Power Network Analysis
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Week - 10
Lecture-46

Hello, everyone. Welcome to the first lecture of week 10 of the course, Power Network Analysis, in which we continue our discussion on fault analysis. And in the previous lecture, we extensively talked about the importance and application of Thevenin's theorem in fault analysis. We also talked about certain assumptions that would help simplify our analysis and keep the computational burden as low as possible while finding out the worst-case fault currents. And in today's discussion, we will take up the application of Thevenin's theorem to understand in detail how balanced faults and threephase faults can be analyzed in a very unique, beautiful, and simpler fashion. So what we have here is a typical N-Bus network.

We have also seen this sort of network while we were discussing power flow analysis. So I have, in fact, chosen and taken up the same diagram from the power flow analysis module. What we have here is an N-bus network where buses are numbered from bus 1 to bus N , and this power network, which I have considered an N-bus power network, is operating in a balanced condition. We have talked about balanced and unbalanced circuits a bit in basic circuit principles.

Those of you who are eager to know more can refer to those particular basic circuit principles lectures, or you can wait until the next lecture, where we will also have an extensive discussion on what is balanced and unbalanced. So essentially, I am assuming that this N bus network is balanced and is operating in a balanced condition. That means all voltage phasors of the three phases are equal in magnitude, and the corresponding phase angle difference between these voltage phasors is 120 degrees apart. The same logic also holds true for the three-phase currents; they could be in phase domain or line to line. So, an important part is that this N bus network is operating in a balanced condition, and hence the single line or per unit based representation is applicable for this N bus network, where instead of representing the three phases separately, just a single phase representation on a per unit basis is good enough to represent the actual three-phase balanced network.

So, with this as our premise, we have an N-bus network that is balanced, can be represented by a single-phase representation, and before the fault has occurred or before any disturbance has occurred, we know for sure what these equivalent single-phase per unit balanced voltages or single-phase voltages are at each of the buses 1 to N . We choose bus k to be ours, so essentially these arrows here indicate the presence of any source or load that might be present in each of these n-bus networks. Depending on whether the source is present, the injection will be positive, so this arrow is applicable. If there is a load present, specifically a rotating load, then the arrow direction would change because the corresponding injection would be negative. So we are not very bothered about what these injections are as of now, but we know that, okay, in the pre-fault condition, before the fault has occurred, the pre-fault voltages are all known at all the buses.

And we choose bus k as our bus where a fault is likely to occur, and we call that fault a three-phase fault. Since the network is balanced in the pre-fault condition and a three-phase fault is now occurring at bus k , how would it occur? If this switch is closed, then it would indicate that bus k is grounded; basically, this is the ground connection. By closing the switch, it would imply that bus k is now grounded through a fault impedance Zf, and since this is a per unit based representation of a three-phase network, balanced three-phase network, on a three-phase basis, it would appear that all three phases, phase a , phase b , and phase c of bus k , which are Vak, Vbk, and Vck in the phasor domain, in the time domain or respective phasor domain, all three buses are equivalently, all three phases of the same bus are equally connected to the ground through this common fault impedance Zf. Since we are talking about a balanced network and balanced fault, we have considered the single-phase representation of this fault impedance Zf, and this three-phase fault is going to occur due to the closure of the switch. So, basically, our points of interest are the bus itself and the ground in which a new event is going to happen compared to the pre-fault condition where everything was perfectly fine.

To analyze the impact of this fault, we will have to find the corresponding Thevenin equivalent circuit between bus k and the ground because it is between these two nodes or points that this three-phase fault is going to occur. So, essentially we have to find the Thevenin circuit of the actual power network as seen from the two points of reference: one bus k and the other being ground z , the ground point itself. The question here is whether Thevenin's circuit has to be found so that Thevenin's theorem could be applied to understand the implications of this three-phase fault. So the point of interest is how do we find this Thevenin circuit, or before we apply Thevenin's theorem, how do we find the corresponding Thevenin circuit? For the time being, we will assume that by some analysis we are able to find Thevenin's theorem, and while finding Thevenin's theorem, which consists of the Thevenin voltage and the Thevenin impedance, it is important to represent all rotating devices-synchronous generators, induction motors, and synchronous motors-by their respective direct axis transient reactances. So in addition to considering the N bus network, which is highly meshed, this circle here basically means that this N bus network is highly meshed; a lot of lines exist inside this encircled network, and while finding the Thevenin impedance Zth, we have to consider all network line impedances.

Basically, if we ignore the shunt, susceptance, and resistance, we will have to consider all line reactances, transformer reactances, which are part of the power network itself, and also the direct access subtransient reactances of the rotating machines, which is an important aspect. And while finding the Thevenin circuit, we have to be careful in shorting all voltage sources, with generators being represented by the subtransient reactance. So this part needs to be considered carefully. We will also understand this aspect as we discuss a numeric example toward the end of this lecture. So suppose, by some analysis, we don't know for sure what that analysis is regarding how to find Zth and how to find Vth.

Suppose we get to know that Vth and Zth pertain to the pre-fault condition of the network. Remember, for analyzing or applying Thevenin's theorem, in fact in the previous lecture as well, the Thevenin circuit is always to be found in the pre-fault condition before the switch is closed. Now once you find Thevenin's theorem, you get to know what Vth and Zth are. We do not know for sure what those values are, but now you can apply Thevenin's theorem and impose the fault event where the switch is closed. So, now when the switch is closed, a current would likely flow from the bus to the ground, and the implication or direction of this fault current is important; we will spend a moment here to understand why the fault current has to flow from the bus to the ground, or for that matter, why Vth polarity is of this particular order: the positive polarity is connected to the ground.

Suppose the Vth polarity is plus and minus compared to minus and plus shown over here, then what would happen? Instead of flowing downwards from bus k to the ground, current would flow from the ground to the bus. Which would mean that, okay, fine, if this phenomenon were to happen, that means if Vth were to be of polarity plus-minus as shown above, then in that case the current would flow from the ground to bus k , which would be a win-win condition for the power network, because without doing anything we are able to get a new additional source of current, which is the ground in this case, and this ground is capable of supplying our loads. Does it sound practical? Definitely not. That is the reason why this sort of polarity is not applicable at all. It is a fault that has occurred; it should not; a fault is anything that interrupts the flow of normal load current.

If the polarity is now reversed from minus plus to plus minus, the fault would help supply loads, and we would be able to cater to loads without generating anything new, and that definitely is not possible. So that is the reason why Vth polarity is such that during a fault, current is drawn from the network to the ground, which otherwise could have gone to some useful load if this fault had not occurred. That is the reason why Vth has the polarity shown like this, so that the current flows into the ground from the actual network. So with this new current that is going to flow, it is definitely going to bring in a few changes in voltages. Again, our question mark points are: what is this Vth, and what is this Zth? Only when we know Vth and Zth can we know what If is.

So, suppose we know Vth, Zth, and that this fault has occurred. So, the fault current is being drawn from the network to the ground, and because of this fault current, the voltages at the buses might have changed. Let us indicate those changes by this vector delta V , which is of the same dimension N cross 1 as V pre, which was the pre-fault voltage.

$$\Delta V = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix}$$

By Thevenin's theorem, if we can know what these delta V values are, then we can apply the superposition theorem and determine our post-fault voltages, which would be the superimposition of the pre-fault voltage and the change in voltages. This change in voltages is dependent upon the Z bus matrix, which was obtained from the Y bus, which is its inverse, and a current injection vector, which is again an N cross 1 matrix.

$$\Delta V = \mathbb{Z}^{bus} \Delta I$$

Remember, this Y bus needs to be evaluated while considering the generators' direct axis sub-transient reactances, along with other network impedances, transformer reactances, etc. So now, if we get to know what delta I is, because Z bus can be known from the inverse of Y bus, then only can we know what delta V is. Now, if we think about what delta I is, which is essentially, what is delta I? Delta I is the change in bus injection currents due to a three-phase fault between bus k and ground. This is basically going to be an N cross 1 vector, which is essentially encapsulating all changes in currents that are going to happen because of the three-phase fault that has occurred between bus k and ground. Now, if we think for a moment, delta I is an N cross 1 vector, so this is essentially equal to delta I1, delta I2, all the way to delta Ik, and the last element is delta I capital N.

$$\Delta I = \begin{bmatrix} \Delta I_1 & 0 \\ \Delta I_2 & 0 \\ & \vdots \\ \Delta I_k & -I_f \\ & \vdots \\ \Delta I_N & = 0 \end{bmatrix}$$

We want to find out what this delta i vector is. If we know delta I , we can find delta V. And possibly, we already know pre-fault voltage V pre, so we can find V post. And then our purpose would be done. Now, if we think for a moment about what delta I is going to be, do you think delta I1 is going to be 0 or non-zero? OK, let's see.

Delta I1 is the change in current at bus 1 due to a fault between bus k and ground. If the fault has occurred between bus K and ground and a current is flowing from bus K to ground, why should the current in bus 1 change at all? The scenario has not actually changed on bus 1 itself. At least directly speaking, bus 1's current has not encountered any change at all. It is only bus K that is experiencing this change in fault current, which is If, and it again depends on what Vth and Zth are. So why should bus 1's current change, and that's the reason why all currents would be exactly equal to 0 except delta Ik? Now remember, when we defined bus injection currents, we inherently assumed these injection currents to be currents flowing from the bus into the network.

During a fault, current would flow from the network to the ground. That means delta I k imperatively, which would mean any injection that is happening from the bus to the network. In case of fault, delta I k is equal to minus delta If because If is flowing from the network to the ground. So,
this current is actually being drawn from the network to the bus. And that's the reason we have a negative sign here.

The directions and notions are different. So delta IK is the only non-zero element present in the delta I vector, as all other generators are unaffected; they are shorted. All generators are represented by the internal EMFs and corresponding reactances of red taxis. While finding Thevenin impedances, we consider all voltage sources to be shorted. So, all other voltage sources are null and void; their voltage sources do not play any role in the overall Thevenin circuit that we are going to look at.

So, all other generators are shorted, and hence all other bus currents are zero; only delta I k is the non-zero current. Now, if we sort of substitute this delta Ik non-zero element, which is minus delta If at the kth position sitting over here, this is the kth element of delta I. And put in our V post voltage, which is N cross one vector in terms of N cross one pre-fault voltage and the N cross N Z bus matrix that we have. What we would see is that all post-fault voltages would only depend on the actual pre-fault voltages and the kth row of the Z bus matrix, because all currents here are 0 except for the kth element, so the contributions of these elements won't come. All post-fault voltages would depend only on the kth column of the Z bus and the corresponding If value that has occurred or changed at bus k .

So, if we look at the bus k equation, then Vk post would be equal to Vk pre plus ZIk multiplied by minus delta If, because all other elements-pardon me, I made a mistake here - instead of the post-fault voltages depending only on the kth row of the Z bus, they would actually depend on the kth column of the Z bus because only the kth element of delta Ik, delta I, is non-zero. So if we look at bus k equation and focus on the kth column of Z bus, only the Zkk term is appearing here which is in product with minus If. So we have the Vk post as this equation. Also, as per this circuit here, if the fault has now happened, the switch is now closed, the voltage of bus k is Vk post itself, and by KVL between bus K and ground, it is nothing but If into Zf. That means this term has to be equal to If into Zf because the post-fault voltage at bus K is the voltage after the switch is closed, so this new bus voltage is Vk post.

From here, if we sort of compare from this equation, If should be equal to Vk pre divided by Zkk plus Zf. That is one equation that we get because after the voltage of the fault has occurred, the post-fault voltage by KVL should be IfZf, and by Thevenin's theorem, it should be the pre-fault voltage plus the change in voltage that might have occurred. If we compare the current with respect to KVL shown over here, then it is also equal to Vth by Zth plus Zf. Now compare these two equations that we have. This is the equation that we are getting by KVL from ground to ground.

Vk post is equal to IfZf by applying KVL between bus k and the ground. If we compare these two equations, we see a great deal of similarity. The similarity is that the Thevenin voltage happens to be the same as the prefault voltage, which is Vk pre, and the Thevenin impedance is nothing but the k -th element of the Z bus matrix; remember that in Thevenin's theorem application, we always focus on the pre-fault condition. We do not consider the impact of faults yet while evaluating the Thevenin circuit, and by comparison of elements, what we see here is that the Thevenin circuit we are getting in terms of Thevenin voltage and Thevenin impedance. They also happen to be the pre-fault situation, not the post-fault situation.

That is where Vk pre is important. Zkk is the k , k element of the Z bus matrix, which was evaluated from the Y bus inverse, where the Y bus was obtained by considering all voltage sources to be shorted, with generator subtransient reactances and other network elements only being present. That's the logic or a sort of reverse engineering process of obtaining or finding a Thevenin voltage and Thevenin impedance. Once you know the Thevenin voltage and Thevenin impedance, you can find the fault current. Once you know the fault current, you can evaluate your delta I vector. Once you know delta I , you can find delta V and superimpose delta V , this entire element on V pre to find the new post-fault voltages.

And essentially, once you know the post-fault voltages in a three-phase fault, you can find the corresponding fault currents and all the necessary evaluations that you need. These are the expressions for post-fault voltages.

$$V_i^{post} = V_i^{pre} - \frac{Z_{ik} V_k^{pre}}{Z_{kk} + Z_f} \forall i = 1, \cdots, N$$

These are the expressions for post-fault currents in three-phase faults. Remember, small Zij is not the i, j element of the Z bus. It is the actual line impedance between buses i and j .

So once you know the post-fault voltages and post-fault currents, your analysis is all done and complete. And the above current line expression can also be used to find the corresponding phase currents. Since we are talking about a balanced fault, the single line per unit representation of current that you would get can be extended to all three phases. Before we go into the example for today's discussion, there is another important aspect or notion
defined as short-circuit capacity, which indicates that at a given bus in a power network, the maximum amount of power that can be exchanged or that can flow in case a dead short-circuit three-phase fault occurs in the network. Essentially, the three-phase faults are the most severe faults, and if they happen to be bolted faults where fault impedance is zero, you can imagine the fault current would be as high as possible.

One simple example indicated here is that if Zf tends to be zero, the denominator would tend to go down, while the corresponding numerator would go up. So for bolted three-phase faults, fault currents would be severe, and short-circuit capacity, in short known as SCC, is the estimation of what the maximum amount of power that would flow during a fault condition is. So, in terms of the three-phase line-to-line pre-fault voltage and short-circuit current ISC, which we have evaluated as If in the previous few lecture slides. In terms of per unit, this is the actual SCC value if we divide it by the rated base power of the given network, where Vll mostly happens to be the same for both pre-fault and post-fault conditions; the difference is not much. Then, in that case, SCC in per unit can be equalized to the inverse of the Thevenin impedance.

Now, why are we neglecting the presence of Zf? Because if we ignore the presence of Zf, SCC will still give us the worst possible values. We were going for the most pessimistic or worst-case condition, and the rating in terms of SCC that we get for every such bus in a given network dictates what the rating or breaking capacity of the associated circuit breaker and isolator is going to be. Because if these breakers and isolators cannot handle such high currents or a high quantum of power flow, they themselves would be at stake, and hence the overall system protection, which would help protect the network from these disastrous faults, would be at stake. So the devices that are useful for protecting the power network should be capable of withstanding or withholding their operation under such high SCC values, and that's where the importance of short circuit capacity comes into the picture.

We'll conclude. Our discussion today, for example, the problem statement looks very long, but don't worry, I'll simply summarize it in the least possible fashion. What we have here is a three-bus network in which bus one and bus two have generators connected to them through three-phase transformers: star grounded, star grounded on bus 1, and delta and star grounded on bus 2. The generators' neutral points are also grounded, which is indicated by this star-grounded connection; they are also neutrally grounded through certain reactants or impedance, which for the problem at hand is not important. We don't have to bother about what these reactance values are as of now for three-phase faults. It would become important when we discuss unbalanced faults.

These generators, which are at bus 1 and bus 2, have their respective sub-transient reactance, not the transient reactance. The sub-transient reactances of the respective generators are given as j0.1pu and j0.2pu.

The transformers' reactances are also given as j0.1 and j0.2 pu. All resistances, susceptances, and capacitances are neglected so that the analysis can become simple and we can find the worst possible conditions of currents. Lines that are present between buses 1,2 , and 3 have their reactances also given in terms of per unit. The question here is that during a no-load prefault condition, because generators are at no load, they are running at their rated voltages with rated frequencies, or that the EMFs are all in phase, we have to find the fault current, bus voltage, line currents, etc. , during a balanced fault at bus 3 through a fault impedance of Zf equal to 0.16 pu . That's what we have to find: in case this switch gets closed, what the currents through these lines would be, what the contribution to the generators is, and the corresponding voltages, and so on. So what do we do? We have to apply Thevenin's theorem, but before we do, let us take certain relevant information from the statement that has been given. The generators are represented by an EMF behind the sub-transient reactor; that is information number 1 .

Information number 2: the system is at no load; that is point number 2. Information number 3. All the generators are running at the rated voltage with the rated frequency, where the EMFs are all in phase. That means if I have to redraw this circuit, which is the one I'll try to draw over here, then I have E1, which is 1 at an angle of 0 per unit. This is in series with its sub-transient reactance Xd equal to jo.

1 , followed by the reactance of the transformer j 0.1 , and then I have bus 1 present over here. Similarly for bus 2, if I draw, I have an internal EMF E2, which is the value I'll mention in a moment. Then I have Xd of the generator 2, which is j 0.2 , and Xt , the second transformer, which is again of reactance j0.

2 per unit. What do you think the value of E2 should be? E2, as per statements 1, 2, and 3, the EMFs are at rated voltage. The rated voltage is going to be 1 pu on a common base. The EMFs are also in phase. That means E2 is 1 because its voltage is the rated voltage. Since they are in phase, the angle here is also 0 degrees if I am choosing 0 degrees as a reference.

And since 100 angle 0 degrees is the per unit voltage of the internal EMF, the same applies if I have to imply, because as per point number 2, there is no load in the network; that means these currents do not exist because there is no load actually happening. For no current to flow, all my buses, bus number 1 and bus number 2, would also be at the same unique potential in the pre-fault condition. 1 and 2 are connected by 0 . 8. And similarly, 1 and 3 , and 2 and 3 are connected by $j0.4$, equivalently 0.4. And bus 3 also has a pre-fault voltage of 1 at an angle of 0 . In case these pre-fault voltages differ, currents would flow, which would violate the condition that the system was at no load. If the system had been loaded, these currents would have been different; the voltages would not have been unique.

So we can safely assume that whatever voltage we have to measure in the pre-fault condition is all one at an angle of zero degrees. Zero being an arbitrary choice for reference, you could choose any other value as well, although zero degrees or zero radians makes our lives simpler. So, with this as our pre-fault condition, we now apply Thevenin's theorem to understand what the implications of this three-phase short-circuit fault would be. So let's do that. So this three-phase short circuit fault can be analyzed by Thevenin's theorem.

So this Thevenin voltage at bus 3 with respect to ground is V30, which has the same value of 0 degrees because it's a pre-fault condition. So the Thevenin voltage can be known. To find the Thevenin impedance between bus 3 and bus K, the points across which the disturbance or fault has occurred, we can simplify this circuit by applying combinations of equivalent resistances and impedances through delta-star and star-delta conversions. So, this essentially figure B here is redrawn as figure A with delta-star conversion applied. And here we see that between S and the ground, there are two similar paths.

One path has j0.4 impedance, the other path has j0.6 impedance, which together when equivalently obtained in parallel would result in j0.4 impedance. And now S and 3 are j 1 connected.

And this goes on. So if we combine j0.4, 2, 4, and 0.1, we get Z33, which is the one shown over here. So this is all possible to do when the network is smaller. Imagine if it had been a 300 bus network where, at some bus, this particular fault occurs; would it be possible to go into all these star, delta convergence, and parallel convergence? Practically impossible. It is practically possible but not feasible in the sense that the meshes would be enormously high and one would get lost. So, is there any other way of finding the Thevenin impedance? Yes, that is where the term Z33 comes into the picture.

Let us now try to find the Y bus matrix, Z bus matrix, and see whether the third comma third element of Z bus turns out to be j .34 . Let's do that. So in the pre-fault condition, while applying Thevenin's theorem, we have to be careful about the pre-fault admittance part only, and our generators are all represented by their internal reactances; transformer reactances are also included.

So that's the reason; if I combine these two together, it becomes j 0.2 , as shown over here. And similarly for the other generator, it is becoming j 0.4 , with j 0.2 being its own internal reactance, the transient reactance, and j 0.2 being the transformer's reactance. So if I combine these two reactances and apply my simple Y-bus evaluation, I'll first try to find my admittances. So Y10, Y20, 0 here refers to the ground. Y10 is j. 2 inverse, Y20, Y13, Y23, and so on; the same usual process.

$$\begin{cases} y_{10} = (j0.2)^{-1} = -j5 \\ y_{20} = y_{13} = y_{23} = (j0.4)^{-1} \\ y_{12} = (j0.8)^{-1} = -j1.25 \end{cases}$$

$$\begin{bmatrix} y_{10} + y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{20} + y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix} = j\begin{bmatrix} -8.75 & 1.25 & 2.5 \\ 1.25 & -6.25 & 2.5 \\ 2.5 & 2.5 & -5 \end{bmatrix}$$

$$= j\begin{bmatrix} 0.16 & 0.08 & 0.12 \\ 0.08 & 0.24 & 0.16 \\ 0.12 & 0.16 & 0.34 \end{bmatrix}$$

From there, I find my Y bus, take the inverse of it, and I'll get my Z bus. Here, the third comma, third element is the same number that I obtained by doing my manual evaluations of star delta delta star convergence. So what I'm trying to say is, to find the Thevenin impedance, please find Ybus of the network, take its inverse, and don't go into impedance evaluation, combination, or parallel combination; no, it's going to be a rigorous process. The simpler exercise is to find the Ybus, keeping in view that the generators are marked by their internal sub-transient reactances; then find the corresponding Ybus, take the inverse, and you will directly get the Thevenin impedance. So once we know the Thevenin impedance and our pre-fault voltage, they are all one at an angle of zero.

I think I have missed that part. Yes, it was given over here. I have also discussed that in slide number 11. So, once my pre-fault voltage is known, I can find the fault current. I3f is minus j 2 pu . The question was to find the fault current, which we have now found.

$$\text{From Fig. } (c), I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2pu$$

Can we find the generator's contribution? Yes. There are two ways of finding the generator contribution. One simpler way is that if this is j 2 pu , it means the current through S and 3 should also be j 2 pu ; that means the currents IG1 and IG2 should together contribute and add up to j2pu. Now, how do I find IG1 and IG2? There are two parallel paths between S and the ground zero. So the currents would distribute among themselves according to the inverse ratio of their respective impedances, and that is the reason why, while finding j 1 , I consider j 0.2 and j 0.4 , which add up to j 0.6 , giving me a current contribution of minus j1.2 pu, and correspondingly, the generator's contribution is minus j0.8 pu. Together, they add up to minus j2, which is the fault current.

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2pu$$
$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8pu$$

Once I know my generator currents, I can also know what my change in injection currents delta I is.

From there, I can find the changes in voltages delta V1, delta V2, delta V3. Superimpose them on the pre-fault voltages, which are all 1,1 , and 1 , and find the actual voltages after the fault. Once I know my actual voltages, I can find the corresponding fault currents after the fault has occurred in terms of the lines I1,2, I1,3, and I2,3, which are differences across the voltages of different buses divided by their actual line impedances.

$$\Delta V_1 = -Z_{13} I_3(F) = j0.12 \times j2 = -0.24pu$$
$$\Delta V_2 = -Z_{23} I_3(F) = j0.16 \times j2 = -0.32pu$$
$$\Delta V_3 = -Z_{33} I_3(F) = j0.34 \times j2 = -0.68pu$$

The bus voltages during the fault

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76pu$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68pu$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32pu$$

The short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{Z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1pu$$
$$I_{13}(F) = \frac{0.76 - 0.32}{j0.4} = -j1.1pu, I_{23}(F) = \frac{0.68 - 0.32}{j0.4} = -j0.9pu$$

So remember, Z1,2 is j 0.8 . This is not equal to the 1st and 2nd elements of Zbus. Please be careful in that regard. It is not the 12th element of the Z bus. It is the actual line impedance.

So what I observe here is that j 12 current is j 0.1 pu , which means I21 would be j 0.1 . I13 is j 1.1 pu . I23 is j 0.9 pu . And after the switch is closed, it is j 2 pu itself, minus j 2 pu . So if I apply KCL at node 3, is my KCL applicable? Yes. Minus j 0.1 added to minus j 0.9 adds up to minus j 1 pu .

Is the KCL applicable to bus 1? Remember, bus 1's generator current was minus j 1.2 . Generator 2's current was minus j 0.8 . So basically, this is the current flowing from the ground to bus 1 during the event of a fault, which is minus j .8 and minus j .1 added to minus j .1 , adding up to j .2 , so KCL here is also true. Similarly, KCL here is also true.

Now the only part that is left to explain is how minus j 1.2 current is flowing over the bus from the ground to bus 1 , where the post-fault voltage at bus 1 is 0.76 pu ; that means the KVL is probably not applied here. Is there something missing? Yes. The part that I am missing here is that I have not considered the internal EMF of the generator before this grounding point. This internal generator E1 was at an angle pu. Now, if I consider the internal EMF together with E2, which is also 1 at an angle of 0pu, 1 minus 0.76 is 0.24 , and 0.24 divided by j 0.2 gives me j minus j 1.2 current. So be careful when finding generator currents. That's the other alternative for finding generator currents. You do not always need to do current division as shown here.

You can avoid this step entirely. Find the post-fault voltage. Find the actual voltages. Go back to the pre-fault circuit where the generators were marked by their internal EMFs. As shown here. Now V1, instead of being 1 at an angle of 0 , is 0.76 pu . Apply KVL between these two nodes, E 1 and bus 1; you would find minus j1.2 as the actual fault current. The same goes for E2 and bus 2. So, that's the entire logic. You can actually sort of avoid these combinations of resistances by trying to find Thevenin impedance through Z bus and Y bus; you can also avoid generator contribution evaluation through this current division manner and actually evaluate them from the actual post-fault voltages.

With this, I conclude today's discussion. In the next lecture, we will talk about Fortescue's theorem and its application to unbalanced networks and sequence components. Thank you.