Power Network Analysis

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Week-07

Lecture-34Lecture 34: Power flow analysis- Effects of mutually coupled impedance

Hello, everyone. Welcome to lecture 4 of week 7 of the course Power Network Analysis, in which we continue our discussion on the third-to-last module, which is Power Flow Analysis. In today's discussion, we will exclusively talk about considering the effect of mutually coupled impedances and transformer taps, specifically off-nominal transformer taps, in the bus admittance matrix. In the previous lecture, we looked at the utility application and the assumptions involved in doing power flow analysis for a given power network. And the basis of power flow starts with a basic understanding of the bus admittance matrix, which in a way says or indicates that if there is a power network with N buses, the buses refer to points of common connection or sources and loads, and all these buses could be connected by several transmission lines. The transmission lines are being modeled as a long line model, short line model, or medium line model depending on the line length.

The operating voltage typically models these lines in the nominal pi model with a series impedance that indicates the line resistance and reactance, and half line charging susceptances indicating the line capacitance. So, all these lines connect these N buses, and for this N bus network, there are a few sets of transmission lines connecting these N buses, which are usually in a meshed pattern for the transmission network. The associated bus admittance matrix, which we denote by the letter Y with subscript bus, is going to be an n by n matrix, indicating essentially that Y bus for an n bus network is going to be a square matrix. In the previous lecture discussion, we have seen that if we consider only the transmission lines with the nominal pi models, then under such conditions the Y bus also turns out to be a symmetric matrix; although we will see a specific case in today's discussion where The Y bus need not be symmetric.

So, in general, Y bus matrices for power networks with off-nominal transformer taps are not symmetric. And we also saw another crucial aspect while evaluating Y bus; in fact, the idea or notion was that Z bus, which is also known as the bus impedance matrix, is nothing but the inverse of Y bus, and the inverse of Y bus, I mean, is this difficult to find in order to find that bus if there is no proper connection between the ground or neutral point. Through which the bus voltages are being measured. So, in a way, the governing

equations were that Y bus essentially correlates all bus currents in terms of Y bus with a function of all bus voltages. The bus voltages are all vector quantities here.

So basically, I and V indicate N cross 1 vectors each, where each of the ith or kth elements of vector I and vector V could refer to the corresponding current and bus voltage at bus K. If the inverse exists, then the associated voltages at all these buses can be represented as functions of bus currents, and this is possible only when the Y bus inverse exists. If this Y bus does not exist, then voltages or unique voltages could not be evaluated even if the bus currents are known, pardon me. Even if the bus currents are known, the bus voltages cannot be known for certain. So, with that as a premise, I would also request the viewers to please go through the previous lecture to better understand how Y-bus is evaluated for a given transmission line.

We will expand that discussion today to consider the effect of mutually coupled impedances and transformer taps. So if we understand or try to understand what this mutual coupling is and why this coupling exists at all. So, I have taken a very simple example. The figure is taken from this particular source, wherein what is shown here is that there is a source that is sort of giving out alternating current, and this current is passing through a solenoid that is wound around a soft iron bar. Because of this alternating current, which changes direction with respect to time due to the time-varying current passing through the solenoid, we can apply our right-hand rule, which we have commonly seen or made use of.

In the discussion of synchronous generators as well as the evaluation of transmission line parameters. This time-varying current is resulting in a time-varying magnetic flux; the solenoid, being a magnetic material, and the soft iron bar are good magnetic materials. This is resulting in a time-varying magnetic flux, and if this flux, which is dependent on the current variation, gets linked with a nearby solenoid. Then this solenoid would also experience this time-varying magnetic flux, as a result of which there would also be current induced or voltage induced, basically. Basically, voltage would be induced across the solenoid because of this time-varying flux, as per Faraday's law and Lenz's law.

And if there is a path for the current to flow, provided there is a load connected across this coil, then the voltage that is induced would result in a flow of current that would also be time-varying in nature because the source generating it is also time-varying in nature. And this basic premise we have also seen, discussed, or understood in the context of inductance evaluation: how nearby conductors can influence magnetic induction. So basically this effect is exactly the same as the electromagnetic induction discussed in terms of transformers or transmission lines. Since mutual coupling can exist with respect to induction, which is electromagnetic induction, it is expected that similar coupling can also exist because of electrostatic induction, which results in respective mutual

capacitance and mutual inductance. That is the basis for why mutual coupling can happen.

Mutual coupling often gets strengthened for high voltages or high operating voltages, usually under low operating voltages, typically for distribution lines. The coupling effect is slightly lesser; although in distribution lines the lines are placed nearby, in transmission lines with transmission towers having high rights of way, the higher operating voltage effect gets minimized or diminished because of large spacing. So, the corresponding coupling effect in transmission lines compared to distribution lines is more or less comparable or similar. So, since the transmission lines or parameters can have a mutual coupling effect thanks to nearby conductors or nearby capacitive effects. So it is important to consider the effect of these mutually coupled impedances in the bus admittance matrix because if the network itself is inducing or exhibiting such behavior, then it is important to model that or consider that model in the bus admittance matrix.

So, how do we go about that? What we consider is that we will start with a very simple case. We have four buses, marked as bus numbers M, N, P, and Q, and between these buses M, N, the pair of M, N, and the pair P, Q, there exist one or two unique transmission lines which could have their own impedances as Z A and Z B. For the time being, we are neglecting the effect of half-line charging shunts; although along similar lines, the mutual coupling effect of half-line charging shunts can also be considered.
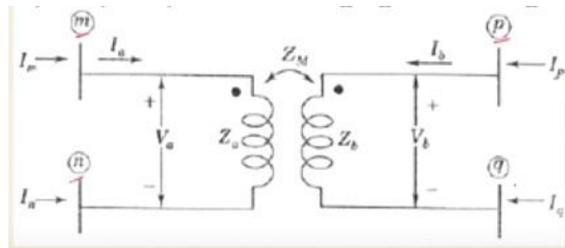
 So, there are two impedances present between two pairs of buses M, N, P, and Q, respectively, and these are mutually coupled with a value equal to Z M. And the dot polarity here refers to or indicates the direction of polarity in terms of how these are coupled, depending on whether the currents are flowing into these dots.

So basically, the dot polarity means that if these impedances carry currents in the same direction as the dot, then the corresponding magnetic flux, which would be created or resulting magnetic flux because of these inductive effects, would both be additive. If the currents are flowing in reverse directions with respect to these dot polarities, then the additive effect is gone; it is actually a subtractive effect. So, with this mutual coupling, let us see how this mutual coupled effect works. So, individually speaking, if Z A and Z B were to be given, then essentially how would we have considered the corresponding bus admittance matrix evaluation? What would we have done? We would have taken the

inverse of Z A, found the corresponding admittance, similarly found the admittance of Z B, and then the M, M element of Y bus, which refers to the diagonal element with respect to bus M, would have a component. equal to y a itself.

Similarly, y_n_n would also have a component equal to y_a, whereas the off-diagonal elements are m_n or n_m. They would be equal to minus y, and similarly, y p p, which is a diagonal term of y bus, would have the effect of y b. The same would be there for y q q also. And the corresponding off-diagonal term YPQYQP would be minus B. If these two lines were not mutually coupled, we would have the effect of considering these impedances or admittances as per the previous lecture discussion.

The difficulty here is that these impedances do not exist in isolation. They are also mutually coupled by this Zm. So how do we consider the effect of Zm? So the idea is very simple. There is no complexity at all in this. What we do is always make use of the first principles and try to see whether the corresponding bus admittance matrix can be evaluated for this mutually coupled consideration.So for the time being, we will ignore or not focus on this aspect; we will go by the first principles and try to see how things are. So let us talk about or see what the corresponding governing equation is behind the operation of this transmission, two transmission lines that are mutually coupled. I have assumed the current directions to be IA and IB for these different transmission lines. So, if I write KVL from bus M to bus N, I would get VM minus IAZA because I am moving along the direction of current, so I would have minus IAZA, and since IB is also flowing into the corresponding other coil, because of this mutual coupling effect, the effect is going to be additive, so I am going to have a drop of IBZM, and eventually, I arrive at bus N, which is VN. VM and VN are voltage phasors of bus M and bus N in steady state with respect to some common reference.

Similarly, if I apply KVL from bus P to bus Q, I would have VP minus IBZB minus IAZM equal to VQ. So if I represent this equation with a little bit of rearrangement, this is what the equation would look like.

$$V_m - V_n = I_a Z_a + I_b Z_M$$

$$V_p - V_q = I_a Z_M + I_b Z_b$$

According to the rod polarities, the inductive effects or voltage effects are additive. So the corresponding voltage drops have their respective signs. Now, if I have to find the Y bus of this particular impedance element, always remember that the Y bus always links a vector of current with respect to a vector of voltages.What I have here is certain voltages as functions of currents, but I need the other way around; I need my currents to be functions of voltages. So, is that possible? So let us do that. If I can rewrite this equation in this form, I will probably be able to guess or estimate what my Y bus would be. So, let us do that. So I have, this is my basic equation and as I told you earlier, I need a vector of

currents and these currents have to be bus injection currents. So in terms of injections, IM, IN, IP, and IQ, with respect to the corresponding line currents, IM and IN are the reverse of each other in terms of IN. Similarly, IP and IQ are the reverse of IB with respect to each other. So I can write IM and IN, the bus currents, in terms of line currents Ia and Ib by making use of this 0 1 matrix, which I call the transfer matrix. So, essentially, I am trying to replace the line currents Ia and Ib with the corresponding bus currents because I have to find the relationship between bus currents and bus voltages, not the line currents or the line voltage drops. So I have a vector in terms of this.

$$\begin{bmatrix} I_m \\ I_n \\ I_p \\ I_q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} \Rightarrow \begin{bmatrix} I_m \\ I_n \\ I_p \\ I_q \end{bmatrix} = A^T \begin{bmatrix} I_a \\ I_b \end{bmatrix}, A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} Z_a & Z_m \\ Z_m & Z_b \end{bmatrix}^{-1} \begin{bmatrix} V_m - V_n \\ V_p - V_q \end{bmatrix}$$

Now this inverse exists only when the Z_a Z_b Z_m Z_m matrix is not a singular matrix. So basically, the determinant of this matrix, which is Z a Z b minus Z m squared, should not be 0. Usually, when the lines or impedances are mutually coupled, mutual coupling itself indicates that the maximum value of Zm that can exist, ideally in terms of physics or in terms of the energy that can be stored, cannot be more than the square root of the product of ZA and ZB; that is the basic philosophy. For those of you who are interested in how this value is obtained, I would request my viewers to please look at the corresponding second-year undergraduate mutual coupling mutually coupled admittance discussion, where you would find that in terms of energy that can be stored, the energy itself indicates this particular limitation. If this is going to be true, then ZM squared can never be more than ZA ZB; forget about ZA ZB being equal to ZM squared.

So, under the condition that is usually true, the determinant need not be 0; this inverse can exist, and given this inverse, I can now find my line currents in terms of line voltage drops, but my Y bus needs to be in terms of not the line currents or voltage drops, but in terms of bus currents and bus voltages. So if I know what this ZA ZMB matrix is, I can evaluate its inverse and associate or find these small admittances; the expression of these admittances can be obtained from a one-to-one mapping with respect to these elements. Furthermore, the line voltage drops again, and the use of a matrix can be expressed in terms of bus voltages. So if I readjust things a bit, I have a I b in terms of this admittance 2 by 2 matrix with multiplication of a, and here I now get the entire 4 by 1 bus voltage phasor. The only thing that I need to do is replace this I a and I b in terms of bus injection current, which is 4 cross 1 current.

So basically, I multiply this entire equation by A transpose, and effectively I get a resulting equation where 4 cross 1 bus currents are expressed in terms of 4 cross 1 bus voltages, and the resulting intermediate equation that appears, or this matrix that appears, is effectively the Y bus matrix for these two mutually coupled admittances or impedances. So if I substitute the values of A transpose and A, where the A matrix was this matrix earlier, and having known the 2 by 2 bus admittance matrix, if I evaluate them, then this is the 4 by 4 matrix that I would essentially get, and this 4 by 4 matrix has a few specific labelings. Now these labels are important to understand why the labeling is coming in. Recollect that when I said that Y bus represents vectors of current in terms of vectors of voltages. So, essentially, if I have to write ith bus current, let us say I_k.Not in terms of vector; if I have to write the kth element of i, which is ik, and I have to find the expression of ik in terms of y bus and b, then essentially it would be the summation of j, not j. Let us say l is equal to 1 to capital N, assuming N buses y bus, comma. K, L into V, L. This is what the corresponding vector equation would result in for individual bus currents and bus voltages. So, essentially, if I have to see the labeling or row comma column index of Y bus, recollect you see that if I K current exists, the first row index is referring to the current itself, whereas the column index is referring to the bus number or the voltage that is being considered.

Similar logic or labeling is applicable here also in terms of 4 cross 1 current and 4 cross 1 voltages. The first current is M, referring to bus current M, the first voltage is bus voltage M, and hence the first element of this 4 by 4 matrix, which is Ymn, should have a row, column label of M, M. The second element in this 4 by 4 matrix refers to I m current with respect to V n voltage. So, that is why I have the second element as m, n, and that is how the 4 cross 4 labeling can come into effect. Once I know this 4 by 4 labeling of which element pertains to which specific row and column position, the only thing left is to incorporate or superimpose this add-on matrix onto the actual Y bus.
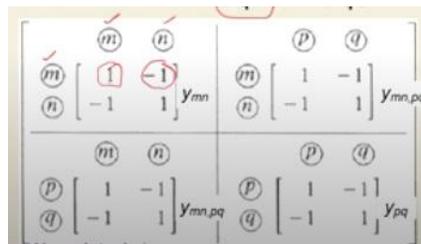
So how do we do that? We evaluate the Y bus, as discussed in the previous lecture, without considering the mutually coupled admittances or impedances at all. In fact, by "without considering," I mean that we consider that those lines or impedances which are mutually coupled do not exist at all. Forget about the mutual coupling; we remove them from the power network and then evaluate the Ybus for every such pair of mutually coupled admittances. Figure out this 4 by 4 matrix and be very specific or clear about the row and column labels. And then you have already evaluated Ybus in step one without

the mutual coupling; for every mutual coupling, you are getting an add-on matrix, which is a 4 by 4 matrix.

The only part left is that each of these elements in this add-on matrix, which is shown over here. They need to be properly superimposed or labeled on this Y bus, which is without mutual coupling. So we superimpose these admittance matrices so that elements with identical row and column labels in the add-on and Y bus are added together according to their respective signs. We will also have an example towards the end of this discussion where this will become clearer as to how the superimposition happens. Now it may also happen that the mutual coupled impedances need not exist between two unique pairs of buses.

That is, bus M can be the same as bus P, and bus Q could be the same as bus N. So there can be repeated labels sitting over here. It need not actually be a four cross four matter. So what is to be done in those cases? If one bus among these four buses acts as the reference or ground bus, remember that for the Y bus there is no incorporation of a reference or ground bus, so the corresponding row and column for the reference node are removed or deleted; it is not considered at all. And if there are two buses among M and PQ that are the same, let us say if M is equal to Q with respect to the previous consideration that we had here.

So, if M and Q are the same, instead of Q, now here we have M; here also we have M; here also we have M; and here also we have M. So, if I have to find what the elements pertaining to label M, M are, I would have 4 elements: element number 1, element number 2, element number 3, and element number 4. Obviously, these elements are respectively multiplied by Y, M, N, Y, M, N peak 2, etc. So, essentially I have this sort of arrangement where m, m are four unique elements. And when I'm doing superimposition, I can either reduce this four by four into unique row and column labels.

I can have a three-by-three representation because essentially there are three unique labels and three unique labels here. Or I should be very careful in mapping or superimposing four elements when I'm considering m, n. m, m in the add-on matrix. Similarly, if I were to look at how many m, n elements exist, there is at least element number 1 sitting over here, and also m, n. There are at least two unique elements that refer to M and N because the N column label has two row labels of M. So I have to be very careful when I map the M, N element; I should consider two elements. That's how these elements are getting added up over here. So let's say this is M, this is N, this is P. YMU is being added to minus YMN. That's how this term is coming in, and so on. The process of superimposing it essentially remains the same. There is no difference. Moving on to consider transformer taps and Ybus.

$$Y = \begin{array}{c} \\ m \\ n \\ p \\ m \end{array} \begin{array}{cccc} m & n & p & m \\ \begin{bmatrix} y_{mn} & -y_{mn} & y_{mu} & -y_{mu} \\ -y_{mn} & y_{mn} & -y_{mu} & y_{mu} \\ y_{mu} & -y_{mu} & y_{pm} & -y_{pm} \\ -y_{mu} & y_{mu} & -y_{pm} & y_{pm} \end{bmatrix} \end{array}$$

$$Y = \begin{bmatrix} y_{mn} + y_{pm} - 2y_{mu} & y_{mu} - y_{mn} & y_{mu} - y_{pm} \\ y_{mu} - y_{mn} & y_{mn} & -y_{mu} \\ y_{mu} - y_{pm} & -y_{mu} & y_{pm} \end{bmatrix}_{3 \times 3}$$

Transformers aid in regulating voltages. As long as transformers have the role of adjusting voltage levels, only then does their effect come into the picture.

If there's a transformer whose transformer tap is exactly equal to 1, it is as good as considering that transformer and replacing it with a transmission line because it is not regulating bus voltages at all. There can also be transformers that can regulate the phase angles between the primary and secondary voltages, and we call these transformers special phase-shifting transformers. So, in essence or in practicality, the transformer step ratio need not always be a scalar number because it can also regulate phase shifters; the transformer tap can also be a complex number, and by using this voltage regulation, as I mentioned yesterday in the previous lecture, real and reactive power flows are significantly affected by changes in tap positions. So, in case the tap is at nominal position in permanent analysis, we have seen that the transformer impedance seen from the primary or secondary remains the same; but usually, the transformer tap is not at its nominal position; it has to be at an off-nominal position; that is how the transformer's role comes into the picture. So, we have a special mechanism for considering how the off-nominal tap position can be regarded; as long as the transformer tap is at the nominal position, it behaves like a usual transmission line where the transmission series impedance is the same as the leakage overall impedance or reactance of the transformer.

So, for off-nominal positions, we have this special mechanism where we have a transformer tap of ratio 1 to A, and A need not be a scalar number; it can also be a complex number depending on whether the transformer is regulating only voltages or whether it can also regulate phase shifters. And when A is equal to 1, we call the transformer tap to be at a nominal position; usually, A need not be equal to 1, and that is how it is called an off-nominal position. Again, in per unit essence, all this discussion is happening on a per unit basis. So, what do we do? How do we consider this off-nominal position? Tap consideration in Y bus. We consider buses I and J, between which this transformer is present, and this transformer has a series admittance Yt, which is essentially the inverse of the primary and secondary impedances at nominal ratio on a per unit basis.

And having considered the leaky or lossy component of the transformer separately, the ideal part of the tap is considered as it is where the ratio A need not equal 1. So, how do we go about considering this specific arrangement? So basically, if A were equal to 1, it would behave like that; oh, BX need not be considered, or in fact, this fictitious node X

need not be considered. This line is directly connected to bus J, and hence, bus Y can be easily evaluated. Since that is not there, there is a transformer tap present; how do we consider its effect? So, what we are going by here is the first principles; the first principle here is that we are assuming the injection current ii into the network, and we are assuming the injection current ij into the network. If I apply KVL between node i and node x, which is the fictitious node, then I get ii is equal to vi minus vx multiplied by the admittance of the transformer, which is equation number 1.

$$I_i = (V_i - V_x)y_t$$

And since this tap position is an ideal transformer. So, whatever the power input to this transformer is, the same is the output of the transformer because the lossy component has been very well incorporated into this admittance. So, if I look at the power that is input to the transformer, the input power is basically complex power. So, basically, Vx into Ii conjugate should be equal to output power, which is minus Vj Ij conjugate. Why do I have a minus sign? Because I have chosen the injection current to be into the transformer tap, that is equation number 2.

$$S_{inp} = V_x I_x^* = S_{out} = -V_j I_j^*$$

The transformer is an ideal transformer; the lossy component has been considered through this admittance And the last equation is that if I have a tap 1 to A between voltages Vx and Vj, then by the voltage transformation ratio Vx to Vj is equal to 1. I have Equation number 3 here. If I have to find the corresponding Ybus for this transformer tap connection, I need a vector that should actually be vector I. Which should actually be IIIJ. IIIJ are the injection currents into the buses between which the transformer is present as a function of some add-on matrix, and the voltages should be VI and VJ VX should not appear at all. So, if I can work out these three equations and try to get an arrangement of this form, my purpose of evaluating Ybus is done. So, let us see how it's done. So I have my first equation, as I mentioned; then, for a power transformer, power loss in an idle transformer does not exist, so input power should be the same as output power, and finally the voltage ratio. From there, I can also find the corresponding current transformation.

$$\frac{V_x}{V_j} = \frac{1}{a} \Rightarrow V_x = \frac{V_j}{a} \ and \ I_j = -\frac{I_i}{a^*}$$

So if I rearrange these equations, basically I have Ii here; I need to substitute Vx in terms of Vj, which I can do by using this equation, so I can get my first row of this Ybus matrix, and if I divide it by minus 1 by A conjugate, I can get my second equation. If I simplify this a little bit, my add-on matrix, which is a 2 by 2 matrix, appears because of the transformer tap, and A here is a complex number; it need not be a real number. It can

be different from 1; it need not just be a scalar. So, to incorporate the effect of transformer taps on Ybus, the first step remains the same; that is, evaluate Ybus without considering the presence of the transformer tap.

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -y_t/a \\ -y_t/a^* & y_t/|a|^2 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

For every transformer tap, evaluate the 2x2 add-on matrix. This add-on matrix will have labels of I, J, and J as discussed earlier because the elements relate current I to current voltage I and current I to voltage J in the second element. So, if you are careful about these labels, the superimposition effect remains the same: elements with similar row and column labels in the add-on matrix and Ybus have to be just added. In case there are several mutually coupled admittances and transformer taps, individually evaluate their Y-bus add-on matrices and superimpose them with identical row and column labels. The example here essentially clarifies that aspect further. We have a four-bus network where bus numbers are labeled as 1, 2, 3, and 4. There are two transmission lines: one between buses 3 and 4, and another between buses 1 and 3.

We have transformer taps whose tap positions are given, and the corresponding series and reactances are also mentioned. We have to find the Y bus for this matrix for this power network, a 4 cross 4 network. Essentially, at the outset, the Y bus should always be a 4 cross 4 matrix for this 4 bus network. So, how do we do that? We convert or evaluate the line impedances into the corresponding admittances first.

We evaluate the admittance matrix. So, since we have two elements between 3 and 4, So, essentially we would have all elements as 0 in this Y bus matrix except for. So, basically, let me expand this a bit; this can be written as Y 1 1, 1 2, 1 3, 1 4, and goes on until 2 1, 2 2. 2, 3, 2, 4, 3, 1, 4, 1, 3, 2, 4, 2, 3, 3, 3, 4, 4, 3, and 4, 4. Since two impedances are present between buses 3 and 4, these elements would have an impact only on these four elements; other elements would all be 0 in the Y bus matrix, and that is exactly what is shown here.

$$Y_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -j9 & j9 \\ 0 & 0 & j9 & -j9 \end{bmatrix}$$

 I have been able to mark the labels with respect to the rows; the columns also have a similar transposed labeling. Two elements, two impedances, have now been considered

through these matrices; now we have to find the individual add-on matrix. So, if we look at the add-on matrix for a transformer between 1 and 3, we first convert this impedance into admittance. So, this is my admission here, and the tap is given as A equal to 1.25; the tap position is placed between 1 and 3. So, I have my labels as 1 and 3 here, and on top, I have 1 and 3; similarly, if I put down those numbers.

So, I have this as my 2x2 add-on matrix with these as the labels; similarly, for 2 and 4, this is my 2x2 add-on matrix with corresponding row and column labels, and now I have only to superimpose these matrices. With similar row and column labels, if you look at the actual full matrix for (1, 1), I have only one (1, 1) element sitting over here, whereas here it is 0, so I have -j 80. Whereas for (1, 3), (1, 3) in this matrix is 0, and here I have one element, which is j 64, which is j 64 here. Likewise, for 3,3, I would have 3,3; one element is here, and the other element is here, which, when added together, gives -60.2j as the final admittance. So, with this, I conclude today's discussion, and in the next lecture, we will look at certain equations that define or create the premise of power flow. These equations are built on top of the bus admittance matrix, and hopefully, by the end of the next lecture, we will understand a few types of buses, their labelings, and classifications in power flow analysis.

Thank you.