Power Network Analysis

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Week-07

Lecture-33

Lecture 33: Power flow analysis- Basic/assumption in power flow, Bus admittance matrix

Hello everyone, welcome to the third lecture of Week 7 of the course Power Network Analysis. In which we continue our discussion on the basics or assumptions involved in power flow analysis. We understood why a power flow analysis is necessary in the previous lecture. The reason is that our sources are not only in terms of voltages and currents; they also have a power rating, and there are the resulting equations they get involved in some nonlinear forms. So, basic critical analysis is not applicable, although network theorems are still true; the resulting equations are non-linear equations. So there have to be special techniques for understanding this power flow analysis.

Power flow essentially tries to find the voltages at all buses in a given network. If we know these voltage phasors with respect to a reference, we can find the currents and powers in steady state conditions for a given power network. So that's what the previous discussion was about. And just to recapitulate the assumptions that we started with in the previous lecture, we assumed that the system is in steady state, it is balanced, and the entire network can be represented as a per-phase single line diagram network on a per unit basis.

Generators; their specifications are in terms of real power and terminal voltage. We will still see why the real power comes into the picture, not just apparent power. Transmission lines are modeled by the nominal pi model, as per the discussion we saw in the previous module. Line shunt conductance sensors are negligible, and thus they are neglected because they are difficult to model. And for power transformers, which could be step-up or step-down, as long as the tap position is in a nominal position, nothing needs to be done because in per unit, the transformer impedance on the primary and secondary sides remains the same; but if it is in an off-nominal position, which is usually the case because transformers also help in reactive power compensation, some special consideration is needed, which we will see in the next lecture.

Coming to the loads, loads are something that people, power engineers, or researchers have not focused on for some time regarding how these load behaviors are. If we were to think that our loads are typical resistive loads, no load in the entire world is a pure resistive load. In fact, those resistive loads that we talk about are probably lighting loads or heating loads, but they also have some sort of non-linearity and non-ideality. So it is also important to understand how the loads are being modeled in powerful analysis because when we say that the sources are in terms of some power and voltage, it is also important to understand how the loads are being modeled. So, in the literature of power networks or power systems, there are different types of load models that are available. We will briefly understand these models and then we will go deep into the power flow analysis module, wherein, depending on the type of load and how it is to be modeled, the corresponding model will be taken up. So the majority of our loads, which we think are all constant power loads, mean that no matter what the voltage or the current at the load end or at the receiving end of the load. The load, if it is rated at a few megawatts or a few kilowatts, will still consume the same amount of power no matter what the voltage or the current is at the load end or at the receiving end of the transmission line. Which is usually the notion, most of our constant power loads, we assume that their power ratings are independent of the voltage and current. Often, such loads do not exist.

And that is where the aspect of constant power factor load comes in, where we assume that the power factor of a given load, which is cos theta, with theta being the load impedance angle, remains constant. From the research perspective, most of us tend to assume that the loads vary in terms of a constant power factor, depending on the type of load; the corresponding application may be different. So, essentially, if the power factor has to remain constant, it would mean that S is actual. So, basically, constant power factor means that if P, which is in terms of apparent power, is multiplied by cos theta, and if cos theta has to remain constant while the load's real power is changing, that means if, let us say, at instant 1 the load has P1 as the real power, then P1 is S1 cos theta; at another prime instant, the same load has a variation and is now changing. Consuming real power P2, which is S2 into cos theta, cos theta cannot change.

So, essentially, if cos theta has to remain the same, P1 by P2 should be equal to S1 by S2, which can also be indirectly written as since S1 is nothing but, or S, basically S is nothing but p square plus q square. So, if we rewrite or rework this expression, it can again be rewritten as p1 by p2 has to be equal to q1 by q2, or essentially what we say is that if the power factor has to remain constant, that means this angle has to remain constant, which would also mean that tan theta will remain constant no matter what the time variation in the load is, and tan theta, as we all know, is nothing but Q by P. So, if tan theta has to remain the same, Q1 by P1 has to be equal to Q2 by P2. which is also indirectly referred to here.

$$\cos\theta \rightarrow power\ factor$$

$$P = S\cos\theta$$

$$P_1 = S_1\cos\theta$$

$$P_2 = S_2\cos\theta$$

$$\frac{P_1}{P_2} = \frac{S_1}{S_2} \Rightarrow \frac{P_1}{P_2} = \frac{Q_1}{Q_2}$$

$$S = \sqrt{P^2 + Q^2}$$

$$\frac{Q_1}{P_1} = \frac{Q_2}{P_2}$$

$$\tan\theta = \frac{Q}{P}$$

So that's the reason why, for a constant power factor load given a base operating condition, PD naught and QD naught, the load has to vary accordingly for the next instant.

We also have certain load models where we tend to represent them as a constant current magnitude load. For current magnitude, if you remember, power is basically power divided by voltage, or let's say power, if I denote it by S, S divided by V is equal to I. So if I has to remain constant, that means S1 divided by V1 should be equal to S2 divided by V2 for the same load that is consuming different amounts of power at different voltages V1 and V2. If we segregate this in terms of corresponding real and reactive power, we get this as our quantity since we are talking about current magnitude. So the corresponding ratios turn out to be simply magnitudes. Often, power electronics loads exhibit this constant current behavior. If constant current behavior can exist, we can also have a constant admittance-based load, where constant admittance essentially refers to Z being equal to V squared divided by S. So if Z has to remain constant, the admittance or impedance has to remain constant. So basically, admittance is Y equal to S divided by V squared.

$$Z = \frac{V^2}{S} \Rightarrow Y = \frac{S}{V^2}$$

If the admittance magnitude has to remain constant, then S1 divided by the square of V1 should be equal to S2 divided by the square of V2, which, when reflected in terms of individual real and reactive powers, gives us these expressions. We can also have a common load model where the common load model can consider a constant power load and can also have a constant current load. It can also have a constant impedance load or admittance load, and the ratio since the same load can have these three different components. So, the ratios in which this load has these attributes are defined as per these factors, and since we are talking about one unique load having these three different modes of operation at any point in time, these factors should always add up to one. The factors to which we are referring pertain to constant impedance or admittance, constant current, and constant power. The ZIP together is also called the ZIP model or the ZIP model.

$$P_d = \alpha_p P_d^0 + \alpha_i \frac{P_d^0}{|V^0|}|V| + \alpha_z \frac{P_d^0}{|V^0|^2}|V|^2$$

$$Q_d = \alpha_p Q_d^0 + \alpha_i \frac{Q_d^0}{|V^0|}|V| + \alpha_z \frac{Q_d^0}{|V^0|^2}|V|^2$$

$$\alpha_p + \alpha_i + \alpha_z = 1$$

ZIP model

ZIP model stands for constant admittance, constant impedance, constant current, and constant power factor load in a composite load model. There is also another form of load model known as the exponential model, in which the factor of NP, and basically NP only, can vary or be any real number greater than or equal to 0. If NP is equal to 0, we essentially have a constant power load; if NP is equal to 1, we have a constant current load; if we have NP equal to 2, we have a constant impedance or admittance load. Any other value of NP can result in any other combination of load, so basically the exponential model is a generic form of representation. Individual ZIP load models.

So, depending on the requirement, our load model can vary based on the type of load meters; it can also change. Typically, distribution or consumer loads, or domestic loads, fall into this category of composite loads, where the ratios of alpha P and alpha Z are typically higher, and for industrial loads, where frequency dependency also comes in, we have only considered the voltage dependency of loads. Our actual loads can also be frequency-dependent, which we have not modeled; in fact, actual industrial loads are highly dependent on frequency, which we are not considering for the sake of simplicity. So industrial loads can have their own ways of defining the load models. By the way, loading models in themselves is a very interesting area of research, and a lot of research has been going on for the last few decades.

Coming to the assumptions involved in powerful analysis of loads and generators, we have understood what the loads are. So basically, if one were to question why we need these load models if we don't understand how our loads are behaving with variation in time or space, then it is difficult to understand the implications of that load on the actual network, and that's where the load models are very important. Since these loads are hungry for power, specifically from the real power perspective, a load is typically an entity that consumes real power. We tend to define the load injections or currents as negative currents or injections because they are trying to consume power from the power network. We will understand this aspect a bit more when we go deeper into the power flow aspect. And similarly, from the generator of sources' perspective, they are those elements that generate real power, and hence their corresponding injections or currents are being driven into the power networks; we consider those injections to be positive. Transformer ratings, reactive power supporting devices, and their ratings are all known. All the above specifications have been considered or modeled on a per unit basis, considering a common system base power and common system base kilovolt as per the relevant section depending on the transformer positions. So, the single-phase single-line diagram in which the corresponding power network has been represented is on a per unit basis with a common power base and a common voltage base depending on the transformer section. So, how do we get this information? I mean, what was the input for the power flow? So, load injections have to be known; generation injections have to be known. So, load injections they are typically known from some historical analysis. And typically we call that analysis a forecasting exercise. Again, load forecasting, or in general, forecasting in the power network is a very broad domain of discussion and research, and it can have its own dedicated course exclusively on forecasting itself. So typically, the power network operators get this load information from some typical historical analysis or forecasting exercise. Generator injections, when we say that they are specified in terms of real power, are known in terms of real power because, if you recollect our first initial basic module, we have talked about it.

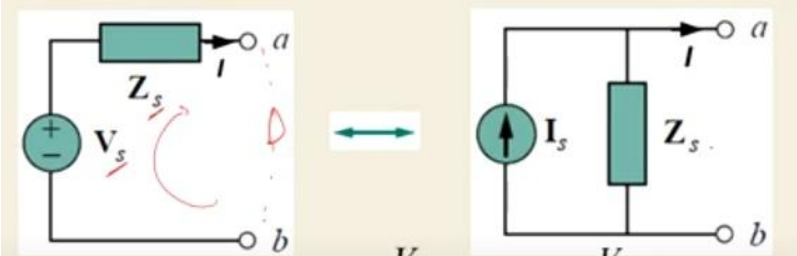
A few good attributes of a power network in which economics was one of those attributes, and by economics, we meant that when power is being generated from generators and transmitted to the loads, it should be done in the least costly manner. The least cost of operation should be involved; the cost of generating this power should be as low as possible. And since the power tariff is usually defined in terms of real power because reactive power is not paid for by exclusive domestic consumers, although there are investment costs or capital costs involved where some ancillary services pricing is done, that's not what we are considering for the sake of our discussion here. The pricing is mostly done from the real power perspective, and that is the reason why generators receive their specifications only in terms of real power. This quantum of real power comes from a well-defined exercise, which we have also discussed in the economic dispatch module, wherein we talked about this economic dispatch in one particular lecture. Just to maintain system economics, we obtain the real power information; the optimum value of real power that a generator can generate or should generate is obtained from some optimization exercise, which is typically some form of economic dispatch. If we consider the power network, that exercise is known as optimal power flow. If one also has to understand how our thermal units, hydro units, and nuclear power plant units are being scheduled, which can have different ramp-up and ramp-down characteristics, then the exercise also becomes known as unit commitment, which is again a very broad topic of discussion, and an entire course can be built on these different italicized terms. The nominal voltage for the generator also comes from another optimization exercise known as the active power loss minimization exercise, which essentially ensures some form of economics while also bringing in some aspects of security, wherein we try to find what the reactive power compensation levels required in the given power network would be and where our reactive power compensation devices should be placed. What should be the settings of those reactive power compensation devices, including generators, so that the voltage profile in the power network is as good as possible and corresponding voltage drops or I squared R losses are as minimal as possible, so that maximum real power or maximum power generated by the generators goes into the loads? The loss component in the network is as low as possible.

So that also involves some form of optimization wherein we try to find the ratings and positionings and specifications of what these devices settings should be from the reactive power perspective. Lastly, system topology and parameters are evaluated again from the optimization problem, which is very well known as state estimation. The term "state" might confuse you with the states of power flow; yes, the states remain the same in state estimation. is basically a real-time monitoring tool, whereas Power Flow is essentially an offline tool that is used for security or contingency analysis. So, essentially, the Power Flow application is what I was trying to talk about in the previous lecture, which is that Power Flow finds its application in an offline model or offline tool, suggesting that if a network or a power network undergoes some disturbance.

Undergoes some change; the change could be in terms of load, in terms of network topology, or in terms of some natural calamity, etc. If such a change were to happen in the power network, then power flow analysis essentially gives a post-mortem tool that shows what the implications would be on the network if this event were to occur. How would the currents change? How would the voltage change? Will generators be able to generate that much power according to system economics? Will reactive power be maintained properly? All those post-mortem analyses essentially come from power flow. On the other hand, state estimation is actually an online real-time tool, and it helps in finding real-time voltages or voltage phases in the power network. Power flow, on the other hand, tends to give an offline measure of finding the bus voltage phases in different operating conditions.

So, given that, let us now move into certain discussions on how we find the bus admittance matrix, which forms the premise for power flow. So, given a source which is, let us say, a voltage source or a generator whose internal impedance Zs is known and Zs is not equal to 0, it is not an ideal voltage source. So Vs is the voltage source with a non-zero internal impedance Zs. And across the terminals A and B, some current might flow out if A and B were to be connected by a particular load. So, if A and B were to be connected by some load, then the current in this particular node would be I, assuming that such a load is connected, if we apply KVL from node B to node A.

We get VB plus VS minus IZS is equal to VA, which can also be rewritten as VA minus VB is equal to VS minus IZS, which can also be rewritten as VAB by ZS equal to VS by ZS minus I, and if I recheck this a bit, let me check, okay, I think I have made a mistake somewhere here, VA minus VB. Vs by Zs is equal to I plus Vab by Zs, which is essentially the term or equation present over here. And VS by ZS can be known by another current known as IS. So, by using this interchange, relationships provided that ZS is not equal to 0, it's not an ideal voltage source; we can re-represent a circuit in terms of a current source where the current source essentially has a source component IS with parallel impedance ZS, with VAB appearing across it. And why is this ZS changing its position from being a series element to a parallel element if we have to replicate a circuit where this KCL has to be true? This KCL node has to be seen from this perspective.



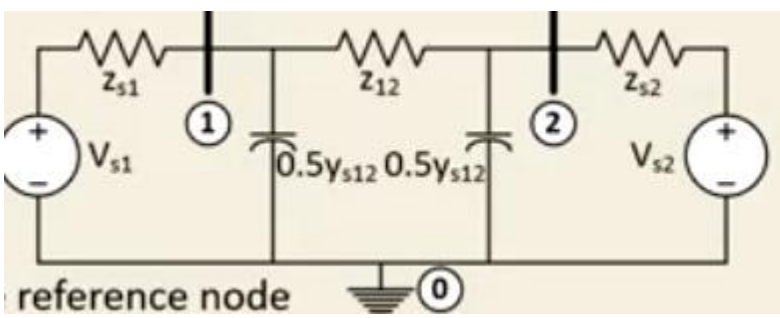
$$V_s = IZ_s + V_{ab} \Rightarrow \frac{V_s}{Z_s} = I_s = I + \frac{V_{ab}}{Z_s}$$

Now, if you apply KCL at node, let us say P, the KCL equation at P is essentially this equation only, and that is how the ZS position changes. Basically, each such non-ideal source can be transformed into a current source or a voltage source with the same internal impedance of the resistance, but with different placements. As I mentioned, the bus intermittence matrix forms the basis for power flow analysis. So at the outset, if I have an N bus network, where the buses are numbered from bus 1 to bus N, and all these buses are interconnected through several transmission lines, step-up transformers, step-down transformers, etc., at these buses we could have either generators or loads connected, then the currents that would flow because of these generators and loads into the power network can be represented in terms of voltages at these buses, which are the voltage phasors measured with respect to some reference.

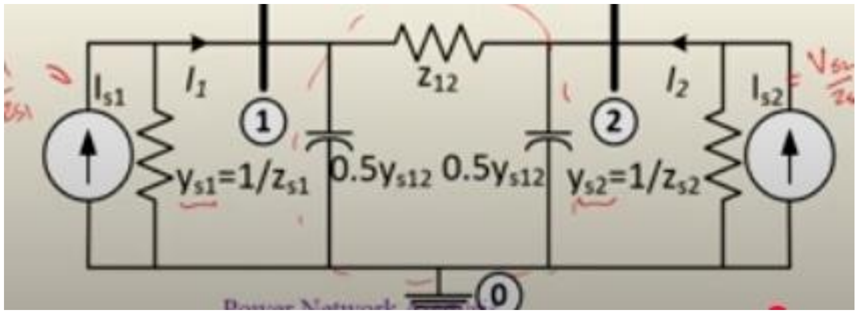
In terms of a corresponding n by n matrix, which is essentially called the bus admittance matrix. So for an N bus network, the node voltage and bus current phasors, again measured with respect to some reference, can be expressed in terms of node voltage phasors and node bus voltage phasors, again with respect to the same reference, through an N cross N matrix, which is known as the bus admittance matrix. And this is the corresponding mathematical expression.

$$I = Y_{bus}V$$

Vector I consists of the currents at all these different buses that are being injected from different nodes. If there is a generator present at a given node, this current typically would be a positive current. If there is a load connected at a typical source at a typical bus, it would have a negative current. For the sake of simplicity, I am assuming all these currents to be positive, but depending on the load or generator connection, the quantities can be negative. And on the other right-hand side, I have an n x n matrix where the elements need not always be symmetric, although in general, Y bus turns out to be a symmetric matrix under special conditions when the transformers are not at their nominal tap positions, and the V vector here refers to voltage phasors for all individual buses. So let's see how we can find this bus admittance matrix, which is going to form the basis for power flow analysis. So what we do is consider a simple single transmission line; at the end of this transmission line, we have two voltage sources connected, V s 1 and V s 2, with internal source impedances Z s 1 and Z s 2, and in between these voltage sources, we have two buses, bus 1 and bus 2, whose potential is to be measured with respect to a ground node 0, and the transmission line here is modeled as a nominal pi model based on the discussion we had in the previous modules.



Now, if we try to find the bus admittance matrix, we essentially have to find a relationship between the node currents or bus currents in terms of bus voltages. And if we do that, the corresponding matrix that would come in would essentially be the bus admittance matrix. So, what we do here is first convert the voltage sources into current sources using the source transformation model, where IS1 is equal to VS1 divided by ZS1 and IS2 is equal to VS2 divided by ZS2 based on the previous two lectures and the previous two slides that we saw. The positions of ZS1, ZS2, the change, and the corresponding admittances can be known. The line parameters are known from network topology estimation or state estimation.



And if we now try to express the currents in terms of the voltages, So by KCL at node 1, we get IS1 equal to V1 multiplied by IS1, which is the current through it. Plus the current flowing across the V1 V2 node, which is the term over here, plus the current also flowing through the half-line charging susceptance, which is the term sitting over here. So, similarly, if we apply KCL at node 2, we get our second equation. We rearrange terms a bit; what we see here is that on the right-hand side, V1 and V2 are the only unknowns, while otherwise, they are all known in terms of the line parameters or the source parameters. So, we have IS1 in terms of V1 and V2; the bus admittance matrix says the source currents should not be considered; what is to be considered is the current that is flowing into the bus apart from the source.

So, essentially the current that we should look for, which is I_1, should be $I_1 = I_1s - V_1Y_1s$. So, essentially, the relationship is only for source currents for node injection currents, which is what I have mentioned over here. The node injection currents are I1, which is the current sitting over here. If we again rewrite this, we get I1 and I2 in terms of V1 and V2. I1 and I2 are basically the currents from the buses into the network.

It does not matter whether I1 is positive or negative; depending on the source or load, I1 and I2 can turn out to be negative or positive. And essentially, this 2x2 matrix that we have here now, because we had 2 buses, bus 1 and bus 2, with respect to the node reference for the Y bus, the bus admittance matrix, the reference node does not come into the picture; we have 2 buses whose voltages have to be measured. So, we have this 2 by 2 matrix which essentially turns out to be our Y bus. If we now try to find the determinant of this y bus, the determinant would essentially be 0.5y_s(1 2) + y(1 2)^2 - y(1 2)^2, which essentially need not be 0 because if y_s(1 2) is not 0, this term would never be 0.

$$I_1 = I_{s1} - V_1 y_{s1} = V_1 0.5 y_{s12} + (V_1 - V_2) y_{12}$$
$$I_2 = I_{s2} - V_2 y_{s2} = V_2 0.5 y_{s12} + (V_2 - V_1) y_{12}$$
$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.5 y_{s12} + y_{12} & -y_{12} \\ -y_{12} & 0.5 y_{s12} + y_{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 0.5y_{s12} + y_{12} & -y_{12} \\ -y_{12} & 0.5y_{s12} + y_{12} \end{bmatrix}$$

So, essentially, the bus admittance matrix depends only on line parameters; it is independent of source impedances, and that is the reason why node injection currents have to be used, not the source currents. For this given two-bus network, Y bus is symmetric; it is also invertible because its determinant is non-zero. If YS12 becomes zero, then the determinant is non-zero. is 0, and it does not become invertible. The inverse of Y bus is also called the bus impedance matrix.

Previous textbooks and current textbooks also tend to discuss methods by which we can find the Z bus directly. But for our discussion here, we would not go into the details of finding the Z bus directly. The reason is that a decade ago, and two decades ago, the computational power and the processor power that were available made it difficult to find Y bus. It was not difficult. It was actually easy to find a Y bus for a large network; let's say a thousand bus network.

For a 1000 bus network, the Y bus would be a 1000 by 1000 matrix to find the impedance matrix, which is Z bus. One had to invert this 1000 by 1000 matrix, and as the system dimension increased, the dimension of this Y bus also increased. The factorization of such large matrices was a tremendous computational burden with the limited computational capability that was available. Thanks to our modern technology, the computational processing speed has improved. So often, one doesn't need to understand how Z-Bus can be directly evaluated.

Earlier, when the computational capability was limited and factorization was difficult, ZBUS techniques and direct evaluation techniques were common. With current computational capability, one can avoid going into the ZBUS discussion directly, although those who are very eager to learn may still wish to understand ZBUS evaluations if they are interested. But we don't need to find or understand the ZBUS evaluation. If we have good computational capability, once we invert Y bus, we can get the corresponding Z bus. Now comes a question: if YS12 becomes 0, the determinant of this matrix is equal to 0, as per the expression shown here.

What does it mean? If Y bus becomes non-invertible, that means that if we have to, so basically, if the Y bus definition is I equal to Y bus into V, and we can invert Y bus and write V equal to Z bus into I, where Z bus is the Y bus inverse, it is a square matrix, so it can be inverted. If the Y bus becomes singular when its inversion is not possible, that means the Z bus cannot exist; then it would mean that no matter what we know about the source or the bus currents, we will not be able to find unique voltages. In fact, the node voltages would be undefined because the Z bus here does not exist. So, V cannot exist here. How is that possible? What is the practical implication behind it? The physical implication is that if ys12 becomes zero, then let's look at the circuit from where we started. If ys12 becomes zero, let's say in this circuit, if ys12 doesn't exist, it is zero; the half-line charging acceptance is zero, and capacitance is very small, basically not appreciable at all. V1 and V2 appear to be hanging with respect to the node voltage. And remember, voltage is always measured with respect to some reference point. What we would not know for a given power network is whether there is a source or a load connected.

For a given power network, the Y bus has to be uniquely defined. If there is no ground connection or no potential connection between the reference and any bus in the power network, then both of these buses would appear as if they are not connected to the reference and are floating. They are floating, which means the voltages can choose any gibberish number, and that is where they are highly undefined. So, in order to find the potential or voltage, there has to be some reference that is uniquely defined; there has to be some connection, and that is the reason why in our nodal analysis in the previous lecture, we chose one of the nodes to be a reference, with the VE node being 0. So, in order to summarize the bus admittance matrix evaluation given an N-bus N-network, we have two unique terms in Y bus; one is called the diagonal term.

If one has to find the ith diagonal term in a Y bus. So, basically, in this 2x2 matrix, the encircled terms are the diagonal terms, whereas the boxed terms are the off-diagonal terms. The diagonal terms essentially are the sum of all admittances connected to bus I. The admittances could be series line admittance, half line charging admittance, or a shunt capacitor or reactor connected at bus I for voltage reactive power compensation. For off-diagonal terms, they are simply equal to the negative of the direct line-to-line admittance between the two buses I and J. So, basically, if bus 1 is not connected to bus 3 in a given N-bus network, Y13 would be 0, and Y31 would also be 0.

$$Y_{ii} = \sum_{j \in nodes} y_{ij} + \sum_{j \in nodes} 0.5 y_{sij} + y_{shi}$$

y_{ij} – series line admittance between nodes i and j

$0.5 y_{sij}$ – half line charging admittance

y_{shi} – shunt capacitor between nodes i and 0

- The *(i,j)* 'off diagonal term'/ 'mutual admittance'/ 'transfer admittance' is $Y_{ij} = -y_{ij} \Rightarrow Y_{ij} = Y_{ji}$

If 1 or 4 are connected, Y14 and Y41 would exist, and they would mostly be symmetric under the special consideration that there is no transformer at off-nominal position. That's all for today's discussion. In the next lecture, we will explore the effects of mutually coupled impedances and transformer taps under off-nominal positions specifically and how they would be considered in the bus head maintenance matrix. We'll understand that. It's a very interesting topic, and hopefully, we'll be able to understand it properly.

Thank you.