Power Network Analysis

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Week-06

Lecture-30

Lecture 30: Transmission line models and performance – surge impedance loading, complex power flow

Hello everyone, welcome to the last lecture of week six of the course Power Network Analysis, in which we will continue our discussion on different transmission line models and the use of these models to understand their performances. In today's discussion, we will specifically look at what is meant by surge impedance loading. When do we say that a line has been ordered to its surge impedance? So, what exactly is surge impedance? We'll understand that. And then we'll also try to understand if there is any similarity at all between the complex power flow expressions in these transmission lines. With respect to the power flow expressions that we have seen in the context of synchronous generators and so on specifically. Till the last lecture, we had discussed the long line model, specifically the lossless long line in which we understood that the voltage and current in a lossless long line can be interpreted as waves, similar to the waves that exist in water or in a pond when a pebble is thrown in So similar is the interpretation about waves; why do they exist there? The similar notation or philosophy can also apply to voltages and currents in a lossless long line. And since these can be interpreted as waves, we can also have definitions of the corresponding wavelength and the corresponding velocity at which these waves propagate. In fact, the waves that propagate in transmission lines are essentially electromagnetic waves, and this is the primary reason how power generated from transmitting stations or sources is fed to the transmission lines and then reaches the corresponding loads; that's why when we turn off or turn on a switch, our appliances get on and off without any delay to the human eye, because the speed at which this energy is propagated is almost equal to the speed of light in a vacuum. And in fact, if this speed were to be associated with the speed of flow of electrons in a particular conductor, and transmission lines are also made of particular conductors, then actual electrons will never travel at such high speed because Einstein's theory of relativity would then Come into effect where the electron will no longer remain as an electron; it would become energy, and in fact, the electrons dissipate around their mean position, and that speed, or the rate

of dissipation, is much, much slower than the value of U, which is there for a practical transmission line, also for a lossless line.

The corresponding attenuation constant would be zero, whereas the phase constant would only depend on the per unit inductance L and per unit capacitance C for a given transmission line. We have also seen these equations for a lossless long line where the reference position of the distance is measured with respect to the sending end; X is the line length.

$$V(x) = V_S \cos(\beta x) - jI_S Z_C \sin(\beta x)$$

$$I(x) = -j\frac{V_S}{Z_C} \sin(\beta x) + I_S \cos(\beta x)$$

At any point x in the transmission line from the sending end, the corresponding voltage and current expressions are in terms of the sending end voltage phasor Vs and the sending end current phasor Is. Vs and Is are phasor quantities. In terms of these phasor quantities, we can also find the corresponding voltage and current at any point x along the transmission line. Zc is the characteristic impedance, and beta is essentially the phase constant, which we saw in the previous slide. So if the line length is indicated by LL and the receiving end is at a distance LL from the sending end, then essentially if we put X equal to LL in the above expression, we would essentially get the receiving end voltage, which is equal to the voltage at X equal to LL, and similarly, the receiving end current phasor IR is I at a value of X equal to LL, so we will be able to get VR and IR, the receiving end phasors with respect to the sending end phasors, and since our transmission line follows the property of symmetricity, reciprocity.

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \cos(\beta ll) & -jZ_c \sin(\beta ll) \\ -j\sin(\beta ll)/Z_c & \cos(\beta ll) \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cos(\beta ll) & jZ_c \sin(\beta ll) \\ j\sin(\beta ll)/Z_c & \cos(\beta ll) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

So the determinant of this particular matrix is not zero; it is not a singular matrix, and hence it can be inverted into this matrix, wherein we can also find the sending quantities in terms of the receiving quantities. Coming to the surge impedance loading, we infer that a line is loaded at surge impedance loading when the load itself is equal to the characteristic impedance of the line. That means if I have a transmission line that is represented by this two-port network, we have talked about two-port networks at length.

And if I denote the sending end voltage as Vs and the sending end current as Is, and similarly the receiving end quantities are Ir and Vr, I would call this transmission line loaded at surge impedance loading if I connect a value of impedance, let's say Zc, where Zc is the characteristic impedance. And if this load is connected to the receiving end, that means the load is feeding a load that is called the surge impedance load. That's the very basic essence. And by KVL across the receiving end, we get this particular voltage equation where the receiving end voltage should be equal to the voltage drop across the connected load, and ZC is the characteristic impedance, which is the surge impedance itself. For lossless lines specifically, in fact for general loading of lines, we always tend to define the line loading in terms of a percentage of SIL, few X percent, Y percent of SIL.

For a lossless line, the characteristic impedance has a very unique property. In the sense of a lossless line, the per unit line resistance is zero, the per unit line conductance is zero; what is not zero is the per unit line impedance. Inductance and the permanent line capacitance, and hence Zc, which is the characteristic impedance, is nothing but the square root of L by C, and it turns out to be a pure resistive number, a pure real number, not a complex number at all. In general, for long line characteristic impedance, it would be a complex number, but for a lossless line, it turns out to be a purely resistive number. Surge impedance loading of a lossless line would imply that ZC, which is appearing here, would be a pure resistor.

$$V_R = I_R Z_C$$

$$Z_C = \sqrt{\frac{Z}{y}} = \sqrt{\frac{l}{c}}\,\Omega$$

It is a pure real number, so there is no complex part associated with it. And for this specific case, for SIL at a lossless long line, there happens to be a very interesting phenomenon, which you will understand. So let's dive deep into that. So for a lossless long line, as per the ABCD parameters of a lossless long line, we can represent VS in terms of VR and IR. We know the values of ABCD, so from there we can obtain these values. And since the lossless line is loaded at surge impedance loading, which is nothing but a ZC load connected to the receiving end here, VR is equal to IRZC as per KVL. So this term here can be rewritten as VR, wherein we get this particular expression, which essentially says that VS is nothing but VR times e to the power j beta LL. Beta is the phase constant, j is the complex operator, LL is the total line length, and beta is $2\pi f\sqrt{(LC)}$, which is again a dimensionless quantity. So, essentially, what we are getting here is that, in fact, beta into LL is a dimensionless quantity. Its unit is in radians or can be converted into degrees for the sake of evaluating e to the power j beta L.

So if we recollect, e to the power j theta, where theta is some angle in degrees or radians, it is equal to cos theta plus j sine theta. And if we have to find the magnitude of e to the power j theta, it would be nothing but the square root of cos squared theta plus sin squared theta, which is always equal to 1 irrespective of the value of theta. So essentially, the mod value or magnitude is always equal to 1. So what I am trying to say is that this quantum over here, sitting here, can have its own phase shift, which is nothing but beta times LL. The value of theta or beta LL can change according to the line length.

But the magnitude will always remain one, which means if the magnitude of the receiving end voltage is known, the same magnitude would apply to the sending end voltage as well, although there could be a phase difference between the sending end voltage and the receiving end voltage, which would be nothing but this angle beta LL. Similarly, for the sending end current, if we substitute those numbers, Vr by Zc is nothing but Ir, so that's how we get Is equal to Ir into j e to the power j beta LL. In fact, if we try to find the voltage and current at any distance x from the sending end, then, as per the lossless long line model expression, Vs and Is can be used to find the corresponding Vx quantity.

$$V(x) = V_s \cos(\beta x) - jI_s Z_c \sin(\beta x)$$

If we substitute these numbers, let's say if we put Vs as Vr e to the power j beta LL over here and if we put Is as Ir e to the power j beta LL, where further ir into zc is equal to vr, we get an expression of this form, which again by using the fact that e to the power j theta is cos theta plus j sin theta, if we combine these two aspects, we get vx in terms of vr, where we again have a Quantity e to the power j some angle, which is beta LL minus x, whose magnitude is always equal to 1; the phase angle can change according to the line length.

$$V(x) = V_S \cos(\beta x) - jI_S Z_C \sin(\beta x)$$
$$\Rightarrow V(x) = V_R e^{j\beta ll} \cos(\beta x) - jI_R Z_C e^{j\beta ll} \sin(\beta x)$$
$$\Rightarrow V(x) = V_R e^{j\beta(ll-x)}$$

Similar analysis or similar expressions can also exist for the current at a distance x from the sending end.

$$I(x) = I_R e^{j\beta(ll-x)}$$

So essentially, since the magnitude of this exponential quantity is always 1, irrespective of the value of x, the magnitude remains 1. That means at any point in the line, which is a lossless line and loaded at surge impedance loading, the voltage magnitude will always

remain the same. The current magnitude will always remain the same. The phase angles of these phasors can differ according to the distance x, but the quantum or voltage magnitude, let's say if the receiving end voltage is at 500 kV, the same 500 kV would be present at every point in the line. The phasor of this 500 kV system may be different depending on the line length. Similarly, if the receiving end current is a few thousand amperes, the same current would flow at every point in the line. The phasor quantity of the phase may be different. So that is what the beauty or specificity is about a lossless long line loaded at surge impedance loading.

$$\frac{|V(x)|}{|V_R|} = \frac{|V_S|}{|V_R|} = 1$$

The magnitude remains the same at all points, irrespective of the length x. And so, because this happens, there has to be a reason why the magnitude of voltage is the same, why the magnitude of current is also the same, and so on.

What happens is when we are focusing or talking about a lossless long line, if you recollect our previous discussions, a lossless long line has a per unit line inductance L and a per unit length line capacitance C. What happens when the line or lossless line is loaded at surge impedance loading, the Inductor of the line, the line inductance which is present at every line length in the distributed model; actually, everything is distributed, so every small minuscule section of the line length has some inductance. The same section might also have some capacitance c. What is happening is that for every such smaller minuscule section or distributed section, the reactive power which is being Generated by the line capacitance, the same power is, in a way, getting absorbed by the line inductance when the line is loaded at surge impedance loading; so, in a way, the actual line is not consuming or delivering any reactive power. The overall reactive power generated by the line capacitance, which is a reactive power source, is entirely getting absorbed in the line inductance, which is a reactive power absorbing quantity. And that's the reason why the net reactive power supplied by line capacitance at each point is equal to the reactive power absorbed by the line inductance. There is no reactive power flow to the load in the situation when the lossless line is loaded at surge impedance loading because, in that case, the load itself is a pure resistor. Zc is a purely resistive quantity in the case of a lossless line.

And hence there is no need for reactive power at the receiving end. The entire line does not consume any net reactive power, nor does it deliver reactive power; furthermore, if we substitute our receiving end voltage and receiving end current into the expression we have seen earlier, we can get the voltage and current at any distance x from the sending end perspective.

$$V(x) = V_R \cos(\beta ll - \beta x) + jI_R Z_C \sin(\beta ll - \beta x)$$
$$I(x) = j\frac{V_R}{Z_C} \sin(\beta ll - \beta x) + I_R \cos(\beta ll - \beta x)$$

So here, in this case, if we have this lossless line, where we have seen what happens when the line is loaded at surge impedance loading, there could also be certain specific cases where the line is open-circuited. That means if the line is open-circuited, there is no load present at the receiving end, which would refer to 0% of surge impedance loading. So for the open circuit case, where the line load is zero, there is no load present at all.

That means the line receiving end is an open circuit. So the receiving end current would become 0. If the receiving end current becomes 0, it means this quantity is gone. This quantity is also gone. And we have only left with Vx equal to Vr cos of beta LL minus beta x.

$$I_R = 0$$

$$V(x) = V_R \cos(\beta ll - \beta x)$$

Now let's look at the value of V of X as X is increasing or decreasing. At a distance of X equal to 0, the X distance is being measured with respect to the sending end. So basically, V of X equal to 0 would refer to the value of V of S, which would be equal to Vr cos beta LL. This is one condition, and then at the extreme condition, we have X equal to LL, which refers to the receiving end voltage being measured from the sending end. So we have VR, which is again equal to VR, because the cosine of 0 is equal to 1, so we have only VR.

Depending on the value of beta and LL, no matter what this beta LL value is, cos of theta is always less than or equal to 1. In fact, its value ranges from minus 1 to 1. So, if we basically take its magnitude, what we see here is that for the sending end voltage when the line is open-circuited, the sending end voltage will be less than the receiving end voltage magnitude in terms of magnitude only if we see because the cosine of the angle will always be less than or equal to one. So if we put that expression here, we have the sending end voltage less than the receiving end voltage. Isn't it a bit strange that our receiving end voltage tends to be boosted compared to the sending end voltage, where the sending end voltage or sending end source is the actual source of power? So this phenomenon of receiving end voltages being boosted or amped up under light loaded conditions is nothing but the Ferranti effect.

What is essentially happening is that under lightly loaded conditions, the receiving end voltage is greater than the sending end voltage, which means that not all of the reactive

power generated by the line capacitance is being consumed by the line inductance because there is no load at the receiving end; it is open-circuited, so more reactive power is being consumed. Generated by the line capacitance, lesser of it is being consumed by an inductance. So essentially, at the receiving end, we have a lot of reactive power that is going nowhere, which in a way results in an increase in the receiving end voltage. That's the basic phenomenon. Similarly, under short circuit conditions where the line is entirely dead short-circuited, the receiving end voltage would become zero, and consequently, we would have the sending end voltage V of X as an expression of I of R.

$$V_R = 0$$

$$V(x) = jI_R Z_c \sin(\beta ll - \beta x)$$

Now, in this case, as we see the value of The impact of x on the voltage itself would show that at the receiving end, the voltage magnitude is at its minimum, while the value is at its maximum at the sending end, meaning that the sending end voltage would always be greater than the receiving end voltage when the line is. Short-circuited, which means that more reactive power is being supplied, so less reactive power is being absorbed for loading less than SIL, and exactly why a silver circuit is happening in the case when the line is loaded more than the SIL or higher loading, which means that more reactive power is demanded from the sending end, so the receiving end voltage has to be lower so that corresponding compensation can happen. This plot here essentially shows the variation of sending end voltage and receiving end voltage as a function of line loading. Usually, the line load is represented as a percentage of SIL. So, the no load condition here would refer to 0% SIL, whereas the short circuit condition would refer to an infinite percentage of SIL. And at SIL, as we discussed, the voltage profile is completely uniform. The magnitude doesn't change at all. Generally, for light load conditions where the load is less than the surge impedance loading, which means more reactive power is being generated by line capacitance and less of it is being consumed by the line inductance, to maintain the Voltage at the receiving end, which is indicating this particular section where the receiving end voltage is more than the sending end voltage, would essentially need some device that can absorb this excess reactive power at the receiving end and try to bring this receiving end voltage closer to the acceptable norms. The voltage at the sending end. So in those cases, we would essentially connect some shunt inductors at the receiving end so that the excess reactive power can be absorbed by these shunt inductors, and vice versa; when the line load receiving end voltage profile is very poor, we might need some device like a capacitor or a reactive power generating device, which can boost the voltage profile at the receiving end, and hence we use shunt capacitors for voltage compensation. Usually, lines are loaded way beyond their thrust impedance loading to maximize their utilization and increase the overall transfer capacity. So essentially, in the majority of compensation aspects, we would often find the use of shunt capacitors mostly being used

in transmission lines because line loading is obviously going to be more than the actual SIL.

So we need some shunt capacitors to increase the voltage profile by adding more reactive power resources at the receiving end. In terms of complex power flow through the line, we have understood different line models. We have figured out ways to find ABCD parameters for short, medium, long, lossless lines, et cetera. One such transmission line where the ABCD parameters are known is as per the discussion that we had with respect to different two-port network parameters. The ABCD parameter is one such representation of a two-port network parameter.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

And we are trying to find the expressions of sending end and receiving end powers. By "powers," I mean complex power, real power, and reactive power. So what I have is a transmission line whose ABCD parameters are known, and I am choosing the receiving end voltage per phase as a reference, while the sending end voltage is also being measured with respect to the receiving end. So the sending-end voltage is leading the corresponding receiving-end voltage by an angle delta. The notion of why delta has been used here is to bring in some correlation with respect to the delta that we saw for the synchronous generator, which was the torque angle there.

But the notation can be different. The meaning or the representation is important here. And we also know our ABCD parameters; usually for a transmission line, these are complex numbers, so we can represent A and B. We can also do the same for C and D, but A and B are complex numbers, so I am representing them through polar coordinates. And remember, by reciprocity or symmetricity, A is equal to D, so if we know the value of A, D would have the same value. And we also know that AD minus BC is equal to 1.

So if we know A, D, and B, we can indirectly also find C, and that's the reason why I have not explicitly defined C and D here because by symmetry and reciprocity we can find those values. And now, if we put in those numbers or the representation here, VR is VR at an angle of 0, and IR is to be known in terms of VS and VR, so we can get a complex expression of IR in terms of A, B, VS, and VR parameters.

Let $V_R = |V_R|\angle 0$ be reference for all phasors and $V_S = |V_S|\angle\delta$,

ABCD parameters are generally complex quantities, hence, let $A = |A|\angle\theta_a$ and $B = |B|\angle\theta_b$

Thus

$$V_S = AV_R + BI_R$$

$$\Rightarrow I_R = \frac{V_S - AV_R}{B} = \frac{|V_S|}{|B|}\angle(\delta - \theta_b) - \frac{|A||V_R|}{|B|}\angle(\theta_a - \theta_b)$$

Similarly, we can also find the sending and current expressions in terms of these A, B, VR, and VS parameters. Receiving end voltage and receiving end current, which are the expressions over here. We can find the corresponding receiving end three-phase complex power, which is 3 VRIR conjugate. So if we plug in those numbers carefully, we would get this expression, which is the complex power expression at the receiving end.

$$I_S = CV_R + DI_R = CV_R + D\frac{V_S - AV_R}{B}$$

$$\Rightarrow I_S = \frac{DV_S}{B} - \frac{(AD - BC)V_R}{B}$$

$$\Rightarrow I_S = \frac{AV_S}{B} - \frac{V_R}{B} \text{ as } A = D \text{ and } AD - BC = 1$$

$$\Rightarrow I_S = \frac{|A||V_S|}{|B|}\angle(\theta_a + \delta - \theta_b) - \frac{|V_R|}{|B|}\angle(-\theta_b)$$

➤ Thus, receiving end three-phase complex power

$$S_R = 3V_R I_R^* = 3|V_R|\left\{\frac{|V_S|}{|B|}\angle(\theta_b - \delta) - \frac{|A||V_R|}{|B|}\angle(\theta_b - \theta_a)\right\}$$

We take the real part and the reactive part out of it, basically the real and imaginary parts. So we obtain the real power expression and the reactive power expression.

$$P_R = 3\frac{|V_R|}{|B|}\{|V_S|\cos(\theta_b - \delta) - |A||V_R|\cos(\theta_b - \theta_a)\}$$

$$Q_R = 3\frac{|V_R|}{|B|}\{|V_S|\sin(\theta_b - \delta) - |A||V_R|\sin(\theta_b - \theta_a)\}$$

 Similar analysis or expressions could also be obtained for sending end complex power, sending end real power, and sending end reactive power.

$$S_S = 3V_S I_S^* = 3\frac{|V_S|}{|B|}\{A|V_S|\angle(\theta_b - \theta_a) - |V_R|\angle(\theta_b + \delta)\}$$

$$P_S = 3\frac{|V_S|}{|B|}\{|A||V_S|\cos(\theta_b - \theta_a) - |V_R|\cos(\theta_b + \delta)\}$$

$$Q_S = 3\frac{|V_S|}{|B|}\{|A||V_S|\sin(\theta_b - \theta_a) - |V_R|\sin(\theta_b + \delta)\}$$

If we reorient or rearrange things a bit and try to bring in the aspect that usually the voltages at the sending and receiving ends can be maintained, regulated, and controlled by proper compensation devices, which we will see in the next lecture.

 So usually, Vs and Vr are controllable, regulatable quantities. So we can assure them that they are often going to be constant. The magnitudes are often going to be constant. What might change is the angle delta, which is between the sending end with respect to the receiving end. For a given transmission line, the ABCD parameters would also usually be fixed.

 So essentially, the only variable that might appear in these different expressions is the angle delta. VS and VR; we generally assume them to be constant, or they can be made constant through proper compensation. So we can rearrange these terms. So that we can get the receiving end powers and the corresponding sending end powers through some simplicity or rearrangement, our unknown term, which is the term present with the angle delta, retains on one side, and all other known quantities are present on one exact side. So essentially, if we rearrange the first two expressions over here, basically I am trying to bring in this negative term on this side, this negative term over this side, and I am only left with the complex term or the cosine theta and sine delta term on the right-hand side, with the fact that I'm replacing the receiving end phase voltage and sending end phase voltage in terms of the receiving end line-to-line and sending end line-to-line voltage.

 So I get this expression.

voltage magnitudes, i.e., $|V_{R,l}| = \sqrt{3}|V_R|$ and $|V_{S,l}| = \sqrt{3}|V_S|$

$$P_R + \frac{|A||V_{R,l}|^2}{|B|}\cos(\theta_b - \theta_a) = \frac{|V_{R,l}||V_{S,l}|}{|B|}\cos(\theta_b - \delta)$$

$$Q_R + \frac{|A||V_{R,l}|^2}{|B|}\sin(\theta_b - \theta_a) = \frac{|V_{R,l}||V_{S,l}|}{|B|}\sin(\theta_b - \delta)$$

Here, if we look at the expression, what we get is that, let's say if I square up this entire term, I square up this entire term and then try to add them up, what I would observe is that I'll have PR plus AVRL squared by B cos(theta B) minus theta A whole squared, similarly for QR plus AVRL squared sine(theta B). Minus theta a by b whole square will essentially be equal to v r l square v s l square by b square because cos square and sin square add up to one. And if I sort of expand my imagination, there is also a similarity between the term over here and the term over there. Essentially, in the complex plane, if I were to mark this equation, which is this locus equation on a complex plane where p is on the x-axis and q is on the y-axis, then this equation essentially represents the equation of a circle whose radius is VRL VSL divided by B and the center is minus A conjugate divided by B conjugate VRL VSL. In the VRL conjugate, I am saying that the conjugate of A and the conjugate of B with a negative sign is because the general expression for a circle is (X minus A) squared plus (Y minus B) squared equals R squared; then, on the XY plane, (A, B) becomes the center and R becomes the corresponding radius.

 That's how the center circle equation looks. Similarly, if I am trying to simplify or re-examine things from a complex perspective, A conjugate, if you remember, was A at an angle theta A, whereas B was the complex number B at an angle theta B. So if I put those numbers here, I have minus A mod by B mod at an angle theta. Theta B minus theta A because B conjugate is mod of B at an angle minus theta B, which when taken up in the numerator becomes plus multiplied by VRL was simple VRL squared, so the mod comes over here, and if I take the cosine of this term, I have the term over here; if I take the sine of this term, then I have the term over here. That's how the circle or the locus looks very similar to the locus of. Synchronous generators, which have a capability curve, show that the locus of complex power at the receiving end is a circle for wye-delta; it's a circle on the pq plane with the center as this coordinate and the radius as this coordinate.

 Similarly, if we look at the sending end quantities, then we have a similar circle with a different center; the radius remains the same. That's all for today's discussion. We will take up the discussion of reactive power compensation, which is going to be the Last lecture for this particular module and the first lecture of Week Seven.

$$P_S - \frac{|A||V_{S,l}|^2}{|B|} \cos(\theta_b - \theta_a) = -\frac{|V_{R,l}||V_{S,l}|}{|B|} \cos(\theta_b + \delta)$$

$$Q_S - \frac{|A||V_{S,l}|^2}{|B|} \sin(\theta_b - \theta_a) = -\frac{|V_{R,l}||V_{S,l}|}{|B|} \sin(\theta_b + \delta)$$

Thank you.