Power Network Analysis

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Week – 01

Lecture-03

Hello everyone welcome to this third lecture of week one which is going to be on basic circuit principles to be precise certain discussion on how power can be calculated in single phase AC circuits. In the last lecture we had started with our first module which was basic circuit principles and we had revisited or sort of recapitulated certain aspects on what phasors are, what do we mean by phasor diagrams, how can we represent time varying AC signals be it current or voltage in terms of phasors where frequency is a important reference depending on what frequency has been chosen as a reference, the phasor diagrams may accordingly change. We had also seen the phasor diagrams for the basic three basic circuit elements which were which are resistors, inductors and capacitors. So, in continuation with that discussion on phasor diagrams in this lecture we will start with what is power, how do we evaluate power, how do we measure power in given AC circuits. So, the first topic which we would discuss today is nothing but the instantaneous power. As the slide shows instantaneous power which is absorbed by an element, by element I will specifically mean a resistor, inductor or a capacitor or combination of R, L and C.

in those elements how is instantaneous power evaluated or defined. So instantaneous power is nothing but the product of instantaneous voltage drop across that element and the instantaneous current which is flowing through that particular element. So if we have a voltage signal time voltage signal defined of this form a cosine wave and a current signal which is also defined as a cosine wave where the difference is that their RMS values are different and the corresponding phase angles may be different may be same depending on what circuit element has this voltage drop across it and what current the corresponding current which is flowing through it. So if i know these time varying AC signals voltage and current across the element or through the element then instantaneous power is nothing but the expression which is defined here which is product of voltage and current in time domain form.

So, having defined these expressions for voltages and currents if we evaluate or put these expressions in the instantaneous power expression We get a expression of this form where there are two distinct terms. The first term is a time invariant term whereas the second term is dependent upon time. and this second term which is dependent upon time has a frequency which is twice the frequency of the actual voltage and current waveforms and essentially its angular frequency is 2 omega. So, as a result if we evaluate the corresponding time period of this instantaneous power waveform in time domain then the corresponding time period is essentially half of the actual time period of the current signal or the voltage signal If we now try to evaluate what is the average or the real power, then the average power is nothing but the average of the instantaneous power. It is very similar to defining average quantities or average values of voltage and current AC signals which we had seen in the previous lecture.

So if we take the average of this instantaneous power P of t, which has two distinct terms one is the constant term whereas other is the time varying function and again this time varying function is a sinusoid which is periodic with time period being half of the actual time period and it is also symmetric along the time axis which if we consider to be the x-axis then this is also symmetric and periodic so as a result the average of P of t would nothing be but the first term itself because it is a time independent term and the second term would essentially have an average value of 0 because it is periodic and symmetric along the time axis. So that is where the average power is nothing but the first constant term of instantaneous power. It's also being referred or called as real power whose unit is watts. And the importance of defining this average or real power is that our analog wattmeters or multimeters, they essentially measure this average value by design. For purely resistive loads where voltage and current signals the phasors are aligned, the corresponding phase angle difference becomes zero.

So cosine of zero is one and essentially the average power is nothing but the RMS values or product of RMS values of voltage and current. If we have the resistance value known which is R, then by Ohm's law V would nothing be I by R. So if you substitute I as V by R, so we get this expression. And if we substitute V as IR into this expression, then we get the corresponding I square R expression. For inductors or capacitors, which are also called as purely reactive loads, by inductors and capacitors I mean lossless inductors and capacitors, the corresponding phase angle difference between voltage and current is either plus 90 or minus 90.

plus 90 for inductors and minus 90 for capacitors. So, in that case cosine of plus or minus 90 becomes 0 and essentially when the cosine terms become 0 the corresponding real power by the absorbed by this reactive loads is purely 0. Going ahead with the definition of instantaneous and average power, we can also define something known as apparent power which is nothing but product of RMS values of voltage phasor and current phasor, voltage across the element and current through the element. Its unit is defined as volt ampere which is mathematically analogous to also watts just to segregate apparent power from real power the units are little technically differently defined although mathematically their units are all similar and we can also define power factor as ratio of average power and the apparent power which leads us to this cosine term cosine of phase angle of voltage minus voltage of the phase angle volt of the current phasor and both these phase angles they are to be measured on a common frequency reference. So, if we were to evaluate power factor for a pure resistor then for a pure resistor as was shown in the previous slide the phase angle difference is perfectly 0 and hence power factor for a pure resistor would simply be equal to 1.

Whereas for pure inductors or capacitors the corresponding phase angle difference is either plus minus 90 degrees which results the cosine to be 0 because cos of 90 is 0 and hence power factor will be 0 for pure inductors and capacitors. And this phase angle, which we have defined as theta V minus theta I, can also be called as the power factor angle. We have already discussed about this in the previous slide. Power factor theoretically varies between 0 and 1. The cosine of phase angle varies between 0 and 1.

So power factor also varies between 0 and 1. And for pure resistors, it's equal to 1, whereas for pure inductors or capacitors, it's perfectly 0. For inductive loads by inductive loads I would mean combination of R and L load where there is a non-zero resistance and there is a non-zero inductance involved. The power factor angle would vary between 0 to 90 and similarly for capacitive loads the phase angle would vary between minus 90 to 0 and coincidentally cosine of this phase angle it is irrelevant or is independent of sign of the phase angle. So, to segregate the power factor the notion of lagging or leading comes in.

Eventually for inductive loads or for capacitive loads which can be replicated by combination of R and C loads, power factor will always be a positive number. To segregate whether this power factor refers to an inductive load or a capacitive load, the notion of lagging and leading concepts also come in. Now, why lagging and leading concepts come in here? We can correlate this with the fact that for inductor, current, lacks the voltage. If it is a pure inductor, then current would lack the voltage by 90 degrees. If it is a combination of RL load, then still the current would lack the voltage.

It need not be 90 degrees and that's where this mathematical expression or range comes in. And similar notion is also applicable for leading loads where current tends to lead the voltage in case of pure capacitors or RC combination capacitive loads. If we know the impedance of the element which let us say if we denote it as Z with Z being the magnitude and phi being the phase angle we will discuss or look more about it how do we define these impedances. If we know the impedance value which is also correspondingly a complex number then the phase angle difference of this impedance which is again being measured with respect to the same frequency reference for voltage and current, it turns out to be same as the power factor angle. How does it come that? We can simply make use of this analogy here where we are comparing the RMS ratio of RMS voltage and current to be equal to capital Z and the phase angle is nothing but phi which is the power factor angle.

So essentially to summarize power factor for a given element, network, load, whatever, it's always going to be a positive number and to segregate whether it refers to an inductive element or a capacitive element, the conditions of leading or lagging come in. So essentially when someone asks what is power factor or what is the typical value of power factor? If the power factor value is different from 1, then and theoretically the power factor can go till a value of 0. Any number of power factor between 1 and 0 would always have this notion of lead or lag associated to it. If power factor is equal to 1, it implies that the corresponding element behaves as a pure resistor and in resistance phase angle differences between voltage and current they are 0. So, there is no need of having this notion of lead or lag when power factor is equal to 1.

Moving ahead with definition of complex power, if we can have a definition for RMS values, product of RMS values of V and I as apparent power, then it also makes sense that there is something which can be defined as complex power. So this complex power which is absorbed by an element or a load is the product of voltage phasor and complex conjugate of current phasor where voltage and current phasors they are being measured on a common frequency reference. Now the question or interesting part to observe here is why we define complex power as V into I conjugate, why do not we define S as simply V into I, why do not we define S or complex power as V into I entire conjugate or why do not we define S as V conjugate I. I will briefly touch upon the reasons behind why this combination is not true while defining complex power. We will talk more about it in the upcoming modules.

So, briefly to talk about or explain why this combinations are not applicable. So, let us go case by case. When we take conjugate of current and multiply it with the voltage or when we take conjugate of voltage and multiply it with the current phasor in both the cases what we would observe is that the phase angle which would come in for this complex power it would always be a difference of the voltage phase angle and current phase angle or in case of V conjugate I it would be theta I minus theta V. we should understand that this is what is going to be the combination of phase angle. Whereas if we talk about the first two combinations which are v into i or conjugate of v into i in both these cases we would essentially have the phase angle in complex power appearing as theta v plus theta i or in v i conjugate entire conjugate it would be minus of theta v plus theta i.

So now let's see which of these combinations actually makes sense. If phase angle of theta v and theta i difference, if it holds true, then probably V conjugate I wherein the sign of the phase angle has reversed. This can possibly also be applicable. Whereas the other two combinations where the phase angles add up to each other, it's not an acceptable value. Now why is that? The reason is for a given element which can be a pure resistor or combination of inductive resistor, capacitive resistor, the average power is always time invariant.

Let us give an example or let us talk about an example. Suppose I have a 25 watt incandescent bulb which is supposed to operate at 230 volts and the desired frequency of this volt is 50 hertz. So, if I have this incandescent bulb and if I connect this to a 230 volt single phase then at 230 volt single phase source this bulb will consume 25 watts no matter what the frequency is. Alternatively speaking, if I take this bulb and connect it to a system where the voltage specification remains the same, but the source frequency is now different which is 60 hertz, this bulb would still consume 25 watts because its element or tungsten filament or the bulb is designed to operate at this 25 watts. So essentially what I am trying to say is for pure resistors or for purely loads which are purely resistors, this notion of frequency is not important.

in order to avoid this notion of frequency wherein the power has to be time invariant there needs to be a mechanism of defining complex power where the frequency reference also becomes invariant of what the complex power is meaning if I choose 50 hertz as my source frequency then corresponding voltage phasors and current phasors they are being defined based on that source frequency. If I define source frequency as 60 hertz I will still get a voltage and current phasor but when I take the difference of their phase angles the importance of reference which is the frequency reference for phasors is being avoided or nullified. So essentially the incandescent bulb which is supposed to consume 25 watt will still consume 25 watt no matter what the source frequency is. That's the basic story behind why V into I conjugate or V conjugate I probably can be true. We'll talk about more why VI conjugate is chosen in the next few modules to come.

If we talk about V into I option or V into I conjugate option the phase angles are getting additive so frequency which was chosen as a reference for voltage and current that also gets additive in this complex phasor and hence if we evaluate or represent this complex phase or complex power in terms of time domain our average power expressions would entirely be wrong. So that's the reason why V into I or V into I conjugate are not the favored options. V conjugate I probably can be an option. We will talk about it in the next few modules. So alternatively we can define our complex power also as in terms of impedances if we know the impedance value.

The unit remains the same as apparent power which is volt ampere. If we know what is apparent power, complex power, then real power, then we can also have a definition of reactive power, which is nothing but the complex part of the, or the imaginary part of the complex power. It essentially is a measure of energy exchange of power between source and the inductive or reactive elements in the circuit. To segregate it little differently from the terms of units, its unit is defined as volt ampere reactive. Mathematically this is same as volt ampere and it is also same as watt, mathematically.

For a given load which has some resistance RL and some inductance XL we can correspondingly figure out given this inductive or given this load which is ZL our complex power would be V into I conjugate where if we apply KVL or KCL across this inductive this given load then V would nothing be I into ZL and now if we substitute I into ZL in V or I in terms of V by ZL we would get S as V into V conjugate by ZL conjugate which if we further expand would result in a complex number from which if we take the real part, we get the corresponding real power and if we take the imaginary part, then we get the corresponding reactive power expression. For pure resistors, the reactive power is always zero because phase angle difference between voltage and current is zero. So the corresponding sign becomes zero for pure resistors. Whereas for inductors, the reactive power is assumed to be absorbed. Whereas for capacitors, this reactive power is assumed to be supplied.

So Q accordingly is positive or negative accordingly for inductive loads or capacitive loads. Again the sign of this reactive power is based on the fact that the load is consuming reactive power. So conventionally in power network when we define reactive power units we associate them with consumption of reactive power not with respect to generation of reactive power. So just to recapitulate our complex power in AC circuits is nothing but product of voltage and current conjugate. I will briefly also probably explain here going ahead with the discussion that we had here that inductors are supposed to again conventionally speaking as power engineers inductors are those elements which absorb reactive power so basically the power factor for inductive loads is lagging whereas for capacitors they are supposed to generate reactive power so the corresponding power factor has to be leading.

Given that discussion we can sort of conclude that complex power is V into I conjugate. If someday the notion of inductors and capacitors change that means if we start associating inductors as leading power factor loads that means inductors are those elements which generate reactive power and capacitors are those elements which sort of absorb reactive powers basically they would have lagging power factor then in that day or in that definition S could also be defined as V conjugate I. Conventionally we stick to the fact that inductors are not leading power factor elements but lagging power factor elements whereas capacitors are leading power factor elements so we stick to this notion of complex power. The corresponding magnitude of complex power is apparent power, the real part of complex power is real power and the imaginary part of complex power is reactive power and this being the definition of power factor wherein the information of lag or lead is also necessary if the power factor is not equal to 1. If we have to represent these phasors on an impedance triangle or on a phasor diagram, then for an inductive load which has a non-zero R and a non-zero X, the corresponding current will lag the corresponding voltage across this RL element or RX element which we can also correlate with an impedance triangle.

And since voltage is nothing but current into the impedance of that complex impedance of the element, so we can also associate this in terms of voltage drops from where we can also get the corresponding power angles. Similar power diagrams or impedance triangles can also be drawn for a capacitive loads, the only difference here being that the current would lead the corresponding voltage across this R and C element where X is equal to 1 by omega C. So if we have a circuit single phase circuit where there are two voltage sources again these voltage sources are being defined on a common frequency reference common frequency phasor reference and in between these voltage sources there is a impedance whose R and X values are known then we can correspondingly write the expression of current I12 as V1 at an angle theta 1 minus V2 at an angle theta 2 by Z at an angle phi and how does this expression come from? It is essentially coming from the KVL Kirchhoff's voltage law which is being applied across the circuit. If we have to find the expression of I21 it would essentially be nothing but the negative of I12 again by KVL. Once we know these currents which are the currents from source 1 to source 2 or current from source 2 to source 1.

We can also find the corresponding complex powers which these different voltage sources are generating or consuming depending on whether they are inductive elements or capacitive elements. So this here is a very generic expression for the corresponding real and reactive power exchanges which are happening through this impedance between these two different voltage sources. The simplification or how do we get these expressions? We simply substitute these expression over here or the expression of I21 over here and then do a simplification where we can get these corresponding expressions. If we go further and try to figure out what are the corresponding real and reactive powers then we take the real and imaginary part of the corresponding complex power and then we would get these expressions. Point to be noted here is that if we also find P 21 and Q21 and add them to P12 or Q12 which are real power flow from 1 to 2 or 2 to 1 and vice versa for reactive power.

This addition would nothing be but equal to the I square R loss or the reactive power loss which is happening in this particular impedance or line having a value of Z at an angle phi. So, coming to three-phase AC circuits, we have talked about single-phase AC circuits extensively, but that is not how the typical power network usually operates at. AC circuits in terms of single-phase are very common in distribution networks, whereas three-phase AC circuits is a notion of how transmission networks operate. So, general practice is to generate, transmit as well as consume power mostly in three-phase form. In three phase circuits, the conductor which is required for setting up the transmission line, it does not require explicit return conductor and hence there is a lot of conductor which is material which is saved, so basically it is highly economic compared to single phase generation or transmission.

The instantaneous power is constant under certain conditions which we call as balanced situations, which we will talk about in the next lecture. And hence our rotating machines like synchronous generators, induction motors, they experience less mechanical vibrations. Three phase AC currents in rotating machines specifically synchronous machines or induction machines, they inherently produce this rotating magnetic field which forms the premise of operation of these machines and it helps in maintaining a constant torque. All these three different aspects of economics, less vibrations and inherent magnetic field, we will discuss this at length in the next few slides or lectures to come in. And coincidentally by design, a synchronous generator or alternator inherently produces three phase voltages.

What is balanced? We will talk about it. in the next particular lecture which is going to be on power in three-phase AC circuits. We will also talk about this three-phase AC generation in the synchronous generator module which would be the next to next module for upcoming discussion. Thank you.