Power Network Analysis

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Week-08

Lecture-40

Lecture 40: Power flow analysis- Fast Decoupled method.

Hello everyone, welcome to lecture five of week eight of the course Power Network Analysis, in which we continue our discussion on power flow analysis. And in today's discussion, we will take up a very interesting technique for solving power flow equations, which we call the fast decoupled method or fast decoupled power flow. Variant names come in, but the important terms in this technique are fast and decoupled. So, this fast decoupled technique, as we will see, is a variation of the variation of Newton-Raphson power flow that we have discussed with voltage phasors represented in polar coordinates. So this is one specific catch in this situation.

The Newton-Raphson power flow discussion that we had in the last lecture with respect to voltages being represented as rectangular coordinates will not be applicable to the fast decoupled method. It is only a variation of the Newton-Raphson power flow with voltages represented in polar coordinates. So, a brief background on why the fast decoupled technique is necessary. So in the previous lecture, we saw that the Newton-Raphson technique, be it in polar or rectangular form of voltage phasors, tends to require the evaluation of a Jacobian matrix, and this Jacobian matrix needs to be factorized at every iteration of the Newton-Raphson power flow.

So, two great gentlemen, one named Ongun Alsak and his PhD thesis supervisor Brian Stott, came up with this problem or challenge in 1973-74 because, 15, 20, 30, or 40 years ago, computational capability was not so great. So the evaluation of this large Jacobian matrix and factorizing it in every iteration of the wraps and power flow was still computationally burdensome in those days. It is still burdensome in current days also because among all the mathematical operations that we have, the basic operations are addition, subtraction, multiplication, division, and the dot product. The division operator is the most cumbersome operation and requires more time. So Alsak and Stott thought about that: oh, can there be a way of avoiding this Jacobian evaluation and associated factorization in every iteration? What they came up with was that, oh, let's try to use our engineering experience of how power systems operate, and through certain experimentation—not through any theoretical proof, but through certain

experimentation—they figured out that there can be a possibility of evaluating the Jacobian only once at the beginning of the power flow solution process.

And once it is evaluated, it needs to be factorized only once. And the same factorized Jacobian can be used in the upcoming iterations of the solution process. That was their great idea. And it took a lot of time for the fast decouple technique that Alsak and Stott had proposed. In 1973 and 1974, to gain traction. For almost two to three decades, the power system network industry was wary of this technique, but given its popularity and the way it has worked, almost all industry-grade power flow solvers are based on this fast decouple technique. So let's dig deep into what this fast decoupling technique is. So, as discussed or mentioned in the Newton-Raphson power flow with voltages in polar coordinates, these equations are also applicable for Newton-Raphson power flow with voltages in rectangular coordinates.

$$\mathbf{x}^{k+1} - \mathbf{x}^k = -\mathbf{J}^{-1}\mathbf{f}(\mathbf{x}^k)$$
$$\Rightarrow \mathbf{J}\Delta\mathbf{x}^k = -\mathbf{f}(\mathbf{x}^k)$$

The kth plus 1 update of the solution or the state depends on the previous state, as well as the Jacobian, the factorized Jacobian, and the function value, which was evaluated or needs to be evaluated at every iteration. So, in a way, this equation can be rewritten in this form where delta x k is essentially x k plus 1 minus x k, or in terms of error, it is equal to e k plus 1 minus e k, where e k plus 1 is x minus x k plus 1 and e k is x minus x k.

$$\Delta x^k = x^{k+1} - x^k$$
$$= e^{k+1} - e^k \quad ; \quad e^{k+1}: x - x^{k+1}$$
$$e^k: x - x^k$$

I probably made a mistake here. So it is actually minus ek plus 1 and ek, and that would make the signs match. So this delta xk is the basic update that comes from every iteration of the Newton-Raphson power flow, which needs to be evaluated from the inverse of the Jacobian matrix and the negative function value present here. In the polar version of voltages, delta xk pertains to the difference of voltage phase angles and the difference of voltage magnitudes in every iteration. Given a power network, assuming that bus 1 has been chosen as the slack bus, which was the case in the previous discussions for the Gauss-Friedel as well as the Newton-Raphson technique.

So the phase angles are unknowns only for the remaining other buses, whereas for voltage magnitudes they are unknown only for the p-q buses, and this J matrix has four subcomponents: $Jp\theta$, Jpv, $Q\theta$, and Qv, whose dimensions depend not only on the

unknown states but also on the function values, which can again be rewritten as Δp and Δq because. Delta P and delta Q essentially represent the mismatch vector in terms of real and reactive power flows or injections for all relevant buses that are present for the P and Q equations. P equations exist only for buses except for the slag bus, whereas Q equations would apply only to PQ buses. These PQ buses may also include PV buses that have lost their voltage control and are now considered PQ buses. So given this, if we focus only on the relevant portion wherein.

$$\Delta x = \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} \Delta\theta_i \forall i = 2,3,...,N \\ \Delta|V|_i \forall i \in PQ \ buses \end{bmatrix}, J = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix}$$

$$-f(x) = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} -P_i + P_{gi} - P_{di} \forall i = 2,3,...,N \\ -Q_i - Q_{di} \forall i \in PQ \ buses \end{bmatrix}$$

$$P_{gi} = 0 \forall i \in PQ \ buses$$

I am going to express this equation in terms of phase angle differences, the voltage magnitude differences, and the delta P delta Q vector from the right-hand side, it would look something like this, wherein there are two sets of vector equations.

The first set, if I write, represents J P theta delta theta plus j p v delta of mod v is equal to delta p, and similarly j q theta delta theta plus j q v mod delta v is equal to delta q. There are two sets of equations; both of these vectorial equations are coupled and dependent on each other.

$$\begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

$$\Rightarrow J_{P\theta}\Delta\theta + J_{PV}\Delta|V| = \Delta P$$
$$J_{Q\theta}\Delta\theta + J_{QV}\Delta|V| = \Delta Q$$

Why are they coupled or dependent? Because these submatrices are functions of both phase angles and voltage magnitudes, they are still decoupled, as both the states delta theta and delta v appear in both equations. The number of unknowns and the number of equations are consistent, and hence these equations can still be solved. ALSA can start to think about a way of avoiding the evaluation of this big J matrix, and once it is stopped, the factorization effort can also be reduced.

So the intent was to improve the computational efficiency and memory requirements to solve this Newton-Raphson power flow technique. So what they thought were their principles was that, based on their physical experience of the operation of the power network, this is mostly true to a certain extent: the change in voltage phase angle, which is angle theta, mostly affects the real power flows in the lines and does not significantly affect the reactive power flow under certain operating conditions. So, in a way, they

emphasize that. Real power, be it real power flow or real power injection, is strongly dependent on the voltage phase angle, and similarly, the reactive power flow is strongly coupled to or dependent on the voltage magnitude. Although voltage magnitude also affects the real power flows, it does not do so to a great extent under certain conditions.

So, with these two principles that P is mostly coupled with theta and Q is mostly coupled with voltage magnitude. They thought about, okay, if this is true according to the power network operation, then they shifted their focus to the Jacobian matrix, which can essentially be considered a sensitivity matrix according to this equation, and that is what Jp theta means. Jp theta is essentially the sensitivity of real power with respect to the phase angle, and similarly, Jpv is the sensitivity of real power injection with respect to the voltage magnitude of PQ buses. So, if P is strongly coupled to theta and Q is strongly coupled to voltage, then these two matrices J P theta, which are the diagonal matrices, are going to have dominant diagonal terms, whereas J P V and J Q theta will have non-dominant terms and hence their values can probably be considered to be 0; that was the idea of Alsek and Stott, as they were trying to make it. Decoupled from these two sets of equations, we were trying to make these equations be solved in a decoupled manner.

By principles, j p theta elements are much larger than j q theta elements, so essentially we can consider this aspect to be zero, and hence j p theta delta theta can be considered to be approximately equal to delta p. Similarly, j q v can be considered to be approximately equal to the product of delta v and delta q, respectively, so with these two. Decoupled equations: Can we start? Can we expect that these two will still give us the same solution? So, ALSA can start in its paper. They have discussed this aspect at length. The problem with these two equations, which appear to be decoupled, is that they are still not independent of each other.

They are still dependent because Jp theta and Qv elements are both functions of phase angles and voltage magnitudes. So, they took one more step, which was the second intermediate step. The first intermediate step was explaining, premising, or proving these two principles: that P theta is strongly coupled to P, which is strongly coupled to theta, and that volt Q is strongly coupled to voltage; that was their first premise. The second premise that they brought in to ensure this independence between these two equations was that usually for a well-designed power network operating under certain good attributes. Usually, it is observed that the voltage phase angle differences across a line or between two buses are very small, and under those conditions, the sine component of this phase angle difference can be approximated to the phase angle difference itself, since the phase angle difference is very small So, this theta j minus theta is almost 0, and the cosine of 0 is almost equal to 1.

$$\sin(\theta_j - \theta_i) \approx \theta_j - \theta_i, \cos(\theta_j - \theta_i) \approx 1$$

that was the second intermediate part principle. The second intermediate principle was usually our transmission network; the lines that we have, which we see, have very high R by very low R by x ratio, meaning the value of R is usually small, the line resistance is usually small, whereas the line reactance is relatively high. So, for a high R by X ratio, what we would observe in the bus admittance matrix is that the line susceptance is much larger than the line conductance because the system or transmission network lines have a low R by X ratio; more inductive effect or capacitive effect comes in rather than the resistance effect, because we wish our line resistance to be much smaller so that I squared R loss is as low as possible. So under that condition, g i j sin theta j is much smaller than b I j cos theta i theta j,

$$G_{ij}\sin(\theta_j - \theta_i) << B_{ij}\cos(\theta_j - \theta_i)$$

and the last part was that the reactive power injected at the bus under normal operating conditions is much smaller than the reactive power generated or induced due to self-susceptance at the bus. This is the basic essential equation: the individual Q i is much smaller than the corresponding V I square negative B I I.

$$Q_i << -|V|_i^2 B_{ii}$$

So under these three principles, that is principle number one, two, If we now evaluate our J P theta and the other 3 submatrices, we can see that delta P theta, which is basically the J P theta matrix here, has J P theta off-diagonal elements since G ij sin theta j is much smaller than B ij cos theta ij. So, this can be approximated to 0. The cosine of theta j minus theta a is almost equal to 1. So, essentially, this element only becomes minus v i v j b i j. Similarly, the diagonal term, which is basically the negative of q i and the negative of v i b i squared, since q i is much smaller, the diagonal term becomes only this particular component.

And similarly, the j, q, v elements, the corresponding off-diagonal terms which are the same as delta p theta a by delta theta j, become the same as this number, and the corresponding diagonal term is also equal to the same term as we have seen above.

$$\frac{\partial P_i}{\partial \theta_j} = -|V|_i|V|_j \{G_{ij}\sin(\theta_j - \theta_i) + B_{ij}\cos(\theta_j - \theta_i)\}, j \neq i \qquad \frac{\partial P_i}{\partial \theta_i} = -Q_i - |V|_i^2 B_{ii} \approx \left(-|V|_i^2 B_{ii}\right)$$

$$\Rightarrow \frac{\partial P_i}{\partial \theta_j} \approx -|V|_i|V|_j B_{ij}, j \neq i$$

$$|V|_j \frac{\partial Q_i}{\partial|V|_j} = \left(\frac{\partial P_i}{\partial \theta_j}\right) \approx -|V|_i|V|_j B_{ij}, j \neq i \qquad |V|_i \frac{\partial Q_i}{\partial|V|_i} = Q_i - |V|_i^2 B_{ii} \approx -|V|_i^2 B_{ii}$$

There is no need to evaluate j p v or j q theta elements because of the strong coupling between p theta and q v, so we are considering these elements to be 0 anyway, or I will start considering them to be 0. So, under these simplifications, the P theta equation, the first decoupled equation which we saw, can be expanded into this form.

$$J_{P\theta}\Delta\theta = \Delta P$$

$$\begin{bmatrix} -|V|_2 B_{22} & -|V|_3 B_{23} & \cdots & -|V|_N B_{2N} \\ -|V|_2 B_{32} & -|V|_3 B_{33} & \cdots & -|V|_N B_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -|V|_2 B_{N2} & -|V|_3 B_{N3} & \cdots & -|V|_N B_{NN} \end{bmatrix} \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \vdots \\ \Delta\theta_N \end{bmatrix} = \begin{bmatrix} \Delta P_2/|V|_2 \\ \Delta P_3/|V|_3 \\ \vdots \\ \Delta P_N/|V|_N \end{bmatrix}$$

where we have all phase angles except the slack bus, and on the right-hand side, we have delta P by delta V terms; essentially, there is a denominator term coming in here because of these two terms, this term as well as this term. We have separated out the voltage magnitude of each individual bus and divided it by the delta P expression over here, and that is how we have only delta V terms for the respective buses, whereas on the right-hand side, we have every term being multiplied by the respective voltage magnitude. Assuming that the volt which comes in is the last principle, principle number 4, which says that assuming the voltage magnitude is not much different from 1 per unit means all these elements can be considered to be 1 on the right-hand side, and let these terms be as they are.

Then what we would see is that the J P theta matrix is essentially a function of the imaginary part of the bus admittance matrix, with the corresponding elements taken from 2 to n because we are choosing bus 1 as slack, and E1 becomes our first set of decoupled independent equations.

$$\text{Let } \mathbf{B}' = -\mathbf{B}(2:N, 2:N); \mathbf{B}'\Delta\theta = \Delta P/|V| \rightarrow (E1)$$

Similarly, the second decoupled equation, which was JQ theta QV delta V into delta Q, is equal to delta Q and can be rewritten as the E2 expression.

$$\text{Let } \mathbf{B}'' = -\mathbf{B}(PQ, PQ); \mathbf{B}''\Delta V = \Delta Q/|V| \rightarrow (E2)$$

So, in these two expressions, what we now see, B dash and B double dash, are functions of the bus admittance matrix only. No voltage phase angles and no voltage magnitudes are coming in as part of B dash and B double dash. So, essentially, if B dash and B double dash are only functions of the admittance matrix, and the admittance matrix is fixed for a given system with a known topology, that means B dash and B double dash

are not going to change throughout the iterative process of solving power flow equations; that was the beauty of what Alsak and Stott had.

Presented, and in fact, once this becomes constant, the Jacobian matrix now becomes constant. So, factorization only needs to be done once, and the same factorized process will continue until the actual phase angles and wordless magnitudes are obtained. So essentially, B dash and B double dash are constant matrices. They depend only on network parameters. So it is only evaluated once, and the factors are stored to be used for all remaining fast decoupled power flow iterations. E1 and E2, which are the equations shown over here, are completely decoupled equations. And solving each equation results in a half iteration. So once we solve the E1 equation, we'll understand what the steps of the E1 and E2 equation-solving process are. Once we solve E1, we call that only 0.5 iterations have been done. After E1 has been solved, when we do the E2 equation, we call that another 0.5 iterations have been done, and hence together they count as one iteration. So, unlike a Newton-Raphson power flow with voltages and polar coordinates where convergence happens in integer iterations, it may happen that in the fast decoupled technique with similar convergence criteria, which we have chosen in the Newton-Raphson power flow. The convergence may happen in half of an iteration; also, that is the reason why fast decoupled methods may have a number of iterations as an integer plus 0.5, where 0.5 pertains to solving each one-half of the equation individually. With these assumptions, Alsakian and Stott also proved that the same solution can be obtained as the one we obtain from the Newton-Raphson power flow. The same sort of solution can be obtained in a fast decoupled power flow. Since there are approximations or computational simplicities involved, the number of iterations in fast decoupled power flow would be higher. Although solving each such iteration would be much smaller, the computational burden would be much smaller than solving a single iteration in Newton-Raphson power flow.

Because the constant Jacobian matrix is factorized and considered only once. Fast decoupled power flow finds its usefulness in exercises done by operators almost every day, every week around the globe. That exercise we call contingency or security analysis, which essentially is an analysis. We will not be considering this analysis as part of this course. The purpose of this analysis, contingency security analysis, is to let the operator know that, okay, if a particular contingency, let's say a disturbance or a fault, were to happen in a power network, then what would be the consequence of that particular disturbance or fault? That exercise is known as contingency or security analysis.

In a typical power network with so many interconnected elements—transformers, buses, lines, generators, and loads—any of these elements may undergo a fault or a disturbance, wherein there may be an interruption in the flow of power from the source to the load. And so the operator needs this analysis to predict way beforehand that, okay, if this disturbance were to happen, if this element were to trip. If this transmission line were

taken out of operation because of maintenance or because of some fault, then what would be the implication of that particular line outage on the remaining network? That exercise is to be done by the contingency of security analysis. And since there are so many elements in a power network with a lot of redundancies, it becomes increasingly important that the operator, when he or she is doing this analysis, does so in a quick manner because the number of contingencies that may happen is also going to be large for a given power network. And that's where the need for fast contingency analysis comes in.

Fast decoupled power flow, since it involves these assumptions, has poorer convergence compared to 100% power flow, specifically in cases where voltage instability may occur, which usually happens when the loading of the system is very high. That is all about decoupled power flow; we still have not explained or understood what is meant by fast. So, Alsak and Stott took one more final step in order to further speed up the decoupled exercise or make it faster; what they did was note that since P is strongly coupled to theta, Q is strongly coupled to voltage magnitude. So why can't the matrices that we are referring to as B dash and B double dash for QV and B dash for P theta? Why can't B dash and B double dash be further simplified? Why should B-dash and B-double-dash be fetched from the same bus admittance matrix when the coupling effects are different? So, what they did was, while evaluating the B dash matrix, they stated that we should consider only those elements that are going to affect the voltage phase angles, and then likely they are going to affect the real power flows. So, essentially, what they meant was voltage regulators, SVCs, and FAC devices, which have a greater role to play in maintaining voltage magnitudes.

Why should their presence be considered at all? They may be present in the network actually. But to simplify or make the process faster, when we evaluate the bus admittance matrix to get the B dash matrix, let us not consider elements that are not going to significantly affect the phase angle and the corresponding real power. That's where another level of beauty lies: neglecting all network parameters that do not significantly affect real power flows while evaluating the B dash matrix. So what is to be done? How do you evaluate the admittance matrix? Consider the elements. In the bus admittance matrix, which affects phase angles, what are those elements? Those elements are significantly line series reactance and shunt susceptance; these are to be neglected.

Line resistance is to be neglected, as well as taps of voltage regulators. All FACTS devices that regulate voltage magnitudes have to be neglected. What is to be considered? Consider only the line series reactants and the taps of phase shifters, if any, while evaluating the Y-bus dash. Y bus dash noted, I mean, intently kept a dash here to make it different from the Y bus that we have been evaluating earlier. Only focus on those elements that affect P and theta.

Which are line series reactants and the taps of phase shifters. Remaining elements, even though they are present, will be considered to be zero while evaluating Vibus. Once Vibus Dash is evaluated, fetch the corresponding elements only for B Dash. Similarly, for B double dash, neglect all network parameters that do not significantly affect reactive power flows. So, in a way, for Y double Y bus double dash, which is again signifying that it is different from the usual Y bus and the Y bus double dash, which we have evaluated here.

We only ignore the effect of the phase shifter while all line resistance, reactance, and shunt elements that affect voltage magnitudes, as well as tap regulators, are to be considered. Phase shifters will only affect the phase angle between two voltages and not the voltage magnitude; that is why this aspect is to be neglected. Resistance and reactance together affect the voltage magnitude, whereas in the case of voltage phase angle, only the reactance plays a strong important role. So we evaluate Y bus double dash and correspondingly fetch the required elements from Y bus to get B double dash.

In this way, the process becomes faster. So the overall process is to evaluate B dash and B double dash, save these factors as discussed in slide number 10, and make an initial estimate of the choices for unknowns similar to flat start in Gauss, Tudor, Newton maps, and power flow. Find the real power mismatch delta P for all buses except the slack bus and the reactive power mismatch delta Q with no need to check PVPQ switching as of now because we are only trying to solve the even equation, the first phase of the equation. So if everything, I mean if the convergence criteria matches, that means delta P change is much small, delta Q change is much small, similar to the discussion that we had in the previous lecture for Newton-Raphson as well as Gauss-Riedel, then nothing needs to be done. If the convergence criteria are not met, then solve each equation.So, find delta theta.

$$\Delta\theta = \mathbf{B}'^{-1}\frac{\Delta\mathbf{P}}{|\mathbf{V}|} \rightarrow (E1)$$

With this delta theta, all voltage magnitude angles of all buses are being evaluated, and using this delta theta, update the voltage phase angle, which means theta k plus 1 is equal to theta k plus delta theta, which is obtained from here. So in this way, 0.5 iteration of FDPF is done after solving the E1 equation. Now, once the E1 equation has been solved, check again if the convergence criteria are being met. If you do not find real power mismatches again, reactive power mismatches.

Now in this case, make sure that the check for PV PQ and PQ PV switching is required because the next set of equations we will solve is equation E2, and E2 would require a voltage magnitude update for PQ buses as well. So if convergence is not met, then

proceed if it is met, and that is how, if that means in step number 8, the stop here indicates that the convergence may happen in integer plus 0.5 iteration 0.5 referring to solving E1. If convergence is not met, proceed to solve E2, where checking for PV, PQ, and PQ PV is required because the delta Q equation and voltage magnitude update would apply only to PQ buses and not to PV buses.

Update the voltage magnitude and repeat from step 3 of slide 11, which is the step shown over here. And that's how the FDPF algorithm works. Step number 8 here results in an integer plus a 0.5 iteration.

$$\Delta V = B''^{-1} \frac{\Delta Q}{|V|} \rightarrow (E2)$$

After E2 has been solved, if we repeat from step 3, we will have an integer number of iterations here if convergence happens after solving E2.

To handle Q limits in fast decoupled power flow, the process is exactly the same for every P-V and Q switching. A dimension of B dash would increase by one, and the corresponding voltage magnitude would become a straight variable. Delta Q evaluation remains the same; QG is limited to its corresponding maximum or minimum limit. If the value is trying to go beyond the maximum or minimum limit, delta Q value is chosen, and B dash needs to be evaluated and factorized again for every PQ PV switching. B dash dimension goes down by one, and hence the overall purpose of retaining B dash, or constant B dash, in fast decoupled power flow for PQ PV PV PQ switching is still not clear.

 Satisfactorily answered, because if the dimension is changing, the corresponding value has to be evaluated again and again. To avoid changing this dimension, there has been some research on how to avoid B double dash evaluation. One way of doing that is by adding a big M number to the diagonal element of the row corresponding to the PV bus, so that if the big M number is present, it means the PV bus remains a PV bus. So let me show you how it is done. Suppose in some network our B double-dash matrix needs to be evaluated.
So let us say this is B double-dash inverse, where B double-dash actually has diagonal elements of 1, 2, and 4. If we factorize this, then this is a number that we would get. Now it may happen that here bus 2 or an element pertaining to bus 2, the corresponding bus here may be a PV bus, and for a PV bus, the B double dash update should not come; delta V should not update. So what is to be done? A big M number, let's say of the order of 10,000, is added to this diagonal element, and now when we take the factorization of this

particular matrix, please verify this. The terms that are relevant for PQ buses remain more or less the same.

The diagonal term here has become zero because of this masking. So by adding this mask or masking this with a large number, the corresponding factors won't come in, and hence, while switching every PV to PQ, only this 10,000 number needs to be added or multiplied, while the remaining other factors can remain as they are with no change in them. That is one of the ways to ensure constant B double dash matrix and fast decoupled power flow, although there can be other different ways. We'll conclude our discussion with an example of a fast decoupled power flow. We have seen this network, the same network, throughout our previous examples as well. Bus 1 is a slack bus, bus 2 is a PQ bus, bus 3 is a PV bus, and the data remain the same.

Line reactances are provided. There are no line resistances. There are no transformer taps and no phase shifters. So if we evaluate its Y bus, this is the 3 by 3 matrix. The summary of all these buses is given here. The unknown states are the two phase angles of bus 2 and bus 3, and the voltage magnitude of the PQ bus, which is bus 2. So we, along with the known variables for bus, slag bus, and voltage magnitude, are given as 1.

We choose 0 degrees or radians as a reference for bus angle. And V3 is given as 1.05 per unit. We choose the initial flat start, and at this flat start, we evaluate the corresponding injections. The injection evaluation remains exactly the same as it has been done earlier for neutral and absent power flow with polar voltage coordinates.QG at bus 3 is within its limits. So, bus 3 can remain as a PV bus. No need to change this voltage magnitude. For P theta half iteration, we evaluate the B dash matrix, which is the negative imaginary component of this Y bus. So, buses 1, 2, and 3, since V2, theta 2, and theta 3 are to be evaluated, we only take these corresponding elements with the negative component multiplied and the imaginary part removed, and that's how B dash is 15 minus 5 minus 5, 7.5. Delta P2 is to be evaluated as per the equation given here; 1.5 is the load at real power load at bus 2, so our mismatch is 1.5. We evaluate, and similarly for P3, we have this mismatch; we get this update of voltage phase angles after half an iteration of FDPA, which is the solution here. And for the next iteration or half-iteration, we evaluate Q2. We also evaluate Q3. Q3 remains within its limit. So the bus voltage of 3 did not change. Now, when we evaluate these limits, we choose this as our initial solution, not the solution that we chose at X0. Please remember that. And for the QV half iteration, only bus 2 voltage needs to be evaluated.

So B22 is to be taken from Vibus, which is 15. In this case, B dash and B double dash are extracted from the same Vibus because there are no taps, capacitances, or resistances considered, as discussed in slide 10. Reactive power mismatch is evaluated; we update the corresponding voltage magnitude. This completes one iteration of the FDPF. The same process, when continued, would converge in 6.5 iterations with the same exact

solution obtained with the Newton-Raphson power flow and Gauss-Tudor method, as discussed in the previous few lectures.

Example

Evaluate all voltage magnitudes and associated voltage phase angles after 1 iteration of FDPF. The line reactances are given in *per unit (pu)*.
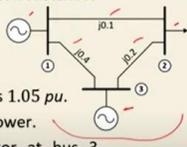
- Bus 1 (1 pu) is the angle reference bus.
- The load at bus 2 is $(1.5 + j1.5)$ *pu*.
- Voltage magnitude at bus 3 is specified as 1.05 *pu*.
- Generator at bus 3 generates 1 *pu* real power.
- Reactive power generation of generator at bus 3 should be within $[-1,1]$ *pu*.

Assume flat start for unknown voltages as initial guess.

The initial choice of variables is (flat start)

$$x^0 = [\theta_2^0 = 0 \quad \theta_3^0 = 0 \quad |V|_2^0 = 1]^T$$

Injections at this solution are

$$P_2^0 = 0, P_3^0 = 0, Q_2^0 = -0.25, and Q_3^0 = 0.39375.$$

$Q_{g3}^0 = Q_3^0 = 0.39375$ is within the limits of $[-1,1]$pu, i.e., bus 3 remains as PV bus and hence $|V|_3 = 1.05pu$.

For P-θ sub-iteration of FDPF, the jacobian is

$$J_{P\theta} = -\begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix} = \begin{bmatrix} 15 & -5 \\ -5 & 7.5 \end{bmatrix}$$

The real power mismatches are $\dfrac{\Delta P_2}{|V|_2} = \dfrac{P_2^0 + 1.5}{|V|_2^0} = 1.5$ and

$$\frac{\Delta P_3}{|V|_3} = \frac{P_3^0 - 1}{1.05} = -0.95238.$$

$$\Rightarrow \begin{bmatrix} \theta_2^1 \\ \theta_3^1 \end{bmatrix} = \begin{bmatrix} \theta_2^0 \\ \theta_3^0 \end{bmatrix} - J_{P\theta}^{-1} \begin{bmatrix} \frac{\Delta P_2}{|V|_2} \\ \frac{\Delta P_3}{|V|_3} \end{bmatrix} = \begin{bmatrix} -0.07415 \\ 0.07755 \end{bmatrix}. \text{ This is } P - \theta$$

half − iteration of FDPF $\Rightarrow x^{0.5} = [\theta_2^1 \quad \theta_3^1 \quad |V|_2^0]^T$.

At this solution, the reactive power injections are

$$Q_2^{0.5} = -0.162228 \ and \ Q_3^{0.5} = 0.461933$$

$Q_{g3}^{0.5} = Q_3^{0.5}$ is within the limits of $[-1,1]$pu, i.e., bus 3 remains as PV bus and hence $|V|_3 = 1.05pu$.

Solution: The bus admittance matrix of the system is

$$Y_{BUS} = \begin{bmatrix} -j12.5 & j10 & j2.5 \\ j10 & -j15 & j5 \\ j2.5 & j5 & -j7.5 \end{bmatrix}$$

➤ Bus 1 is the slack bus with voltage magnitude as 1 *pu*. Bus 2 is a PQ bus with complex power bus injection as $S_2 = P_2 + jQ_2 = -(P_{d2} + jQ_{d2}) = -(1.5 + j1.5)$ *pu*. Bus 3 is a PV bus with real power bus injection as $P_3 = 1$ *pu* and voltage magnitude as 1.05 *pu*.

➤ The unknown state variables are $x = [\theta_2 \ \theta_3 \ |V_2|]^T$.

➤ The known state variables are $|V_1| = 1, \theta_1 = 0, and \ |V_3| = 1.05$.

For Q-|V| sub-iteration, jacobian is $J_{Q|V|} = -B_{22} = 15$

Please note that $\underline{B'}$ in P-θ and $\underline{B''}$ in Q-|V| sub-iteration are to be extracted from the same Y_{BUS} for this system under consideration as there are no taps, shunt capacitances, resistances, etc., as discussed in slide 10.

Reactive power mismatch $\dfrac{\Delta Q_2}{|V|_2} = \dfrac{Q_2^{0.5} + 1.5}{|V|_2^0} = 1.33778.$

$$|V|_2^1 = |V|_2^0 - \frac{J_{Q|V|}^{-1} \Delta Q_2}{|V|_2} = 0.910815 \quad (1 \text{ term } \text{ of } \text{ } 2\text{ } ?)$$

This complete one iteration of FDPF. Overall process converges in 6.5 iterations with solution $V_1 = 1\angle 0, V_2 = 0.89836\angle - 0.0806rad$, and $V_3 = 1.04984\angle 0.085rad$

In the next lecture, we will take up another interesting topic of power flow solution techniques, which we call DC Power Flow, and it gains its motivation from the fast decoupled technique. So we'll see how that DC model is and what its application is all about.

Thank you.