Power Network Analysis

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Week-09

Lecture-41

Lecture 41: Power flow analysis-DC power flow

Hello everyone, welcome to the first lecture of week 9 of the course Power Network Analysis, in which we continue our discussion on power flow analysis, the third-to-last module of this course. Today's discussion will focus on a further extended version of fast decoupled power flow, which we call the DC power flow or the DC model. The previous discussion we had talked extensively about the fast decoupled method. I would request the viewers to please go through the previous lecture on the fast decoupled power flow method before going into the current discussion on DC power flow. That will help in better understanding the discussion we are going to have today. So, DC power flow, or essentially the DC model of the system, is at the outset, I would say, an approximate model.

It's not a practical or accurate model. It's an approximate model that power system engineers or power network engineers have figured out over the years with their experience of how the network operates. If this model were to be used, which is not a realistic model, it's just an approximate, inaccurate model. Even if this approximate model is to be used, the corresponding solution that we would get from this DC model, which is a fictitious model, would still be acceptable or close enough to the actual solutions that we would get from Newton-Raphson power flow. For actual network or fast decoupled power flow in actual AC networks. Essentially, the overall bottom line story is that this DC model of the network is a fictitious network that power network engineers, through their experience of how networks operate, have come up with, and the power flow solution is essentially obtained from this particular model. It is almost acceptable to the solution obtained from actual AC nonlinear techniques for solving powerful equations under certain operating conditions. So, what are those operating conditions? So, in a way, you can understand that if there is a simpler model that can be solved compared to the actual nonlinear model, then obviously people would solve this simpler model, wherein the accuracy is not that bad, and solving the simpler model would definitely require less time, less effort, and the effect of it would still be more or less similar to the actual nonlinear model. That's where the DC model comes in.

It's an approximately inaccurate model. And as I have probably mentioned, it's a linear model of the actual AC network, whereas the previous techniques that we have seen, whether they're Newton-Raphson, Gauss-Seidel, or Fass-DeCouple, all involve a nonlinear model of the AC network. So, this model of the network is acceptable to power network engineers when the operating conditions are such that the network is not loaded in terms of real power or reactive power flows, which means that the line flows are not very high. The line flows will not be very high when the system load is not very high. So, the voltage profile under that condition would be more or less flat because the flows are less, the load is less, and the source is already there.

The corresponding lines do not have to carry a significant amount of current; the potential difference need not be higher across those lines when the flows are lesser because the loads are lesser, and the important part is that the corresponding R by X ratio is very small. This is only for transmission lines, so under such conditions where the system is operating under these two aspects—where the flow is not very high, meaning the load is very small—it is not very high. The network has a higher current carrying capacity. It does not satisfy those conditions. The load is very small. So, basically, it is like a light load condition. Under those conditions, this DC model of the network would give us acceptable and appreciable values. The assumptions involved in this DC model are all elements of the network that regulate voltages or reactive power flows and are to be neglected. Because we are assuming that the voltage profile is not very different, the voltage profile is sufficiently flat, which means almost all voltage magnitudes of buses are equal to 1 per unit. So there is no need to consider any elements like line charging susceptance, shunt factors, regulators, or line resistances that are going to impact these voltages because the voltages are not changing much.

Under light load conditions, only those elements that can affect the real power flows—essentially the phase angles—are to be considered, and that's the reason why phase shifters are only considered. Line reactances are only considered, and if you correlate this, then these assumptions were also applicable while evaluating the B' matrix in fast decoupled power flow. The same assumptions are applicable here, and that is the reason why I mentioned that the DC model or DC power flow is an extension of the fast decoupled method. The voltage phase angle variations are small. The differences are small.

So sine terms are similar to the phase angle differences themselves, whereas the cosine terms are almost all close to one. The network has infinite reactive power generation and absorption capacity, and that's the reason why the voltage profile at every bus in the network is sufficiently flat. So in a way, in DC models of the network, elements that pertain to reactive power injection flows or voltage magnitudes are all ignored. Only those elements that impact the voltage phase angles and the corresponding real power flows are considered, and that is the reason why we would come up with our DC model.

So, in a way, you can expect that in the DC model or DC power flow, there are no equations involved from a power flow perspective that solve for reactive power or voltage equations.

$$P_i = \sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\cos(\theta_k - \theta_i) - B_{ik}\sin(\theta_k - \theta_i)\}$$

Only equations pertaining to real power flows and the corresponding voltage phase angles would be considered. As a result, the voltage at all nodes is equal to 1 per unit because the profile is almost flat, and infinite reactive power capacity exists. And if we look at our real power injection equation at bus I, then since voltage magnitudes are close to 1. So, these terms become close to 1; the cosine of theta k and theta i is equal to 1; g i k is not to be considered. Why? Because all line resistances are neglected, certain susceptances are neglected, and voltage regulators are neglected.

If resistances are neglected, where would we get the real component in the y-bus with respect to conductance g, which would also be 0? So, this is equal to 0. So essentially we are left only with minus BIK, and this assigned term is equivalent to or approximate to theta k minus theta i, which is where this term comes in. If we rearrange our terms, it would be expanded into this form.

$$\Rightarrow P_i \approx -\sum_{k=1}^{N} B_{ik}(\theta_k - \theta_i) = -\sum_{\substack{k=1 \\ k\neq i}}^{N} B_{ik}\theta_k + \theta_i \sum_{\substack{k=1 \\ k\neq i}}^{N} B_{ik}$$

We redefine or change the notation of BIK and the BIK sum over here.

$$\text{Let } B^0_{ik} = -B_{ik} = -x_{ik}^{-1} \text{ and } B^0_{ii} = \sum_{\substack{k=1 \\ k\neq i}}^{N} B_{ik} = \sum_{\substack{k=1 \\ k\neq i}}^{N} x_{ik}^{-1}$$

We denote them as BIK 0, which is minus B of BIK, which is basically a non-diagonal or an off-diagonal term, off-diagonal term. And BII0 is essentially a diagonal term of a matrix. That matrix we will see, we would call that matrix, let's say, the B0 matrix. So, BII0 is a diagonal term of B0, and BIIK0 is an off-diagonal term of B0 where k is not equal to i. So, under this B0 definition, we can express the real power injection at every bus in terms of the voltage phase angles and the corresponding B0 elements.

$$\Rightarrow P_i = \sum_{\substack{k=1 \\ k\neq i}}^{N} B^0_{ik}\theta_k + \theta_i B^0_{ii} = \sum_{k=1}^{N} B^0_{ik}\theta_k$$

If we put it in vector form, then this is the equation that we get, where B_0 is an n cross n matrix, theta is an n cross 1 vector, and P_0 is an n cross 1 vector.

Depending on what type of source or load is connected at the corresponding bus, the terms would be zero or non-zero from a generation perspective. We expect that every bus will have at least a demand on it or at least a non-zero injection on it. This equation is what is called the DC power flow equation. The B0 matrix in this DC power flow equation is a rank-deficient matrix. In fact, its rank is always n minus 1 for an n-bus network, provided that the n-bus is sufficiently connected to the ground from a voltage phase angle measurement perspective.

And that's the reason why the rank deficiency occurs, because our power flow equations are functions of the difference of voltage phase angles. So it is essentially impossible to determine all voltage phase angles uniquely at a given point in time, and that is the reason why one voltage phase angle has to be chosen as a reference. We call that reference bus similar to bus 1 a mass lag bus, angle reference bus, or swing bus, and hence the angle reference bus reference value is chosen, which is usually 0 degrees or radians, although the importance of choosing it to be 0 need not be emphasized; it is not essential, as it can be any non-zero value; correspondingly, the phase angles of other buses would adjust themselves. So, since the reference value is known for the slack bus or angle reference bus. So, theta 0 is essentially the unknown vector, which is theta i for all i ranging from 2 to n if bus 1 is the angle reference bus.

Then that is the reason how the rank deficiency part of B_0 can be handled: because if B_0 is a rank-deficient matrix, then θ_0, which is going to be B_0 inverse P_0, cannot be evaluated because it is rank deficient. To make it in deficient, one of the angle references has to be known, and that is how the B0 matrix becomes N minus 1. So, corresponding theta 0 becomes N minus 1 cross P. P0 also becomes N minus 1 cross 1 vector because, for the slack bus or angle reference bus, the corresponding injection is infinite, similar to the choice that we had for the angle reference slack or swing bus in the initial discussion. Since the orders are changing for phase angles and the known injections, we denote the new matrix as the B dash matrix, which is actually a component or element of the B 0 matrix with the reference bus row and column removed from the B 0 matrix.

And this B dash matrix would trigger or indicate that this B dash matrix is the same B dash matrix that we have seen in the P theta half iteration of the fast decoupled method, and essentially that is how decoupled power flow and DC power flow are interrelated. Essentially, indirectly in DC power flow, we are only solving the E1 equation for the fast decoupled method. The E2 equation in fast decoupled flow is not solved in DC power flow. So once the B dash matrix, which is now an N minus 1 rank matrix because one row and column for the slack bus have been removed from there. So B dash inverse can exist, and we call that inverse X matrix.

$$\boldsymbol{\theta} = \mathbf{B}'^{-1}\mathbf{P} \Rightarrow \boldsymbol{\theta} = \mathbf{X}\mathbf{P}$$

So all phase angles for buses, except the slack bus, can be known with respect to known injections at every bus. It is a linear method. There are no iterations or sorts of dependency. There is no dependency of X on phase angle or voltage. So X is again a constant matrix. B dash again is a constant matrix. DC power flow is solved. When it is solved, it provides a solution only in one go because it's a linear method. There is no iteration involved, and there is no need for any initial choice either. The solution that we get from DC power flow is only the voltage phase angles; voltage magnitudes are chosen to be 1 per unit.

So it is basically a purely reactive model. There is no real power loss involved in DC power flow because line resistances are all neglected. So once we obtain those phase angles, we can put in those assumptions that we had for real power injections for real power flows also, where v squared would be equal to 1; these terms would be equal to 1, and g i k is 0 because line resistance is not considered. So basically, this term entirely is 0; we are only left with.

Let me check, yeah, so sorry. So this term entirely becomes zero; we are only left with this one term. This is approximated as theta k minus theta i, and which is what we get here. So once we get our voltage phase angles, we can put in those numbers here and get the corresponding real power flows. Also, DC power flow, as an extension of the fast decoupled method, serves as a very critical tool for fast contingency or security analysis.

$$P_{ik} = \{|V|_i^2 - |V|_i|V|_k \cos(\theta_k - \theta_i)\}g_{ik} + |V|_i|V|_k \sin(\theta_k - \theta_i)b_{ik}$$

$$\Rightarrow P_{ik} \approx b_{ik}(\theta_k - \theta_i)$$

$$\text{where, } b_{ik} = -x_{ik}^{-1}$$

$$\Rightarrow P_{ik} = (\theta_i - \theta_k)x_{ik}^{-1}$$

It basically gives reliable bus voltage phase angles for nominally loaded or lightly loaded transmission networks.

Transmission networks whose voltage profiles are almost equal to 1. All buses have similar voltages. Line flows are much smaller. We will understand the importance of DC power flow through this example. I have chosen a similar network. The only difference is that in previous networks, we had bus 2 as our PQ bus and bus 3 as our PV bus. Since we are talking about DC power flow, where voltage magnitudes and reactive power flows are not to be considered, I have chosen bus 3 to be now also a load bus; there is no source present here, and that is the only difference. The line reactances remain the same since reactive power injections are not important for DC power flow. So, we are only dealing with real power flow injection. We are considering two conditions: bus 1 is the angle reference bus on which there is a source connected, and buses 2 and 3 have loads of PD 2 and PD 3.

Under light load conditions, PD 2 is 0.4 per unit and PD 3 is 0.2 per unit, whereas for heavily loaded conditions, PD 2 and PD 3 become 5 times their respective values under light load conditions. So, under these conditions, if we were to solve our actual Newton-Raphson power flow, be it in polar version or rectangular version. Process would remain the same. In Newton-Raphson power flow, there would be no PV buses involved. We would treat PQ buses, bus 2, and bus 3 as PQ buses. Qs are essentially all 0. So, QD3 is also 0. QD2 is also zero for Newton-Raphson power flow.

Bus 1 will remain as the slack bus if we solve this network for AC power flow with this information and find the corresponding line flows under two conditions: one is a light load condition and the other is a heavy load condition; then these would be the actual flows. I have not intentionally given the voltage information because that is not important, although that could also be provided. Shown, the important part is having solved the AC power flow solution equations for this given network under this condition; we can use that voltage to find these line flows the way they have been discussed earlier. So let us see if our DC power flow can also provide these flows because, as I mentioned and discussed earlier, DC power flow can give acceptable values of flows under certain conditions.

So let's see it. So, how do we proceed with DC power flow? We find the bus admittance matrix as it is usually evaluated. We take the B0 matrix from here, which is the negative of B. Since bus 1 is the slack bus, we remove row 1 and column 1 from B0, and we get B dash as our 2 by 2 submatrix present over here. The unknowns are the voltage phase angles of bus 2 and bus 3. For known injections at bus 2 and bus 3, both bus 2 and bus 3 are load buses.

So the corresponding injections are -0.4 and -0.2 for the light-loaded condition. Since B dash is known, it isn't a singular matrix. So we can get the corresponding phase angles; these are in radians. Please note these are in radians, not in degrees, and from these angles, we can actually evaluate the corresponding flows through the expression that was discussed earlier.

Now, if we compare these flows, they are more or less equal to the flows that we have obtained from actual AC power flow, to the third or fourth decimal. So, if you compare the numbers given here on slide 9, which was the actual AC power flow solution, with the DC power flows that we have obtained for light load conditions, the solution or the flows match up to the fourth or fifth decimal, indicating that under lightly loaded conditions, DC power flow can also provide similar acceptable real power flows.

$$P_{12,dc} = -P_{21,dc} = 0.457143$$
$$P_{13,dc} = -P_{31,dc} = 0.142857$$
$$P_{23,dc} = -P_{32,dc} = 0.057143$$

As compared to AC power flow, the need to solve AC power flow can be avoided if required for heavy loaded conditions if we follow the same process of solving DC power flow; the flows that we would get would match, I think, until the second or the third decimal compared to the solution which is shown over here, where the accuracy or the error has increased. Hence, the corresponding accuracy has gone down. So the accuracy of DC power flow has decreased compared to AC power flow with an increase in line loading, as discussed earlier.

In fact, these flows that I am stating here are P12 AC, P13, or P23 AC. What are these P12, P23, and P23? P12 is the flow from bus 1 to bus 2 on the line connecting bus 1 and 2. P13 is the flow from bus 1 to bus 3, connecting line bus 1 and bus 3. Similarly, P23 is the flow from bus 2 to bus 3, connecting lines bus 2 and 3. And the other counterflows, they can also be defined.

So, you give here, we can have P31, here we can have P32, and here we can have P21. Since the line resistances are absent, they are not considered if you add up P12 and P21, which should actually be the loss on line 12. Since it is a lossless network, this addition would always be 0, and that is where these negative components or terms mean there is no resistance, so there is no I squared R loss. The same is also being observed in the DC network condition, although the accuracy of the DC power flow solution has gone down because of heavy loading compared to the light-loaded condition.

That's all for today's discussion. In the next lecture, we will continue with the power flow analysis technique or module, and we will discuss the. Technique which is very specific for distribution networks shows how power flow can be solved for typical radial distribution networks where the R by X ratio is high compared to the R by X ratio in transmission networks. We will also discuss why the ratio is high in distribution networks in the next lecture and see that. A technique specific to distribution networks, what is that? We call that the backward-forward technique. And distribution network, Power Flow, is generally not easy to solve using Goff-Fiedel or Newton-Raphson.

Forget about Fast Decay Port or DC Power Flow. They're not even applicable at all. Because the distribution network has its own physics and its own phenomena, we need a dedicated technique for that. That's all.

Thank you.