Power Network Analysis

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Lecture-44

Hello everyone, welcome to Lecture 4 of Week 9 of the course "Power Network Analysis." This lecture will be the second in the module on fault analysis. In the previous lecture, we talked about the variation in synchronous machine reactance immediately after a fault. In the sub-transient period, the transient period, and finally, how it achieves its maximum possible value in the steady state period. We'll discuss or continue along similar lines regarding the variation in the behavior of this machine parameter. In fact, in the previous lecture, we first assumed that the synchronous machine's armature current would have a typical signature where, in the sub-transient period, the current is at its maximum, followed by a dip in the transient period, and finally the lowest possible RMS value in the steady-state period.

Assuming that we understood the variation of the synchronous machine parameters in the previous lecture, we'll discuss that particular assumption of why it happens from the machine armature currents' perspective just after a fault, and we'll cover it in today's discussion. So we have concluded that for a well-designed synchronous machine, when armature resistance is almost negligible, the overall armature circuit is an inductive circuit; hence, the armature current tends to lag the internal EMF or terminal voltage by almost 90 degrees, depending on the loading conditions. And for salient pole machines, the direct-axis reactance is at a maximum, while the quadrature-axis reactance is at a minimum. So we are neglecting the effect of the presence of the quadrature axis current for the sake of simplicity in our discussion.

And because the machine has variations in the corresponding synchronous reactance, Xd double dash, which is a subtransient reactance, it is minimum just after a fault, followed by an increase in the reactance, which is Xd dash. And lastly, the steady-state reactance, which we discussed in the steady-state analysis of synchronous generators, is important. We have not yet understood the importance or application of Xd double dash and Xd dash in any of the modules that we have discussed. So, all this was true, assuming that the armature current of the synchronous generator has some typical signature: the signature is that the current is maximum and then tends to go down as the fault period persists. So why is this an assumption? I mean, what is the basis for this assumption? How do we defend it or understand it mathematically? Can we understand this? Yes, there is a way since our synchronous machine or generator is mostly an inductive circuit, so we can understand the behavior of the armature current, which is fundamental to the synchronous generator.

$$X_d^* < X_d' < X_d$$

The way it behaves during a fault, just after a fault, or in the steady-state period can be understood analogously to the response of a series RL circuit. Why is it an RL circuit? Because our armature circuit is mostly an inductive circuit, resistance is not exactly 0, but for a well-designed machine, the value of R can be considered to be 0 or close to 0. So our series RL circuit here refers to, or is equivalent to, the armature circuit or stator circuit of the corresponding synchronous generator. So if we can understand how the series RL circuit responds to certain switchings, With a terminal with a voltage source V(t) applied to it, the same logic would also be true for a synchronous generator, which is actually unloaded at the beginning. So what we have here is an RL circuit, with some non-zero R and some non-zero L, which is also being excited by a voltage source V of t.

V of t is given as $\sqrt{2}$ V sin(Ωt + α), so this is the time-varying form of V of t. The corresponding phasor form, if you recollect our per unit analysis or basic circuit principle analysis, is. The corresponding phasor representation of this time-domain quantity would be V at an angle of α. Assuming that sin(ωt) is the reference for converting from the time domain to the frequency domain representation, So, alpha is a phase angle with respect to a reference phasor. Alpha could also be chosen as zero.

It could be non-zero depending on which reference has been chosen. And at t equal to 0, basically this is before t equal to 0; at t less than 0, the circuit is unenergized. Because there is no path for this I(t) current to flow through the circuit at t equals 0, this switch is closed, and as the switch closes, the circuit is now energized because of the voltage source present. We wish to find the expression for I(t) as a function of time. We are not retaining or going to the phasor domain, although the circuit can also be analyzed in the phasor domain.

Please refer to the discussion on the basic circuit principles that we had, where you can find details about phasor domain analysis. We are trying to retain the time domain because of a certain behavior that we want to understand, which might not be captured in the phasor domain analysis. So, we want to find the expression for I(t) since, at t less than 0, the circuit current is zero. So, at just t equal to 0, the initial current is 0 because there was no path for this current to flow; it was unenergized. This serves as our initial condition, and the corresponding equation that we make use of is through KVL across the circuit.

$$iR + L\frac{di}{dt} = V(t) = \sqrt{2}V\sin\left(\omega t + \alpha\right)$$

After the switch is closed, we get I into R, which is the voltage drop across this resistor; L di by dt is the voltage drop across the inductor, and since there is no other voltage source present except for the V of t source. So, these voltage drops by KVL should sum up to the voltage applied to the circuit. Now, with this initial condition, this is basically a first-order differential equation, and if we apply closed-form techniques to solve differential equations, we can obtain this expression for I as the closed-form solution. This expression has two unique components: one component, as can be seen here, is an AC component, and I would not say it is in phase with the voltage that was applied. There is now a phase angle that has come in, called gamma.

This gamma depends on the overall power factor angle of the RL circuit, which has an inductance L and a resistance R, as shown here. So, the gamma is the power factor angle, the impedance angle, or the overall circuit, which depends on the tangent inverse of (omega L divided by R), and depending on whether L becomes 0 or R becomes infinity, this gamma can tend to 0; and only in that case will this AC current be in phase with the applied voltage. The RMS value of the AC current is dependent on the applied RMS voltage V and the corresponding impedances that appear in the circuit; this is essentially the Z load equivalent, which is carrying this current I, and this Z load represents the corresponding RL circuit behavior of the overall series circuit. And then the second term that we have here is a periodic component. So, basically, the first component, which is AC, is a periodic component having angular frequency omega; the angular frequency depends on whatever source frequency is applied.

The second term that comes in it is independent; I would not say it is independent, but rather that it does not exist. So, basically, it is an aperiodic term; it does not have periodicity, does not have wave phenomena, and, in fact, it has an exponential component or term with it, which we call a decaying DC component. We will spend some time on this terminology here: why it is called decaying and why it is called DC. The term DC can be understood very straightforwardly because this component does not have any periodicity; it is not a periodic signal. So basically, the DC signals that we see also lack periodicity.

A DC component signal always has the same constant value over time. But since this DC component is not just pure DC, it has an exponential component, which is a function of τ. We call this tau the time constant of the series RL circuit, which is basically the ratio of L to R, and the unit of tau is also seconds. Why seconds? I mean, basically, this unit should match the unit of L divided by R. Remember that for inductance L, the corresponding reactance is omega times L.

So if I rewrite this, omega basically has a unit of ohms; omega has a unit of radians per second, and L has a unit of henries. So, if I rearrange this equation, we will have 1 over omega equal to L over X. So, L has the unit H (henry), X has the unit ohm, and in the denominator of this LHS term, I have radians per second. So, if I work around the units, L by X tends to have a unit of seconds per radian since the radian is a dimensionless quantity. So, L by X essentially has a unit of time, which is seconds, and that is the reason why τ also has a unit of time.

So, this time constant tau dictates what the decay component is and how this aperiodic component decays over time. So, basically, if time tends to infinity, what would happen to e to the power of minus t over tau? This term would tend toward zero. When t is equal to 0, e raised to the power of minus t over tau is equal to 1. So, we would have the maximum DC value of this particular component present at t equal to 0, which is the instant at which the switch has just been turned on in this series RL circuit. And as time progresses, e to the power of minus t over tau starts to diminish, and that is where the terms "decaying" and "diminishing" come in.

And hence we refer to this entire term as the decaying DC component, DC referring to the aperiodic behavior of the signal and decaying because it has an exponential term. So that's what the summary of this particular current is: the current tends to have a steady-state sinusoidal AC component and a decaying DC offset component. We also call it the term "offset" because the overall I(t), which in steady state would be equal to the RMS AC current, is equal to 0 at t equal to infinity when the decaying DC component is 0. At the instant of the fault, or just after the fault, or just after the switching, the actual current I(t) is affected by this decaying DC component, and that is what the term "offset" refers to here. The values of this decaying DC component, as well as the phase angle present in the steady-state financial component, all depend on what this L by R is.

Therefore, the higher the value of L, the longer the time constant will be; for more periods of time and more cycles, the decaying DC component will not vanish. If the inductive part is lower or the corresponding resistive part is very high, the decaying DC component's tau time constant will be lower, and the shorter the period of time, the faster the offset can disappear. Essentially, if I have been able to explain the nature or behavior of this armature current analogous to the armature current in a series RL circuit, I would plot I(t) as a function of time. If the x-axis is time t and the y-axis is I of t at t equal to 0, apart from the sinusoidal component, I will also have the DC component because at t equal to 0, e to the power of minus t over tau equals 1. So, it would start with some peak value I(t).

It would start with some peak value, and as time progresses, this peak value would start diminishing because of the e to the power of minus t over tau envelope, and finally, at t equal to infinity, or t tending to a very high value, I would start observing some RMS component in the steady-state period. So, this particular current signature is exactly the same as the signature that was discussed in the previous lecture and in the current slide. The same armature current behavior is also observed in an unloaded synchronous machine. The unloaded term here is important because the RL circuit that we analyzed was an unloaded circuit at T less than 0; it was unenergized, so the synchronous machine was not carrying any current at all. Thus, the RL circuit was not carrying any current at all under the initial conditions.

So, for an unloaded synchronous generator or machine, if it is subjected to some faults, it could be a three-phase short-circuit fault or any asymmetric faults. The current pattern would still have a similar behavior as discussed here in terms of I of t having some peaky value during the sub-transient period, and gradually the sub-transient period diminishes because of the e to the power of minus t over tau trend; and finally, in steady state, it reaches some RMS value. This same envelope, when compared with the envelope shown here in the sub-transient, transient, and steady-state periods, is identical. Also, before I proceed further, if I really have to understand what the maximum RMS current is that I would expect in the sub-transient periods, basically if I(t) is the y-axis, t is the x-axis, and I have to find the maximum peaky RMS value that can appear in this particular RL circuit, how do I find this peaky RMS value? The answer is very simple since I(t) has two components: one is the AC component, the first term; the second is a decaying DC component.

$$i(t) = \sqrt{2I}\sin(\omega t + \alpha - \gamma) - \sqrt{2I}\sin(\alpha - \gamma)e^{-t/\tau}$$

$$I = \frac{V}{\sqrt{R^2 + (\omega L)^2}}, \tau = \frac{L}{R}, \gamma = \tan^{-1}(\omega\tau)$$

Let me erase all this. The second is a decaying DC component, so I have this AC component here, then I have the decaying DC component (DDC), which is the decaying DC part present over here (DDC). So if I have to find the AC and DDC RMS values, what would I do? I would first focus on what the RMS value of this AC signal is. Recollect the phasor domain discussion we had on slide number 4. If $\sqrt{2}$ V $\sin(\omega t + \alpha)$ is a time-domain representation, the corresponding RMS value in the phasor domain is V. Similarly, for the AC component present here, $\sqrt{2}$ I $\sin(\omega t + \alpha + \gamma)$ will have the AC RMS value equal to I.

Now, this is the corresponding RMS value of the AC component. For the decaying DC component DDC, what is the RMS value? How do we find it? It's a decaying DC component; it doesn't have any periodicity. So essentially, to find its RMS value, what should we do? We would take the maximum possible value of current that this DDC would contribute to that DDC; the maximum particular value is √2 I. √2 I does not have any periodic behavior associated with it. It can only decay; it cannot change or have periodicity with respect to time.

So, DC RMS, which is the maximum possible value that can appear, would be √2 I, and overall, if I have to find the maximum RMS value of I, which is the armature current or series armature current at t equal to 0 in the subtransient period, then this would essentially be equal to the square root of AC RMS squared plus DC RMS squared. Remember why we need to square and add them? RMS itself means root mean square. So, basically, you have to take the square of the individual RMS components. Add them up, and then take the square root. So, if we do that, we have I squared plus 2 I squared DC rms is root 2 I, which essentially gives us I times the square root of 2.

This can be the maximum possible RMS current that can be observed at t equal to 0, just after a fault in the transfer transient period for a series RL circuit or an unloaded synchronous generator. Please do not confuse this with the negative or positive signs present on this particular slide. Depending on the values of alpha and gamma, this negative sign can also become positive, depending on the moment at which the switching occurs. So, having understood this particular behavior, we can now understand the corresponding time periods, or tau, that we discussed, tau being the term sitting over here, the time constant term. So, what is the time period τ that appears in the subtransient period? In the subtransient period, the time constant, which is on the order of a few cycles, depends on the Xd double dash value, and since not much of the filter flux links with the rotor winding in the subtransient period, more of it is linked to the damper winding.

So, Xd double dash is primarily affected by the damper winding reactance; hence, the corresponding time period is minimized during the sub-transient period. Followed by an increase in the time period during the transient period, more linkage now occurs between the stator winding and rotor winding because the Xd' value has increased. And finally, in steady state, the current settles down to a steady-state value, and the value of steady-state reactance \(X_d\) has been achieved, which is the final linkage. So, overall, having understood this, I conclude the logic behind the assumption of why the armature current has this kind of time behavior. The purpose or essence of understanding or learning fault analysis is to find the worst possible currents that can be encountered during a particular fault.

We understood one such fault current in the context of slide 5, which was basically at the maximum asymmetric current component, the maximum RMS that can appear at t equal to 0 in the sub-transient period after a fault. It is equal to √3 I, with √3 I coming in because of the RMS component I of the AC component and √2 I of the decaying DC component. Squaring and adding them up with the square root would give us √3 I. The purpose is to find this maximum possible current that can come or appear in this particular circuit when faults occur. That is the purpose of fault analysis and why it is important to know this.

If we do not know what the maximum possible fault currents would be, how are we going to protect our system? The power system protection has relays, circuit breakers, and switches present in it; the ratings of these devices and the settings of these devices depend on what these maximum short circuit or fault currents that can appear during a fault event are. So, basically, a fault analysis is a subtransient period because only during the subtransient period would you get the maximum fault current. Power flow analysis is a steady-state process. So, essentially, we don't go inside the machine during power flow analysis.

We refrain from using the machines. We pertain only to the terminals of the synchronous generators. So basically, in the context of power flow for synchronous generators, we have been considering PV buses and a slack bus. We understood the logic of converting a PV bus to a PQ bus when the voltage control or reactive power control of the synchronous generator is lost. But we never went inside the synchronous generator; we never made use of the synchronous machine's reactance in a steady state. Essentially, in most of the studies that you would encounter in power network analysis, you would rarely find the application of Xd or Xs during the steady-state period.

Only the values of Xd double dash, which is a sub-transient reactance, find their application in the fault analysis period because using Xd double dash alone enables us to find the maximum possible short-circuit current. This is all about the current module of discussion. The last module, which will be stability analysis, will be a transient period study in which we will understand whether, after a disturbance or a fault, the system is stable enough to reach the steady-state. What we have not sort of understood from the devil's advocate perspective is that instead of this envelope going down, isn't there a possibility that this envelope could go up? In stability analysis, we will try to find those conditions or situations where this envelope will tend to go up, specifically from the fault current perspective, and that is where the important study of the transient period comes in because, essentially, if the envelope goes up, it would more or less start with a similar sub-transient period where the current either has its peak in the sub-transient for a fault or has its minimum during the stability analysis for unstable conditions. Those stable and unstable conditions are what will be important and the focus of our study on stability analysis.

And hence, in that stability analysis, the Xd-dash value is important. For fault analysis, the Xd double-dash values are important. Rarely would you find studies in which Xd and Xs are used. Until you genuinely want to understand machine behavior in a steady state, you will not succeed. So we will conclude our discussion with a numerical example in which we have a 100 MVA, 20 kV synchronous generator.

Connected to a 100 MVA, 20/400 kV three-phase transformer. The machines that react at different points in time are given, and the corresponding time constants are also provided. The transformer reactance is given in per unit as 0.25. The generator is operating at rated voltage, and there is no load connected to it; that means it is an unloaded situation or an unexcited state.

We want to understand the importance of the occurrence of a three-phase fault. We still have not discussed what this three-phase short-circuit fault is. But having understood the impact of symmetric faults, all three phases are affected equally. So if such a disturbance happens where all three phases of the transformer are affected, then under those conditions, we must find the corresponding subtransient, transient, and steady-state currents. We have to find the corresponding maximum asymmetric RMS current that we discussed in slide number five, and we also have to understand that if the circuit is now loaded with a three-phase load of 80 MVA, 400 kV, and a 0.8 power factor, then in the presence of this load, if a short-circuit fault occurs, what the corresponding transient current supplied by the generator would be. So, for the first part, we figured out our baseline values. 100 MVA is common for both generators and transformers. So the same can be chosen as the base apparent power; sorry, the base power. So, if we choose 100 MVA as the base apparent power and 20 kV as the base voltage, which is the machine's rated voltage, the machine operates at the rated voltage.

$$I_{B1} = \frac{S_B}{\sqrt{3}V_{B1}} = \frac{100 \times 10^3}{\sqrt{3} \times 20} = 2886.75A$$

$$I_{B2} = (20/400) \times 2886.75 = 144.3375A$$

So, the base current on the generator side is 2,886.75 amperes. The corresponding current in the transformer's secondary is 144.3575 amperes. We have made use of the turn ratio of the transformer to find the corresponding base current on the secondary side. This current serves as the base current on the primary or low-voltage side of the transformer.

This is basically the HV side of the transformer. The corresponding sub-transient, transient, and steady-state currents that we would find are. So, basically, how would the circuit look? In the first two bits, the circuit is as if there is a synchronous generator connected to a transformer, and there is no other load present in the unloaded situation. For the third bit, we have a generator connected to a three-phase transformer, and then we have a load connected, which is drawing 80 MVA at 400 kV. The rating of this transformer is 20/400 kV, and the generator is also operating at 20 kV. So, for bit A, this is our equivalent circuit; for bit C, this is our actual circuit.

So, in bits a and b, if a fault occurs at the terminal of the secondary of the transformer, which is what is given, then the current contributed by this generator means that the transformer does not contribute current because it is a passive device; active devices can only contribute to fault currents. So, the current that this generator would contribute to it would depend on the reactance of the generator as well as the reactance of the transformer, which is given as 0.25 per unit, and depending on the period of interest, either Xd double dash might be used, or Xd dash would be used, or Xd would be used. So, for the different time periods that the currents are found in slide number 11, you would see Id double dash being the maximum value because the corresponding Xd double dash is at its minimum, which is 0.15 per unit. 0.25 here refers to the transformer reactance; in the transient period, the reactance has gone up to 0.25 per unit, which is Xd dash; here, this is XT. This here is Xd double dash, and finally, in a steady state, you have Xd and XT. Since the generator is unloaded, the internal emf is the rated voltage, which is one pu, and this is what is mentioned here as the rated voltage, so we have considered the internal emf to be 1 per unit in all three cases. This is all about bit a; for bit b, I discussed in slide number 5 that the corresponding RMS currents would depend on I and the square root of 2 times the I value of the DC component.

$I_d'' = \frac{1.0}{0.15+0.25} = 2.5pu \Rightarrow 7216.88$ A on the generator side 360.84 A on the 400 kV side
$I_d' = \frac{1.0}{0.25+0.25} = 2.0pu \Rightarrow 5773.5$ A on the generator side 288.675 A on the 400 kV side
$I_d = \frac{1.0}{1.25+0.25} = 0.6667pu \Rightarrow 1924.5$ A on generator side 96.225 A on the 400 kV side

So, the corresponding asymmetric current would be √3 times I_e double dash. I_e double dash, why? Because Id double dash is the maximum current that can appear during the subtransient period. So, it is 4.33, which is 12,500 amperes on the generator's side. If we compare this current with the rated current, it is almost 5 to 6 times the normal base current of 12,500 amperes that would flow on the generator side, which is the maximum we can achieve.

$$I_{asy} = \sqrt{3}I_d^* = \sqrt{3} \times 2.5 = 4.33pu = 12500A \text{ on the generator side}$$

$$S_L = \frac{80\angle 36.87}{100} = 0.8\angle 36.87pu, V = \frac{400}{400} = 1pu$$

$$\Rightarrow Z_L = \frac{|V^2|}{S^*} = \frac{1}{0.8\angle - 36.87} = 1.0 + j0.75pu$$

So, you can imagine what sort of damage this current can create if it is allowed to flow because the normal generator is operating at a few thousand amperes, and now the current level in the case of a fault has gone up manyfold. For the third bit, we will refer to this particular equivalent circuit; for the third bit, the machine is now loaded. So, we will convert the machine's load power into the corresponding base power of 100 MVA; the voltage remains 1 per unit, and the corresponding load impedance can be evaluated; hence, the load current, I_L, can be calculated. This load current is the current present on the high-voltage side of the transformer. So, in practice or reality, this 1 pu current would practically refer to 144.3375 amperes of current on the HV side of the transformer; this is the steady-state current when the generator is feeding a particular load. Now, in the presence of this load, if a fault occurs, it would essentially be a three-phase fault. That fault would depend on what the internal EMF of the machine was. So, by using the corresponding methods, the question specifically concerns finding the generator's transient current, not the steady-state current. Since the load current is already present during this load presence, we can find what the internal EMF of the generator is; it is a cylindrical pole generator.

Even though it is a cylindrical pole generator, we are neglecting the presence of Xq. So, by using KVL across the terminals of a generator that has a transformer, we finally have a load here. So, we will use X_t and X_d to find the terminal internal EMF with respect to the terminal voltage.

The terminal voltage is 1 per unit, Xd' is 0.25 pu, Xt is 0.25 pu, and IL is the current flowing on the HV side of the transformer, which is 1 per unit. So, you find the corresponding internal EMF of the generator during the transient period. Once we know the transient period internal emf, we can find the corresponding generator current, which would lead us to 2.56. Because it's a short circuit fault, the load current, although it would be contributed to apart from this load, has an additional short circuit path that has occurred here, so you have the load current as well as the Ig dash current flowing because of the transient emf and the transient reactance, which is 2.56 per unit, so you can imagine. Compared to the 1 pu load that was present in steady state during the transient period, you have 2.56 times the actual load. So the actual transient current would be the phasor sum of both currents, which is 3.389 amperes, representing the current contributed by the generator to the load, as well as the actual current. If we want to convert this into actual current on the generator side, we will multiply this by I_B1, where I_B1 is defined on slide number ten.

It would be almost seven thousand to eight thousand, almost nine thousand amperes more, in fact more than nine thousand amperes of current that would flow in case the machine is loaded. So basically, what we have done is superimpose the load scenario with the short circuit fault scenario that was discussed in this particular slide and just added those two currents by superimposition, which is the total transient current from the generator to the load as well as to the short circuit fault, which is very large compared to the normal flow of current. That is all for today's discussion. We will understand the premise behind adding these two currents and why superposition holds true even for the subtransient fault period through the understanding of Thevenin's theorem, and we will first discuss three-phase faults and then follow with other asymmetric faults as time progresses. Thank you.