Power Network Analysis

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Lecture-45

Hello everyone, welcome to lecture 5 of week 9 of the course Power Network Analysis, in which we continue with our discussion on the second-to-last module, which is fault analysis. In this discussion, we will understand a very important aspect of the application of Thevenin's theorem for performing fault analysis. So we will understand the basis of why Thevenin's theorem is important; we all understand very well what Thevenin's theorem is, and we will understand its application from the perspective of fault analysis. Until the previous lecture, we continued our discussion on fault analysis, and we extensively talked about why a synchronous machine, whether it is a conventional synchronous machine, a salient pole machine, or a non-salient pole machine, has a variation in its value of synchronous reactance, Xs for cylindrical pole machines or Xd for salient pole machines. Why does it have a variation, and how does this variation result in distinct behaviors during a fault, specifically in terms of the sub-transient period where the fault current tends to be high, followed by a dip in the transient period? Finally, if things are to remain stable, then the current reaches a steady-state value. So the current value decreases from the sub-transient to the steady state period in terms of the fault current, whereas the value of X tends to be at its minimum in the sub-transient period and has a maximum value during the steady state period.

And we also briefly talked about the difference and purpose behind doing fault analysis and the last module, which is stability analysis. Fault analysis is all about figuring out the worst possible fault currents during faults in a power network, and that is the reason why fault analysis is a sub-transient period: only during the sub-transient period will we be able to know what the maximum current is. In stability analysis, we are all focused on whether the system would remain stable following a fault or disturbance, the system here being the power network, and stability analysis is essentially a transient period study in which the machines' transient reactances would come into the picture. Power flow, which we have talked about extensively in the previous module, basically pertains to a steady-state study wherein we do not go inside the machine terminal at all; we limit ourselves only to the bus, which could behave as a PV bus if the voltage control of the synchronous generator is not lost, or it converts to a PQ bus when the excitation control is lost.

So having said that, let us see what Thevenin's theorem is and how it applies. Faults are any failures, abrupt interruptions, or disruptions in the flow of load current, which are referred to as faults. And those faults essentially appear because of intentional or unintentional operations. Intentional operations occur when a power network element, like a transformer or a line, has to go into scheduled maintenance. Or unintentional in the case of some weather disturbance, aging effects, or unforeseen events, the network element itself might stop working or performing satisfactorily, which might interrupt the flow of current from the source to the load.

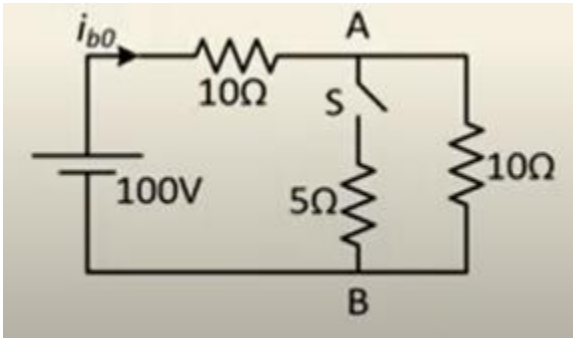
And in general, these faults can be classified as symmetric or asymmetric. Symmetric faults are those faults where all three phases, let's say if you have a transmission line with phases A, B, and C , are equally affected. If one phase current is going down because of this interruption, then a similar quantum of interruption would also happen in the remaining phases. So in a way, the system or the power network still tends to remain symmetric or balanced for a three-phase fault, and that is the term where symmetric terms come in. There are other types of asymmetric faults that we have not talked about yet, but hopefully we will talk about them in the next lecture.

And one important aspect that we discussed in the previous lecture example was that those of you who have not gone through the previous lecture should do so first, and only after that will you be able to understand the aspect that I will talk about. In the previous lecture and example, we considered a numeric study wherein the actual fault current, which was the post-fault current, was essentially a superimposition of the pre-fault current, which was the normal load current, plus the additional fault current that would come in as the fault occurs. So basically, we inherently made use of the notion of superimposition, and we sort of came up with the final fault current to be the addition or superimposition of the previous fault current before the fault and the fault current as and when the fault occurs. Thevenin's theorem, on the other hand, also enables this superimposition, and that is where it would be very useful to find the post-fault currents in all the symmetric or asymmetric faults that we will talk about. So I will start with a very basic discussion of what Thevenin's theorem is and what its applications are.

Thevenin's theorem is a theorem that indicates that given an electrical network with sources of the same frequency, the network can be reduced or an equivalent network can be created for this actual network with several sources and loads operating at the same frequency to a simpler source and load combination where the source is called the Thevenin source and the corresponding impedances that appear across two points of interest from which the Thevenin's theorem or application is to be seen are called Thevenin's impedance. So, Thevenin's theorem is essentially a way of reducing the complex representation of the actual power network. For fault analysis, Thevenin's theorem application indicates or suggests that the change in voltages or currents that would be

observed in the network due to a fault would be equal to the changes which would have been caused by the equivalent Thevenin's pre-fault condition, and the Thevenin's circuit would consist of a voltage source and a corresponding equivalent impedance, which is a Thevenin's impedance in itself. So essentially, if the summary of this statement is that if a fault occurs in a power network and we are interested in finding the change in voltages, the bus voltages, or the change in line currents or the bus currents, then these new changes, which are the fault currents and the corresponding post-fault voltages, should be considered. These can be obtained through a simpler analysis in which there would be just one equivalent Thevenin's pre-fault voltage source and one equivalent Thevenin's impedance.

We don't have to consider the entire 1000 -bus network to find these changes in currents or voltages in the event of a fault; that is what the summary of Thevenin's theorem application is. The purpose of Thevenin's theorem is to reduce the pre-fault network into a simpler network that is easier to handle and that network would consist of equivalent Thevenin voltage and equivalent Thevenin impedance. So if that is the understanding, let us see, before we go into fault analysis, whether Thevenin's theorem actually helps in finding these changes in voltages or currents. Apart from fault conditions, yes, during any other switching it should also apply because this statement is a generic statement. The term "fault" here could be replaced by any other disturbance, for that matter.



So what I have considered here is a very simple circuit, which has a $100-$ volt DC source. All elements here are ideal, and there is a branch A and B connected as shown in the figure, in which there is a switch. At T less than 0 , switch S is open; that means the impedance between A and B is infinite, and there is no current flowing between branch A and B . The actual current that is present in the circuit before the switch is closed is I_b0, which is the source current, and it's very imperative that I_b0, by applying KVL, would be equal to 100 volts divided by 10 ohms (the first series impedance) plus the other 10 ohms (the second series impedance), which is overall equal to 100 by 20 , equal to 5 amperes.

$$i_{b0} = 100V/20\Omega = 5A$$

That means before the switch is closed, that is, at T less than 0 , when the switch is open, the source current is 5 amperes, and we want to find the change in current.

The intent is to find the change in current from the battery or the source, which is Ib_0, after the switch is closed. So before the switch is closed, as I have indicated, 5 amperes is the source current. And if I find the potential difference between node A and node B , then this potential difference is Vab. So basically, if I have to find Va minus Vb , I can apply KVL between A and B, which would tell me that Va is equal to, as I am going along the direction of flow of current, so I_b0 current is flowing over here; then I'll have a voltage drop of 10 I_b0, and finally I have reached point B , which is Vb , so this would give me Va minus Vb, which is Vab equal to 10I_b naught, which is 10 into 5 equal to 50 volts. So, before the switch is closed, the voltage across nodes A and B is 50 volts (Vab), and this Vab also happens to be equal to the Thevenin voltage across nodes A and B before the switch was closed.

$$V_{ab} = 50V$$

So, we will come to this aspect a little later. Now, when the switch is closed, the new current that would flow, I_b, can be found by the corresponding equivalent impedance or resistance of this combination circuit, which is denoted by 5 parallel 10. So basically, 5 parallel 10 is the corresponding equivalent resistance between nodes A and B , which would give us 5 into 10 divided by 5 plus 10 , which is 50 divided by 15 . Also, the same as 10 by 3 ohms. So 10 by 3 ohms is the equivalent resistance between A and B when it is added to 10 amperes; I have 7.5 amperes as the new current after the switch is closed.

$$i_b = 100\sqrt{(10 + 5/10)\Omega} = 7.5 \text{ A}$$

So essentially, the change that has happened compared to the switch open, where I_b0 was 5 amperes and now I_b0, Ib being 7.5 amperes, is because the current has increased; the overall circuit impedance, as seen by the corresponding voltage source, has gone down, so the current change has happened by 2.5 amperes.

$$i_b - i_{b0} = 2.5 \text{ A}$$

Now, is this current change also possible to obtain by applying Thevenin's theorem? Let's see that.

So, by Thevenin's approach, as I mentioned in the previous slide. Vth is the voltage across nodes A and B , which is 50 volts. So, given that my Thevenin voltage source is known, to find the Thevenin impedance, voltage sources would be shorted and current sources would be open-circuited. Voltage sources are being shorted. Why are voltage sources shorted while finding Thevenin impedance? The answer is simple.

When I am finding Thevenin impedance, all my sources should not deliver any amount of energy or power. So basically, when I am shorting my voltage source. The potential across that source itself is zero, so it won't be able to deliver any amount of power or energy, and none of the circuit elements would be consuming any quantum of energy or power from the voltage source. So, basically, I am trying to make my voltage source as passive as possible. Current sources: why are they open circuited? When the current sources are open-circuited, the corresponding current cannot flow, and hence the power or energy cannot be delivered by the corresponding current source, and that is the reason why we are trying to make them passive.

One could also argue why voltage sources are only shorted; why are they opened? If you try to open up a voltage source, potential will still exist, and that's the reason why the circuit won't remain passive. The same thing applies to current sources; they are not shorted because current can still flow and pass through some other circuit element. So that's the reason we are trying to make it as passive as possible. So, to find the Thevenin impedance between A and B, we have only one voltage source, as shown over here. So it is actually shorter.

And when I short this and try to find the Thevenin impedance between A and B , this Thevenin impedance is to be found in the pre-fault condition. Please remember the pre-fault case. So in the pre-fault case, before the fault has occurred, if I have to simulate a similar condition here, then my switch still has to remain open. I don't have to close the switch and find the Thevenin impedance. So in that case, as the switch is open, that means this particular element or circuit is not present.

And 10 ohms, 10 ohms in parallel give me Zth or Zab as 5 ohms.

$$Z_{ab} = 10//10 = 5\Omega$$

So now I can redraw the circuit between nodes A and B while retaining the element or part where the disturbance is going to happen through a Thevenin equivalent source, which is Vab equal to 50 volts in series with Zab, which is equal to 5 ohms. Then I have node A and node B, and the switch here is present, and I have a 5 -ohm circuit resistance in between. So this is my new Thevenin pre-fault condition before the switch S was closed. Now, when the switch is closed, in the Thevenin circuit after S is closed, the current that would flow through branch Iab or between A and B would be equal to Iab, which, again by KVL, would be equal to Vab by Zab plus 5 , which is 50 by 10, equal to 5 amperes.

$$i = 50V/(5+5)\Omega = 5A$$

That means Iab current after the switch was closed; before the switch was closed, Iab is 0 because current cannot flow. After the switch is closed using Thevenin's theorem, the new

current that will flow in branches A and B will be 5 amperes. Now, if 5 amperes have to flow in this circuit between A and B after the switch is closed, which is 5 amperes in this direction, that means this 5 ampere current can be distributed as shown over here. I is 5 ampere current. This 5 ampere current, which is equal to I, can be distributed along the two branches through which this current is coming in.

And since the impedances are equal, it would imperatively mean that 2.5 amperes are coming in this direction on the branch of impedance 10 ohms. And similarly, 2.5 amperes is also flowing into the other 10 ohms of impedance.

And that's how, by KCL, 2.5 and 2.5 would add up to 5 amperes. And that's how the current distribution on the actual network can happen. Now, when this current distribution actually happens, what do we see? What we see here is that let us come back to this circuit, actually. So here, let me take this circuit once again for a moment.

So if Ib was 7.5 amperes, that means the current flowing through this 10 -ohm resistance when the switch S was closed was equal to the inverse proportion. So, 5 -ohm impedance would come in here because of the inverse current distribution, which would give me 2.5 amperes. And similarly, the remaining 5 amperes is coming in here. Why am I getting five here? Because 7.5 times 10 divided by 15. 2 and 3, 3 and 1,3 and 2.5 , which gives me 5 amperes. So, current always distributes in inverse ratio to the actual current distributions. Now, if I compare these currents: 7.5, 2.5, and 5 after the switch is closed, and for a moment observe what is happening over here. What I see here is that this lab current is matching with the 5 ampere current that was flowing. And this circuit here is basically the pre-fault condition. Now, if you look at the pre-fault condition, the prefault condition, the current here was I_b0, which was 5 amperes, and the current here was also 5 amperes.

I_ab did not exist. Now, if I superimpose 5 amperes and 5 amperes with the currents shown over here, the actual currents that would now flow would be 5 plus 2.5 , which is 7.5 , and the current, let's say if I have to find I_10 here, I_10 would be the previous current, which is I_b0. I_b not minus 2.5 because the current direction is reversed, which would give me 5 minus 2.5 to be 2.5 amperes. Now these numbers happen to match the numbers shown over here.

Current from source $i_b = i_{b0} + i/2 = 7.5$ A
Hence, change in current $i/2 = 2.5$ A
Current through parallel 10Ω resistor $i_b - i/2 = 2.5A$
So that means by the current distribution I can easily find the new change in currents because when the switch is now closed, the new change in currents can be obtained through the Thevenin theorem, and then those Thevenin theorem results could be superimposed on the pre-fault condition or the condition where the switch was still open, and correspondingly find my new current: 7.5 amperes, 5 amperes in the switch, and 2.5

amperes to the 10ohm circuit resistance. Essentially, this notion is a very beautiful one, and coincidentally, the same can be applied to the complicated fault analysis of actual power networks.

It can easily help in conducting fault analysis through the extensive use of the system admittance matrix or impedance matrix, basically Y bus or Z bus. And in case we have a network where the machines would usually be loaded, the pre-fault current would be the usual steady-state current, which can be obtained from the power flow analysis. We would also have a similar evaluation done in stability analysis when we deal with disturbances and their impact on stability. And after the fault, we basically apply Thevenin's theorem, find the change in currents, and superimpose or apply it to the pre-fault currents, which are the steady-state currents, to find the overall post-fault current. Remember, Thevenin's theorem is applicable only for circuits with sources of the same frequency because Thevenin's theorem heavily depends on the evaluation of Thevenin impedance, and Thevenin impedance would again be, I mean, in a phasor domain representation where phasors
cannot be defined for circuits with two different sources.
So that is the reason why Thevenin's theorem is not applicable to sources with different frequencies. So coming to balanced fault analysis, the first case, as already explained, occurs when all three phases of a line or bus are equally affected or equally shorted. It is one of the most severe occurring faults, and why it is called severe will be understood when we correspondingly go into the fault current evaluations. It is a severe fault because the change in current in a three-phase fault is significant; the voltage drops are also significant, but it has the least frequency of operation because it is very difficult for such an event to occur.

So basically, these are the most severe. And the least frequently occurring faults in all three phases are equally affected; hence, this fault is called a symmetric fault or a balanced fault. Generally, the magnitude of fault current would depend on the subtransient reactance of the generator and the corresponding equivalent impedance of the overall network. The purpose of fault analysis, as already explained, is to find the worst-case fault currents and voltages so that we can make decisive decisions about the different protection elements that are used for protecting our electrical network from such electrical faults. So in case the fault occurs with fault impedance zero, we would call it a dead short circuit fault or a bolted fault. In case the fault occurs with non-zero fault impedance, then we would call it a non-bolted fault or a non-zero metallic fault.

Usually, the occurrence of bolted short circuit faults is a rare event; usual faults also appear through some non-zero fault impedance. By fault impedance, I mean that if the current which was actually supposed to flow from source to load is getting diverted because of some short circuit, then this short circuit path also has its own impedance. Obviously, this

impedance is lesser than the network circuit impedance; otherwise, the current would have flowed through the network itself. So that's the essence behind having zero or non-zero fault impedance.

Having zero fault impedances is a very rare event. Certain assumptions that would help us simplify or reduce the effort required in fault analysis will be addressed, starting with the three-phase faults first. So, for our faulty network, we will analyze it through Thevenin's theorem. And while applying Thevenin's theorem, we would extensively make use of the Z-Bus matrix, which would depend on what the Y-Bus matrix is. The evaluation of the Y-Bus remains the same as we discussed earlier in the power flow analysis module. Certain assumptions that would help in evaluating or simplifying the fault analysis exercise are that all elements which pertain to reactive power devices like shunt capacitances, admittances, or susceptances have effects that can be considered negligible; their presence can be ignored because even if these elements were to be considered, What would happen is that while finding the corresponding Y bus or Z bus, the overall system impedance would increase in the presence of these elements, and as a result, the corresponding admittance would decrease.

While ignoring their presence, what we are considering is that the fault impedances would go down and the corresponding admittances would go up if these are neglected. In a case where the fault impedance or the Thevenin impedance tends to go down, we would still be able to get the worst possible fault currents, which is an okay estimation or a worst-case estimation, and that is the reason why we go for risk-averse estimation. Similar analysis or understanding also applies to resistances; all line resistances can be neglected, the reason being the same. There is another logic behind why resistances can be ignored for safety purposes: if you recollect the definition of time constant that we had in the two previous lectures, the time constant in an RL inductive circuit was equal to L divided by R . So in a circuit, if we are ignoring the presence of resistance, as indicated here as one of the assumptions in three-phase fault analysis, the overall time constant would tend to infinity, which would mean that the corresponding fault analysis that we are doing with negligible fault resistances would imply that the sub-transient period or decaying DC component would never die off.

That would mean that the current we estimate, which represents the worst-case or risk-averse estimations, would continue indefinitely, and essentially we are not losing anything while ignoring the presence of resistances; that is point number one. Point number two, if resistances had been considered, then the corresponding system impedance would have gone up, and the current levels would have gone down. We are trying to find a higher level of current by ignoring the resistances, which could be the worst possible current, and this difference between the actual current with impedances considered and the current without considering impedances provides a sufficient safety margin for different protective

elements like circuit breakers and relays to work with. So these assumptions, they essentially help in providing additional margin. They also help reduce the complexity involved by not considering these elements.

All synchronous machines, whether generators or motors, since it is a subtransient period, are represented by their internal EMF and the direct axis sub-transient reactance. All static devices, that is, non-rotating devices, can have their presence ignored because, as seen in the previous lecture example, even if these loads were to be considered, which could be non-rotating in structure, the steady-state load current quantum is much smaller than the current that we would encounter during faults. So, basically, we are trying to get rid of the pre-fault condition in itself; that is point number 1. Point number 2: non-rotating loads do not have any magnetic flux. So, essentially, they would not have any experience of sub-transient, transient, or steadystate variation in them at all because there is no magnetic flux that is varying with time.

Obviously, in the case of transformers, the machine has a magnetic behavior, but there are no rotating devices involved at all. So transformers also experience these inrushes or surges, but there the behavior is completely different compared to synchronous machines. Rotating loads like synchronous and induction motors can contribute to fault current, and such devices have to be represented carefully through their internal EMF and the least quantum of direct access sub-transient reactances. Going ahead with this logic of ignoring non-rotating faults, pre-fault conditions can also be chosen to be at 1 at an angle of 0 PU for all bus voltages because if we consider this condition, then essentially we are ignoring the flow of normal pre-fault current. If, as engineers, we want to find that we are very sacrosanct and should not make the assumption of having a flat voltage profile across all buses in the network, we should go ahead, find no issues, perform an actual non-linear power flow analysis, find voltages, and find the pre-fault currents.

But the overall actual fault current that we would observe without performing power flow and while doing power flow, the difference in currents would not be significant, and that's the reason why this sort of assumption works well from a practical consideration perspective. In the next lecture, we will extend this discussion of Thevenin's theorem to understand how balanced faults can be analyzed, see a particular corresponding example if possible, and continue with the unbalanced faults in the upcoming discussions. Thank you.