**Power Network Analysis**

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**Week-08**

**Lecture-36**

Hello everyone, welcome to the first lecture of week 8 of the course Power Network Analysis, in which we continue our discussion on power flow analysis, specifically the classification of buses involved in power flow equations and the corresponding equations that we would need. In fact, we have seen some of these equations in the previous lecture as well. And at the end of the previous lecture, we were discussing one such classification of a bus, which is a load bus. So, basically, why do we need this classification? What we have observed is that every bus in the power network will have at most two equations, which are alpha and beta. But every such bus can have four unknowns: two independent unknowns, which are voltage magnitude and the voltage phase angle with respect to a reference, and the corresponding two dependent controls, which are $P_i$ and $Q_i$, the injections at these buses.

So we have to ensure consistency between the number of equations and the number of unknowns before we solve the power flow equations, and that's where the classification of buses is needed. And in fact, we have also seen that the demand $P_{di}$ and $Q_{di}$ for a given load can be known from the load forecasting exercise. $P_{gi}$, which is the generation of I with the generator connected to bus i, can be known from economic dispatch. The corresponding voltage magnitude phasor can also be determined from active power loss minimization via reactive power control, active power loss minimization, power factor control, or excitation control.

So these quantities can be known for the generators, and these could be known from loads. So can this information not be used to understand the classification of buses and bring consistency between the number of equations and the number of unknowns? That's where the first load type or bus type, the load bus or PQ bus, comes in, where there is definitely no, I mean, generator connected to such a load bus; a load may or may not be connected. The bus may only have a few incoming lines and a few outgoing lines through which the power is getting transferred, and since there is no generator connected to such a load bus, the injection at the i-th bus becomes simply the corresponding negative of $P_{di}$ and $Q_{di}$. Remember, injection from a real power perspective is $P_{gi}$ minus $P_{di}$. So if the generator is not present, this is 0 .

So $P_i$ is simply equal to minus $P_{di}$, and similarly, $Q_i$ becomes simply equal to minus $Q_{di}$. And since the load bus results in two knowns, which are P and Q themselves, that is the reason why it is also called a PQ bus. So every such PQ bus has two equations and two unknowns, the unknowns being the voltage magnitude and phase angle for the corresponding i with the bus voltage phasor. Before I go into the next classification of bus, I will just briefly explain this part a bit. A load may or may not be connected.

So essentially what I mean is there is one such bus in the network which has few incoming lines and few outgoing lines; there is no other device connected to this bus, then this bus could also be called a load bus and/or a PQ bus, and since there is no injection at all, no external injection for this particular bus, the corresponding injections would simply be zero, and we would also call them zero injection buses or ZIB, often also known in abbreviated form in the literature. If there is any other device connected to such a zero injection bus, for example, a capacitor or a synchronous condenser, then the notion becomes a little different because this bus, in a way, now becomes a controllable bus wherein the voltage can be controlled, which is what leads us to the next category of buses. It has various names; we will start with the name of generator bus and then understand in a few moments why it is also called a PV bus and why it is also called a voltage control bus. So, a generator bus, as the name suggests, is the bus in the network that has at least one generator connected to it and this bus, through which the generator is connected, may or may not have a load connected to it. So, essentially, if I talk about it from the perspective of real power injection, where $P_i$ is $P_{gi}$ minus $P_{di}$. $P_{di}$ may be 0 or may not be 0 , but definitely $P_{gi}$ should not be 0 for a generator bus. $P_{di}$ may exist or may not exist, but as long as the generator is connected, this quantity should not be 0 , and then those buses would be called generator buses. And as we have seen in the previous lecture, for generators, specifically synchronous generators, the terminal voltages of those buses can also be known through active power loss minimization, and this terminal voltage can be controlled or regulated through corresponding reactive power control or power factor excitation control. So that notion of a voltage-controlled bus would also become evident because at such a bus, where a generator is connected, specifically a synchronous generator whose reactive power control can be regulated, it can also have variation or controllability from voltage, so it is also called a voltage-controlled bus. Lastly, since the voltage specifications can be known from active power loss minimization, the quantities that are known for such a generator bus essentially are the real power injection and the terminal voltage.

So, since P and V are known, it is also often referred to as a PV bus. So, that is the reason why P and V in blue are known from corresponding economic dispatch, load forecasting, or active power loss minimization. And since the reactive power equation is when reactive power injection is not known a priori and only the real power injection is now known here. So, we have the alpha equation, which is the real power equation, and the subscript PV essentially means that it is referring to the PV bus. The reactive power, so for a bus to remain a PV bus, the voltage magnitude should remain fixed.

That means the reactive power generation should be within its limits. Reactive power generation, if it is within its limit as per the capability curve, allows the generator to maintain a terminal voltage while regulating its internal EMF through power factor control or excitation control; for that, the reactive power limit should be within the respective minimum and maximum limits as per the capability curve. In case the PV bus or generator loses its reactive power control capability, that means the capability curve boundary points have been reached. Terminal voltage will no longer remain a fixed quantity, and it should now become a different type of bus; we will see what that different bus is. For a PV bus, P and $V_i$ are known; the only unknown is $\theta_i$, so essentially for every PV bus we have only one equation and one unknown.

We do not exclusively solve the reactive power equation because the reactive power equation can be solved only when the injection at the PV bus is known. The injection can be known only when the mean $Q_{di}$ can be determined from load forecasting, but $Q_{gi}$ has to be known, and $Q_{gi}$ has to be within a limit; it should not necessarily be a fixed number. So how do we ensure that we sort of cater to the reactive power equation? So we will see that on the next slide. So just to summarize, for every PV bus, we only have the real power equation because P is known, and terminal voltage is known from active power loss minimization. We have one equation and only one unknown for the PV bus.

And as mentioned, terminal voltage is sort of known from active power loss minimization; it is assumed to be fixed. So, for this voltage to remain fixed, as I stated, the reactive power generation should be within its limit. So, how do I find the reactive power generation? I can rewrite this equation as $Q_{gi}$ equal to $Q_i$ plus $Q_{di}$. $Q_{di}$ is known from load forecasting; I also know the expression of $Q_i$ from the beta equation. So, in order to ensure that $Q_{gi}$ remains within its minimum or maximum limit as defined by the capability curve of the generator or the reactive power control available in the synchronous generator.

I can probably make use of the latest voltage solution to find the reactive power injection using equation beta, and then put this number here to find $Q_{gi}$. If $Q_{gi}$ happens to be within this limit, the voltage magnitude can remain fixed. If not, then something else needs to be done. So, what do we do for known voltages? These could be intermediate voltages, final voltages, or any intermediate solution that has come up while solving power flow. For known voltages, we find the injection using equation beta and using $Q_i$ plus $Q_{di}$ as the injection as the generation.

We then can find the reactive power generation at every such bus if it is within the limit as defined by the capability curve specified by the manufacturer. Then the voltage magnitude can remain fixed; nothing else needs to be done. The PV bus would remain as a PV bus; if it doesn't remain within its limit, that means the generator or the synchronous generator has lost reactive power control, which means that either $Q_{gi}$ is more than $Q_{gi}^{max}$ or $Q_{gi}$ is less than $Q_{gi}^{\min}$. As per this equation that we have evaluated, these limits, which are the maximum limit and minimum limit, are as per the capability curve. And it's unsafe for the generator to generate this quantum of reactive power, so what do we do? We don't allow the generator to generate beyond the maximum limit or minimum limit.

What we do in this case is cap the generation to its maximum value or to its minimum value, depending on the violation that is going to happen. And these points, when they are set, essentially determine or indicate points on the boundary of the capability curve, wherein no more voltage control can be done. So, if voltage control can't be done, then it is expected that the terminal voltage will no longer remain fixed. In a way, by capping these generation values to a minimum or maximum limit, we are indirectly fixing the reactive injection at every such PV bus, and remember that in a PQ bus, P and Q are known quantities; if voltage is no longer fixed in a PV bus, it has to become a new variable. So, we have two equations coming in for every such PQ bus or converted PV to a PQ bus.

Which are the alpha and beta equations, and we get two more unknowns, $V_i$ and $\theta_i$, like the PQ bus. That is where the PV bus gets converted to a PQ bus because the reactive power control is lost; the generator should not generate beyond the maximum value or below the minimum value for safety reasons. The third category of bus, which is usually one bus or a combination of buses, is called the slack bus. Remember when we said that the voltage has to be measured with respect to a reference; the reference was inherently chosen as the neutral point or the ground, which is all okay from the perspective of evaluating these equations. However, if we look a bit more specifically, that is, if you look at the alpha and beta equations and think about what the arguments of the sine function and the cosine function are. Both these arguments are the same number, which is essentially the difference of the phase angles and the difference of the voltage phase angles of two connected buses. When every alpha and beta equation consists of the difference of voltage phase angles, and if we try to solve these voltage phase angles, we would find that at best we can find the value of $\theta_k - \theta_i$; this can be known from power flow. That means the difference of voltage phase angles can be known; it is almost difficult to find the actual $\theta, \theta_k$, and $\theta_i$. The difference of $\theta_k$, or basically, if I were to say what we are trying to get here, is that we are able to find the difference of two numbers from power flow analysis. Our purpose is to find these two numbers uniquely.

To uniquely find these numbers, one has to choose one of these phase angles again as a reference. This reference need not be the same as the reference of the ground or neutral. So, basically, we choose a bus whose voltage phase angle is chosen as some fixed quantity. For convenience's sake, that number often turns out to be 0 degrees or 0 radians, although the phase angle could be any other real number. Then that choice of angle for a particular bus helps in finding the remaining phase angles and not just the difference in phase angles.

And that is the reason why the last category of bus is also known as the angle reference bus. Power flow analysis, as I have given here, can provide values of phase differences and not actual voltage phase angles. The voltage phase angle of the slack bus is chosen to be a fixed number, and once the phase angle of the reference bus is fixed, it helps in evaluating other voltage phase angles through these differences; that is one purpose of the angle reference bus. It is also called the slack bus; basically, for every such slack bus, swing bus, or angle reference bus, the phase angle is fixed or chosen; it is not known from some numerical analysis. We, as power engineers or operators, choose it to be some number; generally, 0 is a convenient number.

Although it need not always be 0 , it could be any other real number like 5 degrees, 1 degree, minus 2 degrees, minus 3 degrees, etc. The terminal voltage, or the voltage magnitude of this angle reference bus, is also known. In fact, the choice or the notion of calling it a swing bus is because among the generator buses or among the PV buses, we choose one of those buses to be an angle reference bus or swing bus. The value of $P_{gi}$ is intentionally not specified for this swing bus, which has a generator connected to it. And that is the reason why voltage magnitude becomes known: because from active power loss minimization, voltage settings could be obtained.

But the $P_g$ value is not chosen from economic dispatch for this swing bus or angle reference bus because. If we were to get all our generators or generation values from economic dispatch for different demands, then since we have done economic dispatch, load forecasting has provided us with all demands. So, for this particular value of demand, economic dispatch has given us the best possible value. But is it always feasible to carry out economic dispatch? Probably not. But on the other hand, the load can always change over time.

It is a time-varying quantity. Load doesn't remain fixed. So does it mean that one has to do economic dispatch every microsecond, millisecond, or nanosecond possible? No, it's not possible. If the load is changing and economic dispatch cannot be done, there has to be one generator that can account for this change in demand. That is point number one. The other aspect is that since the load is changing, the active power loss would also change.

So there has to be one generator, which is the generator of the swing bus, which is intentionally not specified, because if all Pgs were to be given based on the fact that economic dispatch was recently done. Then, for a change in load, there is no guarantee that all generation will match the load entirely. Second, system loss is not a known quantity. I mean by system loss both the active power loss and the reactive power loss. Active power loss and reactive power loss tend to happen in transmission lines.

So, the active power loss is basically $I^2R$ loss, where R is the line resistance, and reactive power loss could be inferred as $I^2X$ loss, where X is the line reactance. In fact, active power loss is the most critical loss that needs to be considered. Before we solve the power flow equations, we do not know what the voltage phasors are. If we do not know what the voltage phasors are, we will not be able to know what the current flowing through a transmission line is. If the current cannot be known, how do I know what the value of $I^2R$ is? And remember, a generator needs not only to satisfy the load generator portion; some portion of generation also gets consumed as this active power loss before solving power flow.

If we don't know what this loss is, then how do we ensure total generation is equal to demand plus loss? So that's the reason why system loss is also not known. The slack bus, or the swing bus, accounts for any generationload mismatch and any system loss that would happen as the load changes with respect to time. I hope I have explained the notion behind the angle reference bus because the angle is chosen for the sake of finding other voltage phase angles. The bus is also called a swing bus because we assume one of the generator buses to be this swing bus or slack bus, and slack comes in with the fact that the slack bus accounts for any power mismatch between generation and load and system losses. So essentially, $P_{gi}$ and $Q_{gi}$ are not specified for a slack bus or angle reference bus.

In fact, ideally speaking, the slack bus should have infinite reactive power generation capacity. Generally, one of those buses, which is the largest and connected to the largest generator, is chosen as the slack bus. Min and max limits of $Q_g$ are assumed to be infinite so that the voltage magnitude can always remain fixed. If it is not chosen, then what would be the problem? We would have difficulty finding the actual phase angles from voltage phase angle differences if one of the voltages is not chosen as a reference. Usually, what would we expect? We would expect that in per unit analysis, if our voltage bases are chosen properly, the voltage magnitude should be a number close enough to one per unit.

If we do not specify this or let Power Flow know, okay boss, what is the one per unit reference that you are trying to find? So, basically, if the voltage reference is also not known, like the angle reference, we would have a hard time finding voltages in power flow analysis because there would be no reference of one per unit going into the power flow module. One per unit is not a sacred number. I am just trying to convey that if the power flow knows what the typical voltage numbers should be, then probably the power flow equation would be easier to solve. Similar is the case, so basically, with no voltage reference, it would be like all bus voltages are floating with respect to ground or neutral, and there is no unique voltage magnitude that can be obtained for such cases. So, essentially to summarize, the slack bus, or angle reference bus, or swing bus has no power flow equations associated with it in power flow analysis because, for the slack bus or angle reference bus, the voltage magnitude and phase angle are known a priori; there are no unknowns involved at all.

So this table here, or this slide here, essentially summarizes the equations and unknowns involved for an N bus power network. For an N bus power network, if there are $N_d$ number of PQ buses and $N_g$ number of generator buses, out of which one bus is acting as the slack bus or swing bus or angle reference bus, essentially the effective number of PV buses is $N_g - 1$, where $N_g + N_d$. It is equal to capital N , capital N being the number of buses in the power network. Then, for each of these types and numbers of buses, the associated known quantities or specifications are given in the third column of the table. The equations to be solved for power flow are also mentioned along with their numbering.
The number of unknowns, if we count, that is, for a PQ bus, there are P and Q equations, so each P and each Q result in $N_d$P and $N_d$Q equations; the number of unknowns is the voltage magnitude and phase angle, which are again $N_d$ cross one vector and $N_d$ cross one vector pertaining to every such PQ bus, so there is a sort of consistency between the number of equations and the number of unknowns for every PQ bus. For a PV bus, we have only one equation for each PV bus. So basically, we have $N_g - 1$PV bus equations. The number of unknowns is only the voltage phase angles because the voltage magnitude is known. So here again it is $N_g - 1$ cross 1 vector, which is consistent with the number of equations present for every such PV bus.

So, basically, there is a matching between the number of unknowns and the number of equations that can be solved by any technique at hand. And after we solve these equations and get these unknowns, we can then use them to evaluate the remaining quantities like reactive power generation for the PV bus, real and reactive power generation at the slack bus, real power flows, active power loss, etcetera, etcetera. The next few sets of slides essentially enumerate what is done after the power flow is solved. We still have not seen how power flow equations are to be solved. We have only looked at the structure of these equations, and we will see from the next lecture onwards how these equations can be solved.

By the way, just to sort of emphasize another aspect before I forget, in the context of slack bus or swing bus, we said that if one of the voltage magnitude references is not available, then power flow will have a hard time figuring out what the voltage magnitude should be. An indication or another analogy of this comes from the fact that if I look at my individual power flow equations, in each of these power flow equations with voltages as polar coordinates, the equations are quadratic equations in terms of voltage magnitude. So, these are actually non-linear equations; they are not linear equations. And the moment I have non-linear equations or quadratic equations, every quadratic equation can have two distinct roots. By root, I mean the value of voltage or unknown at which the effective equation becomes equal to zero.

In a quadratic equation where there can be two possible roots, the logical value of those voltages could be any two random numbers; typically, we call those low voltage solution and high voltage solution. Probably in the last module, if time permits, I will explain why low voltage solution is not a desired solution and high voltage solution is the expected natural operating point for every power network, and this low-high voltage solution is a solution wherein voltages are around 1 per unit. There is a typical range in which these voltages can vary, typically around 0.8 to 1.2 or 1.9 to 1.1 per unit. So, with the high voltage solution acting as a reference for the corresponding power flow equations, we can expect our solution to be a high voltage solution, which is the practical operating condition. If the voltage reference is lost, then the power flow equations will not be able to dictate whether the solution is going towards a low voltage solution, which is an impractical solution; theoretically it is possible, but practically it is not going to operate because the network will not be able to operate at a low voltage solution. So by low voltage, if I were to say, let's say our voltage becomes equal to zero, zero per unit. So basically, our entire power network is solidly grounded. If the generator is trying to feed power from generating stations to the load ends, the power will not go to the loads.

They would actually flow into the ground or terminal with no overall effective utilization. So, once the power flow analysis is done, voltage phases are known, and line flows can be evaluated: line flow $S_{ik}$, where. So, basically, this is the complex power flowing from bus i to bus k on the line connecting buses i and k . This is the complex power, which can again have its real and imaginary parts P and Q , which are equal to the product of the voltage of bus i and the conjugate of the current through buses i and k via line ik. That current is again a function of the voltage difference plus the component of half line charging susceptance, which is included because of the nominal pi model of the line between buses i and k . $Y_{ik}$ is the series line admittance, which again has a real part and an imaginary part, and $b_{sik}/2$ is the half line charging susceptance. This expression can again be expanded in terms of actual voltage magnitudes and phase angles, which are known from the corresponding power flow solution. Please verify the correctness of these equations. Along similar lines, slack bus power generations can also be evaluated, given that everything is now known, so we can find the corresponding real power and reactive power slack generations at every bus. The other injections should then equal the generation minus demand, so we can alternatively make use of generation values to find the corresponding generation at the slack bus generators.

Similarly, we can also find the corresponding currents through every such line, using which we can find the corresponding $I^2R$ loss. Although $I^2R$ loss has not been mentioned, it can be found at every line, and when added together, they give us the system loss. The system loss is also equal to basically, if I were to mention here that by power flow we are ensuring that $P_i$ should be equal to $P_{gi} - P_{di}$ for all buses in the power network. And if I add all these equations, So here I would get $\sum_{i=1}^{N} P_i$ is equal to $\sum_{i=1}^{N} P_{gi} - \sum_{i=1}^{N} P_{di}$.

This term here, in a way, represents total real power generation. What is happening in the network? Whereas this term here represents the total real power load in the network. And if total generation is not exactly equal to the total real power load, then the difference here must be the total real power loss that is occurring in the transmission lines as $I^2R$ loss.

By different methods, we can also find the power loss occurring in both the real and reactive perspectives in the power network because we know all the voltage magnitudes and the currents, allowing us to exclusively determine these corresponding loss components. Similar equations also exist for rectangular coordinates; I will just cursorily go through them. Voltage is now expressed in terms of rectangular coordinates, so therein you can sort of separate the corresponding real and reactive power injection equations. Associated real and reactive power flows on lines can also be evaluated along similar lines. In fact, if you compare the previous slide, which is slide number 10, you can substitute these numbers, so $V_i^{\ 2}$.

Or in an alternative way, if you recollect $e_i$, which was the real part of the $V_i$ phasor, it was equal to $V_i \cos\ \theta_i$, as per the initial few slides that we saw, and $f_i$ was the imaginary part of $V_i$, which is $V_i \sin\ \theta_i$. If I square $e_i$ and $f_i$, they essentially should be equal to $V_i^2$ because $\cos^2\ \theta_i + \sin^2\ \theta_i = 1$. So $V_i^2$ can be replaced as $e_i^2 + f_i^2$. Is there a similar term appearing in the rectangular version of real power flows? Yes, it is appearing similarly here. So essentially, you can substitute $V_i \cos\ \theta_i$ and $V_i \sin\ \theta_i$ with $e_i$ and $f_i$, respectively, and also get these numbers.

Slack bus power generation can also be evaluated. Since in slack bus power, the voltage magnitude is specified and the voltage phase angle is zero, for slack bus which is $V_s \sin\ \theta_s$, if $\theta_s$ is chosen as zero degrees, $f_s$ would be zero, and that's the reason why $f_s$ representing slack bus becomes zero. The corresponding real part of voltage is the same as the voltage magnitude, and on similar lines, you can find the corresponding real electric power generations of the slack bus in rectangular coordinates, and the same also holds true for the active and reactive power loss. That is all for today's lecture. We will take up one common or well-known technique known as the Gauss-Seidel method to understand how it can solve our nonlinear power flow equations. Thank you.