuming phase a as the reference for sequence components Ia0, Ia1, and Ia2. So I can eventually convert these phase quantities into corresponding sequence components and do some mathematical observations, which is where I get this as my corresponding matrix; the value of capital A has already been discussed or given in the previous few lectures.

$$V^{a012} = A^{-1} \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix}$$

$$\Rightarrow Z^{012} = \begin{bmatrix} Z_s + 3Z_n + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

And now, if I evaluate this expression, I would get this 3 by 3 matrix, and I would call the first element here the zero sequence impedance, the second component the positive sequence impedance, and the third component, which is coincidentally the same as the positive sequence impedance, we will call the negative sequence impedance. How do I get there? The arrival at this conclusion comes from the definitions of Va012 and Ia012. If I expand Va012, it is Va0, Va1, and Va2. And similarly to Ia02, I have Ia0, Ia1, and Ia2. And if I go back and look at my definition of impedances as per this statement, the impedance offered to the flow of zero sequence current because of zero sequence voltage would be called the zero sequence impedance, and that is the reason why the first term is zero sequence impedance.

Similarly, the second term, which relates Va1 to Ia1, is positive sequence impedance, and lastly, the third term correlates Va2 and Ia2. So, respective impedances lead to corresponding drops in their respective components only; they do not mingle or cross each other. I intently chose this three-phase balanced load with mutual impedances, where the resulting equations, if I consider them in terms of impedances, formed a full 3×3 matrix. The moment I converted into the sequence impedance part, I only saw a 3×3 diagonal matrix.

The other off-diagonal terms are all zeros. Relating to that, positive sequence has nothing to do with zero sequence. Negative sequence has nothing to do with the positive sequence. And that's where the decoupled part comes in. For a balanced load, the three sequence networks appear to be independent, as per this 3×3 matrix. In similar lines, we can have definitions of impedances for transmission lines, transformers, and static devices.

The orientation or notion of positive and negative sequence doesn't matter because of geometry; it does not change irrespective of the sequences, and hence for transmission lines, positive and negative sequence impedances are the same as the actual line impedance, which we have discussed and evaluated in module number 4, which was, I think, transmission line parameter evaluation. The same is also true for transformers, except that instead of line impedance, here the transformer's leakage impedance comes into the picture, which is the same as the positive and negative sequence impedance of the transformer. Zero sequence impedance significantly depends on the way the ground is connected or whether the shielding wires are present. So, the zero sequence impedance of lines is usually 3 to 5 times that of the corresponding positive sequence or negative sequence line

impedance. For transformers, it all depends on the configuration of the transformer, whether it is star connected, star ungrounded, or delta. The configuration of the transformer may or may not allow the flow of zero-sequence currents, and it is all because of the fact that if current has to flow or appear in the primary winding, it gets linked due to MMF balance resulting from the flow in the secondary winding. This aspect we have extensively discussed when we were discussing transformers: how the primary gets to know about the change in secondary current and why the primary also responds. It is all because of the MMF balancing aspect, the physics that goes behind the operation of transformers. So in the case of transformers with different configurations, if the secondary circuit has to carry current, the primary might carry current depending on the configuration, thanks to the MMF balancing aspect. If zero sequence current flows, then the zero sequence impedance would be the same as the leakage impedance of the transformer.

Also, in terms of transformers for similar configurations, we have seen this 30 -degree phase shift between star and delta transformers thanks to the way they are connected. So, in order to understand the impedance and zero-sequence impedance for transformers, we would actually make use of this MMF balancing act and understand whether zero-sequence impedance can exist or be present for three-phase transformers. So for three-phase networks or three-phase transformers, we have four possible configurations of transformers: star grounded and star grounded. If zero sequence current can flow, it will be the current I_n, which will be three times the zero sequence current on the primary side. Here we might have I_n2, which is not I_n2; let me say I_n HV and I_n LV, which is three times the zero sequence current on the LV side and the HV side.

If this current can exist because of the neutral current being available, then here also the zero-sequence current can flow because we also have a grounding connection, and that's the reason why this circuit pertains to the associated zero-sequence representation of a star-grounded transformer. If zero-sequence current can flow on the HV side, a similar current can also flow on the LV side. There is a possibility of zero-sequence current flowing through the zero-sequence impedance. In case the neutral is ungrounded, the current might exist here, but if there is no path for the neutral current to flow to the ground, then zero-sequence current has to flow to the ground. Then here InLV is exactly 0 , and if the secondary current is 0 by the MMF balancing act, this current, although there is a path, would also be 0 , and that is what indicates this break between a and a'. a is, let's say, the primary side, which could be the HV side or LV side, and a' here refers to the secondary terminal. The primary and secondary terminals in the zero sequence circuit for a three-phase transformer are unconnected for star grounded and star ungrounded because zero sequence current cannot exit out of the terminals. For a star grounded in delta configuration, the zero sequence current can exist because there is a path for the current to

flow. But on the terminal or the secondary side, zero-sequence current cannot escape from the terminals. It can flow, but essentially, the zero sequence current flows as a circulating current.

It does not escape on the terminals. And that is the reason why we have this sort of configuration for star-grounded delta connection: because on the primary or HV side, zero sequence current can exist, which can go through the ground, indicating this component, but it cannot exceed the terminal on the secondary side because of the delta connection. And lastly, for star ungrounded and delta, the current cannot exist, so the series break indicates that break, whereas on the delta side there is no escape possible. Although this part is grounded, the corresponding break here indicates that delta connection. For a delta-delta transformer, none of the terminals can have the presence of zero-sequence currents because there is no path for the zero-sequence current to escape. So, in a way, the zero-sequence currents won't exist on the primary or secondary side of a delta-delta transformer.

In case the neutral ground is connected, which is indicative of this neutral impedance, then, as we have seen in terms of three-phase balanced load, this was the term over here. Three times the neutral impedance was appearing as the zero sequence impedance. The same is also applicable for transformers. If the transformer is grounded through some impedance Z_n , then it will have three times its component present over here. For synchronous machines, we will discuss the corresponding sequence components in the next lecture.

Since we're talking about fault analysis, synchronous machines have to be represented by the internal EMF and the subtypes in the reactance. So we will see with Xd double dash how the sequence components or impedances would look. So that's what we will continue in our next part of unbalanced fault analysis. Thank you so much.