Power Network Analysis

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Week-07

Lecture-32

Lecture 32: Power flow analysis- Basic circuit and analysis

Hello everyone, welcome to lecture 2 of week 7 of the course Power Network Analysis, in which we will start with a new interesting module, which is going to be Power Flow Analysis. In the first lecture of Power Flow Analysis, we will try to understand the basic circuit principles that we have been so well aware of, specifically the use of Kirchhoff's law, nodal analysis, and mesh analysis. Why can't the same basic circuit analysis be applied to circuits where the sources are in terms of voltage and current? Why can't the same be done for the actual power network? Why do we need this power flow analysis at all? So that we will discuss in the current lecture. So far, we have been able to cover five initial modules, starting from basic circuit principles to transformers, the importance of per unit analysis, and from the perspective of transformers, we understood how to model or understand single-phase transformers and what three-phase transformers are. Then we went into synchronous generators, and finally, the last two modules were all about transmission lines. How do we evaluate the corresponding parameters of a transmission line and make use of those parameters to develop certain steady-state models for the transmission line that can be used to understand its performance under different operating conditions? So, basic circuit analysis, when viewed from the perspective of the power network or power system, shows that our power system or network is a highly interconnected manmade network consisting of several electrical components ranging from generators to step-up transformers.

The generators are the sources of power that are essentially consumed by the loads or the consumers here. And since power cannot be transmitted to far-off lengths at low voltages, we typically generate this power in a few tens or twenties of kVs. The same power, if it were to be transmitted to far-off distances to the load points, would result in very high losses, the voltage drops would be significant, and hence we need step-up transformers to boost the supply voltage. The source voltage is stepped up, and then the stepped-up voltage or power flows through these transmission lines.

The devices here refer to the reactive power compensation devices, FACTS devices, STATCOMs, SVCs, etc. which helps in proper reactive power compensation and maintaining the voltage profile along these three-phase transmission lines, where the operating voltage is in the order of a few 765 kV, 400 kV, etc., for the Indian system, and at the same voltage level, these loads are consumers that cannot consume power. So, we need step-down transformers to reduce the voltage, and then the distribution network comes from the substations, where the loads get their power. So, essentially, this complicated network can again be represented as a basic circuit whose topology is known, which means how the lines are connected, where the sources are present, and where the loads are present.

And if we know the network parameters in terms of line parameters and also the fact that the sources are known in terms of voltages and currents. Then the resulting equations that would be involved in analyzing this given circuit would essentially turn out to be a few sets of linear equations based on Kirchhoff's current law, Kirchhoff's voltage law, Thevenin's theorem, nodal analysis, mesh analysis, and so on. So, is that still true for the given power network? Let us see that. So, to begin with, I will try to explain the basics of nodal analysis and mesh analysis, and then we will see whether these can be used for the actual power network. So, given the circuit, if we have to analyze it by nodal analysis.

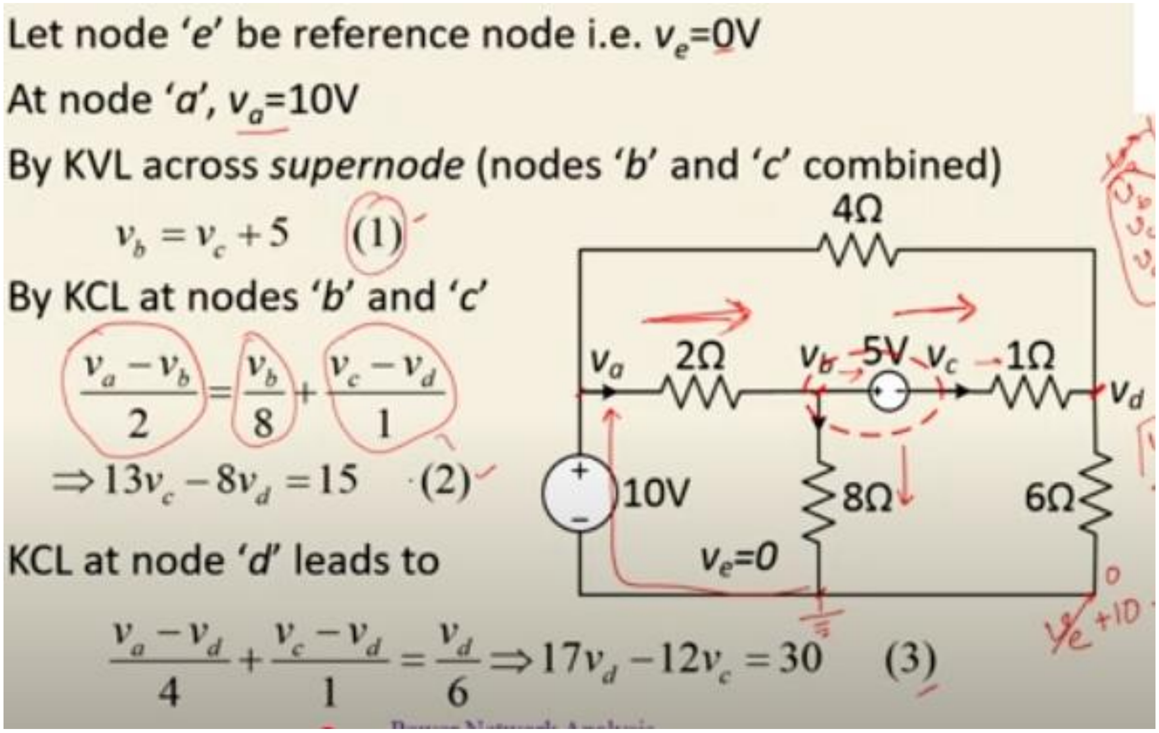
So, I have considered one such circuit where there are 5 distinct nodes: node A, whose voltage is VA; node B; node C; node D; and node E. And if I understand or figure out the voltages of all these nodes, then basically I can find the currents through each of these circuit elements, which could pertain to a typical conductor or a transmission line connecting to such nodes or buses A and B in the actual power network. And in this circuit, there are also sources present whose specifications are known in terms of the specified voltage. By nodal analysis, we can find the unknown node voltages, which would be obtained from a set of linear equations derived from the application of KCL and KVL, if required. Since we have 5 nodes, our aim in node analysis is to find these 5 node voltages.

Now, when we say that the circuit is known to us or given to us, where the resistive elements are all known, a 10-volt source is given, and a 5-volt source is given. We also have to understand that the node voltage, let us say VA, if we have to find its voltage, is always a form of potential difference, which means there have to be two points between which this voltage or potential can be measured. So, in this given circuit, if we have to find these five unique voltages, we have to choose a reference for measuring these voltages first. Given the circuit, it is convenient to choose node E as the reference, neutral point, or ground point from which the potential can be measured. And if we choose node E to be itself the reference, that means the node voltage V_E is equal to 0 volts, considering it to be equal to the ground voltage or the neutral voltage.

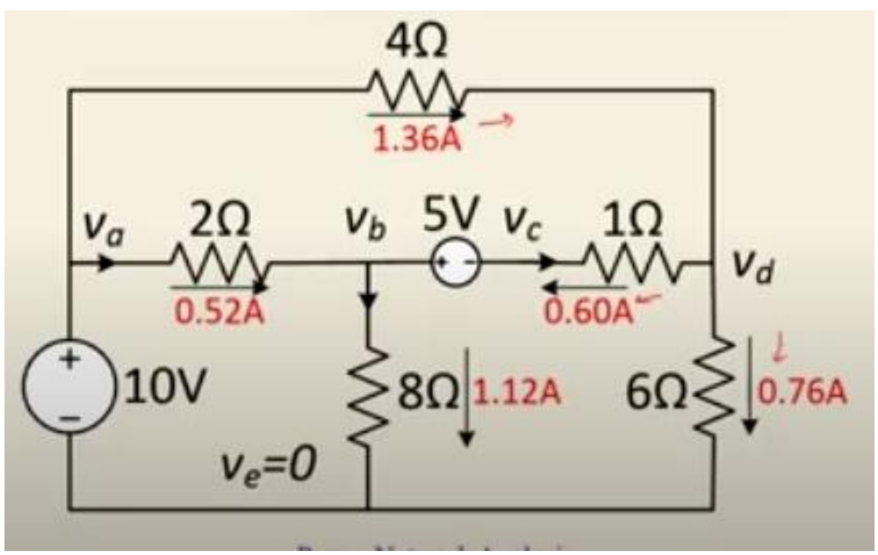
And essentially now we are only left with four unknowns: VA, VB, VC, and VD. If we have four unknowns, we need at least four equations—four unique independent equations—to find these four unknowns, and those four equations would be obtained by the use of nodal analysis. Also, if we see, there is some specificity about node B and node C. Node B and node C actually form a super node, and the reason why node B and C together form a super node is that this encircled dotted portion refers to the super node, because in this super node, there is only a voltage source which correlates node B and node C voltage. So essentially, if we apply KVL across the super node, we would get VB as VV minus 5 volts, or let us say everything is in voltage. So VB minus 5 is equal to VC. We are starting from node B, traversing along the node voltage potential. So we get a minus 5 here because we are crossing from plus to 5. And essentially, when we reach the other node C, we get VB minus VC minus 5 is equal to VC. So, this could probably be one of our equations from which we could find the corresponding other four unknowns.

So, given that node E is the reference, if we apply the same KVL between node E and node A across this path, then essentially, if we start with VE, we jump from a minus to a plus sign, so we gain a potential of 10 volts, and essentially, we reach node A, which is VA, so this is the KVL equation. Since VE is equal to 0, VA turns out to be 10 volts. And, as I mentioned, B and C together form a supernode. So we get this equation in the form of equation number 1. So we now have three unknowns left. VA is also known as 10 volts, VB, VC, and VCD; we need two nodes and three equations. If we apply KCL at nodes B and C, that means if we have nodes B and C, we apply KCL at node B first; then there are three branches pertaining to node B. This is one branch, then we have the other branch from B to C to D, and lastly, the third branch is from B to E. So if we apply KCL at node B and Va is the potential at node A and Vb is the potential at node B, across this particular Branching the current that would come in would actually be VA minus VB divided by 2; 2 ohms is the impedance, and since this is the incoming current by KCL, the sum of currents going out should be equal to the sum of currents coming in. So, the sum of the current going out from branch B to D would be VC minus VD, which is the current here.

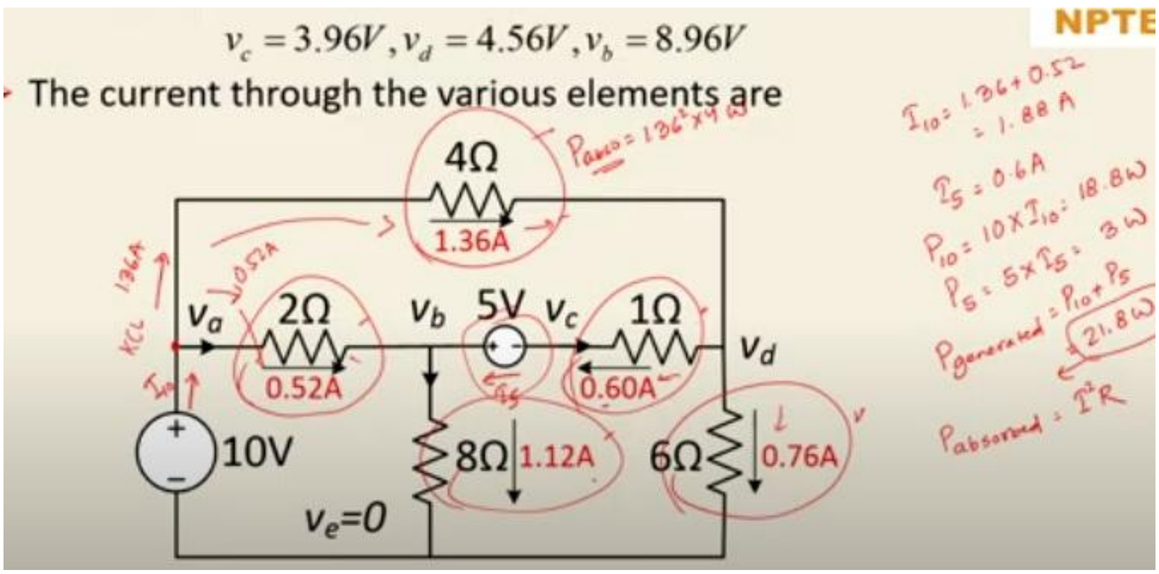
The same current would also flow through this voltage source. It's not important to find the current in terms of VB and VC, but VC and VD would give us that current. And lastly, we have the final current, which is VB minus VE divided by 8.VE is 0, so we have VB as the last term by 8 only. So, if you rearrange the terms, we get our second equation. Similarly, if we apply KCL at node D, we get the third equation. We have our equations 1, 2, and 3. Essentially, we are left with only three unknowns.



We solve these three simultaneous equations. We will get our remaining node voltages, and from there, if we apply KVL, we could find the corresponding currents through individual resistances, impedances, and so on.



It's important to note that if one were to figure out what the current being delivered by this 10-volt voltage source is. So essentially, we would apply KCL at node A. When we apply KCL at node A, we see that there are two currents going out of node A. One current is 1.36 amperes, which is the current evaluated over here, and the second current is 0.52 amperes, which is the current across the two-ohm resistor. So, essentially, the current I from the 10-volt source by KCL should be 1.36 plus 0.52, which is 1.88 amperes. And similarly, if we see what the current is from the positive potential terminal of the 5-volt voltage source I5, and using similar logic, if we apply KCL, since the 5-volt voltage source between B and C creates a super node. So the same 0.6 current has to flow, so we have 0.6 amperes of current, and given these currents, we can probably also find that this is a DC circuit where the voltages are all DC sources; they are ideal independent sources. So, if we want to find the power delivered in this DC circuit by the 10-volt voltage source. It would be nothing but 10 into 10, which is going to be 18.8 watts. And similarly, if we want to find the power from the 5-volt voltage source, this would be 5, the potential difference across the voltage source, multiplied by I, which would be 3 watts. So, essentially, the real power being generated together by the sources is P10 plus P5, which is 21.8 watts. Please check that the same quantum of power is also being consumed by these resistors, which are carrying this amount of current. Please verify that for a given resistor, if the current is known, then the power absorbed is nothing but individually it is.



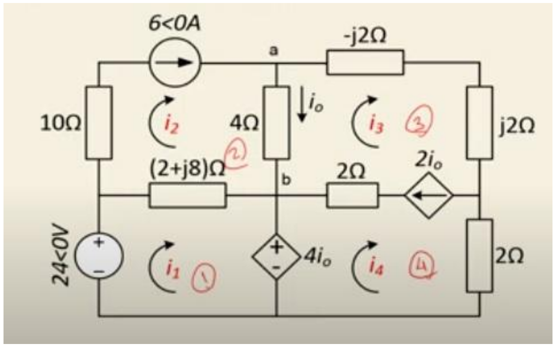
I square I into R, where I is the current through the resistor and R is the resistance value itself. So, basically, if you want to find the power that is being consumed or absorbed by this particular 4 ohm element, it would be 1.36 squared times 4 watts. Please evaluate this element or quantum for all such resistors; 5 remaining resistors add them up, and it should match with 21.8 watts because, again, by conservation of energy, power or energy cannot be generated or destroyed. Everything has to be properly accounted for. So whatever is being generated, the same amount of power is also being absorbed by these particular elements. Please keep this idea in mind. This accounting of power generated, power consumed, power lost, power absorbed, etc., would become very important when we understand or delve deep into power flow analysis, where we would actually understand the differences between power generated and power consumed. There are these two quantities that would hardly ever match for a given AC network because of the inherent I squared loss that would happen in the transmission network. So please keep that in mind. The next analysis, which is well-known, is a mesh analysis.

Here I have considered a slightly different circuit. Essentially, it's an AC circuit where you can see the sources in terms of the voltage source as a reference. We also have a current source that is also considered to be a reference. Both are being measured at the same reference point. So these are all phasors, and instead of having pure resistors as impedances or branch impedances, we also have capacitive elements as well as inductive elements.

I have just marked them through certain blocks. I have not shown the corresponding capacitor or inductance symbols. And we also have a current-dependent voltage source. We also have a current dependent current source. So basically, this is a current-dependent capacitor.

A voltage source, and this symbol here refers to a current-dependent current source, and since these two are dependent on the current i naught, which is the current through the four-ohm impedance, that's where the dependency comes in. Again, these sources here are ideal sources; their internal resistances are zero, so all that has not been considered for the sake of simplicity. We want to find or apply mesh analysis, and we want to find the current in each of the four meshes. Now, if we observe the circuit or understand the circuit, mesh analysis also involves finding unknown mesh currents through a set of certain linear equations, which are obtained by applying KVL and sometimes KCL for super meshes. So the four unique meshes that this circuit has are the ones shown here: mesh number 1, mesh number 2, mesh number 3, and mesh number 4.

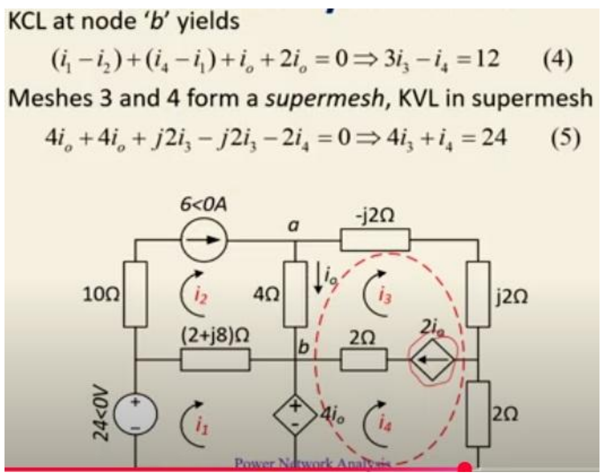
In mesh analysis, we first assume what the current or the fictitious current present in these meshes is, and then we try to understand if there is a way to find the I1 to I4 currents.



So, if we understand these or know these meshes, then it is very clear that in mesh 2, if this current I2 has to flow, this current cannot be different from the source current which is present in that particular mesh. That is the reason why, if I2 has to flow through this particular current source, I2 has to be equal to 6 amperes, and that is the reason why in mesh 2, I2 is known. Similarly, if we look at the current. In node A and B, wherein we want to find I naught, I naught can be found basically by applying KCL at node A.

So, if we apply KCL at node A, we have to first figure out what the branches are and in which these currents are flowing. So, the incoming current at node A is I_2. The outgoing current over here is I3, and at the bottom part from A to B, it is I0. So if you apply KCL, I2 is equal to I3 plus I0, which can be written as I2 minus I3, and since we know I2, we can find I0 in terms of I3. And similarly, if we see meshes 3 and 4 together, they form a super mesh.

So, if we apply KVL, why do these form a supermesh? Because there is a current source present between these two meshes. This current source essentially represents the fact that these two meshes are interconnected or interlaced. And that's the reason why it is called a super mesh; meshes 3 and 4 together form a super mesh. And if we apply, we have to find a relevant number of equations. So, if we use KCL at node B, we get Equation 4.



If we apply KVL across the supermesh, we get equation number 5. We have our three equations, which are basically 4 and 5. So we have the third equation, which is the equation presented here. We know what I2 is; we still want to find what I1, I3, and I4 are. So if we solve equations 4 and 5, we get I3 and I4, and lastly, for I1, we apply KVL in mesh 1, which is this KVL, and from there, we can also find the corresponding current I1.

So, given this particular mesh, we can apply mesh analysis and find all these individual currents. In this perspective, you can also figure out what the total complex power generated by each of the sources is. Which you would basically make use of the fact that the voltage from the particular voltage source or current source into the corresponding current conjugate is the net complex power generated. This should match the complex power that is being generated.

Absorbed together by elements encircled here. All these elements, if we try to find their corresponding complex power combination, because now we have an AC circuit, all these six elements' complex power absorbed together should match the complex power generated by the sources of 24 volts and 6 amperes, the current-dependent voltage source, and the current-dependent current source. So, please do ensure that I mean the way we did it for nodal analysis. Coming to power flow analysis, the question that remains is: can a similar circuit analysis be applicable for the interconnected power system? Probably, yes, it should be possible. But the major hurdle that comes in applying circuit analysis to interconnected power systems is that, unlike in the previous circuits where we had assumed them to be voltage or current sources, in an actual power network, our sources are not known solely in terms of voltage and current sources. In fact, they are specified in terms of real and reactive powers, basically not purely as a function of current and voltage, and to be specific, most of our nameplate rating specifications that we see for electrical equipment include two quantities common to all nameplate specifications.

They are the corresponding apparent power S that the machine or equipment can handle or generate and the corresponding rated voltage at which it should ideally operate. So, typically the sources are known in terms of power and voltage, not purely in terms of current. And since power and voltage are interrelated in some form, as power is equal to the product of voltage and current. So, the resulting equation that comes from the powers perspective inherently tends to be non-linear, and hence the equations no longer remain linear. The laws of KCL, KVL, Thevenin's network theorem, and Norton analysis still hold true, but the application of these corresponding theorems doesn't result in linear equations; they result in non-linear equations, and that is where the importance of power flow analysis comes in.

Power flow analysis also tries to achieve the same objective, which is to solve these non-linear equations to find steady state system states. We call these system states because if we know the voltage phasor of every bus in a given power network, we can find the currents through the branches and the power consumed or absorbed, etc. That’s why these are called system states, which consist of the bus voltage magnitudes and phase angles with respect to a particular reference. Basically, we want to find the bus voltage phasor. All buses in the given power network must be analyzed, and we have to solve certain nonlinear equations; the process of solving these is essentially understood through the notion of power flow analysis.

For understanding power flow analysis and for applying power flow analysis, there is certain information that is needed. The information that is needed is the bus injections that should be known. That means the corresponding source power should be known, the load power should be known, the power network parameters should be known, the line parameters should be known, how the line is connected, whether it is an 80-kilometer line, a 120-kilometer line, or if it is getting tripped off in between; the network topology also has to be known. So, certain assumptions which must be made before one understands why a power flow is to be solved or how it is to be solved have been briefly touched upon; the answer to that is that we want to find all system states, and how it is to be solved we still have to determine. However, where this power flow analysis solution helps, we will see in a few moments or probably in the next lecture.

So certain assumptions that precede our understanding of how powerful analysis is conducted are that we assume the system to be in a steady state. There are no faults, disturbances, or transients happening. The network is perfectly balanced, and the entire network, with all its elements, can be represented in a per-phase analysis through a single-line diagram; generators are represented as constant real power sources with specified terminal voltage via real power. Please have patience; we will understand that in a moment. Transmission lines are transposed, balanced, and typically represented through the nominal pi model with series impedance and line charging admittance, as we have seen in the previous module.

Line shunt conductances are negligible and hence are neglected. Transformers also need to be represented, and there needs to be special consideration for these transformers. Tap positions are called the off-nominal tap. We'll see what is meant by this nominal and off-nominal tap in probably the next few lectures. So, these transformers, as long as they are represented on a per unit basis—if you recollect the per unit analysis we saw in our second main module—the impedance of the transformer, be it seen from the primary or secondary side, remains the same.

So, we call the tap position a nominal position when the tap setting is equal to 1 per unit. One per unit tap setting does not mean that the turn ratio of the actual transformer is equal to one. Basically, if this is a transformer whose turn ratio is equal to one, it would mean that there is no need for a transformer at all. In per unit, this ratio is equal to 1, which means let's say if we have a 33 by 415, 33 by let's say 220 kV transformer, and if we choose the base voltage on the low voltage side to be 33 kV, recollect that in current analysis, the HV side voltage has to be the same as per the nameplate rating, so it has to be 220 kV. If we convert this low voltage into the corresponding per unit, we would have 1 pu. Similarly, VHB would be 220 V by VB, which is again 1 pu. And if we take this ratio now, the tap ratio turns out to be 1 per unit. So as long as the tap ratio is equal to 1 per unit, which is based on the voltage settings chosen as the basis for a given transformer, no special consideration is needed. This tap position need not be equal to 1 per unit in practical considerations because transformer taps also help in regulating the reactive power flow through the transmission line. They also support the reactive power compensation part, which we understood in the previous module. So often you would find taps to be in an off-nominal position where the tap is not equal to 1 PU, and in such cases, some special consideration is needed before power flow analysis can be applied.

So, that's all for today's lecture. In the next lecture, we'll continue with these assumptions in power analysis and understand a critical backbone of how power analysis is to be done by understanding what the bus admittance matrix is.

Thank you.