Power Network Analysis

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Week-08

Lecture-39

Lecture 39: Power flow analysis- Newton- Raphson method (rectangular coordinates)

Hello everyone, welcome to lecture four of week eight of the course Paranormal Analysis, in which we continue our discussion on the Newton-Raphson technique for solving power flow equations. And in today's discussion, we will specifically consider the application of Newton-Raphson for solving power flow equations when voltage phasors of every bus are in rectangular coordinates. What I mean by rectangular coordinates is that if I have to indicate or write the voltage phasor at bus i, then it has a real part, which is e_i. With an imaginary part f i, where j is the complex operator minus 1, and analogously, these e i and f i are nothing but the magnitude of voltage with cos theta i; theta i is the phase angle, and f i, the imaginary part, is with respect to v i sin theta i, where again. The importance of reference or angle reference is significant, so if phase angle theta i is being measured with respect to some reference, which is usually the slack bus or the angle reference bus or swing bus, then all these imaginary parts and real parts are also being measured with respect to this particular angle reference bus or slack bus. So we would assume our voltage phasors to be of this form and then see how the Newton-Raphson technique can be used to solve power flow equations in rectangular coordinates.

$$V_i = e_i + jf_i \qquad j = \sqrt{-1}$$

$$e_i = |V_i| \cos\theta_i \qquad f_i = |V_i| \sin\theta_i$$

In the previous lecture, we discussed at length the application of the Newton-Raphson technique for power flow equations in polar coordinates, where voltages had both a magnitude and a phase angle, and we also examined the quadratic convergence and mathematical rigor behind it. Proving the quadratic convergence of the Newton-Raphson technique. So, we will start this discussion with an example of understanding how the

Newton-Raphson technique works. So, basically, I mean the terminologies are a little wrong here; this is actually one iteration of the Newton-Raphson method.

Pardon me for using the same example from the goth riddle slides or goth riddle lecture. So, the problem statement remains the same. We have a three-bus network where bus one is the angle reference bus with a voltage magnitude of one pu. Bus 2 is a PQ bus or a load bus, whereas Bus 3 is a PV bus with a voltage magnitude of 1.05. PG has 1 PU, and reactive generation should be within minus 1 and 1 PU. We will have to see how the Newton-Raphson technique works for one iteration of solving the corresponding problem. Voltage phasors in polar coordinates. That means the voltage magnitudes and the corresponding voltage phase angles are included. The line reactances are given in per unit.

So basically, we can find the corresponding y-bus. Please cross-check that this bus is correct. And since we have three types of buses, for every such bus we get to know what the corresponding knowns and unknowns are. Bus 1 is a slack bus, so there is actually no unknown in terms of voltage states for the slack bus. Its magnitude is 1 PU.

Angles can be chosen as 0 degrees or 0 radians for the sake of convenience. Bus 2 is a PQ bus; it's only a load bus, so basically minus 1.5 and minus 1.5. The values given over here are the respective real and reactive injections at this corresponding bus 2 PQ bus. Bus 3 is a PV bus, so its P is given, the voltage magnitude is given, and the unknowns are the voltage phase angles of bus 2 and bus 3 and the voltage magnitude of bus 2. Voltage magnitudes of bus 1 and bus 3 are specified. If we have three unknowns, we choose the corresponding three equations and the prior information known to us, which are the voltage and phase angles of bus one and the voltage of bus three. We have three unknowns, so we need three equations. Those three equations pertain to the real power equations of all buses except the slack bus; basically, the real and reactive power equations exist for PQ buses, whereas we only have the PV bus 3, which is that PV bus, so we have only the P equation for bus 3, whereas for bus 2 we have both P and Q equations.

$$f(x) = \begin{bmatrix} P_2 + 1.5 \\ P_3 - 1 \\ Q_2 + 1.5 \end{bmatrix} = 0$$

You have three equations, three unknowns, the corresponding function forms of P2 and P3, depending on the corresponding Y bus element shown here, and the values that are known above; those function values have been given.

$$P_2 = 10|V|_2 \sin\theta_2 - 5|V|_2|V|_3 \sin(\theta_3 - \theta_2),$$
$$P_3 = 2.5|V|_3 \sin\theta_3 - 5|V|_2|V|_3 \sin(\theta_2 - \theta_3), \text{ and}$$
$$Q_2 = 15|V|_2^2 - 10|V|_2 \cos\theta_2 - 5|V|_2|V|_3 \cos(\theta_3 - \theta_2)$$

Once we know these function values, we can evaluate the corresponding derivatives for the initial choice of values that we select. Injections at this initial solution, basically with voltage phase angles of 0, you can easily cross-check; if angles are 0, the corresponding sine values would also be 0, so P2 and P3 for the initial values of theta 2 and theta 3 as 0 turn out to be perfectly 0. Whereas for Q2, the cosine values they turn out to be equal to 1, cosine of 0 is 1, voltage magnitude of V2 is chosen as flat start which is 1. So essentially this term is 15; here we have minus 10, and here we have minus 5 into 1 into 1.05, and not 1 per unit. Basically, this Q2 value is 15 minus 10 minus 5.25, which is minus 0.25, which is what is shown over here in terms of Q20. Similarly, we can also find the injection at bus 3 because bus 3 is a PV bus, so we have to ensure that its generation is within the specified limit.

So, for the initial choice of solution, bus 3 remains a PV bus. We find the corresponding function forms, which are these numbers here. So 0 plus 1.5 is 1.5. 0 minus 1 is -1. Minus 0.25 plus 1.5 is 1.25. So f of x at the initial value of x0 is this function's order.

- PV bus 3 reactive power injection expression is
  $Q_3 = 7.5|V|_3^2 - 2.5|V|_3 \cos\theta_3 - 5|V|_2|V|_3 \cos(\theta_2 - \theta_3)$
- The initial choice of variables is
  $x^0 = [\theta_2^0 = 0 \quad \theta_3^0 = 0 \quad |V|_2^0 = 1]^T$ (given as flat stat)
- Injections at this solution are $P_2^0 = 0$, $P_3^0 = 0$, $Q_2^0 = -0.25$ and $Q_3^0 = 0.39375$. Please note that $|V|_3 = 1.05$. $Q_{g3}^0 = Q_3^0 = 0.39375$ is within limits of $[-1,1]$ *pu*, thus bus 3 remains as PV bus and hence $|V|_3 = 1.05$.
- Thus, PF eqn. vector at this solution is
  $$f(x^0) = [1.5 \quad -1 \quad 1.25]^T$$

We can then find the corresponding Jacobian elements. The corresponding function forms were discussed in the previous slides.

$$\frac{\partial P_2}{\partial \theta_2} = -Q_2 + 15|V|_2^2 = 15.25,$$
$$\frac{\partial P_2}{\partial \theta_3} = -5|V|_2|V|_3\cos(\theta_3 - \theta_2) = -5.25,$$
$$\frac{\partial P_2}{\partial |V|_2} = \frac{P_2}{|V|_2} = 0, \quad \frac{\partial P_3}{\partial \theta_2} = -5|V|_2|V|_3\cos(\theta_2 - \theta_3) = -5.25,$$
$$\frac{\partial P_3}{\partial \theta_3} = -Q_3 + 7.5|V|_3^2 = 7.875,$$
$$\frac{\partial P_3}{\partial |V|_2} = -5|V|_3\sin(\theta_2 - \theta_3) = 0, \quad \frac{\partial Q_2}{\partial \theta_2} = P_2 = 0,$$
$$\frac{\partial Q_2}{\partial \theta_3} = -|V|_3\frac{P_2}{|V|_3} = 0 \text{ and } \frac{\partial Q_2}{\partial |V|_2} = \frac{Q_2}{|V|_2} + 15|V|_2 = 14.75$$

Once we get this Jacobian matrix form, always ensure or remember what kind of Jacobian we are looking at. We have two P equations and one Q equation. We have two voltage phase angles and one unknown voltage magnitude. So JP theta is our two-by-two matrix. QV is a one by one matrix. Q theta is a two cross one matrix. Whereas JPV is a one by two matrix depending on the order of the knowns and unknowns. So we have this three cross three matrix as the Jacobian matrix.

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial |V|_2} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial |V|_2} \\ \frac{\partial Q_2}{\partial \theta_2} & \frac{\partial Q_2}{\partial \theta_3} & \frac{\partial Q_2}{\partial |V|_2} \end{bmatrix} = \begin{bmatrix} 15.25 & -5.25 & 0 \\ -5.25 & 7.875 & 0 \\ 0 & 0 & 14.75 \end{bmatrix}$$

- The PF eqn. vector at this solution is

$$f(x^0) = [1.5 \quad -1 \quad 1.25]^T$$

We take its factorization and get the value of X1. So this is our initial X1, which corresponds to theta 2, 1, theta 3, 1, and V2, 1. One is the iteration number. We put in those values and get the next iterative values of all voltages.

- Hence, after first iteration of Newton-Raphson method, the solution is

$$x^1 = x^0 - J^{-1}f(x^0) = \begin{bmatrix} -0.070922 \\ 0.079703 \\ 0.915254 \end{bmatrix}$$

which implies that after first iteration, $V_1 = 1\angle 0, V_2 = 0.915254\angle -0.070922\text{rad} = 0.91295 - j0.06486$, $V_3 = 1.05\angle 0.079703\text{rad} = 1.04666 + j0.08359$

- The converged solution in 4 iterations is

$$V_1 = 1\angle 0$$
$$V_2 = 0.89836\angle(-0.08055\text{rad})$$
$$V_3 = 1.04984\angle(0.085\text{rad})$$

And this sort of process continues until convergence. The process converges in only four iterations compared to 19 recursions in the Gauss-Friedel method in the previous lecture. which you can always go back and cross-check with the final solution remaining the same as that obtained in the goth riddle method. So, you can just have an estimate that the number of recursions involved here is very high. The number of iterations involved in Newton-Raphson is very small, but every iteration of Newton-Raphson involves factorization of this Jacobian and further evaluation of this Jacobian. And this value of j is likely going to change in every iteration because the corresponding values of v2 and theta2 are changing. So that's where the burdensome part is, and that's where the trade-off is. But still for a smaller system, the number of iterations involved is much slower in neutron absent and hence the convergence is faster and number of iterations involved is lesser. For rectangular coordinates, as I have already mentioned in the opening slide, we are choosing the voltage of bus I to be of the form EI plus JFI, where J is equal to the complex operator, EI is the real part of VI, FI is the imaginary part of VI, and if we use this notation and re-express our real and reactive power equations, these all turn out to be quadratic equations and not transcendental equations. And similarly, all our real and reactive power equations, bus injections at the slack bus, real and reactive power generation, and system loss— we've already seen these expressions, which we can evaluate once the power flow solution has been obtained.

So, with this as our premise of equations, the real and reactive power equations being quadratic functions, let's see how the Newton-Raphson technique can be used to find the unknowns, which are the real and imaginary parts of voltages at every such bus. So what we do is have the same convention that we choose bus 1 as our slack bus. We have our buses numbered from 1 to capital N. And for the sake of convenience, we are choosing bus 1 as the slack bus, which means it's the real part. I mean, the reference is the slack bus.

So for the slack bus, when you choose the phase angle as 0 degrees or radians, it is implicit that the corresponding real part, mod of V cos theta, would be simply equal to mod of V because cos theta for 0 degrees is equal to 1. F, on the other hand, is mod of V sin theta; theta being 0, sin would be 0. So, irrespective of the mod of V, F would always be 0.

$$\theta = 0^{\circ}$$

$$e_i = |V_i| \cos\theta = |V_i|$$

$$f_i = |V_i| \sin\theta \ = 0$$

And that's the reason why, for the bus, slack bus, we have E1 as the specified voltage magnitude, F as 0 degrees, choosing theta of the slack bus as 0 degrees or radians with respect to potential or reference. For PV buses, the voltage magnitude is specified, which means that for every such PV bus, we know what the specified value of voltage is. But unfortunately, in the Newton-Raphson method, the polar rectangular coordinate voltage magnitude is not an explicit variable; the variables are all E's and F's. So, how do we handle these E's and F's in terms of voltage magnitude, and remember, when voltage magnitude is specified, it implicitly means that for every PV bus, the sum of the squares of the real and imaginary components will be equal to the voltage magnitude squared.

$$e^2 + f^2 = |V^{sp}|^2$$

Why? Because E is V cos theta and F is V sine theta, if we square them and add them, cos square theta plus sine square theta is equal to 1, so we have this unique equation. So essentially for every such PV bus, if it remains as a PV bus, this specified value is specified or fixed, so it has to be there and E's and F's, they should ensure together that their square sum is equal to mod of V square. So how do we handle that in this situation? Because in rectangular coordinates for Newton-Raphson, voltage magnitude is not an explicit variable.

The getaway or get around for this is, okay, if the equation doesn't exist, let us include that equation, and then we'll see what happens. And that is the reason why this equation is inherently present or written here.

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_i - P_{gi} + P_{di} \forall i = 2,3,\ldots,N \\ Q_i + Q_{di} \forall i \in PQ\ buses \\ e_i^2 + f_i^2 - |V^{sp}|_i^2 \forall i \in PV\ buses \end{bmatrix} = 0$$

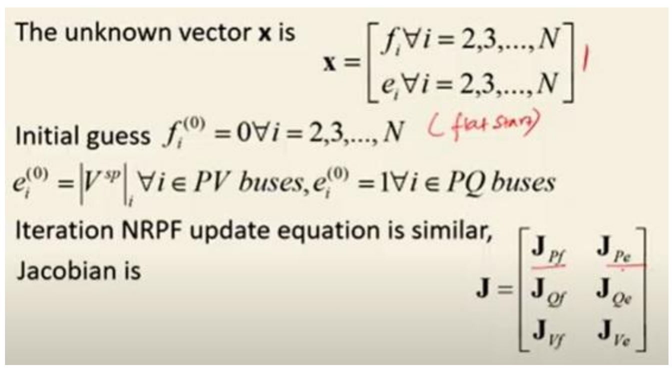
$$P_{gi} = 0 \forall i \in PQ\ buses$$

In polar coordinates, the powerful equations that we saw were limited only to the first two equations. For Newton-Raphson in rectangular coordinates, where voltage magnitudes are specified for the PV bus, we also have to ensure that this third equation comes in so that the PV bus remains as a PV bus. So basically we have three sets of equations for Newton-Raphson in rectangular coordinates.

The first set refers to real power equations for all buses except the slack bus. So basically, P is n minus 1 equations. We have the same Q equations as we saw in the Newton-Raphson method for polar coordinates whose dimension is nd. or NPQ the number of PQ buses in the network, and then we have certain voltage magnitude expressions that pertain only to PV buses. So, essentially, this number is N minus 1 minus ND or NPQ; N minus 1 is the number of buses except the slack bus.

If you subtract the number of PQ buses from there, then we have this as our number of voltage equations. The unknowns here are E's and F's, so let's also look at what the unknowns are. The unknowns will be all F's pertaining to all buses except the slack bus by except slack bus before for slack bus it is already chosen as 0. So, except slack bus which means the number of unknowns in terms of imaginary components of these would be N minus 1 in number and ease again would be for all buses except slack bus even for PV buses we would have ease.

So, this is also N minus 1 in number. Why? Because in the Newton-Raphson rectangular coordinate system, voltage magnitude is not a variable. It is a known value. And E's and F's together have to ensure that they satisfy the voltage magnitude specification. So we basically have, in rectangular coordinates, 2n minus 2 unknowns. And do we have the same number of equations for these unknowns? Yes. If we add these three terms here, we have the same n2n minus 2 equations from which we can find these corresponding unknowns. So this is what the... Unknown vectors, initial choice again is equivalent to flat start in Gauss-Thudel method or Newton-Raphson polar version. The set of equations here is a little different. So we have six submatrices,



J, P, and F, pertaining to the partial derivative of real power with respect to the imaginary part of voltages. JPE pertains to the partial derivative of real power equations with respect to the real part of voltages. And since P, Q and the voltage expressions their forms are known in terms of quadratic expressions we can easily find those derivatives the convergence criteria remains the same. The corresponding dimensions of these six sub matrices they are all given over here and I request all to please cross check and verify these expressions with respect to the previous forms of P's and F's and the voltage expressions which are given over here. So basically, if I have to, the first two sets are pretty easy to handle and pretty simple to form.

- Dimension of J_{Pf} is *(N-1)X(N-1)*
- Elements of J_{Pf} are

$$\frac{\partial P_i}{\partial f_j} = G_{ij} f_i - B_{ij} e_i, j \neq i; \frac{\partial P_i}{\partial f_i} = G_{ii} f_i - B_{ii} e_i + \sum_{j=1}^{N} (G_{ij} f_j + B_{ij} e_j)$$

- Dimension of J_{Qf} is *(N_d)X(N-1)*
- Elements of J_{Qf} are

$$\frac{\partial Q_i}{\partial f_j} = -G_{ij} e_i - B_{ij} f_i, j \neq i; \frac{\partial Q_i}{\partial f_i} = -B_{ii} f_i - G_{ii} e_i + \sum_{j=1}^{N} (G_{ij} e_j - B_{ij} f_j)$$

How do I get the corresponding JVF matrices? So if I look at JVF matrix, it is essentially partial derivative of V equation with respect to imaginary component of f. Now, if we look at the voltage equation, this is true only for PV buses. If I have to express this function as let us say some m of x or let us say some m of e and f where the i-th equation is e i square plus f i square minus v specified i square and i want to find the derivative of the m-th i equation with respect to the j-th f The jth f, or let's say the kth f, maybe k is some other bus number which is not the same as bus i, since fk is not present here at all; it is only the imaginary component of that same bus which is present over here, so this derivative would always be zero. For the mth i derivative with respect to fi where or let me if I say k where k is equal to i means k is same as i so fi is appearing here.

This derivative is 2 times fi, which is coming in because of this quadratic form. And that is the reason why the off-diagonal terms are 0 for JVF, whereas the diagonal terms are the non-zero terms. The same logic is applicable for the E partial derivative forms. The first two forms are pretty simple. The third form is exactly similar to the form that we discussed for the F matrix here.

The handling of Q limits, which is the last part, remains exactly the same. PV to PQ switching and PQ to PV switching will continue to happen depending on whether the reactive power injection equation is satisfied for every such PV bus. So during PV to PQ switching, the corresponding voltage equation is not to be considered because the voltage magnitude that was defined for the PV bus is no longer fixed. It has violated its reactive power control.

So voltage now becomes free. The reactive equation is now frozen, so the reactive power injection equation with new reset values of QG comes in. So the voltage magnitude is free to change for PV to PQ switching. For PQ to PV switching for a specified PV bus instead of reactive power injection equation voltage equation would come in and voltage magnitude is now a fixed value. So essentially, compared to Newton-Raphson in rectangular coordinates and polar coordinates, the dimension of the requirement matrix in rectangular coordinates remains the same irrespective of pv to pq switching or pq to pv switching, which is 2n minus 2 cross 2n minus 2.

The order of equations changes. while the Jacobian dimension doesn't order of equations changing means the Jacobian elements also have to change but then compared to polar version the Jacobian dimension was also changing additionally compared to the in addition to the number of equations being changing so that's the big difference as compared to polar version in rectangular coordinate irrespective of bus switching the Jacobian elements change but the order doesn't change or the dimension of j doesn't change We already talked about the quadratic convergence of Newton-Raphson in the previous lecture for a well-defined system. Generally, the Newton-Raphson method converges in a very limited number of iterations. It has robust convergence and mostly gives the high voltage solution wherein high voltage means the solutions or voltage magnitudes are close to 1 per unit and phase angles are in order of few plus minus 45, 60 degrees so on for well-defined systems. For a low voltage solution, the voltage magnitude itself would tend towards zero no matter what the phase angle is. The Newton-Raphson method usually provides the high voltage solution, although it can provide a low voltage solution depending on the initial choice.

For poor systems or ill-conditioned systems where the load is very high, the distribution line R by X ratio is very high, or the network structure is very radial, the corresponding Y bus becomes diagonally non-dominant. And that is a big link here: the Vibus becoming diagonally non-dominant, which would mean that the Jacobian also becomes diagonally non-dominant. In all these expressions that we have seen here, you wonder or see that the Jacobian elements are functions of the Vibus elements. So, if Vibus elements are diagonally non-dominant, the Jacobian would also become diagonally non-dominant, and that is what creates ill conditioning, meaning the corresponding inverse of J is difficult to evaluate. Overall nutshell, Newton-Raphson although has quadratic convergence is iterative in nature.

Every such iteration involves the factorization or evaluation and factorization of a large Jacobian matrix for a large power network, which can become computationally burdensome. And in this regard, several researchers have spent their entire lives on how to mitigate or reduce the challenge of the computationally burdensome aspect of the Newton-Raphson technique. Since this is a very basic course on how power networks work, I won't be able to discuss the details of those research works, but I can always introduce what those topics or contents would look like. There is something known as the use of the optimal multiplier, wherein researchers have successfully demonstrated that with the use of this optimal multiplier technique, Jacobian factorization can be avoided in every iteration. In fact, with use of optimal multiplier under certain operating conditions for a power network, Jacobian needs to be factorized only once, that too at the beginning of Newton-Raphson technique.

And the same factorized Jacobian can then be used to solve the Newton-Raphson equation, whether in polar form or rectangular form; that is one idea. So when you limit the number of factorizations of the Jacobian, the Newton-Raphson method, which is already faster, becomes even faster and can still provide us with the same solution that Newton-Raphson would have given with multiple factorizations of the Jacobian involved. The other technique is known as the sparsity technique, in which, instead of storing or saving the entire sparse Jacobian matrix, Jacobian is heavily dependent on the fibers. Typically, for large power networks, fibers are also a sparse matrix. What is sparse? The meaning of sparse is Most of the elements in a particular matrix if they turn out to be zero with very few limited terms being non-zero we call that to be a sparse matrix.

The larger the number of zero elements in Y bus, the larger the sparsity, and why would elements in Y bus be zero? It would refer to the corresponding element, let's say the M, N element in the Y bus; if it is 0, it would mean that bus M is not connected to bus N directly. That's how sparsity comes in. If a network is highly meshed with strong interconnections, the Y bus would also become a non-sparse matrix. It is not always possible to have a highly interconnected power network. The power network also has limited interconnection, so the larger or lesser the interconnection, the more sparseness appears in the fibers; more sparseness in fibers means the Jacobian also becomes more sparse, and the sparsely stored technique does not enable storing or saving this Jacobian matrix with only limited non-zero solutions or elements being stored.

The remaining zero elements are not stored at all, and through the use of proper reduction and inversion techniques, the stored sparse or non-zero dense matrix can be easily factorized. That is what sparsity techniques refer to. eople who are highly interested or motivated to understand more about optimal multiplier or sparsity techniques, please give a search and you would be able to find corresponding references wherein you can go through these techniques. There are several textbooks which probably I will provide or I'll give reference to in the next lecture and you can go through those details at length. The next lecture, we will spend time on another technique, which is an industry-grade solution.

In fact, it is a variant of the Newton-Raphson technique known as the fast decoupled method. which came up somewhere around in the 1980s, to be precise, I believe 1979 or 1981. I will have to check, but it is somewhere around the 1980s where one PhD student, along with his supervisor—the student is ALSEC, and the supervisor is Brian Stott. Alsak and Stott together came up with this technique, a variant of the Newton-Raphson method, which is the fast decoupled method. And they, I mean, there is no theoretical basis for this, but they were able to numerically prove that the fast decoupled method is actually a better method than Newton-Raphson, and the problem that they were trying to attack was whether the number of Jacobian factorizations can be limited in Newton-Raphson.

They were able to come up with this fast decoupled method, which showed that with one factorization of the Jacobian, the same solution as in Newton-Raphson can still be obtained with a lot of time being saved. So, we will talk about this interesting technique which was thanks to Alsak and Stott and then we will see how it works in the next lecture.

Thank you.