Power Network Analysis

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Week-07

Lecture-35

Lecture 35: Power flow analysis- equations, classification of buses

Hello everyone, welcome to lecture 5 of week 7 of the course Power Network Analysis, in which we continue our discussion on the third last module, which is Power Flow Analysis. Today, we will discuss what these power flow equations are and how we understand a step before solving these equations, so that we can find the associated bus voltages, to be precise, bus voltage phasors in steady state for a given power network with respect to a reference terminal, for example, ground or neutral. And before we start solving this, there are certain classifications of buses involved, which essentially come from these basic equations that are part of power flow analysis. So that is all we will discuss in today's discussion. In the previous lecture, we finished our discussion on the evaluation of the bus admittance matrix, which forms a premise for power flow analysis, and we also considered the effect of mutually coupled impedances or admittances and transformer taps when they are in an off-nominal position in the evaluation of the bus admittance matrix. So I request everyone to please go through the previous few lectures in order to have a better understanding, and then the upcoming discussion will become much clear. So, in a way to summarize, for a given n-bus power network where the network parameters are known, by network parameters I mean the line parameters, to be precise. The line parameters are evaluated or known according to the discussion held in module 4. Which was the transmission line parameter evaluation. The line models are also known, and these parameters can be evaluated; the models can be estimated using some special exercise known as state estimation, which we briefly discussed in the previous lecture.

The transformer tap positions, all of which are known, are available for the reactive power compensation devices; their settings are also known to be precise. By reactive power compensation, I mean the capacitor settings, the capacitor bank positions, their values, the inductors, etc. So for a given N bus network where all this information is known, the corresponding Y bus is evaluated as per the discussion we had earlier in the last two lectures, and this Y bus turns out to be an N by N matrix. It is not always symmetric, specifically in the case when transformer taps are at the off-nominal position;

only when the transformer taps are at the nominal position, or basically to be much more precise, if the transformer taps act only as voltage regulators and not as phase shifters, then the corresponding Y bus turns out to be a symmetric matrix. And this Y bus, as discussed, correlates all N bus current injections in terms of the N voltage phasors being measured with respect to a reference node, and this reference node does not exist; its presence, numbering, or labeling is not present in the Y bus evaluation that we have seen earlier.

So given this Y bus, our job is to find a certain set of equations, which we call powerful equations, and from them we will be able to find the corresponding bus voltage phasors. If we know the bus voltage phasors, then we can use the Y bus and voltage phasors to find the corresponding currents at all buses, from which all other relevant quantities can be determined. So when we talk about these voltages, as I have already mentioned, these are voltage phasors. So a phasor is actually a complex quantity or a complex number in the time domain, which can have two forms of representation. One is a polar representation, and the other is known as a rectangular representation.

These representations vary according to this notation, where in polar form every phasor has a magnitude that we refer to as the RMS voltage value, and the corresponding phase angle theta i is measured with respect to a reference, which is the reference node that is not present at all in the Ybus. Now, this polar quantity, or voltage phasor, to be very precise in terms of mathematical notation, vi mod at an angle theta i, is actually equal to vi mod e to the power j theta i, where e is the exponential number, not the e which is present over here. Please do not confuse this e with this e; this is the actual exponent or the natural exponent based on which the natural logarithmic terms are evaluated, and j here is the complex operator, which is minus the square root of minus 1. This number can also be expanded in terms of cos(θ) i + j sin(θ) i, wherein we had a polar representation of a complex number that, when segregated into relevant real and imaginary parts, gives us the analogous rectangular representation of the same complex number.

$$V_i = |V|_i \angle \theta_i = |V|_i (\cos\theta_i + j\sin\theta_i) = e_i + jf_i = |V|_i e^{j\theta}$$

So, depending on how we are representing our voltage phasors—whether they have a pure magnitude or the phase angle, or they have the relevant real component and imaginary component—E I is the real part of the V I phasor, whereas F I is the imaginary part of the V I phasor. The complex number which is v i and e i is actually v i mod cos theta i, whereas f i is v i mod sin theta i. So depending on whether the polar representation is being used or the rectangular representation is being used, the corresponding voltage variables that would come in may be different in the power flow equation. So basically, in polar form, you would only have a magnitude and a phase angle, whereas in rectangular form, you would have the real component and the imaginary component, which again can be correlated with the voltage magnitude and voltage phasor correspondingly. So essentially, depending on what type of representation for the voltages is being used, the corresponding equation that we would see would also vary accordingly. So, given that we will usually start with the polar form of representation in today's discussion, and towards the end, we will see how the rectangular version of voltage phasors leads to relevant equations.

So, assuming that this voltage has the form of this polar representation where the magnitude and angle need not be known, the purpose of power flow is to find these unknown quantities. If we suppose we know those quantities, then essentially we can find the currents at every bus again based on the Y bus definition for the kth bus, if we have to find—or sorry, for the ith bus—if I have to find the bus current, which again is a complex number; actually, it is a phasor. This can be represented in terms of the corresponding Ybus elements and all the N voltage phasors in this form. Where every such Ybus element, which again is a complex matrix, can have the associated real component. Which is the real part of YIK and the corresponding B matrix, where I, K refers to the imaginary BIK, which refers to the imaginary part of YIK.

$$I_i = \sum_{k=1}^{N} Y_{ik} V_k \text{ where } Y_{ik} = G_{ik} + jB_{ik}$$

So, essentially we can have G and B matrices, which are again n by n matrices in themselves, and these are actually equal to the real part of Y bus. whereas B is equal to the imaginary part of Y bus. So, we can also separate these two parts: real and imaginary. So, essentially, G and B are actually all real numbers; there is no imaginary or complex component present in the capital G and capital B matrices. So if we know these current injections, then we can also evaluate the complex power at every node. We have seen this expression of complex power in our first module, which was basic circuit principles, where we have seen that complex power is the product of the voltage phasor and the conjugate of the current phasor.

$$S_i^* = V_i^* I_i = \sum_{k=1}^{N} V_i^* Y_{ik} V_k$$

For the sake of our convenience, I have intently evaluated the conjugate of complex phases. So, the conjugate, instead of appearing over current, is essentially coming over here. So, basically, if SI is VI into II conjugate, then SI conjugate would be VI II conjugate whole conjugate, which when broken up becomes VI conjugate II conjugate; double conjugate doesn't change the sign of II, actually, so II double conjugate is simply equal to II. So, that's how the current expression is, and I have simply substituted the II current expression into this expression over here, where I can get a summation form. Now, YIK can also have its own individual real and imaginary parts, and depending on whether voltage is being considered in polar form or rectangular form, the expression of SI conjugate in polar form looks something like this.

$$\Rightarrow S_i^* = \sum_{k=1}^{N} |V_i||V_k|(G_{ik} + jB_{ik})\angle(\theta_k - \theta_i) \text{ [polar]}$$

$$\Rightarrow S_i^* = \sum_{k=1}^{N} (G_{ik} + jB_{ik})(e_i e_k + f_i f_k + j e_i f_k - j e_k f_i) \text{ [rect]}$$

Whereas, if VI and VK have rectangular forms as EI plus JFI, VK as EK plus JFK, then if we substitute these numbers into this expression and expand them, we would get a complex form of this expression, which is where the rectangular coordinate for complex conjugate power can be evaluated, and remember SI is actually PI plus JQI, where PI is real power. Q i is reactive power. So, essentially, what I am trying to say is we can take the corresponding real and imaginary parts of these entire summation terms and also analogously get P_i and Q_i expressions, which we will see in the next few slides. Before we do that, I will also briefly discuss why S i should be equal to V i conjugate, and why not any other combination like only V i or V into i whole conjugate or V conjugate into i.

The answer to that lies in the discussion that we will have in this particular slide about why S i is not a simple product or a entire conjugate of a product. The reason for this, which I had already mentioned in the first module, was the intent of having this conjugate appear in either voltage or current to ensure that the power we are evaluating, be it real power, reactive power, or complex power, are all average quantities. If you recollect the definition of real power, it is actually the average of instantaneous power, and when we say average power, it has to be a quantity that is going to be independent of time. Remember our voltage and current phasors; they can have their own time domain representations, and if we want to convert a time domain signal into the frequency domain, we have to choose a frequency component or a frequency signal as a phasor, wherein actually sin(omega t) or cos(omega t), where omega is that frequency, the source frequency based on which the reference is to be chosen. So VI and II are actually time-varying quantities; in the phasor domain, they appear to be steady-state quantities or time-invariant quantities.

But when we are trying to find average power, the average power should be independent of time to ensure that this independence is maintained. When we are evaluating the average power, there is a need to sort of get rid of this frequency domain reference, which is the basis for evaluating V and I. When we take the conjugate either on VI or ii, the conjugate part is sort of, so what does the conjugate do? Conjugation essentially changes the sign of the phase angle of the phasor. If the angle is plus theta i for current, i conjugate would have minus theta i as the angle, and the same i conjugate when it is represented in the time domain quantity. The corresponding sine omega t, which was chosen as a reference, would now become sine of minus omega t.

So, to get rid of the frequency dependency or the phasor dependency, one of these quantities should have a conjugate. So, the power that is measured as average power remains time invariant, and to give you a more precise example. So, basically, that is the reason why one of these combinations could be a possible combination for complex power. And definitely not these would serve because here the frequency dependency, which is the reference for the voltage and current phases, is not getting rid of. The other secondary example is that if we have a 1 kilowatt resistor, let's say a purely resistive bulb or incandescent bulb, if it is rated at 1 kilowatt at, let's say, 230 volts, and this 230 volts at one point in time, let's say in India, is at a source frequency of 50 hertz, or let's say in the US or Canada, the source frequency is 60 hertz.

The bulb, no matter whether it is connected in India or the US, will still consume 1 kilowatt if the voltage rating is 30 volts, which is its rated voltage. So that gives another notion or example of why the average power in actual quantities needs to be time-invariant. And that's how the notion of rated power, rated quantities, or rated MVA, et cetera, is defined. So one of these combinations has to be true, which is the conjugate present here. The secondary reason why SI is often equal to the VI into II conjugate is as per the definition of power factor.

A day if it comes that our inductors would be called those devices which deliver reactive power and capacitors would be called those devices which consume reactive power, essentially changing the notation of power factor, then on that day may SI also be equal to VI conjugate into II. For the time being, our power factor convention has not changed. We still associate inductors with being reactive power-absorbing devices and capacitors with being reactive power-delivery devices. So SI is called VI into II conjugate.

Going ahead. If we assume our voltages to be in polar coordinates, we have seen this expression of complex power, which is the conjugate complex power. From there, we can separate out the real part as well as the imaginary part, and we call these equations alpha and beta. I am not going to spend much time on how these are to be evaluated, but I can just. give you a brief indication of how this can be done.

- Real power injected from node '*i*' into network

$$P_i = \sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\cos(\theta_k - \theta_i) - B_{ik}\sin(\theta_k - \theta_i)\} -- (\boldsymbol{\alpha})$$

- Reactive power injected from node '*i*' into network

$$Q_i = -\sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\sin(\theta_k - \theta_i) + B_{ik}\cos(\theta_k - \theta_i)\} -- (\boldsymbol{\beta})$$

So, V_i mod V_k mod they are in themselves scalar quantities; they are not complex numbers. We have two complex numbers here; one is this complex number, and the other one is inherently sitting here as one at an angle theta k minus theta i. So, one at an angle theta k minus theta i, when expanded, would become cos of theta k minus theta i plus j sin theta k minus theta i, and when this complex number gets multiplied with this entire complex number, it would give us g i k cos of theta k minus theta i minus, or let me expand it further, plus plus j g i k. $\sin(\theta_k - \theta_i) + j\, b_i$ k $\cos(\theta_k - \theta_i) + j^2\, b_i$ k $\sin(\theta_k - \theta_i)$; remember j is the square root of -1, so j^2 is -1; thus, this term here actually becomes b_i k - and overall the product has this as the real component, and if I take out j separately, then this becomes the imaginary component. So the associated terms actually sit in the real part, and the associated terms appear in the imaginary part. So, that is how you can get real and reactive power expressions.

We would like to note these equation numbers: alpha refers to real power equations, and beta refers to reactive power equations. And since S_i conjugate is p_i minus j q_i, this minus sign is actually appearing here in the minus sign. So given a bus, bus I, which can be connected to several other buses like bus K, bus M, and bus N, because our power network is actually a mesh network, wherein bus I has a generator connected to it and also a load connected to it, remember our generator injections are considered to be positive because the generator is feeding power into the network from the source end, whereas a load injection is considered negative because it is consuming power from the network into itself. So, because of the change in notation or sign of current, the way they deliver and absorb power, the corresponding injections are respectively positive and negative for the generator and the load. Given a bus, if I have a bus where there is a generator generating PGI real power, there is a demand consuming PDI as the real power demand.

Then, by KCL—KCL might be a wrong word—but if we apply the analogy of Kirchhoff's current law, let's say Kirchhoff's equation, which says that whatever the net current into a bus is, the same current should go out of the bus, maybe through different parallel paths. If the same logic is applied here for power, then what we see is that the generator is generating PGI power into this bus, and at the same bus, there is a demand that is consuming PDI power. PGI and PDI need not be the same. They may be different; in fact, PGI may be less or may be more compared to PDI. Then the resultant difference in power, which is, let us say, PGI minus PDI, if PGI is positive—that is, it is generating more power than the demand itself—where can this power go? The excess of real power, which is PGI minus PDI, has to go somewhere because power.

Like energy, which cannot be created or destroyed, there is no other storage device that we are considering. So, this power has to go somewhere; it cannot be stored itself at the bus because the bus itself is not a power-storing device. So, this power actually goes to neighboring buses through these transmission lines, which are marked as I, K, I, M, and I,

N. And the corresponding real power flows, which are PIK, PIM, and PIN, should together add up to this particular injection because, as per the conservation of energy or conservation of power, excess power, if it is present, or deficit power, if it is present, has to be accounted for as flows into the network because we are not considering any storage device at all on bus I. So what happens is that As per the notation or thought process of solving power flow equations, P i is always equal to P g i minus P d i.

By solving power flow, we are ensuring that P i should become equal to P i k plus P i m and P i n. So maybe if I were to simplify this, there are two boxes that I am marking. This is inherently true. Nothing needs to be done exclusively before solving power flow. This has to be either naturally true or inherently true.

What we are trying to serve in power flow is to ensure that these equations become true, and this is done via power flow analysis. This does not become true naturally; it has to be done through the power flow equation, and that is where power flow actually finds what the corresponding voltage phasor at bus i, bus k, bus m, and bus n should be so that this equation becomes true. This is only from the perspective of real power flows. The same logic would be applicable if we talked about reactive power at the respective buses. Given this understanding that injections are naturally or inherently true, power flow essentially tries to find the bus voltage phasors at all buses so that the corresponding conservation of power aspect becomes true from the real power and reactive power perspectives; that is the basic premise of power flow analysis. So, if we were to sort of see what type of equations we are handling, if we are talking only about the polar coordinate version of voltages, then every such bus will have two equations: alpha and beta.

$$P_i = \sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\cos(\theta_k - \theta_i) - B_{ik}\sin(\theta_k - \theta_i)\} -- (\boldsymbol{\alpha})$$

$$Q_i = -\sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\sin(\theta_k - \theta_i) + B_{ik}\cos(\theta_k - \theta_i)\} -- (\boldsymbol{\beta})$$

When we say equations are equations, they naturally should be in the form of real numbers; voltages, however, are actually complex numbers. So, every complex number has two real parts: one real part and the other part is the imaginary part. So if we remove the J part from the imaginary part, it also becomes a real part. We are trying to solve real equations to find real solutions.

So alpha and beta can exist for every such bus or node where capital G and capital B are in terms of network parameters and voltage phases, which are unknown. The parameters are known from manufacturer data, and there is an optimization exercise that also helps determine the network topology and states, which is known as state estimation; the purpose of power flow is to find all these voltage phasors. So if we were to look at every such bus, let's say bus I, then each bus has only two equations, alpha and beta, and each bus also has only four variables. What are those variables? Those four variables are the bus voltage itself, the bus angle itself, and the corresponding injections, which have to be equated with respect to their respective line flows. So there is a dilemma in this situation that v_i and theta_i are the first two independent variables, and p_i and q_i are the consequent dependent variables, which should be equated with the corresponding real power flows.

So every such bus has two equations and four variables, and it is pretty common or obvious that if the number of unknowns is more than the number of equations. We cannot find a unique set of solutions for those unknowns. So, there has to be some other mechanism or process that has to be followed. The process is that for two available equations, we can at best uniquely solve for only two variables, provided the other two are fixed or known in some way. So in general, if we talk about load PDI and QDI, they are uncontrollable quantities because they depend on the consumer's choice to consume power or not.

So PDI and QDI are not actually under operators' control. Often, PDI is known, and using the respective load model, one can estimate what the corresponding QDI would have been. PDI and QDI are the demands they are generally known from the load forecasting exercise, and our generations tend to follow this load because the purpose of generation is to meet the power demand. So, if the load is known from forecasting, generations also have to follow this load, and hence they are called controllable quantities, whereas the load is called an uncontrollable quantity. PGI is usually known from the economic dispatch problem that we discussed in the previous lecture.

Real power generation is kept to a minimum, and it is further fine-tuned by automatic load frequency control or generation control, which we also discussed during the synchronous generator discussion for small load variations. This is all for real power generation. Usually, the same is not true for reactive power generation. In fact, specifications for reactive power generation are very rare. What is actually the reason for that is that by using power factor control or excitation control of a synchronous generator, the reactive power generation can be regulated.

The generator may absorb reactive power as well as deliver reactive power, which may, in a way, regulate the terminal voltage of the generator. So indirectly, the terminal voltage of generators is specified in terms of set points other than the reactive power generation. And these terminal voltage set points are also obtained from an optimization exercise known as active power loss minimization. Which we discussed in the previous lecture, and the purpose of this objective or exercise is to find voltages or have voltage profiles in the network that are almost flat or uniform with each other so that the corresponding potential difference across lines in the power network is as minimal as possible. So, the corresponding power loss in a way that the power flow and the bus voltages regulate the power flow, but with the intent to have minimum active power loss in the network.

So, as I mentioned, active power loss minimization is a separate exercise similar to economic dispatch, which helps in determining what the terminal voltages of the synchronous generators should be; these are again regulated or controlled by another automatic loop known as the automatic voltage regulator, which we have not discussed at length in this particular course. It is a control loop or feedback loop, a closed-loop feedback very similar to automatic load frequency control or generation control. So, essentially for buses where generators are connected, what we can know from the optimization and related optimization exercises is that the terminal voltage of a generator can be specified, and its set point can be changed according to the reactive power control or acceleration control of the synchronous generator. The corresponding real power output can be known depending on the economics involved. And based on this terminal voltage, I mean there is always an upper cap on what can be the minimum or maximum power factor control that is available.

For a given synchronous generator, which in a way comes from the capability curve that we have discussed. So every synchronous generator based on the capability curve or the available power factor control exercise can be replicated or stated in terms of minimum and maximum reactive power generation. How much is the minimum reactive power that the generator can generate or absorb? Or deliver, and vice versa, for the maximum number depending on the capability curve of a synchronous generator. So that leads to our premise for classification, to sort of summarize before I explain what type of buses we have.

We have many unknowns. We have fewer equations in power flow. So there has to be a way of fixing some of these variables for every such bus so that the number of equations to be solved becomes the same as the number of unknowns. And that leads to this discussion of what can be known and what cannot be known in power flow analysis. In a nutshell, PDI demand, real power demand, and reactive power demand can be known from the load forecasting exercise. PGI can be known from this, so let us call it load forecasting.

Exercise PGI can be known from economic dispatch exercise as per system economics; terminal voltage of generator buses can also be known from active power loss minimization via voltage profile control. If these quantities can be known by some judgment, some mathematics, some optimization, then maybe we should make use of this information and try to have a match between knowns and unknowns in power flow. And that, in a way, leads to the part of the classification of buses in power flow analysis, wherein we have our most common bus, which is called the load bus or PQ bus. As the term says, load bus or PQ bus, there is definitely no generator connected to such a bus. If there is a generator connected to such a bus, then in that case PGI and voltages become known, and hence it doesn't remain a load bus anymore. It might happen that there is a bus which has nothing connected to it. It has just a few incoming lines and a few outgoing lines. There is no generator connected to it.

There is no load connected to it. Those buses could also be called load buses, PQ buses, or, in fact, zero injection buses. And the reason why we call them PQ bus we will see in a moment. The explanation for load bus, I think, is pretty evident. It only has a load connected to it; the load itself could be zero. There is no generator connected to such a bus, and once the load is known, I mean the load bus type is known, then in that case PDI and QDI are known from load forecasting.

So, for such a load bus, if we have to see what the injection is at the corresponding i-th load bus. Since there is no generator connected to the load bus, PGI is not present; it is actually zero. So PI is simply equal to minus PDI. And in similar lines, the injection at the i-th load bus is only minus QDI because QDI is a demand. So if the injections are now known, which are derived from the load forecasting exercise, and we put them in equations alpha and beta, then essentially we have a P equation and a Q equation, and this P and Q, when combined, give it the name of a PQ bus because for a PQ bus the P injection and Q injection values are already known. So the LHS part here is known. What is unknown is the quantity sitting in the RHS, and essentially, for two equations, we have, in a way, reduced these two unknowns. We have only two unknowns. Those unknowns are the voltage magnitude and the corresponding phase angle with respect to a reference phasor. So, we have two equations, alpha and beta, to segregate or signify those equations for a PQ bus.

I have used a subscript PQ, with PQ sitting over here. Just to ensure that these refer to PQ buses only. There are two equations for every PQ bus, and there are only two unknowns for every PQ bus. So, in a way, we have tried to ensure consistency between the number of equations and the number of unknowns for one such classification of a bus, which is known as a load bus.

$$-P_{di} = \sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\cos(\theta_k - \theta_i) - B_{ik}\sin(\theta_k - \theta_i)\} - (\boldsymbol{\alpha}_{\mathbf{PQ}})$$

$$-Q_{di} = -\sum_{k=1}^{N} |V|_i |V|_k \{G_{ik}\sin(\theta_k - \theta_i) + B_{ik}\cos(\theta_k - \theta_i)\} - (\boldsymbol{\beta}_{\mathbf{PQ}})$$

We will continue this discussion in the next lecture, where we will expand on the classification of buses, examine what other buses can exist, and also continue with the equations part of rectangular equations, and so on.

Thank you.