Power Network Analysis

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Week – 01

Lecture-04

Hello, welcome to the fourth lecture of week one of the course Power Network Analysis. In today's discussion we will continue our topic of basic circuit principles. In the last two lectures we had talked about power in single phase circuit and some aspects of phasor diagram. In today's discussion, we will focus on evaluation of power for three-phase AC circuits. So towards the end of the last lecture, we discussed the nature of three-phase AC systems, how the natural form of power generation is inherently done through three-phase AC circuits or networks because it provides certain advantages, one of them being there is no need of an explicit return conductor. the instantaneous power remains constant for balanced operating conditions and hence our rotating machines like synchronous generators, induction motors undergo less mechanical vibrations.

Also, the rotating magnetic field produced in the air gap between stator and rotor of rotating machines turns out to be rotating at a constant speed or at a synchronous speed. So, this three-phase AC currents, the way the alternators are designed, They inherently produce this rotating magnetic field which aids in operation of rotating machines. And thanks to our design of synchronous generators, it inherently happens. Nothing exclusive needs to be done from the perspective of creating these rotating magnetic fields.

So the last lecture we ended up with this notion and in today's discussion we will try to talk about the reasons behind why these three-phase AC systems attribute these three distinct advantages and what is meant by balanced operation, balanced voltages, et cetera, et cetera. Before I go deep into what exactly I mean by balanced voltages, I'll just try to give a definition first. So for a three-phase system, balanced voltages would mean that there are three phase quantities or there are three voltages which can be represented again in frequency domain or phasor domain or they may be represented in time domain. The associated discussion regarding phasor diagrams would come into assistance here. So I have three balanced voltages in which each of these phase voltages can be represented as a phasor or in time and the important part is these phasors if I talk about these phasors are equal in magnitude and they have the same phase angle difference between each other.

That means if I have, let's say, a three-phase system in which I have three-phase voltages, VA, VB, and VC, that's the common convention of how phase A, phase B, phase C are defined in three-phase networks. In certain references, you would also find VR VB and VY as the common reference, R by V referring to red, blue and yellow. We'll try to strict to ABC, small ABC notation. So those discussions are still applicable for VR, VY VB discussion. So what is meant by definition of three-phase balanced voltages are that I have three phase voltages, VA, VB, VC.

Each of these phase voltages, if they are represented in phasor domain on a phasor diagram, then the magnitudes of or the RMS values of the corresponding time domain signal, which essentially turns out to be the magnitude of the phasor quantity, these amplitudes or magnitudes would be equal to each other and VA, VB, VC would have common or uniform phase angle difference among each other. Now, what is the premise or basis for this phase angle difference? What should be the value of this phase angle difference? We'll talk a bit about it when we talk about the fault analysis module. Thanks to Fortescue's theorem, C.L. Fortescue was a renowned power system engineer who came up with a theoretical basis of defining unbalanced systems in terms of balanced systems and from that theorem essentially comes the notion of balanced voltages or to be specific the phase angle difference for balanced voltages.

Fortescue's theorem is not only applicable for three-phase systems, it can be applicable for any n-phase system where n is more than 1. So basically it can be applicable for 2 phase system, 3 phase system, 4 phase system and so on. So Fortescue's theorem from there we can figure out what is going to be the phase angle difference. So coming to specific about 3 phase systems where we have 3 phase voltages. If I have to figure out what is going to be this phase angle difference, the term same here doesn't indicate what should be the phase angle difference.

So as per Fortescue's theorem, Fortescue stated that this phase angle difference can have two unique possibilities. One possibility is that the phase angle difference remains zero. Zero is also a common phase angle difference which can exist among all the three phasors. Or the other possibility is 360 degrees by n, where n is greater than equal to two. So if it is a three-phase system, then for three-phase system, n is equal to three.

So this number would turn out to be 120 degrees when n is equal to three. For a value of n different from three and more than one, the corresponding phase angles can vary. So for three-phase balanced voltage, we can have two such unique phase angle differences. One could be 0 degrees, the other could be 120 degrees. So how do we associate or visually present this in terms of phasor diagrams? That's where the next slide comes in.

We can have 120-degree phase angle difference among the three phase voltages, VA, VB, VC, in a clockwise fashion. and we tend to associate this as positive sequence voltage. If positive sequence voltage is a valid balanced system or balanced voltages, there can also be a negative sequence voltage where VA, VB, VC still have 120 degree phase angle difference among them, but the orientation is now anti-clockwise. And the third or the last possibility is where all the phasors are aligned to each other with common phase angle difference being zero. So as per the definition of balanced voltages, in all the three phasor diagrams, be it positive sequence voltage, negative sequence voltage, or zero sequence voltage, again the terms positive, negative, and zero sequence, they are just definitions and notations.

in practice or reality there for an N phase system where let's say N is four, five, these terminologies may accordingly change. The important part is in all these positive, negative and zero sequence voltages, all the three phasors In every diagram, they are equal to each other among a set, and all the three phases have equal phase angle difference among each other. That is what the important part is, that for a three-phase system, there can be three definitions of positive sequence voltage. So now the question is, okay, fine, three single-phase or three valence voltages are possible in three-phase system. Which one of them actually exists in practice? The answer is simple.

In practice or reality by design of alternators or synchronous generators, our machines tend to generate positive sequence voltages and consequently our loads, transmission lines, they also tend to experience this positive sequence voltage that's inherently by design. Someday the orientation of positive sequence reverses to negative sequence, that day the negative sequence voltage would become a convention for generation, transmission and distribution. Zero sequence voltage has its own issues, probably that's the reason why it won't be considered as a common way of generating voltages or generating power. because there is no uniformity in terms of rotating magnetic fields. Basically these voltages, they'll have a hard time generating rotating magnetic field for three-phase machines, three-phase rotating machines, whereas the same is not true in positive sequence or negative sequence voltages.

So alternator, they inherently generate this positive sequence voltage and if a three-phase balanced load is connected to such three-phase synchronous generator, then the currents that would be flowing from the source to the load would also be called as three-phase balanced currents. We'll talk about more of definition of balanced currents, balanced loads in the next few slides to come. Unlike balanced currents and voltages where the magnitudes have to be equal and phase angle difference among the phases have to be equal, balanced voltages definition is little different. Balanced voltage states that each of the phase of a n-phase system should have the same value of load. The notion of magnitudes being equal still is applicable for balanced load definition, but the difference is the phase angle difference The notion of having the same phase angle difference for balanced voltages and currents, that is not applicable for balanced loads.

Balanced loads, they mean that the same quantum of load be present in each phase of the network or each phase of the system. So for three-phase circuits, we can have two possibilities, two common possibilities. One is called as the star configuration. The other is called as the delta configuration. Usually in three-phase machines, the stator windings, which are again three-phase, again we will discuss about the nature of stator windings when we discuss more about synchronous generators module.

By practice or the natural practices that three-phase stator windings of three-phase synchronous generator. They are usually connected in star, not in delta. Reason being, in delta configuration, there is always a possibility of having circulating stator current in case the loads are unbalanced. And the insulation level requirement for windings to be operable in the three-phase synchronous machine there, the insulation requirement for star configuration is lesser compared to delta configuration. We will come back to this aspect in a moment.

Please have patience. Coming to three-phase instantaneous power, as per definition of balanced load, the same quantum of load is present in each phase of the three-phase system. So if I know the voltage in a particular phase, then I can use simple KVL and try to figure out the corresponding phase current. So let's say if my phase voltages are known, which are balanced and that's how the corresponding phase angle differences come in, this is the positive sequence voltage. And since I know the value of impedance or the load which is Z at an angle theta, so I can also find what are the corresponding phase currents that are likely going to flow in the respective phases to satisfy this three-phase balanced load which is having same quantum of load across each phase. And having known these voltages and currents, the important point to note among voltage and currents is The initial phase angle part, it is similar to the phase angle that existed in the voltages.

Since the impedance is having a phase angle theta L, so the corresponding load impedance angle or power factor angle is also appearing in the corresponding phase currents. And from there, if we try to evaluate the instantaneous power, so these are all instantaneous powers across respective phases, which is nothing but the product of instantaneous voltage and current, and then try to add them up. The beauty which appears is that in three-phase balanced circuits, the instantaneous power is independent of time. In single-phase AC circuits when we evaluated the instantaneous power, it always had a constant term which was the first term and then the second term was double the frequency current and it was function of time. In three-phase circuits, if we add up all the three instantaneous powers, the overall instantaneous power, which is sum of individual phase powers, it's invariable of time.

The corresponding time-dependent function is nullified because, and that can be verified by addition of these cosine terms sitting inside the brackets here. So that's one favorable advantage of how for balanced operation three-phase instantaneous power is time constant, it is invariant and that is the precise reason why average power has been evaluated or the notion of average power is defined to explain the notion of real power. Coming to voltage and current specifications for three-phase AC circuits, in single-phase AC circuits, the voltage and current specifications are typically defined in terms of RMS values. In three-phase circuits, the difference comes is that it is defined on a line-to-line basis, or I would say phase-to-phase basis. In single-phase AC circuits, the voltage and current specifications they inherently refer to phase quantities whereas in three phase AC circuits the specifications would refer to phase to phase or line-to-line quantities.

What are these phase-to-phase line-to-line? We'll talk about it in the next few slides. And the power which is defined in three-phase AC circuits, that's also the three-phase total apparent power, which is sum of individual phase apparent power. So for example, if I have a three-phase voltage source of 110 volt operating at 50 hertz, then it means that this three phase voltage source, the RMS voltage across phase to phase or line to line is 110 volt. It's not the phase voltage which is equal to 110 volt. Phase voltage and line to line voltage, there would be a difference, specifically for star connection.

Similarly, for a machine of 30 kVA supplying a balanced load, it would mean that the maximum single phase power which this machine can deliver under balanced condition is not going to be more than 10 kVA. 30 kVA by 3 is 10, so that's how individual phase power or capacity is defined for balanced operations. We now come to our first configuration so I have a synchronous generator here to be specific which is connected in star and the reason why it is connected in star or why it is called as star connected is because if I look at this particular circuit from the aerial perspective it represents the symbol Y which is the alphabet Y in English and that's the reason why it is called as Y connected or star connected and in this three-phase AC synchronous generator Suppose I know the individual phase voltages which are positive sequence voltages by design and if I know these phase voltages then the line to line voltages or also called as phase to phase voltages would be nothing but the voltage which is appearing across the two terminals of a given synchronous generator. So we can have three possibilities of terminals. One possibility is figuring out the voltage across terminal A and B.

The other possibility is figuring out the voltage across terminal C and A which we call as VCA. The other could be figuring out the possibility of finding voltage across terminals B and C which we call as VBC. VAB, VCA, VBC are nothing but the line to line voltages. Here line refers to the terminal of the synchronous generator. So we are trying to measure potential difference across two lines and hence the term line to line voltage comes in.

Now if I know my phase voltages, I can always figure out these potential differences across lines by simply applying KVL. So if I let's say start from node A traverse all the way from here to here and reach node B. Then analogously, I would have VAB as difference of phase voltage of phase A minus the phase voltage of phase B, wherein if I substitute the known values of VAN and VBN, I would get the expression of VAB of this form. And similarly, I can also find the corresponding phase to phase or line to line voltages across BC and CA. Now there's an important observation here, the observation is my initial phase voltages, they were all balanced, they were positive sequence oriented and if I now see the line to line voltages, they also appear to be balanced because the magnitudes or RMS values are all the same and the phase angle difference between any two such phasors or adjacent phasors is still 120 degree, that too again in the clockwise or positive sequence orientation wise.

So the beauty of three-phase AC circuit star connection is that the line-to-line voltages also turn out to be balanced three-phase voltages. This phasor diagram here, essentially the ones which are in black, they are the phase voltages of the star-connected synchronous generator Whereas the red ones are the line-to-line voltages of the star-connected synchronous generator. And either way I look at it, be it clockwise or anti-clockwise, all the three or all the two sets of voltages, phase voltages and line-to-line voltages, they're still balanced. The line to line voltage in sort of to generalize if I were to choose VAB and compare it with VAN then VAB is having a magnitude of root 3 times of VAN and the line to line voltage it leads the corresponding phase voltage which is VAN here by 30 degrees. The same analogy is applicable for VCA and VCN and the same combination is applicable for VBC and VBN.

So to generalize or summarize we can define this relationship which is line to line voltages root three times of phase voltage magnitude and it leads the phase voltage by 30 degrees. Now couple of slides back we talked about that in synchronous generators most of the setter currents are connected in star. The reason is now evident here. The reason is if I were to operate or connect my stator windings in star connection then the individual phase voltages which would appear they would be root 3 times or they would be 1 by root 3 times less than the voltage which is appearing on the terminal and remember For synchronous generators for three-phase circuits, the voltage specifications refer to line-to-line RMS voltages. So basically for a 20 kV synchronous generator where 20 kV refers to line-to-line RMS voltage, the corresponding phase voltage which will be observed in the filter windings would actually be 20 by root 3 kV.

So essentially, my windings, they need not operate at higher voltages. The terminals automatically have those higher voltages, and that's how insulation shaving can be done. Suppose now we have a balanced load connected to a balanced star-connected generator. So the idea here is to find the corresponding phase current which are Ia, Ib and Ic. So we can apply a simple KVL and the load here is balanced because all the three phases are having the same quantum of load.

So I can apply KVL and simply write that, so to find out these currents first of all, I have to know what my voltages are, source voltages are. So if I now choose line-to-line voltages as a reference for phasors, for phasor diagrams, then from there I can reverse track and try to find what are the corresponding phase voltages which is nothing but 1 by root 3 times of line-to-line voltage and now it is lagging the corresponding line-to-line voltage by 30 degrees. So now if I know my corresponding phase voltages and suppose I focus on phase A current. So what do I get if I apply KVL from this node to this node? I would see that let's say V capital N is the potential of the neutral point of the source. I'm traversing towards phase A so VN minus VA small n minus IAZ at an angle theta should be equal to V of small n where V of small n is the potential of the neutral point of balanced load and the potential of the neutral of generator and load are being measured with respect to some common ground or some common reference.

So from here if I try to find what is phase A current, phase A current would nothing be V capital N minus V N minus V A N by z at an angle theta. And if this is true, then I can also have a similar expression of phase B and phase C currents, where most likely the first term, which is encircled here, would remain the same. The voltage, however, would change from phase A to phase B to phase C. So if I summarize this in this particular slide, there is a difference or change in notation here because in my previous slide, I had chosen this terminal to be negative and this to be positive, whereas in this slide, the corresponding terminal orientations have been different. But anyway, that's not important for AC circuit because in AC circuit, the voltages or currents, they always form a symmetry.

So that's where you would find a little difference in the currents here. If I know these currents, the only unknown available here is it is not known for sure what is going to be this term because other than this term, VAN, VBN, VCN are known, the phase balanced voltages are known. So if I know what is V capital N minus V small n, then only I can be able to find what is IA, IB and IC. Now let's focus on the neutral point of this star-connected balanced load. If I apply KCL at the neutral point of the star-connected balanced load, then by KCL it would appear that Ia plus Ib plus Ic should be zero.

This is by KCL at node small n, because at node n there is no other branch through which the sum of these currents can escape. So if this has to be true, there is only one possibility under which this has to be true, this is possible when, so if I substitute these phase currents Ia, Ib, Ic into this expression, the only possibility under which it can be true is only when V of capital N minus V of small n is equal to zero or indirectly saying the neutral voltages of source and load, they are equal to each other. Why this has to be true? Because V a n, V b n, V c n are all finite quantities it's a source voltage the impedance is also finite it's not non-zero so it's a finite load and ia plus ib plus ic can only be zero only when this neutral voltage is equal to zero so with this We come to our second advantage that since the neutral points under balanced operations are equal, even if there was a physical wire which was connecting these two neutral points, since this wire does not have any potential difference across it, no matter whether this wire was present or not present, the potential difference never exists. So as a result, the current through this wire will not exist, it will always be zero. That's where the advantage of saving return conductor is applicable for three-phase circuits.

So discussion that we had in the previous one that has been summarized here and even though the nodes are open circuit, potential difference is always zero. So it's similar to short-circuiting the two terminals. Even if there was a wire connected, that wire would never carry any current under balanced condition. So unlike in single-phase systems, for three-phase systems, there is no need of an explicit return conductor as was explained in the previous few slides. Even if the loads are not balanced, which is often true, Because by design we can ensure that the sources are balanced but the loads are uncontrollable entities.

They don't adhere to certain norms and conditions. So ensuring that loads remain balanced is a difficult task. Even under such conditions when the network is experiencing unbalances, still potential differences between neutral points is made zero. The reason for that, it helps substantially in fault analysis to be specific. The application of fault analysis is about system protection. Usually during system protection, which we'll talk about in fault analysis module, as long as balanced operation is ensured, Then only during non-fault conditions we will specifically be able to clearly figure out these unbalanced operations.

We'll talk more about it when we talk in detail about the balanced fault analysis or unbalanced fault analysis which is going to be part of the second last module. Just to summarize, so if then the node potential differences Vn is same as V of small n, so the second terms they all vanish and from there we can find the corresponding phase currents Coincidentally in star connection the phase current is same as the line current because the line is essentially defined by the terminal of the load or the source and from there if we try to find the corresponding complex power total complex power the expressions are all given here. They are again based on the discussion that we had in the previous single phase AC circuit discussion. This slide sort of enumerates what is the definition of apparent power which is nothing but the sum of the individual phase powers and One thing to note here is that The three-phase apparent power, it is definable in terms of line-to-line quantities.

It is also definable in terms of face-to-face quantities. As long as line-to-line quantities are being used, you would always experience a factor of root 3 while defining apparent power, whereas a factor of 3 would come in when you define apparent power using face quantities. The same is also applicable for complex power. And it is this power which is usually used as the specification of power for three-phase machines. If we take the corresponding real part of apparent power or complex power, that would give us the real power and correspondingly the imaginary part would give us the notion of reactive power where theta is the corresponding power factor angle or load impedance angle. If we now talk about balanced load, the source remains star connected, the load is now connected in a delta connection.

So we use reference of line to line voltage here as reference for all phasor quantities. And from there we can find the individual phase currents which are IAB, ICA because they are the currents which are flowing through the individual phases. Since we know the corresponding line-to-line voltage, so from here we can easily find the corresponding phase-to-phase currents. And once we know the phase-to-phase currents, now we can also find what are the line currents. So in this case, line current IA by KCL at this particular node is nothing but IAB minus ICA.

Since I know the expressions of IAB and ICA, I can put this over here and get my phase currents or line currents and these line currents similar to line to line voltages in star connection, they also appear to be balanced, they have the same quantum of magnitude and the phase angle differences are 120 degree apart. So this is a similar phasor diagram which was drawn to show voltages for star connected line to line voltage and phase voltage. Similar reasoning is also applicable for currents in a delta connection. And the notion is a little different here. The line-to-line currents, the magnitude is similar to line-to-line voltages.

It is root three factor comes in. But unlike line-to-line voltages advancing phase angles or phase voltages by 30 degree, the line-to-line currents here, they lag the corresponding phase currents by 30 degrees. And on similar lines, if we now try to find the corresponding apparent power, complex power, the notion still holds true where theta is the corresponding phase angle. And that gives us one idea that probably when we have to analyze balanced three-phase systems, We can probably analyze them on a single phase basis and then once our analysis is done specifically for balanced operation, we can eventually scale up those per phase or single phase analysis to a three phase analysis by properly incorporating the factors of three if phase quantities are being considered or factor of root 3 when line to line quantities are being considered for the phase analysis. We will also talk about it in detail when we go into our next module which is essentially going to be on transformers and per unit analysis. Thank you.